18. EXPERIMENTAL TESTS OF GRAVITATIONAL THEORY

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Einstein's General Relativity, the current "standard" theory of gravitation, describes gravity as a universal deformation of the Minkowski metric:

$$g_{\mu\nu}(x^\lambda) = \eta_{\mu\nu} + h_{\mu\nu}(x^\lambda) \ , \ \text{where } \eta_{\mu\nu} = \text{diag}(-1,+1,+1,+1) \ . \tag{18.1}$$

Alternatively, it can be defined as the unique, consistent, local theory of a massless spin-2 field $h_{\mu\nu}$, whose source must then be the total, conserved energy-momentum tensor [1]. General Relativity is classically defined by two postulates. One postulate states that the Lagrangian density describing the propagation and self-interaction of the gravitational field is

$$\mathcal{L}_{\text{Ein}}[g_{\mu\nu}] = \frac{c^4}{16\pi G_N}\sqrt{g}g^{\mu\nu}R_{\mu\nu}(g) \ , \tag{18.2}$$

$$R_{\mu\nu}(g) = \partial_\alpha\Gamma^\alpha_{\mu\nu} - \partial_\nu\Gamma^\alpha_{\mu\alpha} + \Gamma^\beta_{\alpha\beta}\Gamma^\alpha_{\mu\nu} - \Gamma^\beta_{\alpha\nu}\Gamma^\alpha_{\mu\beta} \ , \tag{18.3}$$

$$\Gamma^\lambda_{\mu\nu} = \tfrac{1}{2}g^{\lambda\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}) \ , \tag{18.4}$$

where G_N is Newton's constant, $g = -\det(g_{\mu\nu})$, and $g^{\mu\nu}$ is the matrix inverse of $g_{\mu\nu}$. A second postulate states that $g_{\mu\nu}$ couples universally, and minimally, to all the fields of the Standard Model by replacing everywhere the Minkowski metric $\eta_{\mu\nu}$. Schematically (suppressing matrix indices and labels for the various gauge fields and fermions and for the Higgs doublet),

$$\begin{aligned}\mathcal{L}_{\text{SM}}[\psi, A_\mu, H, g_{\mu\nu}] = &-\tfrac{1}{4}\sum\sqrt{g}g^{\mu\alpha}g^{\nu\beta}F^a_{\mu\nu}F^a_{\alpha\beta}\\ &-\sum\sqrt{g}\,\overline{\psi}\,\gamma^\mu D_\mu\psi\\ &-\tfrac{1}{2}\sqrt{g}g^{\mu\nu}\overline{D_\mu H}D_\nu H - \sqrt{g}\,V(H)\\ &-\sum\lambda\sqrt{g}\,\overline{\psi}\,H\psi \ ,\end{aligned} \tag{18.5}$$

where $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$, and where the covariant derivative D_μ contains, besides the usual gauge field terms, a (spin-dependent) gravitational contribution $\Gamma_\mu(x)$ [2]. From the total action follow Einstein's field equations,

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu} \ . \tag{18.6}$$

Here $R = g^{\mu\nu}R_{\mu\nu}$, $T_{\mu\nu} = g_{\mu\alpha}g_{\nu\beta}T^{\alpha\beta}$, and $T^{\mu\nu} = (2/\sqrt{g})\delta\mathcal{L}_{\text{SM}}/\delta g_{\mu\nu}$ is the (symmetric) energy-momentum tensor of the Standard Model matter. The theory is invariant under arbitrary coordinate transformations: $x'^{\mu} = f^{\mu}(x^{\nu})$. To solve the field equations Eq. (18.6), one needs to fix this coordinate gauge freedom. *E.g.*, the "harmonic gauge" (which is the analogue of the Lorentz gauge, $\partial_\mu A^\mu = 0$, in electromagnetism) corresponds to imposing the condition $\partial_\nu(\sqrt{g}g^{\mu\nu}) = 0$.

In this *Review*, we only consider the classical limit of gravitation (*i.e.* classical matter and classical gravity). Considering quantum matter in a classical gravitational background already poses interesting challenges, notably the possibility that the zero-point fluctuations of the matter fields generate a nonvanishing vacuum energy density $\rho_{\rm vac}$, corresponding to a term $-\sqrt{g}\;\rho_{\rm vac}$ in $\mathcal{L}_{\rm SM}$ [3]. This is equivalent to adding a "cosmological constant" term $+\Lambda\, g_{\mu\nu}$ on the left-hand side of Einstein's equations Eq. (18.6), with $\Lambda = 8\pi G_N\, \rho_{\rm vac}/c^4$. Recent cosmological observations (see the following *Reviews*) suggest a positive value of Λ corresponding to $\rho_{\rm vac} \approx (2.3 \times 10^{-3}{\rm eV})^4$. Such a small value has a negligible effect on the tests discussed below. Quantizing the gravitational field itself poses a very difficult challenge because of the perturbative non-renormalizability of Einstein's Lagrangian. Superstring theory offers a promising avenue toward solving this challenge.

18.1. Experimental tests of the coupling between matter and gravity

The universality of the coupling between $g_{\mu\nu}$ and the Standard Model matter postulated in Eq. (18.5) ("Equivalence Principle") has many observable consequences. First, it predicts that the outcome of a local non-gravitational experiment, referred to local standards, does not depend on where, when, and in which locally inertial frame, the experiment is performed. This means, for instance, that local experiments should neither feel the cosmological evolution of the universe (constancy of the "constants"), nor exhibit preferred directions in spacetime (isotropy of space, local Lorentz invariance). These predictions are consistent with many experiments and observations. The best limit on a possible time variation of the basic coupling constants concerns the fine-structure constant $\alpha_{\rm em}$, and has been obtained by analyzing a natural fission reactor phenomenon which took place at Oklo, Gabon, two billion years ago [4]. A conservative estimate of the (95% C.L.) Oklo limit on the variability of $\alpha_{\rm em}$ is (see second reference in [4])

$$-0.9 \times 10^{-7} < \frac{\alpha_{\rm em}^{\rm Oklo} - \alpha_{\rm em}^{\rm now}}{\alpha_{\rm em}} < 1.2 \times 10^{-7}\,, \tag{18.7}$$

which corresponds to the following limit on the average time derivative of $\alpha_{\rm em}$

$$-6.7 \times 10^{-17}{\rm yr}^{-1} < \dot{\alpha}_{\rm em}/\alpha_{\rm em} < 5.0 \times 10^{-17}{\rm yr}^{-1}\ . \tag{18.8}$$

The second best limit on the variability of $\alpha_{\rm em}$ comes from analyzing isotopic measurements of some meteorites dating back to the formation of the solar system (about 4.6 Gyr ago). The ensuing determination of the lifetime of Rhenium 187 can be interpreted in terms of the following bound: $(\alpha_{\rm em}^{\rm 4.6Gyr} - \alpha_{\rm em}^{\rm now})/\alpha_{\rm em} = (8 \pm 8) \times 10^{-7}$ [5]. Recent measurements of absorption lines in astronomical spectra also give stringent limits on the variability of $\alpha_{\rm em}$ [6], which disagree with an earlier claim of a non-zero effect [7]. Direct laboratory limits on the time variation of $\alpha_{\rm em}$ (based on monitoring the frequency ratio of several different atomic clocks) are [8]: $\dot{\alpha}_{\rm em}/\alpha_{\rm em} = (-0.9 \pm 2.9) \times 10^{-15}{\rm yr}^{-1}$, which is less stringent than Eq. (18.8), but less model-dependent. See Ref. 9 for a general review of the issue of "variable constants."

The highest precision tests of the isotropy of space have been performed by looking for possible quadrupolar shifts of nuclear energy levels [10]. The (null) results can be interpreted as testing the fact that the various pieces in the matter Lagrangian Eq. (18.5) are indeed coupled to one and the same external metric $g_{\mu\nu}$ to the 10^{-27} level. For astrophysical constraints on possible Planck-scale violations of Lorentz invariance, see Ref. 11.

The universal coupling to $g_{\mu\nu}$ postulated in Eq. (18.5) implies that two (electrically neutral) test bodies dropped at the same location and with the same velocity in an external gravitational field fall in the same way, independently of their masses and compositions. The universality of the acceleration of free fall has been verified below the 10^{-12} level for laboratory bodies [12], notably earth-core-like and moon-mantle-like bodies [13],

$$(\Delta a/a)_{\rm ECMM} = (3.6 \pm 5.0) \times 10^{-13} \ , \tag{18.9}$$

as well as for the gravitational accelerations of the Earth and the Moon toward the Sun [14],

$$(\Delta a/a)_{\rm EarthMoon} = (-1.0 \pm 1.4) \times 10^{-13} \ . \tag{18.10}$$

See also Ref. 15 for *short-range* tests of the universality of free-fall.

Finally, Eq. (18.5) also implies that two identically constructed clocks located at two different positions in a static external Newtonian potential $U(\boldsymbol{x}) = \sum G_N m/r$ exhibit, when intercompared by means of electromagnetic signals, the (apparent) difference in clock rate,

$$\frac{\tau_1}{\tau_2} = \frac{\nu_2}{\nu_1} = 1 + \frac{1}{c^2}[U(\boldsymbol{x}_1) - U(\boldsymbol{x}_2)] + O\left(\frac{1}{c^4}\right) \ , \tag{18.11}$$

independently of their nature and constitution. This universal gravitational redshift of clock rates has been verified at the 10^{-4} level by comparing a hydrogen-maser clock flying on a rocket up to an altitude $\sim 10,000$ km to a similar clock on the ground [16]. For more details and references on experimental gravity see, *e.g.*, Refs. 17 and 18.

18.2. Tests of the dynamics of the gravitational field in the weak field regime

The effect on matter of one-graviton exchange, *i.e.*, the interaction Lagrangian obtained when solving Einstein's field equations Eq. (18.6) written in, say, the harmonic gauge at first order in $h_{\mu\nu}$,

$$\Box h_{\mu\nu} = -\frac{16\pi G_N}{c^4}(T_{\mu\nu} - \tfrac{1}{2}T\eta_{\mu\nu}) + O(h^2) + O(hT) \ , \tag{18.12}$$

reads $-(8\pi G_N/c^4)T^{\mu\nu}\Box^{-1}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu})$. For a system of N moving point masses, with free Lagrangian $L^{(1)} = \sum_{A=1}^{N} - m_A c^2 \sqrt{1 - \boldsymbol{v}_A^2/c^2}$, this interaction, expanded to order v^2/c^2, reads (with $r_{AB} \equiv |\boldsymbol{x}_A - \boldsymbol{x}_B|$, $\boldsymbol{n}_{AB} \equiv (\boldsymbol{x}_A - \boldsymbol{x}_B)/r_{AB}$)

$$L^{(2)} = \frac{1}{2} \sum_{A \neq B} \frac{G_N\, m_A\, m_B}{r_{AB}} \left[1 + \frac{3}{2c^2}(\boldsymbol{v}_A^2 + \boldsymbol{v}_B^2) - \frac{7}{2c^2}(\boldsymbol{v}_A \cdot \boldsymbol{v}_B) - \frac{1}{2c^2}(\boldsymbol{n}_{AB} \cdot \boldsymbol{v}_A)(\boldsymbol{n}_{AB} \cdot \boldsymbol{v}_B) + O\left(\frac{1}{c^4}\right) \right] . \tag{18.13}$$

The two-body interactions (Eq. (18.13)) exhibit v^2/c^2 corrections to Newton's $1/r$ potential induced by spin-2 exchange. Consistency at the "post-Newtonian" level $v^2/c^2 \sim G_N\, m/rc^2$ requires that one also considers the three-body interactions induced by some of the three-graviton vertices and other nonlinearities (terms $O(h^2)$ and $O(hT)$ in Eq. (18.12)),

$$L^{(3)} = -\frac{1}{2} \sum_{B \neq A \neq C} \frac{G_N^2\, m_A\, m_B\, m_C}{r_{AB}\, r_{AC}\, c^2} + O\left(\frac{1}{c^4}\right) . \tag{18.14}$$

All currently performed gravitational experiments in the solar system, including perihelion advances of planetary orbits, the bending and delay of electromagnetic signals passing near the Sun, and very accurate ranging data to the Moon obtained by laser echoes, are compatible with the post-Newtonian results Eqs. (18.12)–(18.14).

Similar to what is done in discussions of precision electroweak experiments, it is useful to quantify the significance of precision gravitational experiments by parameterizing plausible deviations from General Relativity. The addition of a mass-term in Einstein's field equations leads to a score of theoretical difficulties [19] which have not yet received any consensual solution. We shall, therefore, not consider here the ill-defined "mass of the graviton" as a possible deviation parameter from General Relativity (see, however, the phenomenological limits quoted in the Section "Gauge and Higgs Bosons" of this *Review*). Deviations from Einstein's pure spin-2 theory are then defined by adding new, bosonic light or massless, macroscopically coupled fields. The possibility of new gravitational-strength couplings leading (on small, and possibly large, scales) to deviations from Einsteinian (and Newtonian) gravity is suggested by String Theory [20], and by Brane World ideas [21]. For compilations of experimental constraints on Yukawa-type additional interactions, see Refs. [12,22,23] and the Section "Extra Dimensions" in this *Review*. Recent experiments have set limits on non-Newtonian forces below 0.056 mm [24].

Here, we shall focus on the parametrization of long-range deviations from relativistic gravity obtained by adding a massless scalar field φ coupled to the trace of the energy-momentum tensor $T = g_{\mu\nu}T^{\mu\nu}$ [25]. The most general such theory contains an arbitrary function $a(\varphi)$ of the scalar field, and can be defined by the Lagrangian

$$\mathcal{L}_{\rm tot}[g_{\mu\nu},\varphi,\psi,A_\mu,H] = \frac{c^4}{16\pi G}\sqrt{g}(R(g) - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi) + \mathcal{L}_{\rm SM}[\psi,A_\mu,H,\widetilde{g}_{\mu\nu}] \ , \qquad (18.15)$$

where G is a "bare" Newton constant, and where the Standard Model matter is coupled not to the "Einstein" (pure spin-2) metric $g_{\mu\nu}$, but to the conformally related ("Jordan-Fierz") metric $\widetilde{g}_{\mu\nu} = \exp(2a(\varphi))g_{\mu\nu}$. The scalar field equation $\Box_g\varphi = -(4\pi G/c^4)\alpha(\varphi)T$ displays $\alpha(\varphi) \equiv \partial a(\varphi)/\partial\varphi$ as the basic (field-dependent) coupling between φ and matter [26]. The one-parameter (ω) Jordan-Fierz-Brans-Dicke theory [25] is the special case $a(\varphi) = \alpha_0\varphi$ leading to a field-independent coupling $\alpha(\varphi) = \alpha_0$ (with ${\alpha_0}^2 = 1/(2\omega + 3)$).

In the weak-field slow-motion limit appropriate to describing gravitational experiments in the solar system, the addition of φ modifies Einstein's predictions only through the appearance of two "post-Einstein" dimensionless parameters: $\overline{\gamma} = -2\alpha_0^2/(1+\alpha_0^2)$ and $\overline{\beta} = +\frac{1}{2}\beta_0\alpha_0^2/(1+\alpha_0^2)^2$, where $\alpha_0 \equiv \alpha(\varphi_0)$, $\beta_0 \equiv \partial\alpha(\varphi_0)/\partial\varphi_0$, φ_0 denoting the vacuum expectation value of φ. These parameters show up also naturally (in the form $\gamma_{\rm PPN} = 1 + \overline{\gamma}$, $\beta_{\rm PPN} = 1 + \overline{\beta}$) in phenomenological discussions of possible deviations from General Relativity [17,27]. The parameter $\overline{\gamma}$ measures the admixture of spin 0 to Einstein's graviton, and contributes an extra term $+\overline{\gamma}(\boldsymbol{v}_A - \boldsymbol{v}_B)^2/c^2$ in the square brackets of the two-body Lagrangian Eq. (18.13). The parameter $\overline{\beta}$ modifies the three-body interaction Eq. (18.14) by an overall multiplicative factor $1 + 2\overline{\beta}$. Moreover, the combination $\eta \equiv 4\overline{\beta} - \overline{\gamma}$ parameterizes the lowest order effect of the self-gravity of orbiting masses by modifying the Newtonian interaction energy terms in Eq. (18.13) into $G_{AB}m_Am_B/r_{AB}$, with a body-dependent gravitational "constant" $G_{AB} = G_N[1+\eta(E_A^{\rm grav}/m_Ac^2 + E_B^{\rm grav}/m_Bc^2) + O(1/c^4)]$, where $G_N = G\exp[2a(\varphi_0)](1+\alpha_0^2)$ and where $E_A^{\rm grav}$ denotes the gravitational binding energy of body A.

The best current limits on the post-Einstein parameters $\overline{\gamma}$ and $\overline{\beta}$ are (at the 68% confidence level):

$$\overline{\gamma} = (2.1 \pm 2.3) \times 10^{-5} \ , \qquad (18.16)$$

deduced from the additional Doppler shift experienced by radio-wave beams connecting the Earth to the Cassini spacecraft when they passed near the Sun [28], and

$$4\overline{\beta} - \overline{\gamma} = (4.4 \pm 4.5) \times 10^{-4} \ , \qquad (18.17)$$

from Lunar Laser Ranging measurements [14] of a possible polarization of the Moon toward the Sun [29]. More stringent limits on $\overline{\gamma}$ are obtained in models (*e.g.*, string-inspired ones [20]) where scalar couplings violate the Equivalence Principle.

18.3. Tests of the dynamics of the gravitational field in the radiative and/or strong field regimes

The discovery of pulsars (*i.e.*, rotating neutron stars emitting a beam of radio noise) in gravitationally bound orbits [30,31] has opened up an entirely new testing ground for relativistic gravity, giving us an experimental handle on the regime of radiative and/or strong gravitational fields. In these systems, the finite velocity of propagation of the gravitational interaction between the pulsar and its companion generates damping-like terms at order $(v/c)^5$ in the equations of motion [32]. These damping forces are the local counterparts of the gravitational radiation emitted at infinity by the system ("gravitational radiation reaction"). They cause the binary orbit to shrink and its orbital period P_b to decrease. The remarkable stability of pulsar clocks has allowed one to measure the corresponding very small orbital period decay $\dot{P}_b \equiv dP_b/dt \sim -(v/c)^5 \sim -10^{-12}$ in several binary systems, thereby giving us a direct experimental confirmation of the propagation properties of the gravitational field, and, in particular, an experimental confirmation that the speed of propagation of gravity is equal to the velocity of light to better than a part in a thousand. In addition, the surface gravitational potential of a neutron star $h_{00}(R) \simeq 2Gm/c^2R \simeq 0.4$ being a factor $\sim 10^8$ higher than the surface potential of the Earth, and a mere factor 2.5 below the black hole limit ($h_{00} = 1$), pulsar data have allowed one, as we discuss next, to obtain several accurate tests of the strong-gravitational-field regime.

Binary pulsar timing data record the times of arrival of successive electromagnetic pulses emitted by a pulsar orbiting around the center of mass of a binary system. After correcting for the Earth motion around the Sun and for the dispersion due to propagation in the interstellar plasma, the time of arrival of the Nth pulse t_N can be described by a generic, parameterized "timing formula" [33] whose functional form is common to the whole class of tensor-scalar gravitation theories:

$$t_N - t_0 = F[T_N(\nu_p, \dot{\nu}_p, \ddot{\nu}_p)\,;\,\{p^K\}\,;\,\{p^{PK}\}]\ . \tag{18.18}$$

Here, T_N is the pulsar proper time corresponding to the Nth turn given by $N/2\pi = \nu_p T_N + \frac{1}{2}\dot{\nu}_p T_N^2 + \frac{1}{6}\ddot{\nu}_p T_N^3$ (with $\nu_p \equiv 1/P_p$ the spin frequency of the pulsar, *etc.*), $\{p^K\} = \{P_b, T_0, e, \omega_0, x\}$ is the set of "Keplerian" parameters (notably, orbital period P_b, eccentricity e and projected semi-major axis $x = a \sin i/c$), and $\{p^{PK}\} = \{k, \gamma_{\rm timing}, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\}$ denotes the set of (separately measurable) "post-Keplerian" parameters. Most important among these are: the fractional periastron advance per orbit $k \equiv \dot{\omega} P_b/2\pi$, a dimensionful time-dilation parameter $\gamma_{\rm timing}$, the orbital period derivative $\dot{P}_b$, and the "range" and "shape" parameters of the gravitational time delay caused by the companion, r and s.

Without assuming any specific theory of gravity, one can phenomenologically analyze the data from any binary pulsar by least-squares fitting the observed sequence of pulse arrival times to the timing formula Eq. (18.18). This fit yields the "measured" values of the parameters $\{\nu_p, \dot{\nu}_p, \ddot{\nu}_p\}$, $\{p^K\}$, $\{p^{PK}\}$. Now, each specific relativistic theory of gravity predicts that, for instance, k, $\gamma_{\rm timing}$, $\dot{P}_b$, r and s (to quote parameters that have been successfully measured from some binary pulsar data) are some theory-dependent functions of the Keplerian parameters and of the (unknown) masses m_1, m_2 of the pulsar and its companion. For instance, in General Relativity, one finds (with $M \equiv m_1 + m_2$, $n \equiv 2\pi/P_b$)

$$\begin{aligned} k^{\rm GR}(m_1, m_2) =& 3(1-e^2)^{-1}(G_N M n/c^3)^{2/3} \ , \\ \gamma^{\rm GR}_{\rm timing}(m_1, m_2) =& e n^{-1}(G_N M n/c^3)^{2/3} m_2(m_1 + 2m_2)/M^2 \ , \\ \dot{P}^{\rm GR}_b(m_1, m_2) =& -(192\pi/5)(1-e^2)^{-7/2}\left(1 + \tfrac{73}{24}e^2 + \tfrac{37}{96}e^4\right) \\ & \times (G_N M n/c^3)^{5/3} m_1 m_2/M^2 \ , \\ r(m_1, m_2) =& G_N m_2/c^3 \ , \\ s(m_1, m_2) =& n x (G_N M n/c^3)^{-1/3} M/m_2 \ . \end{aligned} \tag{18.19}$$

In tensor-scalar theories, each of the functions $k^{\rm theory}(m_1, m_2)$, $\gamma^{\rm theory}_{\rm timing}(m_1, m_2)$, $\dot{P}^{\rm theory}_b(m_1, m_2)$, *etc.*, is modified by quasi-static strong field effects (associated with the self-gravities of the pulsar and its companion), while the particular function $\dot{P}^{\rm theory}_b(m_1, m_2)$ is further modified by radiative effects (associated with the spin 0 propagator) [26,34,35].

Let us summarize the current experimental situation (see Ref. 36 for a more extensive review). In the first discovered binary pulsar PSR1913 + 16 [30,31], it has been possible to measure with accuracy the three post-Keplerian parameters k, $\gamma_{\rm timing}$ and $\dot{P}_b$. The three equations $k^{\rm measured} = k^{\rm theory}(m_1, m_2)$, $\gamma^{\rm measured}_{\rm timing} = \gamma^{\rm theory}_{\rm timing}(m_1, m_2)$, $\dot{P}^{\rm measured}_b = \dot{P}^{\rm theory}_b(m_1, m_2)$ determine, for each given theory, three curves in the two-dimensional mass plane. This yields *one* (combined radiative/strong-field) test of the specified theory, according to whether the three curves meet at one point, as they should. After subtracting a small ($\sim 10^{-14}$ level in $\dot{P}^{\rm obs}_b = (-2.4211 \pm 0.0014) \times 10^{-12}$), but significant, Newtonian perturbing effect caused by the Galaxy [37], one finds that General Relativity passes this $(k - \gamma_{\rm timing} - \dot{P}_b)_{1913+16}$ test with complete success at the 10^{-3} level [31,38,39]

$$\begin{aligned} \left[\frac{\dot{P}^{\rm obs}_b - \dot{P}^{\rm galactic}_b}{\dot{P}^{\rm GR}_b[k^{\rm obs}, \gamma^{\rm obs}_{\rm timing}]}\right]_{1913+16} =& 1.0026 \pm 0.0006({\rm obs}) \pm 0.0021({\rm galactic}) \\ =& 1.0026 \pm 0.0022 \ . \end{aligned} \tag{18.20}$$

Here $\dot{P}^{\rm GR}_b[k^{\rm obs}, \gamma^{\rm obs}_{\rm timing}]$ is the result of inserting in $\dot{P}^{\rm GR}_b(m_1, m_2)$ the values of the masses predicted by the two equations $k^{\rm obs} = k^{\rm GR}(m_1, m_2)$, $\gamma^{\rm obs}_{\rm timing} = \gamma^{\rm GR}_{\rm timing}(m_1, m_2)$. This experimental evidence for the reality of gravitational radiation damping forces at the 0.3% level is illustrated in Fig. 18.1, which shows actual orbital phase data (after subtraction of a linear drift).

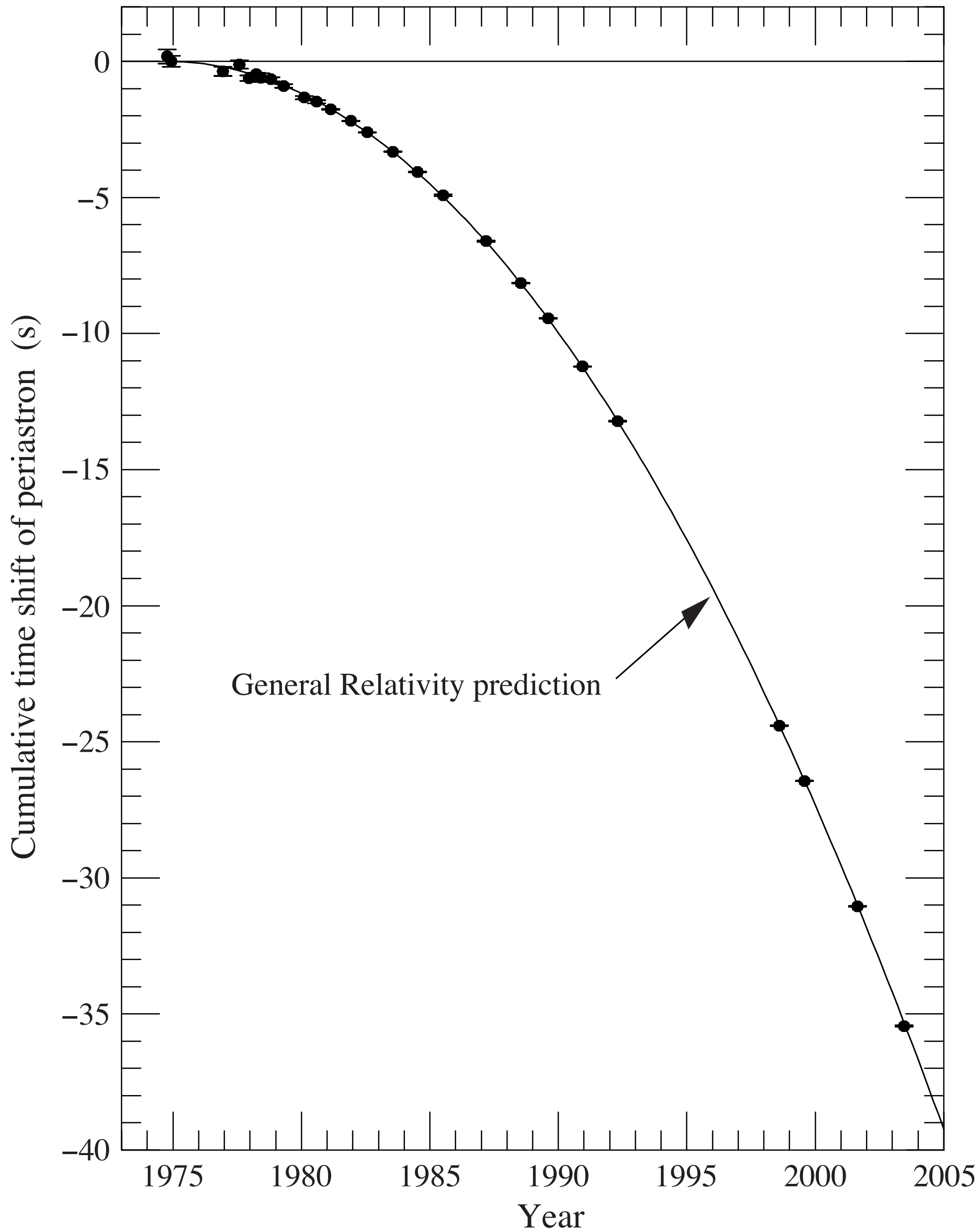

Figure 18.1: Accumulated shift of the times of periastron passage in the PSR 1913+16 system, relative to an assumed orbit with a constant period. The parabolic curve represents the general relativistic prediction, modified by Galactic effects, for orbital period decay from gravitational radiation damping forces. (Figure obtained with permission from Ref. 39.)

The discovery of the binary pulsar PSR1534 + 12 [40] has allowed one to measure the four post-Keplerian parameters k, $\gamma_{\rm timing}$, r and s, and thereby to obtain *two* (four observables minus two masses) tests of strong field gravity, without mixing of radiative effects [41]. General Relativity passes these tests within the measurement accuracy [31,41]. The most precise of these new, pure, strong-field tests is the one obtained by combining the measurements of k, γ, and s. Using the most recent data [42], one finds agreement at the 1% level:

$$\left[\frac{s^{\rm obs}}{s^{\rm GR}[k^{\rm obs},\gamma^{\rm obs}_{\rm timing}]}\right]_{1534+12} = 1.000 \pm 0.007 \ . \tag{18.21}$$

It has also been possible to measure the orbital period change of PSR1534 + 12. General Relativity passes the corresponding $(k - \gamma_{\rm timing} - \dot{P}_b)_{1534+12}$ test with success at the 15% level [42].

The discovery of the binary pulsar PSR J1141 − 6545 [43] (whose companion is probably a white dwarf) has led to the measurement of the three post-Keplerian parameters k, $\gamma_{\rm timing}$ and $\dot{P}_b$ [44]. As in the PSR 1913 + 16 system, this yields *one* combined radiative/strong-field test of relativistic gravity. One finds that General Relativity passes this $(k - \gamma_{\rm timing} - \dot{P}_b)_{1141-6545}$ test with success at the 25% level [44].

The discovery of the remarkable *double* binary pulsar PSR J0737 − 3039 A and B [45,46] has led to the measurement of *six* independent timing parameters: five of them are the post-Keplerian parameters k, $\gamma_{\rm timing}$, r, s and $\dot{P}_b$ entering the relativistic timing formula of the fast-spinning pulsar PSR J0737 − 3039 A, while the sixth is the ratio $R = x_B/x_A$ between the projected semi-major axis of the more slowly spinning companion pulsar PSR J0737 − 3039 B, and that of PSR J0737 − 3039 A. [The theoretical prediction for the ratio $R = x_B/x_A$, considered as a function of the (inertial) masses $m_1 = m_A$ and $m_2 = m_B$, is $R^{\rm theory} = m_1/m_2 + O((v/c)^4)$ [33], independently of the gravitational theory considered. These six measurements give us *four* accurate tests of relativistic gravity [47]: one test is a new, precise confirmation of the reality of gravitational radiation (obtained after only 2.5 years of timing)

$$\left[\frac{\dot{P}_b^{\rm obs}}{\dot{P}_b^{\rm GR}[k^{\rm obs},R^{\rm obs}]}\right]_{0737-3039} = 1.003 \pm 0.014 \ , \tag{18.22}$$

while the other three (obtained from combining the measurements of k, $\gamma_{\rm timing}$, r, s and R) are new, accurate tests of strong-field gravity. General Relativity passes all those tests with flying colors. The most precise new strong-field confirmation of General Relativity is at the 5×10^{-4} level:

$$\left[\frac{s^{\rm obs}}{s^{\rm GR}[k^{\rm obs},R^{\rm obs}]}\right]_{0737-3039} = 0.99987 \pm 0.00050 \ . \tag{18.23}$$

In addition, data from several nearly circular binary systems (made of a neutron star and a white dwarf) have led to strong-field confirmations (at the 5.6×10^{-3} level) of the 'strong equivalence principle,' *i.e.*, the fact that neutron stars and white dwarfs fall with the same acceleration in the gravitational field of the Galaxy [48,49].

The constraints on tensor-scalar theories provided by the various binary-pulsar "experiments" have been analyzed in [35,50] and shown to exclude a large portion of the parameter space allowed by solar-system tests.

Finally, measurements over several years of the pulse profiles of various pulsars have detected secular profile changes compatible with the prediction [51] that the general relativistic spin-orbit coupling should cause a secular change in the orientation of the pulsar beam with respect to the line of sight ("geodetic precession"). Such qualitative confirmations of general-relativistic spin-orbit effects were obtained in PSR 1913+16 [52], PSR B1534+12 [53] and PSR J1141−6545 [54].

The tests considered above have examined the gravitational interaction on scales between a fraction of a millimeter and a few astronomical units. On the other hand, the general relativistic action on light and matter of an external gravitational field on a length scale ~ 100 kpc has been verified to $\sim 30\%$ in some gravitational lensing systems (see, *e.g.*, Ref. 55). Some tests on cosmological scales are also available. In particular, Big Bang Nucleosynthesis (see Section 20 of this *Review*) has been used to set significant constraints on the variability of the gravitational "constant" [56]. For other cosmological tests of the "constancy of constants," see the review [9].

18.4. Conclusions

All present experimental tests are compatible with the predictions of the current "standard" theory of gravitation: Einstein's General Relativity. The universality of the coupling between matter and gravity (Equivalence Principle) has been verified around the 10^{-13} level. Solar system experiments have tested the weak-field predictions of Einstein's theory at the 10^{-4} level (and down to the 2×10^{-5} level for the post-Einstein parameter $\overline{\gamma}$). The propagation properties of relativistic gravity, as well as several of its strong-field aspects, have been verified at the 10^{-3} level in binary pulsar experiments. Recent laboratory experiments have set strong constraints on sub-millimeter modifications of Newtonian gravity.

Several important new developments in experimental gravitation are expected in the near future. The improved lunar laser ranging experiment APOLLO [57] is currently accumulating data with a range accuracy of about one millimeter. The approved European Space Agency's Atomic Clock Ensemble in Space (ACES) Mission [58] should provide improved tests of gravitational redshift and of the 'variability of constants' by comparing several types of ultrastable clocks in space. The universality of free-fall acceleration should soon be tested to much better than the 10^{-13} level by some satellite experiments: the approved CNES MICROSCOPE [59] mission (10^{-15} level), and the planned (cryogenic) NASA-ESA STEP [60] mission (10^{-18} level). The recently constructed kilometer-size laser interferometers (notably LIGO [61] in the USA and VIRGO [62] and GEO600 [63] in Europe) should soon directly detect gravitational waves arriving on Earth. As the sources of these waves are expected to be extremely relativistic objects with strong internal gravitational fields (*e.g.*, coalescing binary black holes), their detection will allow one to experimentally probe gravity in highly dynamical circumstances. Note finally that arrays of millisecond pulsars are sensitive detectors of (very low frequency) gravitational waves [64–66].

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