Astrophysical Applications of the Maestro Code

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Convective velocities in a white dwarf leading up to a Type Ia supernova or in the accreted layers on a neutron star preceding an X-ray burst are very subsonic. Under these conditions, sound waves can be neglected, but capturing compressibility effects due to nuclear reactions and background stratification is critical to accurately model the flow. We have developed a new algorithm, MAESTRO, based on a low Mach number formulation that exploits the separation of scales between the fluid velocity and the speed of sound. Here, we provide a brief overview of MAESTRO and present the initial astrophysical applications of the algorithm.

Low Mach Number Hydrodynamics

- Reformulation of compressible Euler Equations for highly subsonic flows [1,2,3]
  - Pressure decomposed into dynamic and thermodynamic components, with $\pi/p_0 \sim O(M^2)$
  - Thermodynamic pressure in hydrostatic equilibrium—defines base state, $\nabla p_0 = \rho_0 g$
  - Dynamic component only appears in momentum equation—filters out sound waves
- Timestep constraint based on bulk fluid velocity instead of sound speed
- Compressibility effects due to background stratification, thermonuclear reactions, compositional mixing, and thermal diffusion retained.
- Self-consistent evolution of background state due to heat release and large-scale mixing.

$$\frac{\partial(\rho X_k)}{\partial t} = -\nabla\cdot(\mathbf{U}\rho X_k) + \rho\dot{\omega}_k \ ,$$

$$\frac{\partial(\rho h)}{\partial t} = -\nabla\cdot(\mathbf{U}\rho h) + \frac{Dp_0}{Dt} + \rho H_{\rm nuc} + \nabla\cdot\kappa\nabla T \ ,$$

$$\frac{\partial \mathbf{U}}{\partial t} = -\mathbf{U}\cdot\nabla\mathbf{U} - \frac{1}{\rho}\nabla\pi - \frac{(\rho-\rho_0)}{\rho} g\mathbf{e}_r \ ,$$

$$\nabla\cdot(\beta_0\mathbf{U}) = \beta_0\left(S - \frac{1}{\overline{\Gamma}_1 p_0}\frac{\partial p_0}{\partial t}\right) \ .$$

with

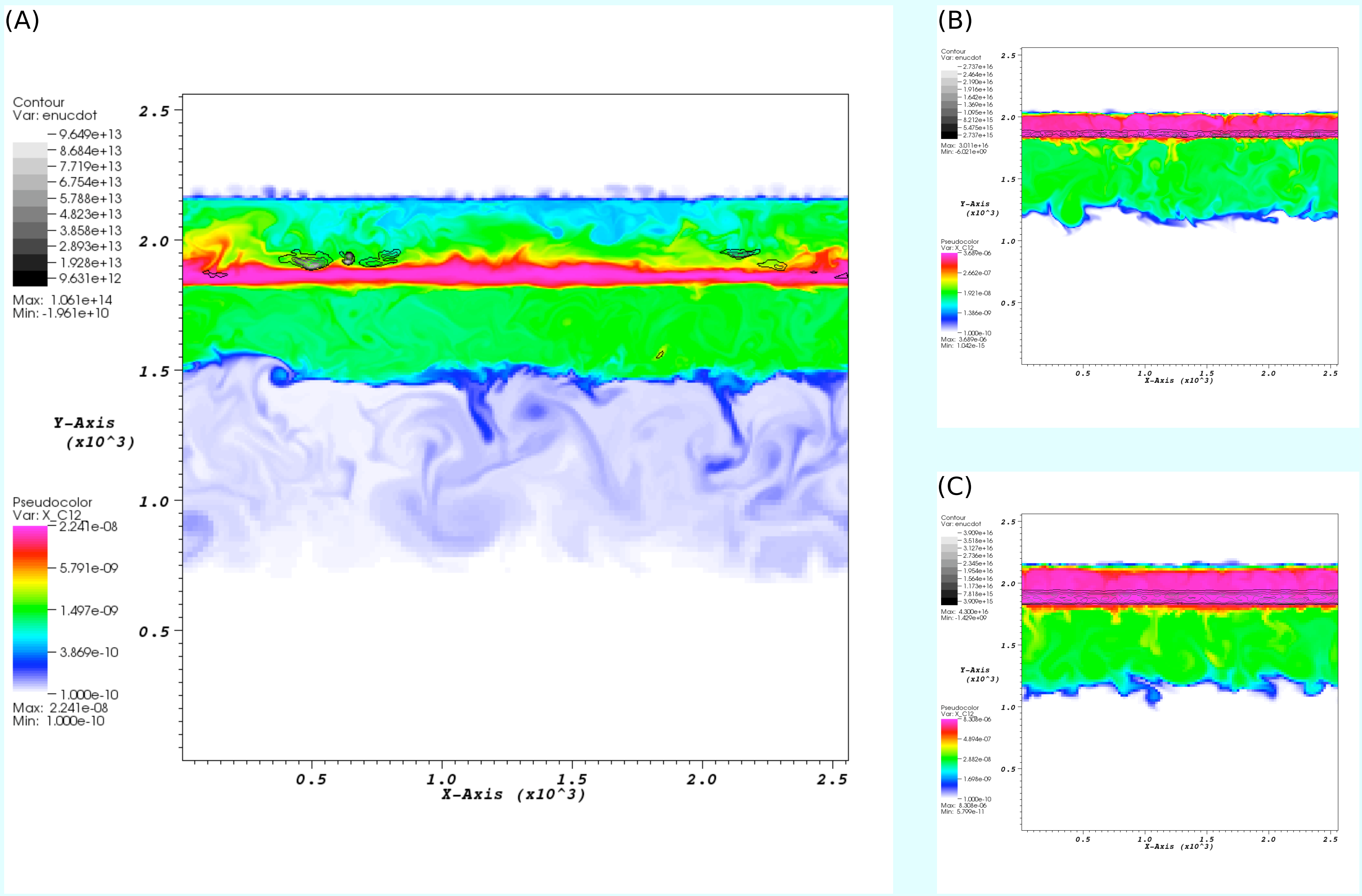
$$\beta_0(r,t) = \beta_0(0,t)\exp\left(\int_0^r \frac{1}{\overline{\Gamma} p_0}\frac{\partial p_0}{\partial r'}\,dr'\right)$$

$$S = \sigma\left[-\sum_k \left.\frac{\partial h}{\partial X_k}\right|_{T,p}\dot{\omega}_k + H_{\rm nuc} + \frac{1}{\rho}\nabla\cdot\kappa\nabla T\right] + \frac{1}{\rho p_\rho}\sum_k \left.\frac{\partial p}{\partial X_k}\right|_{T,\rho}\dot{\omega}_k$$

$$\sigma = \frac{p_T}{\rho c_p p_\rho} \qquad p_\rho = \left.\frac{\partial p}{\partial \rho}\right|_{T,X_k} \qquad p_T = \left.\frac{\partial p}{\partial T}\right|_{\rho,X_k}$$

X-ray Bursts

- Thermonuclear explosion of accreted hydrogen/helium layer on the surface of a neutron star.
  - Reactions at the base of the layer drives convection throughout the layer, leading to the consumption of nearly all the accreted fuel.
  - Ultimate goal: understanding lateral propagation of burning front through fuel layer.
- Long timescale evolution is needed to capture the initial convection.
  - Plane-parallel approximation used to model the accreted layer.

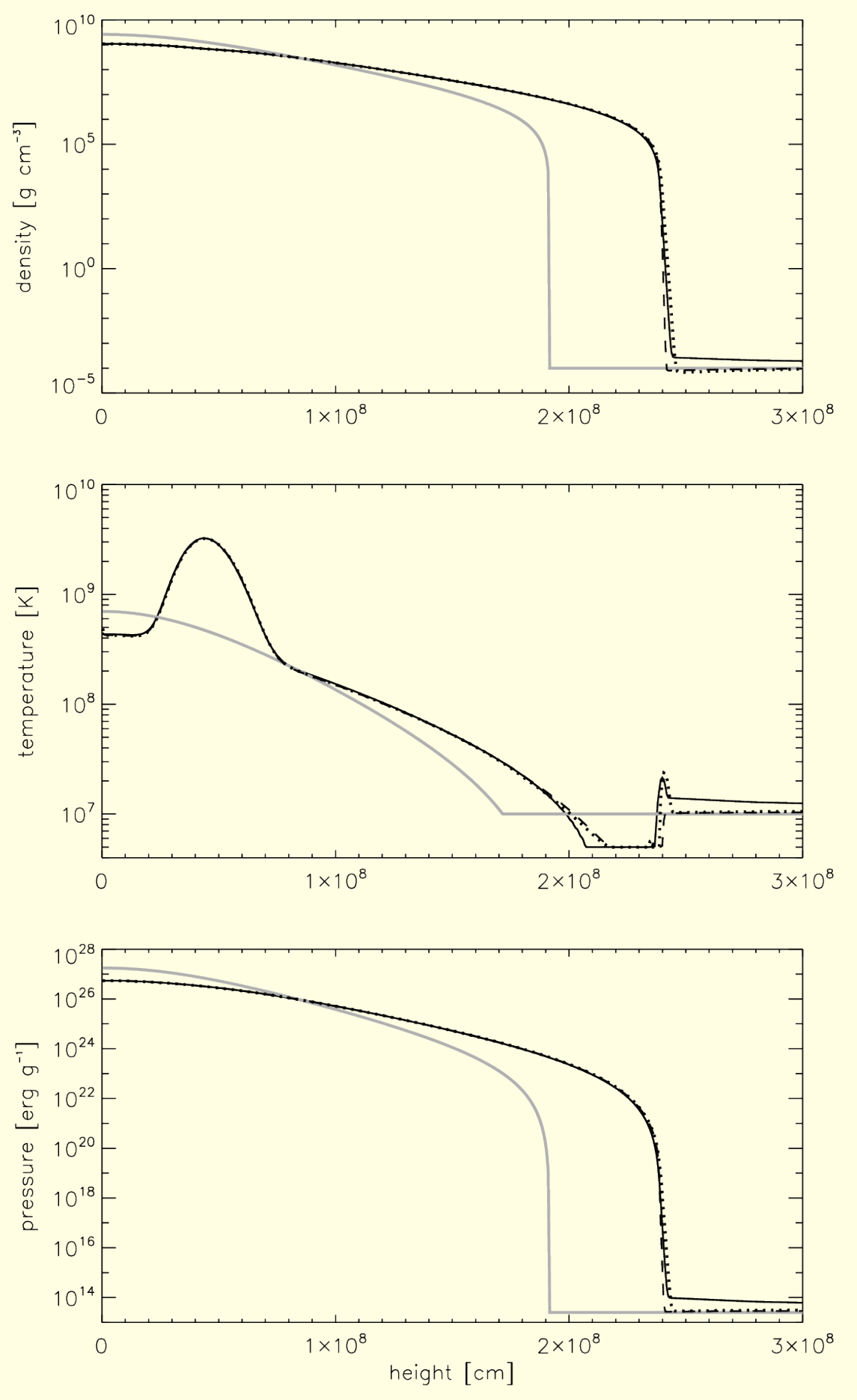


Preliminary Maestro simulations of helium burning on the surface of a neutron star, showing carbon mass fraction (colors) and energy generation rate (contours) 2.5 ms after mapping onto the 2-d grid. The 5 cm resolution case (A) shows significantly less overshoot at the base of the reactive layer than the 10 cm (B) or 20 cm (C) cases.

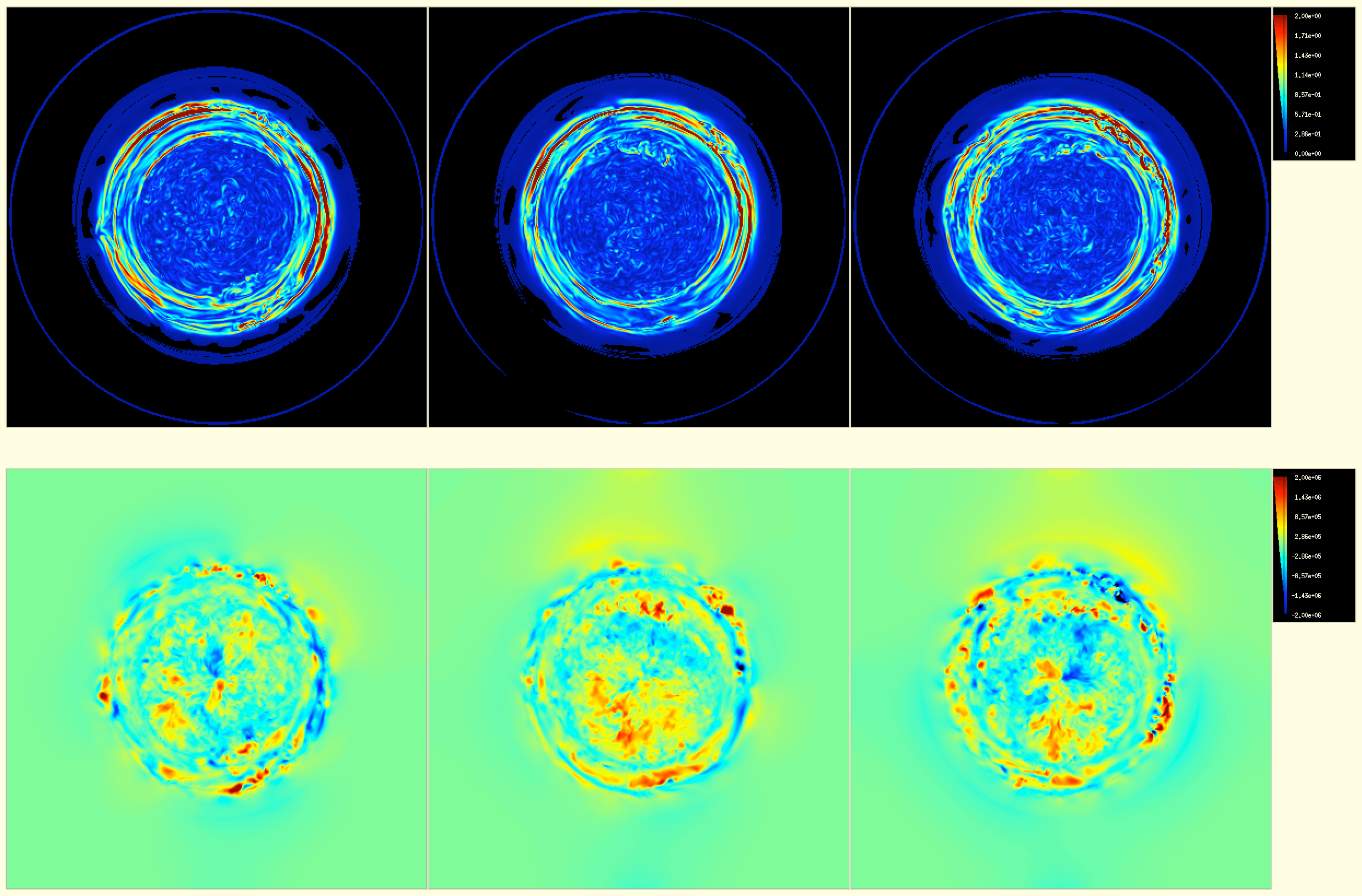
- Current challenges / future improvements
  - Understanding overshoot at the base of the convective layer—physical or numerical?
  - Further development of mesh refinement to better capture the thin reaction zone.
  - Adding rotation to the low Mach number hydrodynamics model.

Type Ia Supernovae

- Thermonuclear explosion of a carbon/oxygen white dwarf.
  - Accretes from a companion until reaching the Chandrasekhar mass.
  - Convective period (~ 100 years long) precedes ignition and explosion.
  - Explosion releases enough energy to unbind the star.
  - Location, number, and distribution of hot spots that seed the explosion is uncertain.
- Long timescale evolution is needed to capture many convective turnover times.
  - Requires reformulation of our method to work with spherical self-gravitating stars.
- Current challenges
  - Mapping between the 1-d radial base state and 3-d Cartesian grid.
  - Initializing a 3-d velocity field that captures existing convection.



Hydrostatic adjustment of a 1-d spherical white dwarf in response to heating. Initial model (gray), fully compressible solution (black), and low Mach number solution (dotted and dashed—different CFL numbers) are shown. This is the basis for the base state evolution in Maestro.



Preliminary Maestro convecting white dwarf calculations showing vorticity (top) and radial velocity (bottom) at three orthogonal slice planes through the center of the star (data range compressed to show detail). A 384^3 grid was used. The vorticity plots show the inner region of the star is convectively unstable whereas the outer portion is stable. The radial velocities show an asymmetry to the convective flow. These images are 120 s after the model was put onto the 3-d grid. Transient behavior resulting in the cooling of the center of the star just after this mapping is still being investigated.

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