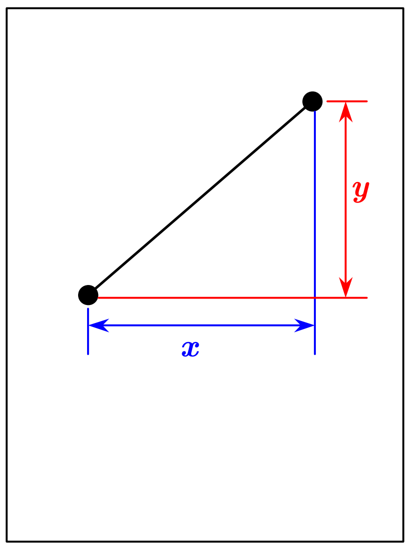
Leo Bellantoni
FermiLab

Introduction

This is an introduction to some of ideas of relativity. Along the way we will shoot at a stop sign, take the square root of minus one, derive $E = mc^2$, and stumble across antimatter.

Dots on a piece of paper

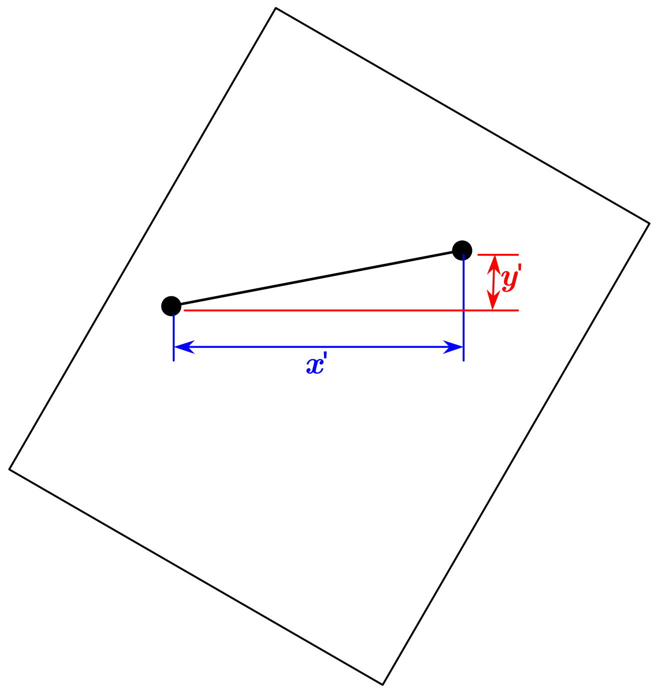
Let me start by drawing a couple of dots on a piece of paper, and then - wait for it - connect the dots. Sort of like this:



The distance between the two dots is given by the Pythagorean theorum: the square of the hypotnuse is the sum of the squares of the sides. Or, to put it differently,

$$R^2 = (x)^2 + (y)^2 .$$

Now imagine that this paper is sitting on the table in front of me, and you are sitting just a bit to my right. This is what you will see:



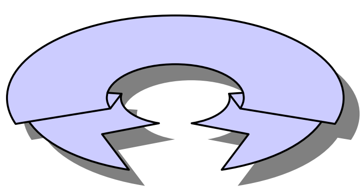
Again, the square of the hypotenuse is the sum of the squares of the two sides - but the sides are different from this other point of view. I've drawn the sides to indicate the distance from left to right and from top to bottom; since you are looking over my shoulder, your left and my left are in slightly different directions. But the distance between the dots is the same:

$$R^2 = (x')^2 + (y')^2 .$$

This is all you need to understand the principle of relativity.

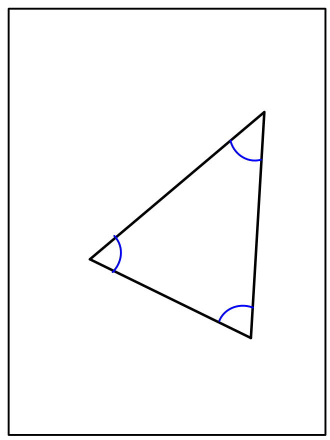
The distance between the two dots is the same, independent of viewpoint. As such it is, in a certain sense significant. The span of the figure from left to right or top to bottom does depend on your viewpoint; it is relative while R is absolute. In dweeb, we say that R is an invariant, or that it is conserved. Not only is R absolute, the form of the equation is also absolute; the Pythagorean theorum is true from both points of view.

Another terminology, which is more popular among dweebs nowadays than in Einstein's time, is that of symmetry. Here is a picture of a symmetric object:

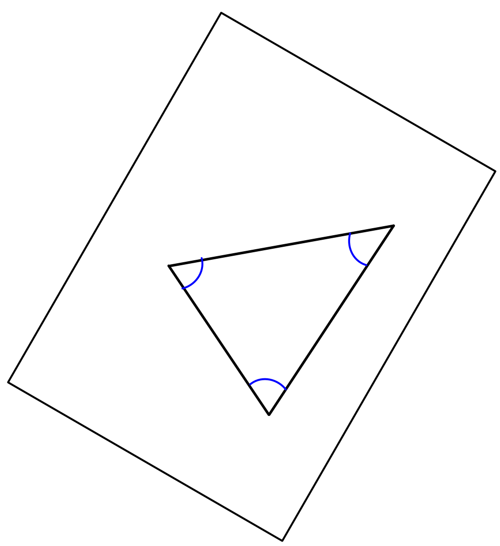


To be precise, to say that this picture is symmetric is to say that if we flip this drawing over horizontally, it will look the same. In the example of the dots on the paper, the Pythagorean equation looked the same from the two points of view, and the distance between the dots was also the same from both viewpoints. Dweebs call these things symmetric under rotations.

There are reasons why symmetry as we call it has become such an important idea in 20th century physics. The first reason is that symmetries provide a simples test that we can apply to theories to see if they could even possibly be true. Suppose for example, that I draw a triangle on a piece of paper



and that I try to remember some of that old geometry class, and that I work with this triangle and come up with a proof that the sum of those three angles is 173 degrees. That is my theory. Right, you say. Since you are looking over my shoulder, you have a different view of the paper



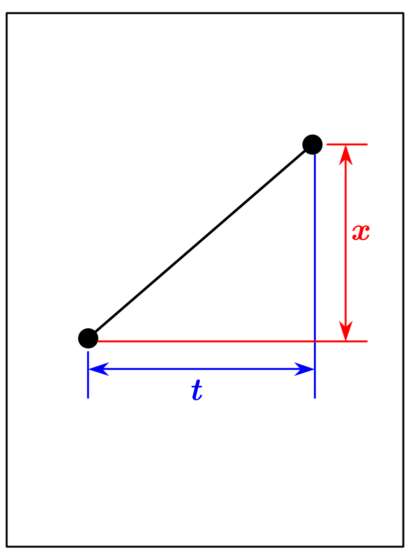
but you can still go through the steps of my proof of my theory, from this different point of view. And you do. But you get a different result: you conclude that the sum of these three angles is 184 degrees. Now neither of us believe my theory anymore! It failed to pass the symmetry test. My "proof" has some flaw of some sort, and it doesn't work for everyone. It is relative, and does not tell us anything that we could possibly think of as being absolutely true. It still might not be a valid proof even if it does pass the symmetry test, but it certainly will not tell us anything if it fails this test.

This is one of the reasons why dweebs like symmetry; it helps us get rid of bad proofs and other dumb ideas quickly. By disregarding things that are relative, we can focus on what might be absolutely true. In a way, the term relativity is unfortunate; the idea is really to classify things as either relative to your viewpoint or else absolute, and then to focus on the absolutes. It might have saved some of us some confusion if it had been called absolutivity rather than relativity!

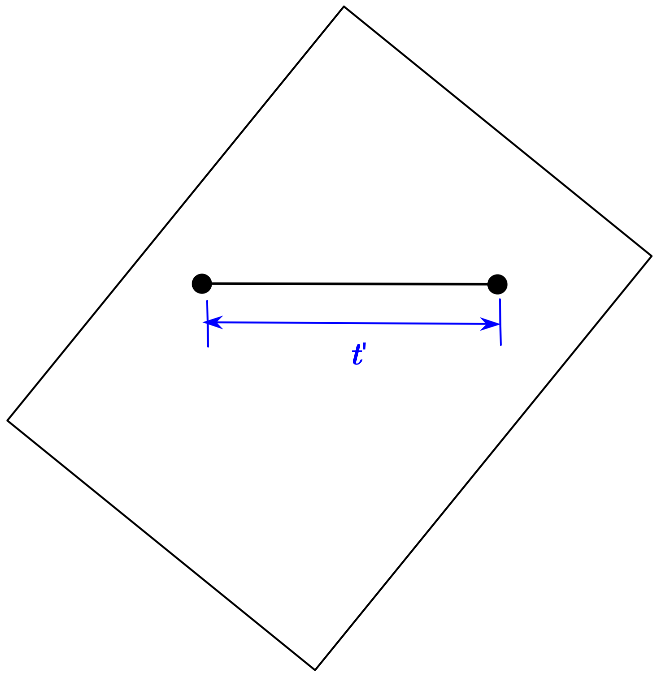
The other reason why dweebs like symmetry are more mundane. There is a lot of mathematics available to throw at problems when they have some kind of symmetry in them.

The principle of absolutivity was not invented by Einstein. It dates back to Newton at least. What Einstein did was apply it to the laws of electricity and magnetism that were discovered in the mid-19th century. Although by 1905, when Einstein published his theory, these laws were being used to invent all sorts of things (Marconi had managed to put England and the U.S. in radio contact in 1903), the dweebs were still puzzled by them. The things for which Einstein are famous are the result of applying the principle of absolutivity to these laws, rather than the priniciple itself.

One of the famous discoveries is that in some ways, time is like space. Here is how that works. Instead of labeling our dots with x and y, use x and t, where x is a spatial dimension and t is time.



This picture is a way of representing an object which is at one place at one time and another place at another time. Say, a car. The change in the position is x and the change in time is t, and the speed is (x/t). Now, speed is a relative quantity. From the point of view of someone in the car, it does not appear to be moving at all. From the point of view of someone in the car, all that stuff outside the car is moving backwards but the car is stationary. So from that viewpoint, time passes, but there is no change in position; x' is zero.



By putting a time dimension on a piece of paper with a space dimension, we can easily represent changes in the speed of the viewer. Changing from the viewpoint of someone standing by the side of the road to the viewpoint of someone moving along the road is as easy as rotating the paper.

Again, it didn't take Einstein to figure this out. This method of plotting things is very old. But next we will do the kind of thing that it did take Einstein to do, at least the first time. We look for absolute quantities. We look for numbers or equations that will be true no matter how the paper is turned.

First guess: use the Pythagorean theorum. Heck, it worked the last time, so start with

$$(t)^2 + (x)^2 .$$

Is this an absolute quantity or a relative one?

Wrong! Trick question! It isn't either. It is gibberish. You see, x is a distance and so x^2 is an area - square meters or square feet or some such. But

t is a time and so t^2 is square seconds. You can't add square meters to square seconds any more than you can add apples to oranges.

So we need to turn the seconds into meters, or vice versa. To turn seconds into meters, you need a speed, a velocity; multiply time by speed and get distance. But which velocity? Since we are trying to build a formula for an absolute number, it is probably a good idea to use an absolute number in the recipe. There is one, and only one, absolute speed: the speed of light. From every viewpoint, the speed of light (c = 299,792,458 meters/second = 670,616,629 miles per hour) is exactly the same.

All of the wierd stuff in the theory of absolutivity is contained in that statement: the statement that the speed of light is always the same. Here is an example of just how wierd that really is.

Suppose I have a 0.308 which fires bullets at 1900 miles per hour. I don't, but just say. And suppose further that we are flying down the highway at some totally unsafe speed, like 100 miles an hour, and I lean out the window and start shooting. In real life, I drive a bit on the slow side while listening to Twila Paris, but just say. Those bullets are gonna hit that stop sign at 1900+100 = 2000 miles an hour. They'll go right through, too. Now suppose further that this rifle has a laser sight. The laser light leaves the rifle at 670,616,629 miles per hour and it hits the stop sign at the exact same speed. It does not hit the sign at 670,616,629+100 = 670,616,729 miles an hour. It hits the sign at 670,616,629 miles per hour.

Between the time when the first clues appeared that light does this, and the time that we dweebs really settled down about the idea, there was about a half a century of experiments and studies. The story of that half century is very interesting, although I won't go into it here. But I will say that when you read today about Einstein's results of a century ago, you are probably reading a very sharply edited version of a very long history.

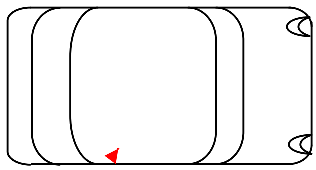
OK. So now we have an absolute number to convert seconds into meters, and we are ready for a second try at trying to construct an absolute quantity for space-time:

$$(ct)^2 + (x)^2 .$$

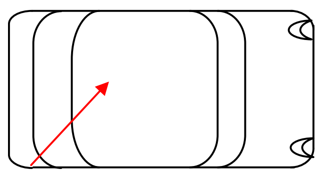
Now we are adding apples to apples. But we are still not creating an absolute number; this quantity is relative and will come out to a different number depending on one's viewpoint.

Here is the simplest way to construct an absolute number for space-time. Imagine a car of width w moving along a road at a speed v. Inside the car, on one side of the car, a little light turns on. The light goes across the car and gets to the other side in a certain time, t, which is the width of the car divided by the speed of light: $t = w/c$. What does the situation look like from the point of view of someone outside of the car? Well, the arial photography will look like this:

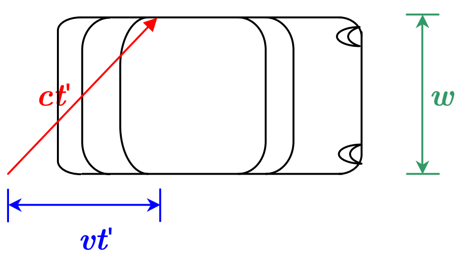
Here is the car, and the light is shown in red. It is just starting its trip.



After the light is about half way across the car, and the car has moved some down the road, the situation looks like this:



And by the time the light is all the way across the car, the car has moved still further down the road:



The light has traveled a distance ct', and the car has travelled a distance vt'. Notice that I write t' rather than t. If we used t here, it would mean that we are assuming that time is absolute. But we need more imagination than that to solve this problem. We have to imagine that time, or indeed anything, is relative until we can prove that it is absolute. So we explicitly allow that the nature of time will seem to be different depending on ones point of view. Now ct', vt', and $w = ct$ are distances in ordinary space and we can apply Pythagorus' theorum:

$$(ct')^2 - (u\ t')^2 = (w)^2 = (ct)^2 .$$

Things will be a little simpler later if we swap the names. From now on, what used to be t is t' and vice-versa.

$$(ct)^2 - (u\ t)^2 = (w)^2 = (ct')^2 .$$

What is the absolute quantity here? The absolute truth, the thing that is true both from the viewpoint in the car and from the viewpoint outside the car, is that the car has width w. The equation above gives three ways to write the number that is the car's width squared, and it shows that the time for the light to travel across the car is not an absolute quantity.

This equation is physically correct - light really does work like this - but it does not look like what we were expecting. The term (vt) is a distance, so we could call it x, but we don't have anything that looks like $(ct)^2 + (x)^2$; we have $(ct)^2 - (x)^2$. Actually, since the width of the car is a distance like x, but in a different direction, it is a distance in a second space direction. If we call the distance in the second direction by its usual name, which is y, and subtract it from the equation we get

$$(ct)^2 - (x)^2 - (y)^2 = zero\,.$$

What about the third spatial dimension, z? As long as the shocks are OK and the road is smooth, the car does not move up and down, and neither does the light. Since there is no change in the altitude of the car, $z = 0$ and we can subtract $(z)^2$ and still have an absolute number:

$$(ct)^2 - (x)^2 - (y)^2 - (z)^2 = zero\,.$$

Now we have more minus signs than plus signs! And besides, the original form of the Pythagorean theorum, before we started to try to get time involved, had $(x)^2 + (y)^2$ in it. Out of desperation, we multiply this absolute number (zero) by minus one

$$(x)^2 + (y)^2 + (z)^2 - (ct)^2 = zero$$

and by golly, now we have something. We have the Pythagorean theorum for four dimensions. The factor c is still in there to turn seconds into meters, and for some reason there is a minus sign in front of the time coordinate, but otherwise this is some distance, squared. Because we constructed this expression by adding, subtracting, and multiplying absolute quantities together, so we know that this thing, whatever it is, will be the same from all points of view.

What we have here is useful enough to be given a name. The proper distance Δ is defined by

$$(\Delta)^2 = (x)^2 + (y)^2 + (z)^2 - (ct)^2\,,$$

and it is the distance between two points in four dimensional space-time. It also, as in the example of the light in the car, going to be zero for light or for anything travelling at the speed of light. Sometimes, when we look at an object from a viewpoint where it does not seem to move, Δ^2 can be negative!

The square root of minus one

What is the square root of minus one? One squared is one, so one is a square root of one. Minus one squared is one also, so minus one is also a square root of one. What number is there that has a square of minus one? Good question. Let's go back to the dots.

Here are four dots. You can see them, they are right there on the paper or the computer screen or whatever. There are the dots. Where is the four?

The four is not in, on, under or near any of the dots. It is between your ears. "Four" is a word we use to describe the world, but it is not a word for any specific thing in the world. Maybe there might even be languages in which there is no word for four, and people who speak those languages will have a harder time to describe those dots than you or I. A number is an abstract mental object that we can use to describe the universe. An equals sign means that the abstract mental object on the left side of the equals is the same abstract mental object as the one on the right side of the equals. All that a mathematician really requires of numbers is that they follow some consistent rules. Apart from that, you can make up any number you want. In fact, all the numbers are made up.

And right now, we are going to make up another number. This number has one key property: if you multiply it by itself, you get minus one. We will call the number i, and actually it turns out to be a fairly useful number. The square root of -1, and numbers which are proportional to it, are called imaginary numbers. That is not a really good name in that all of the numbers are imaginary. Even real numbers.

I will show you some things that you can do with i. Since I am not a real mathematician, I am not show you a real proof. Instead, I am going to ask you to take out your calculator and check a few things.

My first claim is that if θ is an angle in radians, and it is pretty small, (like 0.02 or -0.004) then $sin(\theta)$ is very close to

$$\sin(q) \cong q - q^3/6 + q^5/120 .$$

So for example, let's try an angle of -1.75 degrees. In radians, that is (-1.75) times (π /180) = -0.030543262 radians. Then θ^3 is -0.000028494 and θ^5 is -0.000000027, and the thing on the right is -0.030538514, which is indeed the sin(-1.75 degrees). Each of the three terms on the right is smaller than the one before it, and there are even more terms which are even smaller which I have not written out. The thing on the right side of the equation is called a series expansion. In this, as in most series expansions, there are an infinite number of terms but they become infinitely small so that the sum is finite.

My second claim is that if θ is again an angle in radians, and it again pretty small, ***cos***(θ) is very close to

$$\cos(q) \cong 1 - q^2/2 + q^4/24 .$$

You may haul out your calculator, pick a few small numbers for θ, and check me.

My third claim is that if θ is again a pretty small number, then the exponential e^{θ} is very close to

$$e^{q} \cong 1 + q + q^2/2 + q^3/6 + q^4/24 + q^5/120 .$$

So for example, if θ is 0.1, then I claim that e^{θ} will be

$$1 + 0.1 + 0.01/2 + 0.001/6 + 0.0001/24 + 0.00001/120 = 1.105170917$$

Is it true? If you are good with a spreadsheet program, you can make plots of the ***sin***, the ***cos***, and the exponential functions and compare them to the series that I have written out.

Now here is my last claim:

$$e^{iq} = \cos(q) + i\sin(q)$$

And I will try to convince you using these series expansions. From the series expansion for the exponential,

$$e^{iq} \cong 1 + (iq) + (iq)^2/2 + (iq)^3/6 + (iq)^4/24 + (iq)^5/120 .$$

but since i^2 = -1, this is

$$e^{iq} \cong 1 + (iq) - q^2/2 + (iq)^3/6 + (iq)^4/24 + (iq)^5/120 .$$

Now if i^2 = -1, then $i^3 = i\,(i^2) = i\,(-1) = -i$, and $i^4 = (i^2)\,(i^2) = (-1)(-1) = 1$. Therefore,

$$e^{iq} \cong 1 + (iq) - q^2/2 - iq^3/6 + q^4/24 + (iq)^5/120 .$$

I will leave it to you to figure out what i^5 is, but the next step is to re-arrange the terms:

$$e^{iq} \cong \left[1 - q^2/2 + q^4/24\right] + \left[iq - iq^3/6 + iq^5/120\right].$$

and if you factor the i out from the second square brackets, voila! Now, since I only showed you that these series expansions work for small numbers, you have no reason to believe that it will work for bigger numbers, like for $\theta = \pi$ radians. Actually, it will work, but I will let you try to prove that yourself.

Here is what we will do with the square root of negative one: we re-write the formula for the proper distance to have all plus signs.

$$(\Delta)^2 = (x)^2 + (y)^2 + (z)^2 + (ict)^2 .$$

When you hear that in Einstein's theory, time and space are the same, this is what it means: if you take time and multiply it by the speed of light and the square root of -1, then you have a number that looks like a distance in the more general form of Pythagorus' theorum. There are other ways in which time is still quite different from space. Our minds - which probably are not the same as our brains - perceive time as flowing past us but do not see space as having a past or a future. The theory of absolutivity does not really address that. The theory of absolutivity puts times in the past and times in the future on the same footing at the very outset; it ignores the one-directional flow of time entirely.

Speeding right along

In everyday language, we have the words speed and velocity. We tend to use them interchangeably and by them we mean how much distance a moving object will cover in a certain time. Let us look at the proper distance and take the terms like x, which indicate a change in position, and divide them by time t, the span of time involved in that particular proper distance.

$$\left(\frac{\Delta}{t}\right)^2 = \left(\frac{x}{t}\right)^2 + \left(\frac{y}{t}\right)^2 + \left(\frac{z}{t}\right)^2 + (ic)^2 .$$

This thing has three numbers that look like what a dweeb would call the velocity: they are $v_x = x / t$, $v_y = y / t$, and $v_y = y / t$. These three numbers, taken together, give not just how much distance the object has moved in the time t, but they also say how much of the motion was in the x direction, how much was in the y direction and how much is in the z direction. Unlike in everyday use, a dweeb would mean a slightly different thing when he uses the word speed; precisely speaking, he means v, as defined by $v^2 = v_x^2 + v_y^2 + v_y^2$. I will just use speed here and not spend any more time hashing about the differences between speed and velocity.

$$\left(\frac{\Delta}{t}\right)^2 = (u)^2 + (ic)^2 .$$

Here is an equation for this thing that looks kind of like a velocity with an extra fourth dimension. But it has a problem. We made it by dividing the proper distance Δ, which is an absolute number, by ordinary time t, which is a relative number. The result is a relative number. So while we can write a formula for (Δ / t), it is not a very interesting thing.

Another word that is used both in everyday language and by dweebs is momentum. The dweeb means mass times velocity. I had better take a few minutes and talk about the word mass. In physics, there are several different things that are very similar, and not all dweebs mean the same thing when they use the word.

The first kind of mass is called the gravitational mass. It is the mass that creates the gravitational field of an object. It is conceptually different from the inertial mass, which is the mass that keeps a body at rest still in a state of rest unless acted upon by an outside force. Or if you exert a force $\boldsymbol{F}$ on an object, and it then gets some acceleration $\boldsymbol{a}$, the inertial mass is the mass that you use for $\boldsymbol{F} = \boldsymbol{ma}$. These two kinds of mass do not have to be the same - but the universe is constructed so that they are. We do not know why. It is easy to imagine a universe in which they are different.

In four dimensions, with the principle of absolutivity, we will find that inertia is a relative quantity,[†] but it is possible to divide out the relative part, leaving a quantity which is a mass of some kind and which is an absolute quantity.

Some dweebs call the first quantity the relative mass and the second quantity the invariant mass, but not me. When I say mass or write $\boldsymbol{m}$, it is the second, absolute, quantity that I mean. It is both how much the object responds to the force of gravity, and also is how much inertia the object has when you look at it from its own rest frame - *i.e.*, a point of view in which the object is not moving.

Now we can figure out the absolute momentum. We have a velocity equation, and a carefully defined mass $\boldsymbol{m}$, so the momentum $\boldsymbol{p} = \boldsymbol{mv}$ would look like

$$\left(\frac{m\Delta}{t}\right)^2 = (p)^2 + (imc)^2 .$$

[†] Actually, it turns out that not only does the amount of inertia that an object has depend on how fast it is moving, it also depends on the direction that the force is applied; so it seems from certain viewpoints that the direction of the force and the direction of the acceleration is different. But this is too hard for me to try to derive here.

Well, not quite. Remember, this (Δ / t) is not an absolute quantity. This number, $(p)^2 + (imc)^2$, is relative is because it is proportional to $(v)^2 + (ic)^2 = v^2 - c^2$, which is relative. If we divide out that $v^2 - c^2$, the result will be an absolute quantity.

$$\frac{1}{u^2 - c^2}\left(\frac{m\Delta}{t}\right)^2 = \frac{(p)^2 + (imc)^2}{u^2 - c^2}.$$

Multiply this equation by c^2, and you will have something interesting. Well, assuming that you are interested in this kind of thing. Which you must be, or you would not have gotten this far, right? Right. And the reason why the thing you get is interesting? It has mc^2 in it.

$$\frac{c^2}{u^2 - c^2}\left(\frac{m\Delta}{t}\right)^2 = \frac{(mc^2)^2 - (cp)^2}{1 - \left(u/c\right)^2},$$

and therefore,

$$\frac{(mc^2)^2}{1 - \left(u/c\right)^2} = \frac{c^2}{u^2 - c^2}\left(\frac{m\Delta}{t}\right)^2 + \frac{(cp)^2}{1 - \left(u/c\right)^2}.$$

The stuff on the right is kind of complicated, but this quantity must be positive. That is because (as we will soon see and you have no doubt heard) no object can go faster than the speed of light. So (v / c) is less than one and $1 - \left(u/c\right)^2$ is positive and so is m^2c^4 divided by $1 - \left(u/c\right)^2$. The set of all the positive real numbers is the set of all the squares of the real numbers, so the quantity given by this equation is some real number, call it E, squared. You can tell where I am going here.

$$\frac{(mc^2)^2}{1 - \left(u/c\right)^2} = E^2,$$

so

$$E = \pm mc^2 \sqrt{1/\left(1 - \left(u/c\right)^2\right)}$$

where I have included the ± to remind us that there are both positive and negative square roots to a number. This is the full form of the famous equation.

The famous equation

Now I must justify why this particular E really is energy. Look at the case where the velocity of the object is small compared to the speed of light. In that case, we know from Newton that the kinetic energy is ½ mv^2. This proof needs an other series expansion.

$$\sqrt{\frac{1}{1-b^2}} \cong 1 + \frac{b^2}{2} + \frac{3b^4}{8}$$

At a low enough speed, we can forget the third term, and also all the even smaller terms which I did not write, so

$$E = \pm\left(mc^2 + \frac{1}{2}mu^2\right).$$

There is the Newtonian ½ mv^2. There is no doubt that E is the energy, somehow. But there is an additional energy in an object equal to mc^2, that is there even when the object has no kinetic energy at all. This energy is like a potential energy in that it is sort of hidden, but it can be released under certain conditions, such as in nuclear reactions.

And there still is that $\pm$ sign, too. What could it possibly mean that the energy is negative? A moving object has positive kinetic energy and there is the rest energy of mc^2, but what is a negative energy? It is like the opposite of moving but also not at rest. Makes no sense!

Between about 1905 and 1929, "makes no sense" is about the best answer that anyone had. Sometimes equations have solutions that do not correspond to what happens in the real world, and we just ignore them. But between 1927 and 1929, quantum mechanics was worked out and suddenly, negative energy began to make sense.

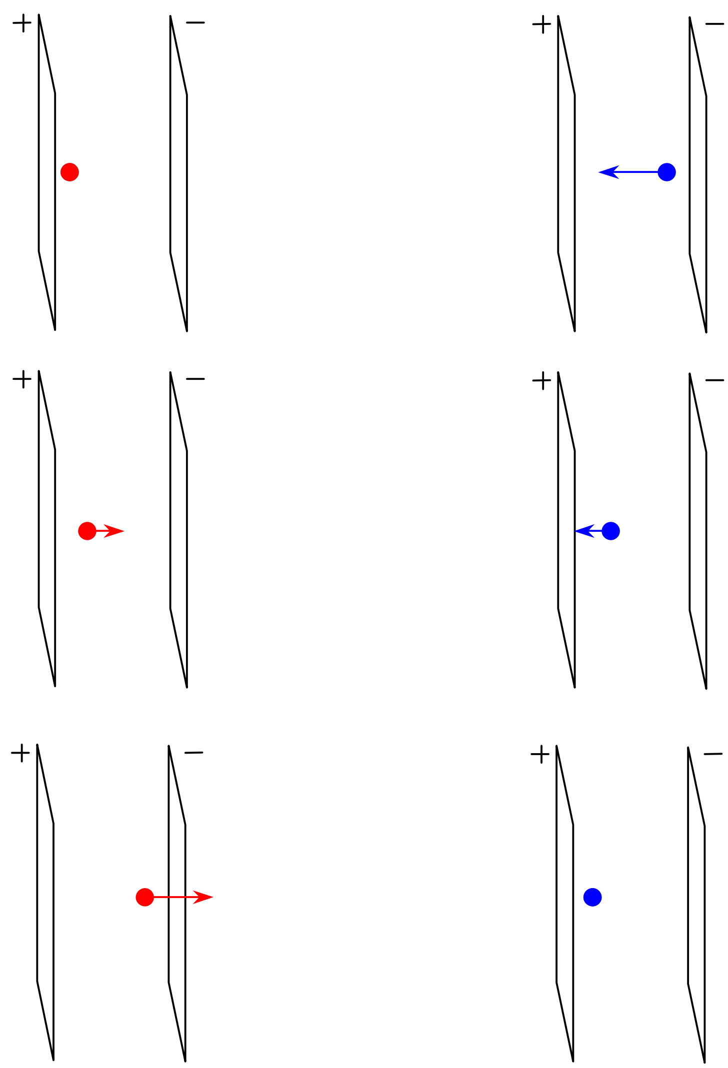
Quantum mechanics is about the behavior of extremely small objects - electrons and protons and such. The first example of quantum mechanics that most of us learn is the Bohr atom. Niels Bohr suggested that maybe a hydrogen atom is a positively charged lump with an electron spinning around it - and that the angular momentum of the electron around the lump has to be a multiple of $\hbar = h/2p = 1.055 \times 10^{-34}$ kg-m^2/s. The very small size of $\hbar$ is why quantum mechanical effects only matter for very small objects. But the thing I want to focus on is the units. Angular momentum is a radial distance times a momentum: (meters)(kilogram-meters/second). Energy, as you can tell from the form mc^2, is kilograms by (meters/second)^2. Energy multiplied by time is the same units as angular momentum.

In quantum mechanics, what we do with the energy once we have it is multiply it by time and compare it to $\hbar$. Everywhere we have E, we have $Et/\hbar$. Which means that negative energy is the same as negative time; a particle with the negative sign in the famous equation is a particle moving as if time is going backwards for it. That is what antimatter is - matter for which time seems to be going backwards.

Remember! All of our theoretical machinations here ignore the existence of past and present at the very outset. So we can not conclude that a human being made of antimatter rather than matter would remember tomorrow and look forward to yesterday.

A particle of antimatter has the same amount of inertia as the same particle of matter. In otherwords, the antimatter form of a proton has the same m as the matter form of a proton, and $mc^2 \Big/ \sqrt{1-\left(\frac{u}{c}\right)^2}$ will be the same for a proton and an antiproton at any given speed. But the charge will be different. Here is why.

Imagine two plates of metal with some electric charge on them and a proton in between. The proton, being positively charged, will accelerate away from the positive plate and towards the negative plate. If it starts at rest, it will soon have some speed, call it V, towards the negative plate. Now imagine the process happening in reverse: a particle starts out with speed V towards the *positive* plate slows down to a stop.



In the figure, the red dot shows the motion of a positively charged particle going forward in time as you read down the page. The blue dot is a particle of the same mass, but which moves as if time were reversed. The red dot ends with a velocity towards the negative plate, and the blue dot starts with a velocity towards the positive plate.

Now, if a particle starts moving towards a positive plate and slows down to a stop, what charge does it have? Positive. The antiproton is a particle with the same mass as a proton, but with the opposite charge. More generally, antimatter has the same mass as matter, but has the opposite electrical charge. It also has the opposite "charge" for other forces, like the nuclear binding force.

Now here is an interesting question. Does antimatter have a gravitiational mass opposite to matter? Remember, we said that the gravitational mass is the one which decides what the force of gravity is on an object. The inertial mass is the one which is m in mc^2; and that the universe is created somehow so that they are actually the same. But (for no reason that anyone can explain) the universe is made out of matter, mostly. So all of our experiments that show the equivalence of gravitational and inertial mass are done with matter. We have no experiments for antimatter; maybe it has an opposite gravitational mass. Maybe antimatter falls up!

Well, probably not. Here is why. When an electron and an anti-electron collide, they become a particle of light, called a photon. Similarly, a photon can turn into an electron and an anti-electron[†]. Suppose that happens, and the photon comes out going down. As the photon falls, it gains energy from the gravitational field. Then imagine that the photon splits again into an electron and an antielectron. It will take some amount of energy, call it U, to lift the electron against the force of gravity back to the initial point. If antimatter falls up, then one gets U energy out by moving the antimatter up. So if antimatter falls up, there is no net energy needed to lift the electron / anti-electron pair; but then the process can be repeated, putting still more energy into the system. If antimatter falls up, maybe we can get infinite amounts of energy out of nowhere. That seems pretty unlikely. So, although there are no experiments to prove it, we expect that anti-matter falls down.

Breaking the speed limit

With the famous equation (in its full form) we can also see that objects can not go faster than the speed of light. Start at $v = 0$. Then $E = \pm mc^2$. neglecting our wierd little $\pm$, the energy of the object is at its smallest possible value. As the speed increases, the energy increases also. At $v = c / 2$, the

[†]I am simplifying. At best, one gets two photons. And probably other things have to happen. But the soundness of the argument is unchanged.

energy is $\pm 1.41421 mc^2$; at $v = 0.9c$, it is $\pm 2.29416 mc^2$. As the speed approaches the speed of light, the energy that you need to put into the object increases infinitely. At $v = c$, in fact the energy goes straight up to infinity:

$$E = \pm mc^2 \sqrt{\frac{1}{1-(1)^2}} = \frac{\pm\, mc^2}{(zero!)}$$

Unless you have an infinite supply of energy, you can not get an object to move faster than the speed of light. You can keep pushing on it, but it will not move any faster. In other words, inertia is relative - if you take a viewpoint where an object is already moving quickly, then you will see the object be very resistant to further acceleration.

So then how can light go at the speed of light? Any real number, like mc^2, when divided by zero, is infinity. But zero divided by zero is another matter. Zero divided by zero calls for highly trained specialists - but it can be done. Light can travel at the speed of light because it has $m = 0$. That is, it has no inertia and does not create a gravitational field. Any massless object will also travel at that same speed - it is the maximal possible speed that the universe allows.

So what this shows is that an object going slower than the speed of light can not get to above lightspeed. What about the possibility of objects that start at above the speed of light? The idea exists; such things are called tachyons. But although they have a name, they probably do not exist.

In quantum mechanics, we have wavefunctions; in your chemistry class, they may have been called orbitals and used to help picture the shapes of molecules. We do not really know the exact location of a particle quantum mechanically, but the wavefunction tells us the probability of finding the particle at any given place.

If you write down a mathematical expressions for a wavefunction, it will have a factor $e^{iEt/\hbar}$ in them.

Suppose that somehow we had an object that got to be moving faster than the speed of light. Then the energy becomes an imaginary number:

$$E = \pm mc^2 \sqrt{\frac{1}{1-\left(\frac{u}{c}\right)^2}} = \pm mc^2 \sqrt{\text{a negative number}} \propto \pm imc^2 .$$

For the positive root, $iE/\hbar$ is a negative number, so the wavefunction is proportional to $e^{(\text{a negative number})t}$. As time goes on, t gets bigger, and the wavefunction gets smaller and smaller. The probability of finding the particle diminishes with time - the particle disappears! The negative root is even stranger. For the negative root, the wavefunction is proportional to

$e^{(\text{a positive number})t}$. Your odds of finding the particle increase infinitely as time goes on! Not much chance of that. The idea of tachyons does not mix well with quantum mechanics.