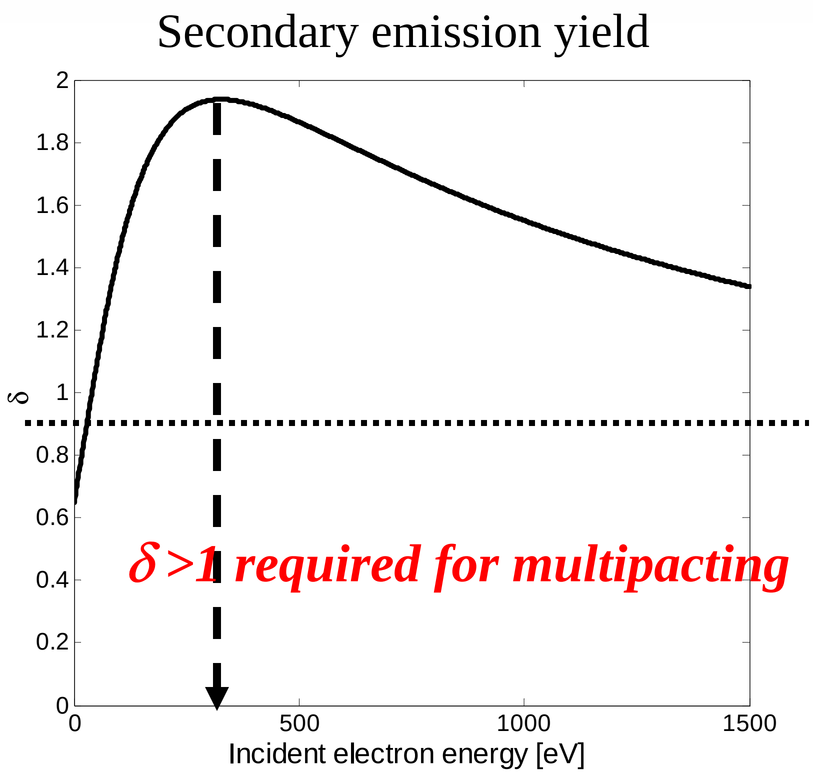
Mechanism and Important parameters of multipacting

L. Wang, BNL

Secondary electrons



Key parameters for Multipacting (Strong energy and SEY dependence)

- SEY depend on the material property of the chamber surface (peak SEY and energy at peak SEY)
- Beam-electron interaction dependence (beam pattern, bunch current, bunch shape, bunch length, chamber size…)

Beam space charge field

SNS beam transverse profile shape

- **Square shape** resulting from correlated painting during the injection.
- Inclusion of the space charge causes rapid diffusion in azimuthal direction and results in **round beam shape**
- **Electron Multipacting (energy at the wall surface) does not depend on transverse profile**

Space charge field of uniform cylinder beam

$$E_r(r,t)=\begin{cases}\dfrac{\lambda(t)}{4\pi\varepsilon_0}\dfrac{2}{r} & (\mathrm{r}>a)\\[2ex] \dfrac{\lambda(t)}{4\pi\varepsilon_0}\dfrac{2r}{a^2} & (\mathrm{r}<a)\end{cases}$$

$$U(r,t)=\begin{cases}\dfrac{\lambda(t)}{4\pi\varepsilon_0}\left(1+2\ln\dfrac{r}{a}\right) & (\mathrm{r}>a)\\[2ex] \dfrac{\lambda(t)}{4\pi\varepsilon_0}\dfrac{r^2}{a^2} & (\mathrm{r}<a)\end{cases}$$

Nonlinear Hamiltonian of the radial motion

$$H=\frac{p^2}{2m}+eU(r,t)$$

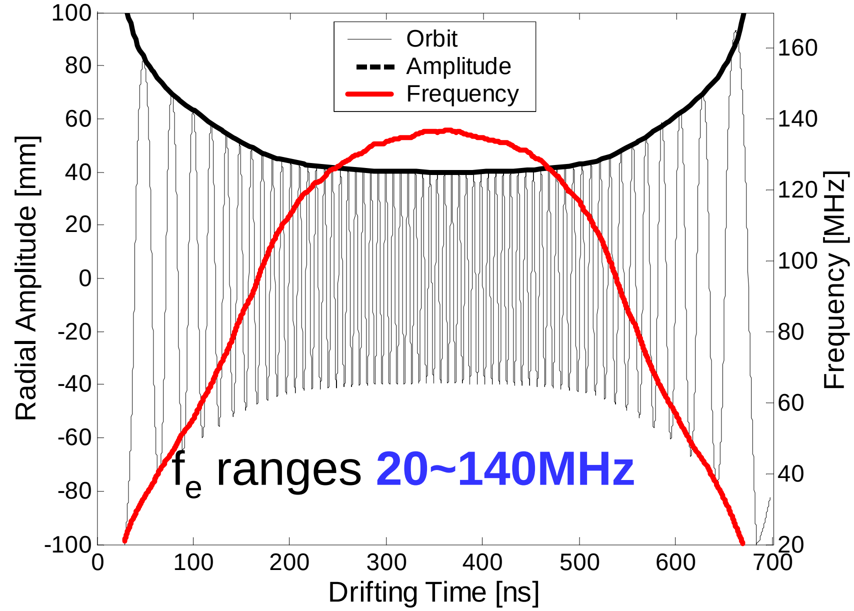
The longitudinal beam force is neglected

Nonlinear Oscillation Period and Adiabatic invariant

Nonlinear Oscillation Frequency

$$T = 4.0\int_0^{r_{amp}} \frac{dr}{\upsilon(r)} = 4.0\int_0^{r_{amp}} \frac{dr}{\sqrt{2\Phi e/m}}$$

$$T = \begin{cases} 4.0\sqrt{\dfrac{\pi\varepsilon_0 m}{\lambda e}}\left(\sqrt{2}a\arcsin\dfrac{1}{\sqrt{1+2\ln(r_{amp}/a)}} + \displaystyle\int_a^{r_{amp}} \dfrac{dr}{\sqrt{\ln(r_{amp}/r)}}\right) \\ 2\pi a\sqrt{\dfrac{2\pi\varepsilon_0 m}{\lambda e}} \qquad (r_{amp} \le a) \end{cases}$$



Oscillation amplitude and frequency

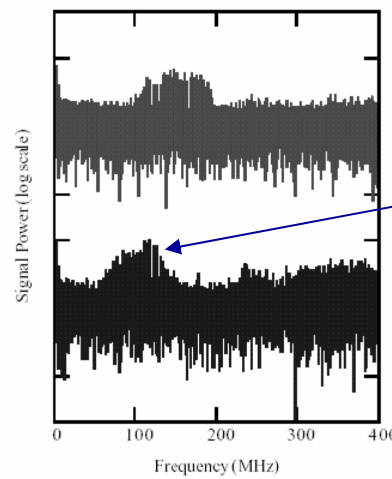
Adiabatic invariant

$$\frac{1}{\omega_e^2}\frac{d\omega_e}{dt} << 1$$ *(if t>20ns and t<680ns for SNS)*

$$J = \oint p\,dq$$

$$J = \begin{cases} \dfrac{\pi r_{amp}^2}{a}\sqrt{\dfrac{me\lambda}{2\pi\varepsilon_0}} \qquad (r_{amp} < a) \\ 4a\sqrt{\dfrac{me\lambda}{2\pi\varepsilon_0}}\left(\dfrac{\sqrt{2}}{2}x^{1/2} + \dfrac{1+2x}{2}arctg\dfrac{1}{\sqrt{2x}} + \dfrac{\sqrt{2}}{a}\displaystyle\int_a^{r_{amp}}\sqrt{\ln\dfrac{r_{amp}}{r}}dr\right)(r_{amp} > a) \end{cases}$$

$$x = ln(r_{amp}/a)$$



LANL PSR

beam spectrum

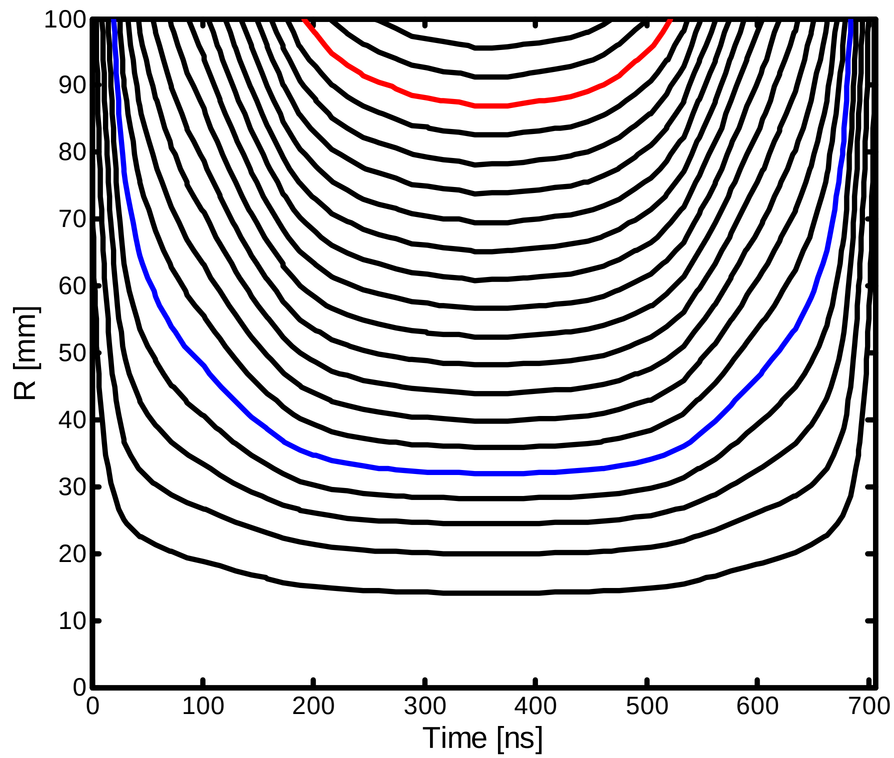
50~150MHz

courtesy

Robert J. Macek

Oscillation amplitude from adiabatic invariant

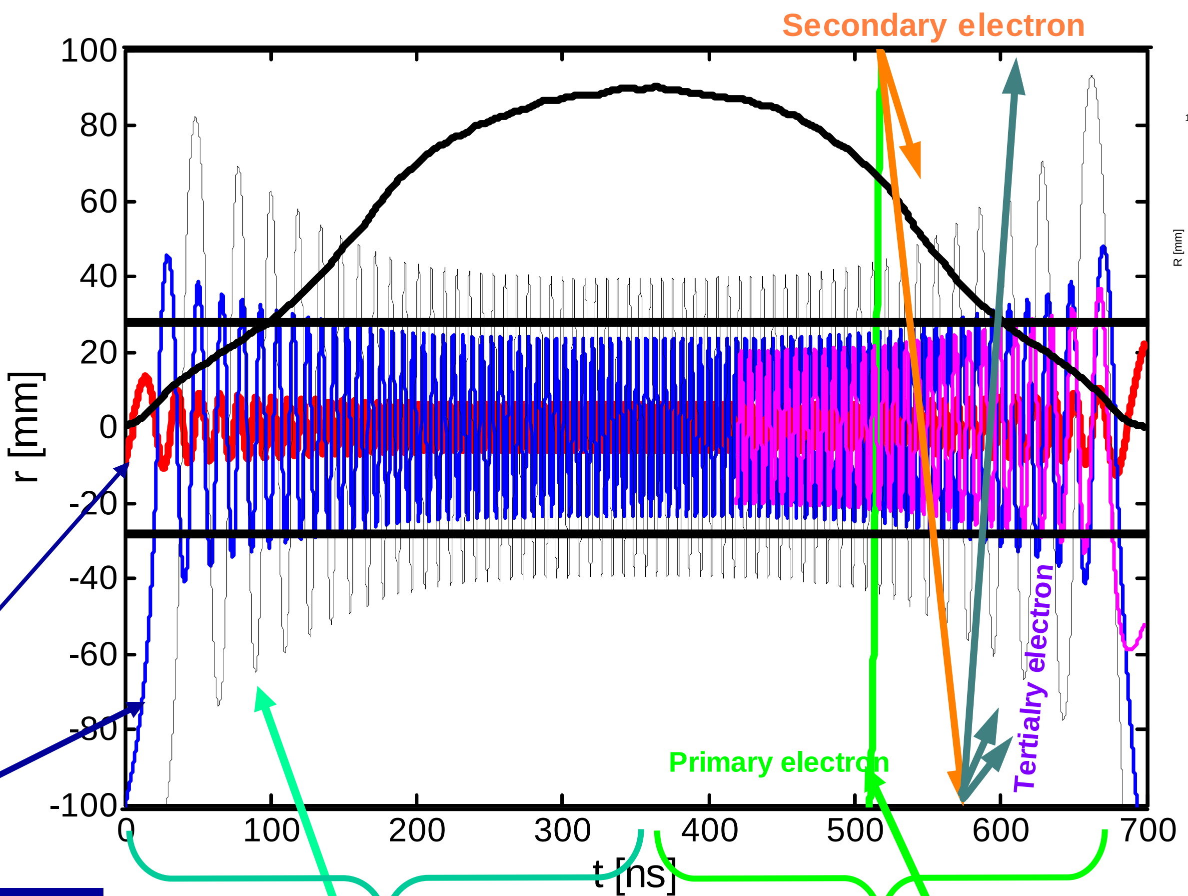
- Contour plot from adiabatic invariant can clearly descript the electron orbit
- All electron emitted (including gas ionization) before the bunch center or survived from last bunch gap can be trapped (inside beam for the survived electrons) during the bunch passage and are released at the bunch tail. The trapped electrons, most of them are the survived electrons from the last bunch gap, contribute to beam dynamics (instabilities)
- All electrons which emitted from the wall after bunch center will directly drift to the opposite of wall surface. The straight drifting electrons contribute to multipacting due to their short drifting time &high energy when they hit the wall surface.

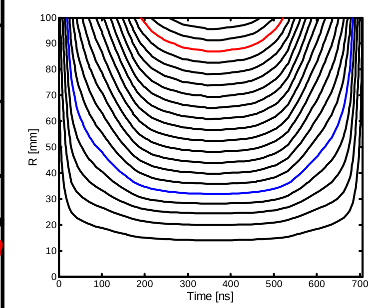


Contour plot of the oscillation amplitude resulting from adiabatic invariant for SNS beam

Particle motion *vs.* instability & multipacting

Typical orbits of various electrons trapped by SNS beam, bold solid line shows the longitudinal beam profile shape and the dashed back lines show the beam size





Electrons by ionization

(beam instabilities)

Electrons survived from the bunch gap(beam instabilities)

Electrons emitted before bunch center (trapped and lost after bunch center, multipacting)

Electrons emitted after the bunch center(multipacting)

Energy Gain of straight drifting electron & Mechanism of trailing edge multipacting

Assuming the beam line density is a linear function of time during the short electron drifting time(~10ns)

$$\Delta\lambda \approx \frac{\partial\lambda}{\partial t}\Delta t = \frac{\partial\lambda}{\partial z}c\beta\Delta t$$

$$\Delta \mathrm{E} = -\frac{1}{2}\sqrt{\frac{me}{2\pi\varepsilon_0}}\beta c\left(a(2\zeta-1)\arcsin\frac{1}{\sqrt{\zeta}} + a\sqrt{2\ln\frac{b}{a}} + \sqrt{2}\zeta\int_a^b \frac{dr}{\sqrt{\ln(b/r)}} - \frac{1}{\sqrt{2}}\int_a^b \frac{1+2\ln(r/a)}{\sqrt{\ln(b/r)}}dr \right)\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}$$

$$\zeta = 1 + 2\ln(b/a)$$

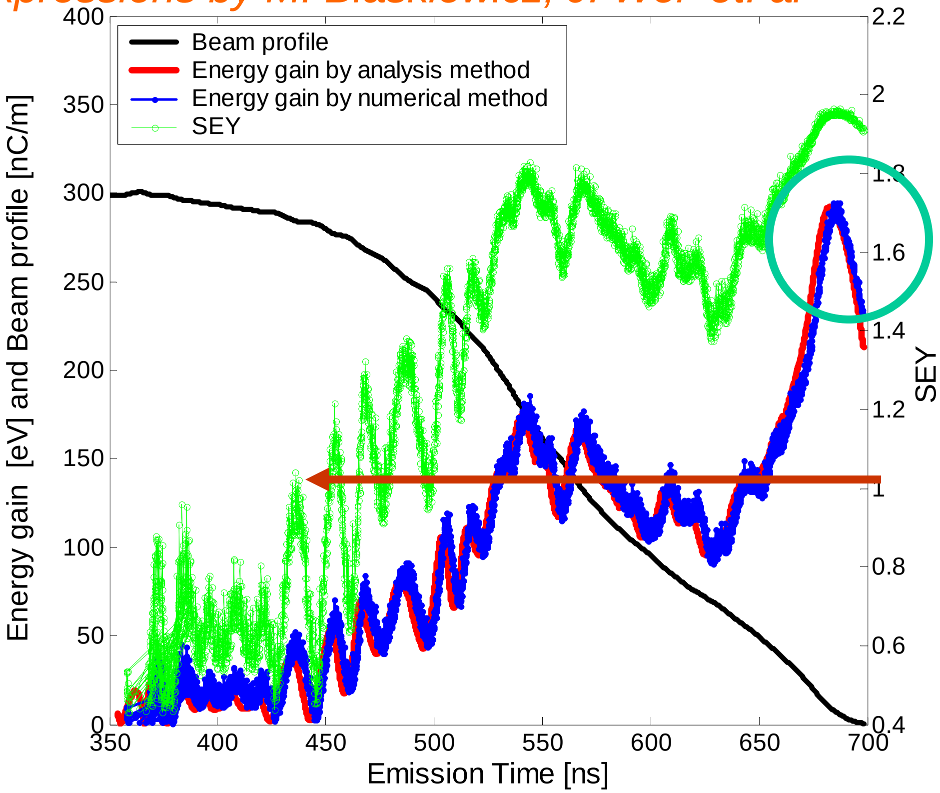
Also see other expressions by M. Blaskiewicz, J. Wei et. al

a: beam size, *b*, chamber radius, λ is beam line density

Longitudinal beam profile factor

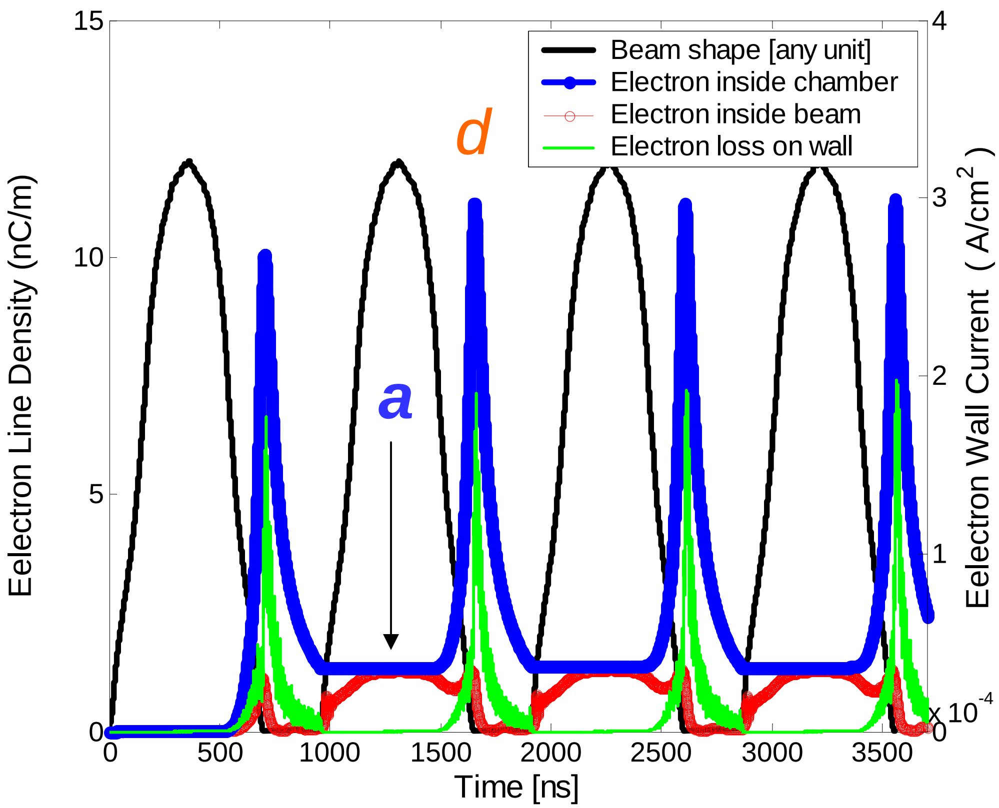
$$Factor_{profile} = -\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}$$

- Good agreement with numerical method
- Calculated SEY can be used to predict the multipacting directly
- Adiabatic motion and Energy gain can explain the mechanism of "trailing edge multipactor"

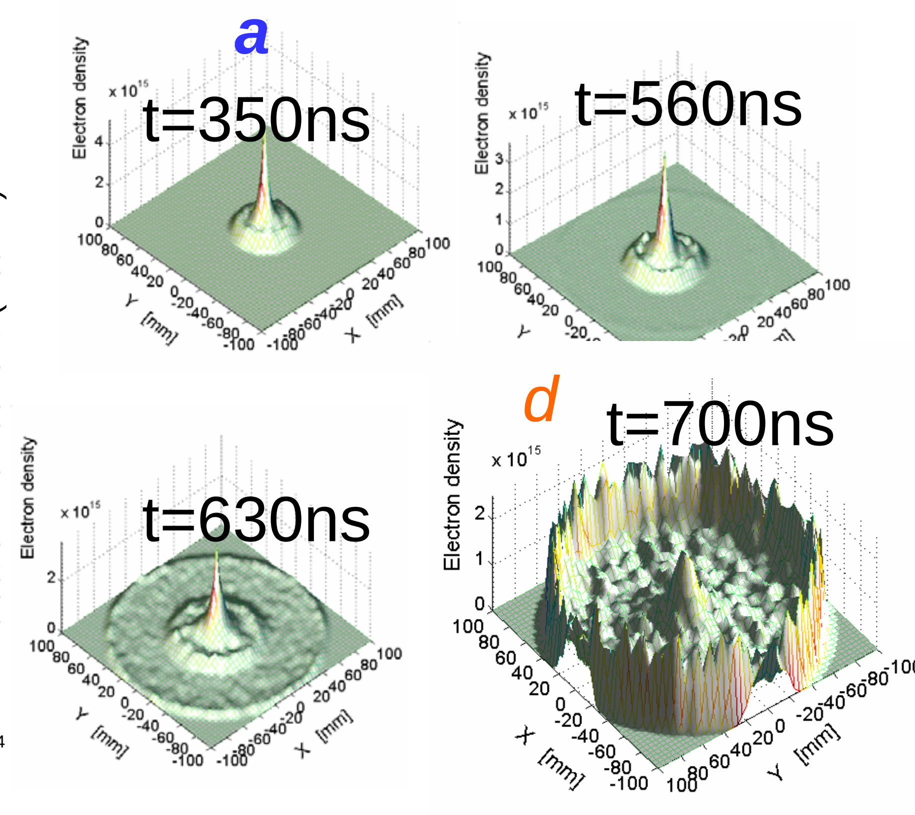


E-cloud in drift region

- **Single bunch multipacting & Trailing Edge Multipacting**
- **All surviving electron from the last gap are trapped inside beam during the bunch passage (Contributing to beam instabilities)**
- **Bunch gap is important for *beam dynamics***

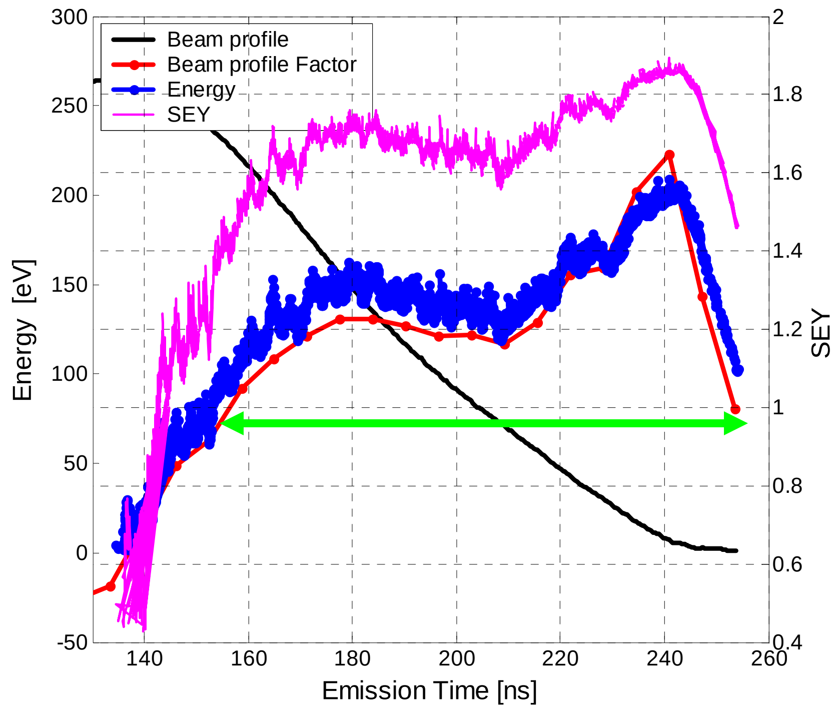


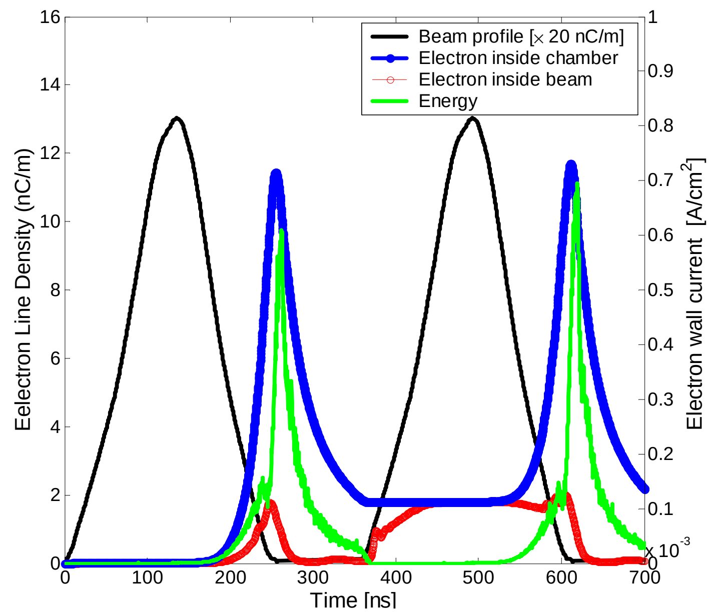
E-cloud build-up in SNS drift region



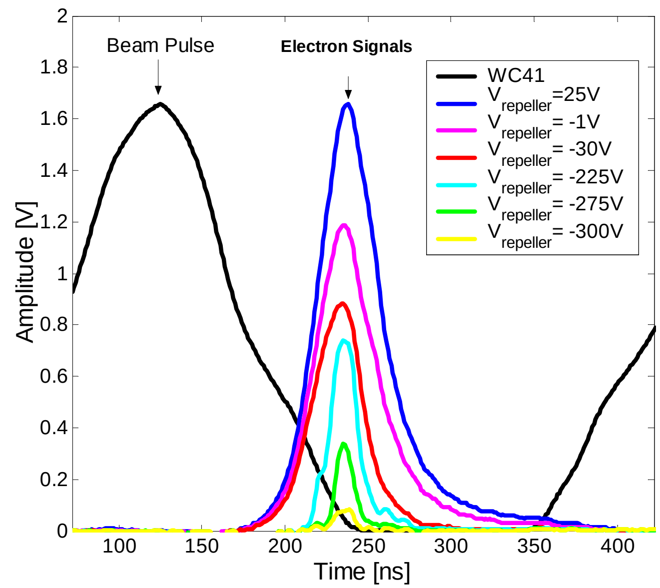
E-cloud distribution in different time

Ecloud in PSR



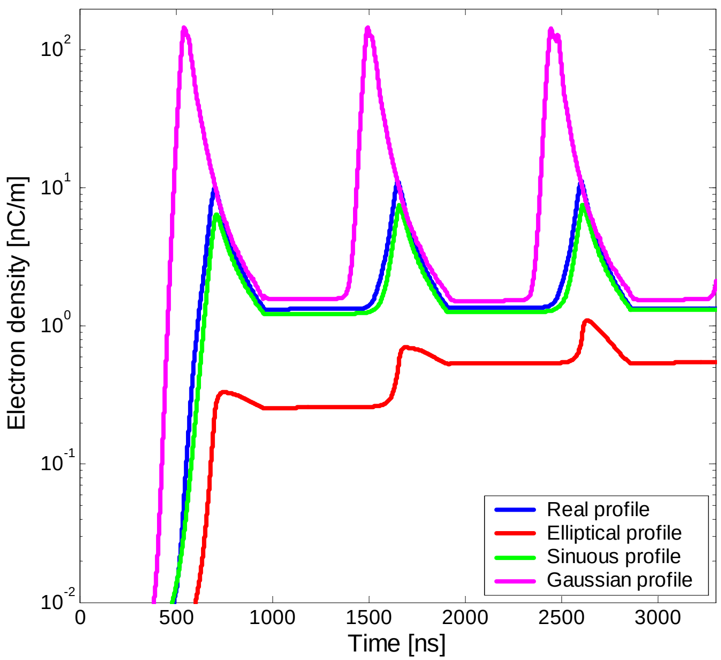


ED42Y Signals for various repeller voltages



courtesy **Robert J. Macek**

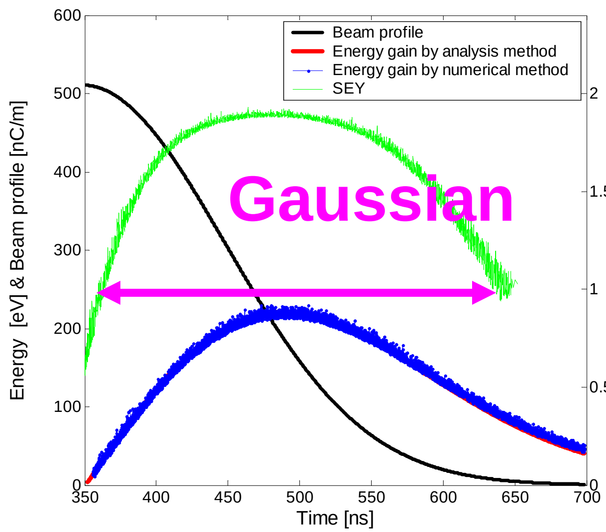
Important Factors Related to Electron multipacting(1)

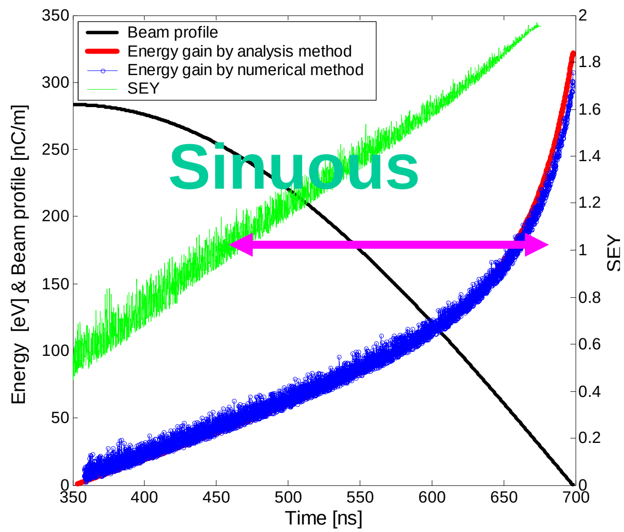


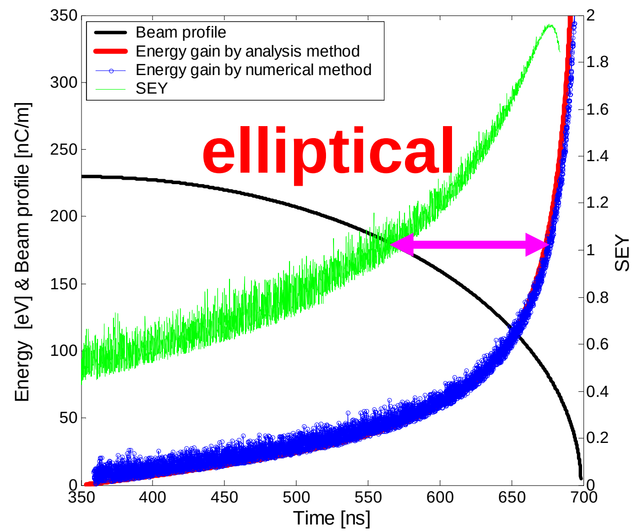
Longitudinal Beam Profile

For assumed Gaussian, sinuous & elliptical beam profile:

- Gaussian profile excites the strongest multipacting due to long bunch tail
- Elliptical profile has the weakest multipacting
- Electron cloud of the real profile is close to that of the sinuous profile

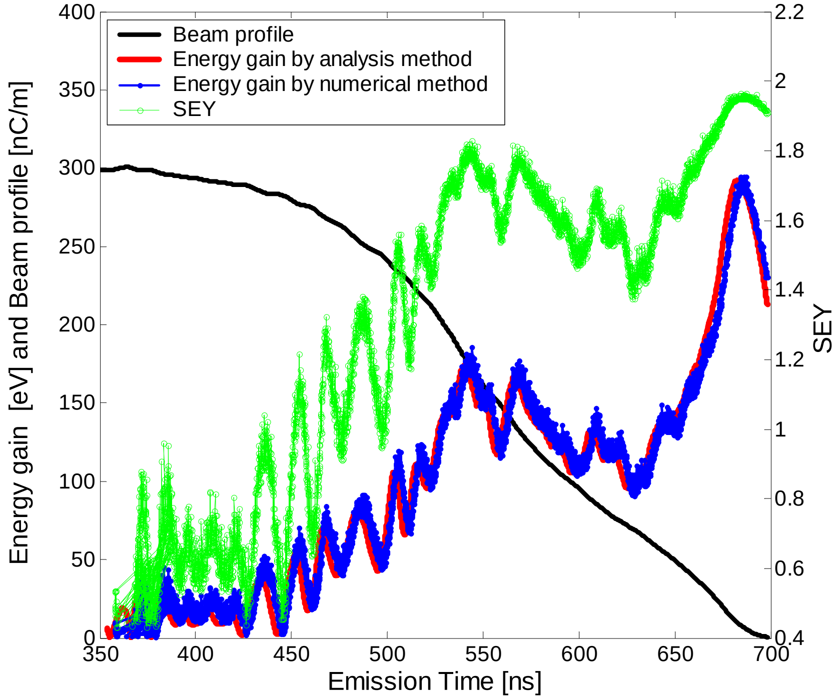


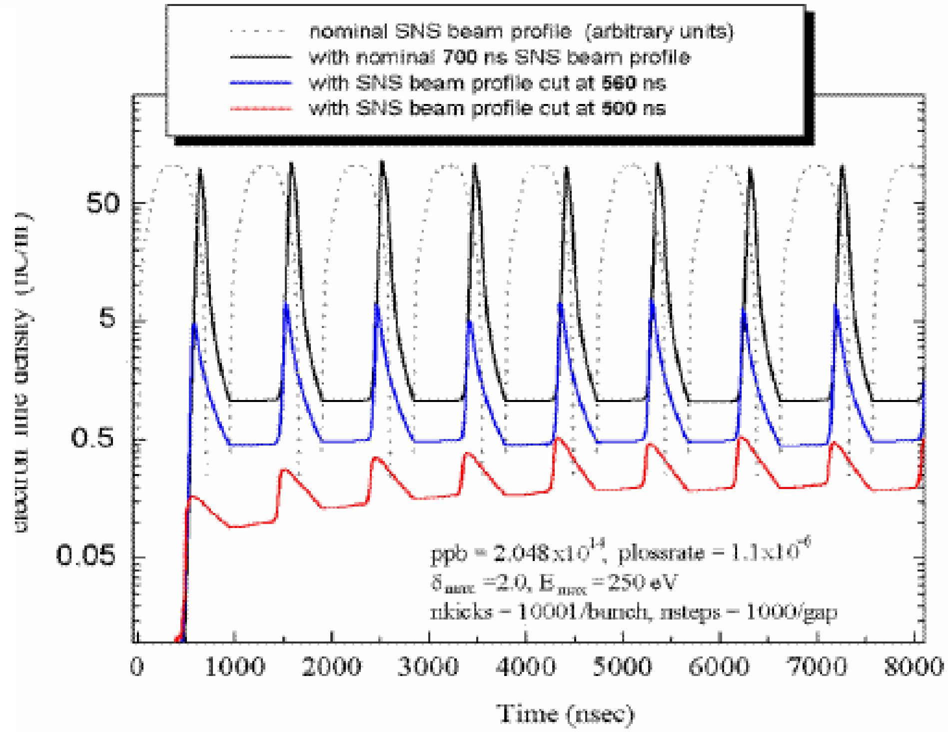




Longitudinal Profile effect, simulation

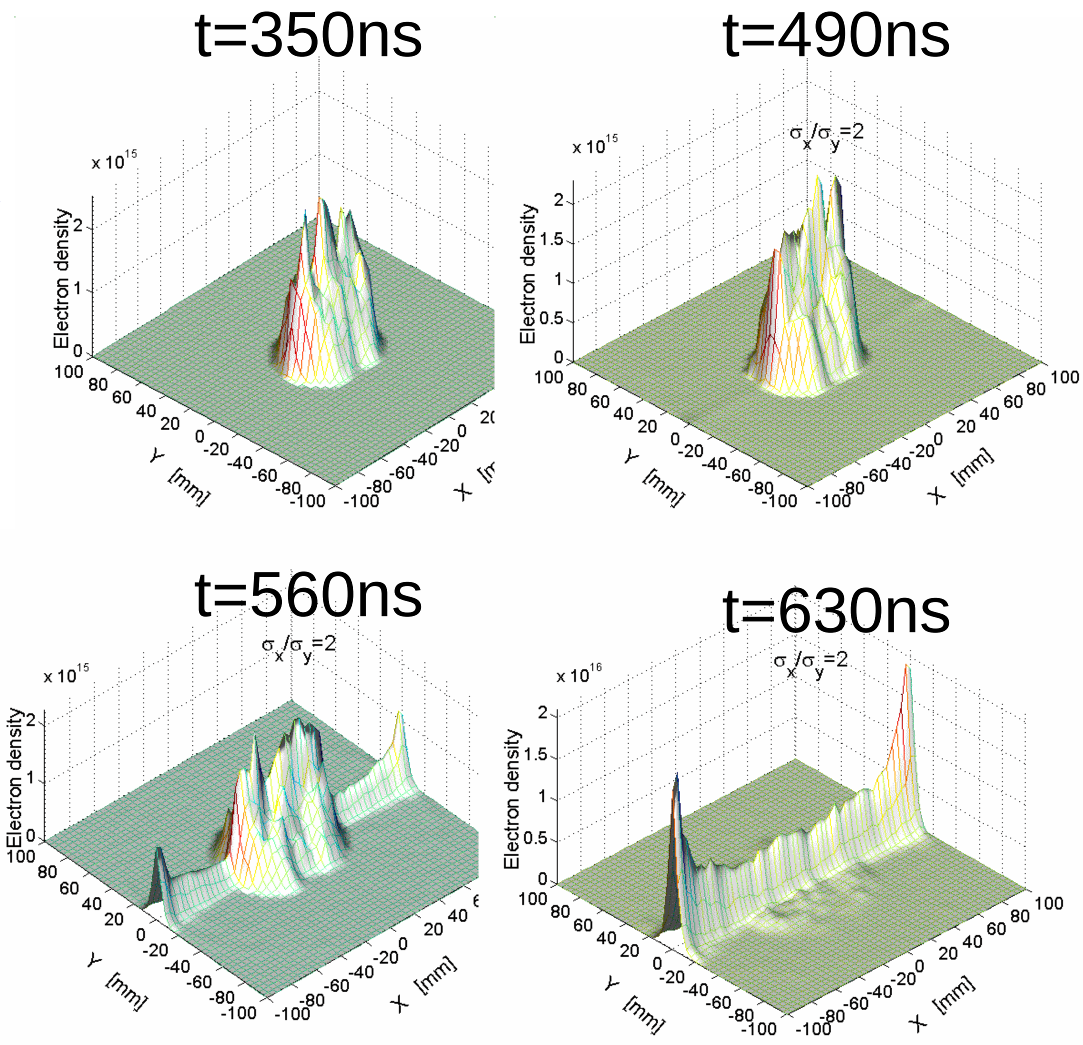
M.T. F. Pivi and M. A. Furman
PRSTAB Vol. **6**, 034201 (2003))

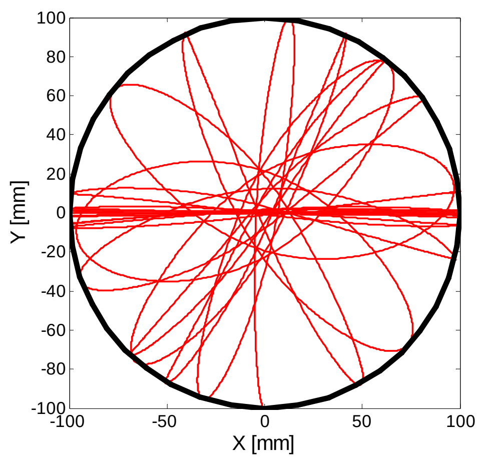




Flat beam effect on EC distribution

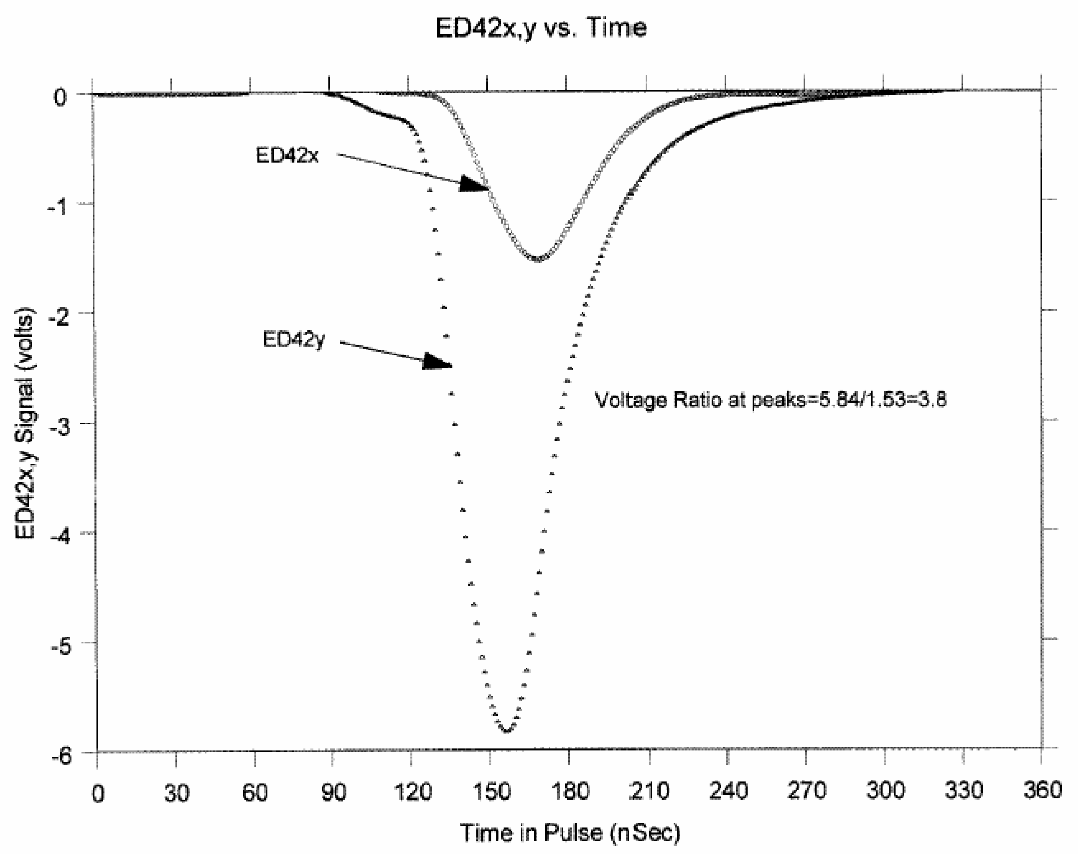
- **Flat beam $\sigma_x : \sigma_y = 2:1$**
- **Stronger multipacting in larger beam size direction due to the "polarization effect" of strong beam space charge force**





PSR experimental study----flat beam

- **Qualitatively agrees with LANL PSR observation**
- **Instability & detector**



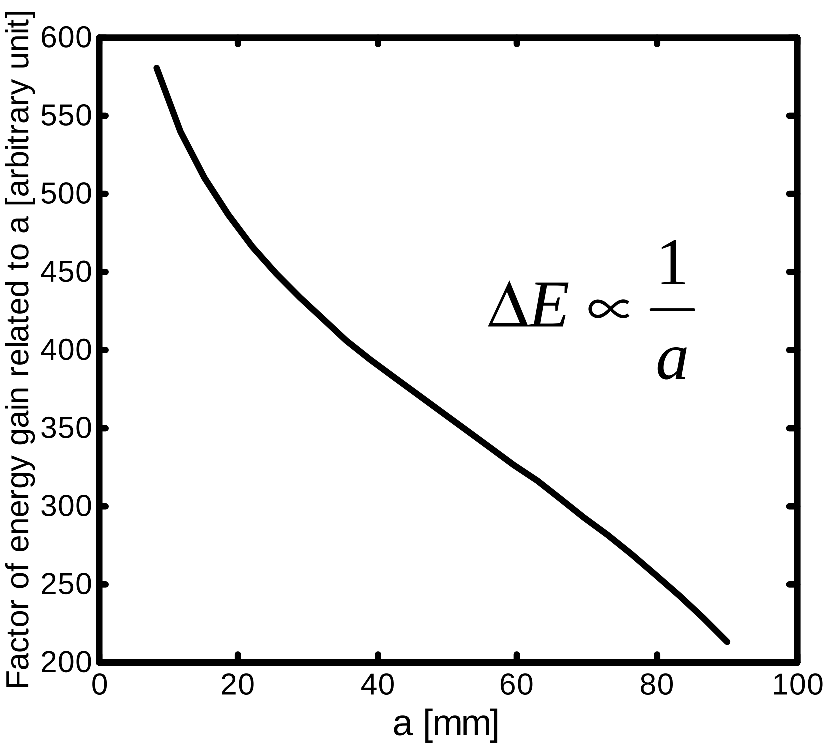
A. Browman Two-stream-2000

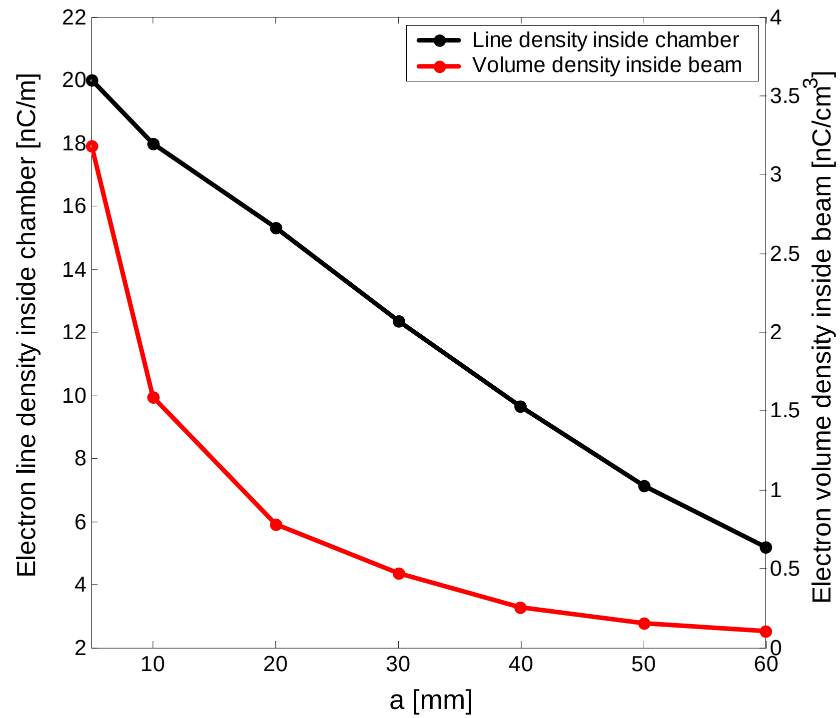
Transverse Beam Size Effects

- **A smaller beam size contributes to stronger space charge field and hence larger electron energy gain and stronger multipacting.**
- **Instabilities is sensitive to *a*. Small *a*, strong instability**

$$\lambda_{chamber}[nC/m] = 21 - 0.27a[mm]$$

$$\rho_{cen}[nC/cm^3] = 4.9e^{-0.1a[mm]}$$

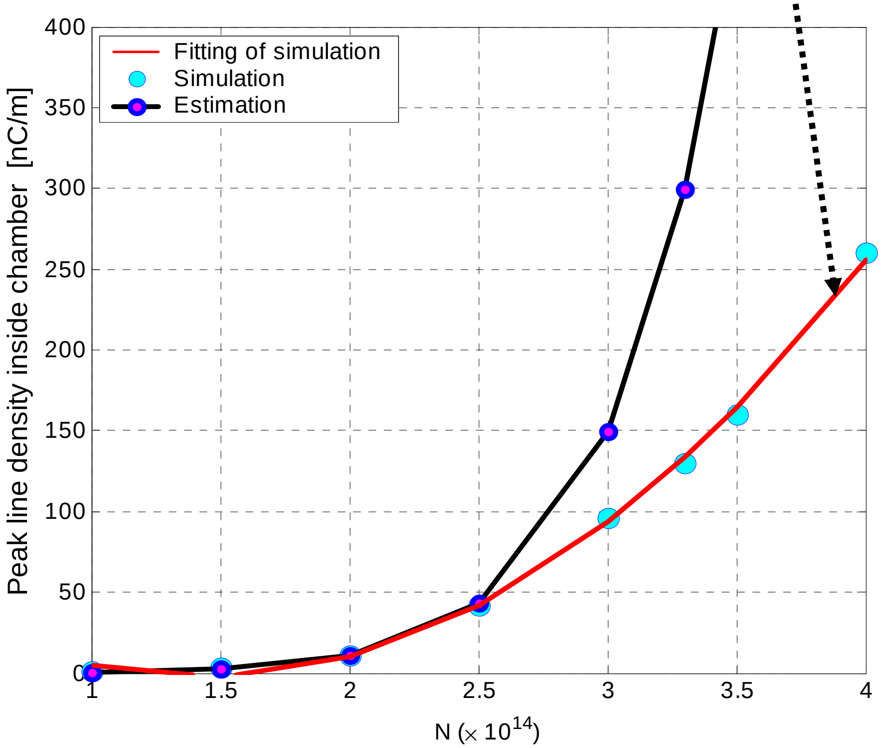


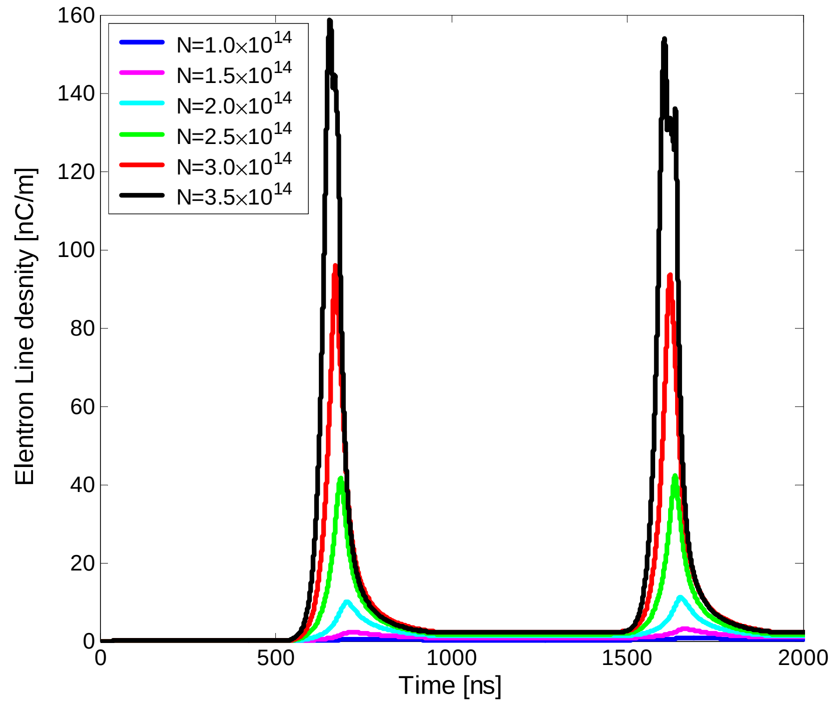


Beam intensity effects (I)

- **High beam intensity causes high electron energy gain** $\Delta E \propto \sqrt{\lambda}$
- **High beam intensity increases multipacting frequency** $f_{multipacting} \propto \sqrt{\lambda}$
- **Space charge slows the growth of electron density inside chamber when strong multipacting case happens**

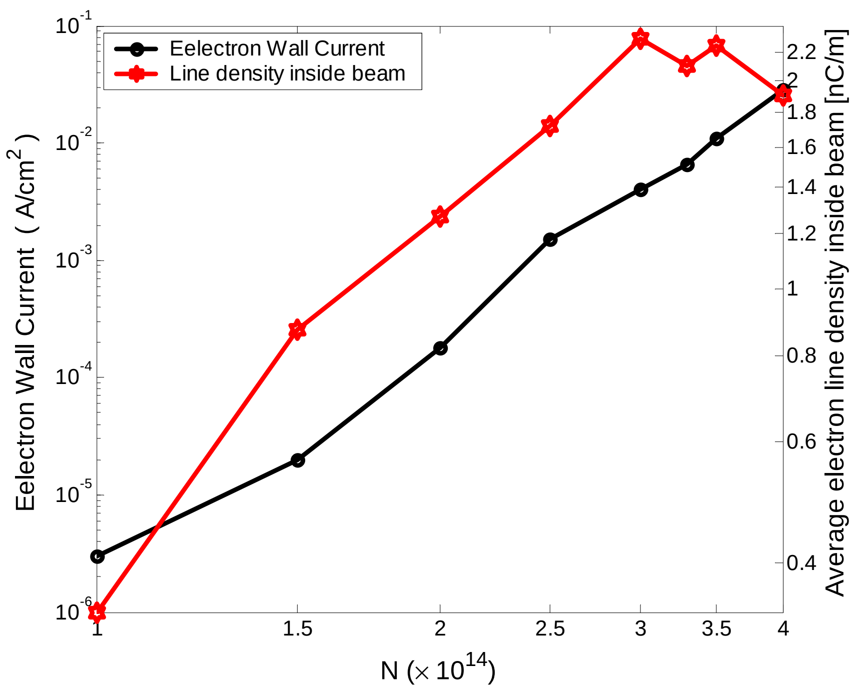
$$\lambda_{chamber}[nC/m] = 78 - 112\times1.0^{-14}N + 39\times1.0^{-28}N^2$$

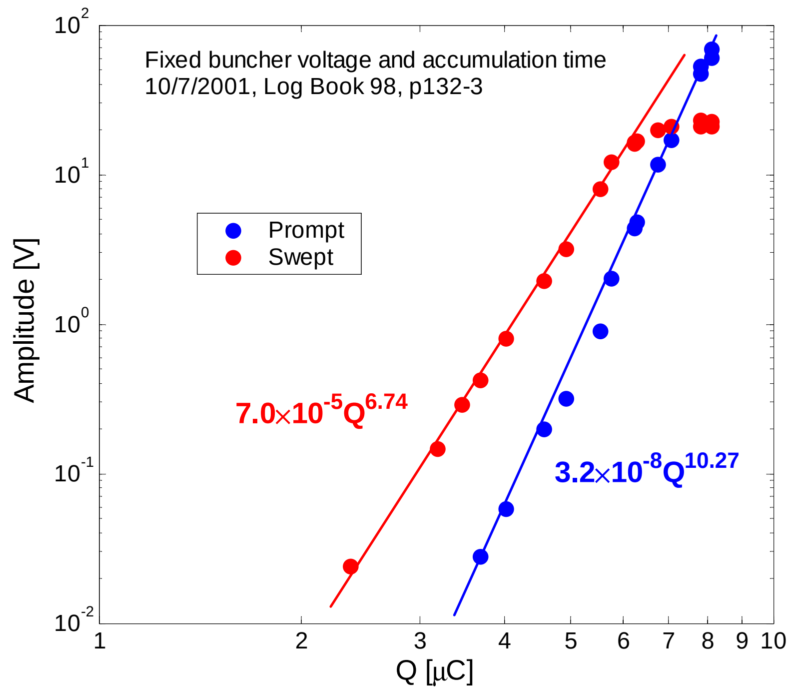




Beam intensity effects (II)

- **Space charge makes the electron density inside beam saturated when strong multipacting case happens**



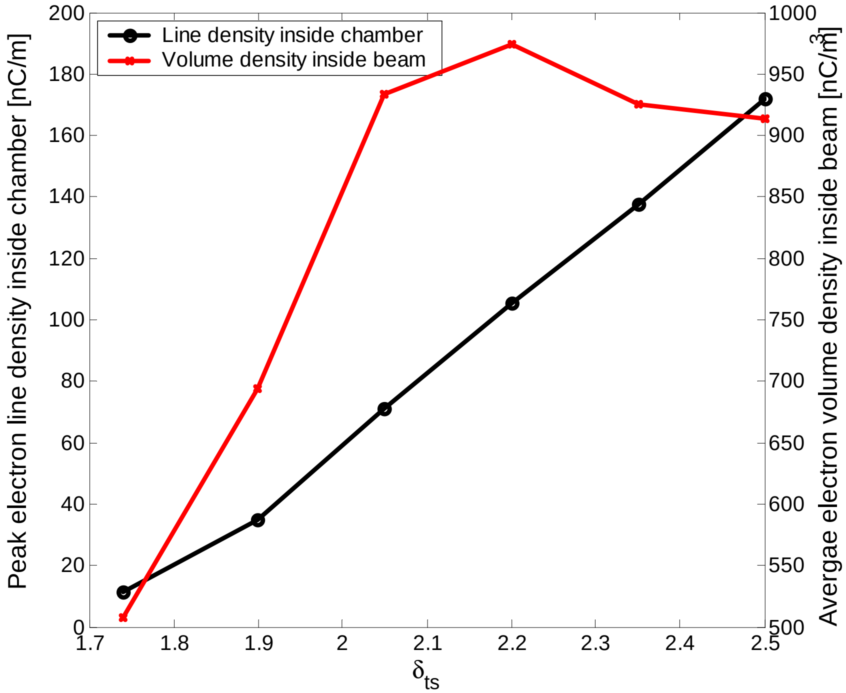


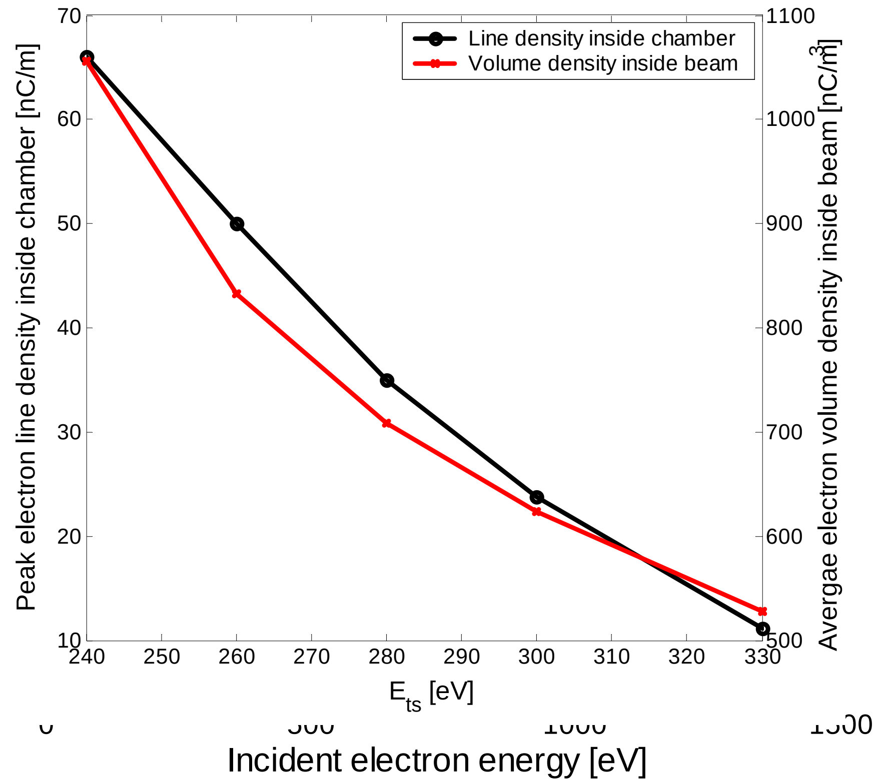
SNS, simulation

LANL PSR, Experiment, R. Macek

Peak SEY and Energy at Peak SEY

- E-cloud density inside chamber is a linear function of peak SEY and energy at peak SEY.
- E-cloud density inside chamber (and hence Instability growth rate) increases linear with peak SEY and finally saturates at some level.
- E-cloud density inside chamber (and hence Instability growth rate) decreases with the energy at peak SEY.





Electron motion in dipole magnets

- **Dipole magnetic field B**=(0,By,0)
- Beam electric field **E**=(Ex,Ey,0)

$$\frac{d\upsilon_y}{dt} = eE_y / m$$

$$\upsilon_x = \upsilon_{x0}\cos\omega t + \upsilon_{z0}\sin\omega t + \frac{E_{x0}}{B}\sin\omega t + \frac{1}{\omega B}\frac{dE_x}{dt} - \frac{1}{\omega B}\left(\frac{dE_x}{dt}\right)_0 \cos\omega t$$

$$-\frac{1}{\omega B}\cos\omega t\int_0^t \frac{d^2E_x}{dt^2}\cos\omega t dt - \frac{1}{\omega B}\sin\omega t\int_0^t \frac{d^2E_x}{dt^2}\sin\omega t dt$$

$$\upsilon_z = \upsilon_{z0}\cos\omega t - \upsilon_{x0}\sin\omega t + \frac{E_{x0}}{B}\cos\omega t - \frac{E_x}{B} - \frac{1}{B}\cos\omega t\int_0^t \frac{dE_x}{dt}\cos\omega t dt + \frac{1}{B}\sin\omega t\int_0^t \frac{dE_x}{dt}\sin\omega t dt$$

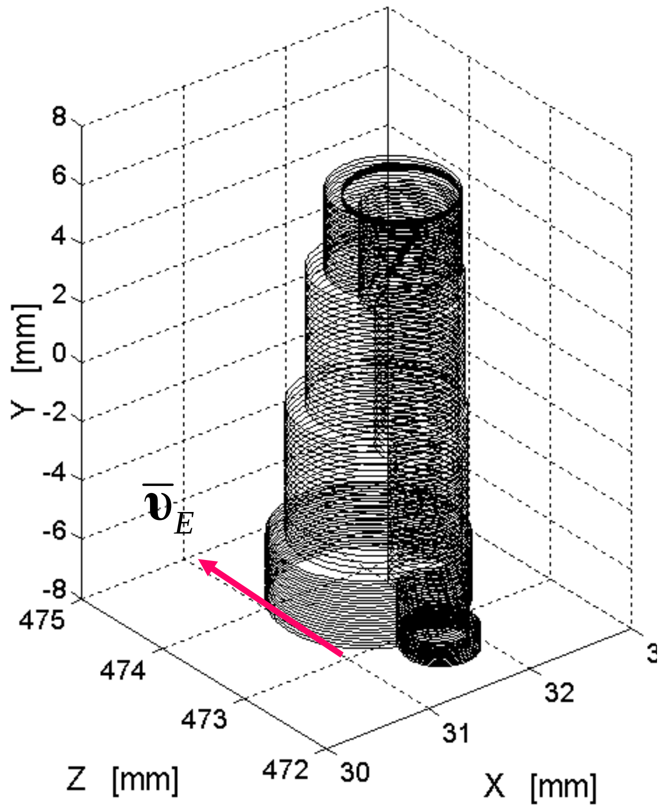
- In strong dipole magnet(B~1T for SNS) $\left|\frac{1}{\omega E_x}\frac{dE_x}{dt}\right| << 1$
- Electron motion ≅ **gyration motion + translation** (cross-field drifting) + movement along B-field lines
- The kinetic energy of gyration motion and cross-field drifting is **smaller** comparing with the kinetic energy in B-field direction.

Electron Orbit in Dipole Magnets (Short bunch case)

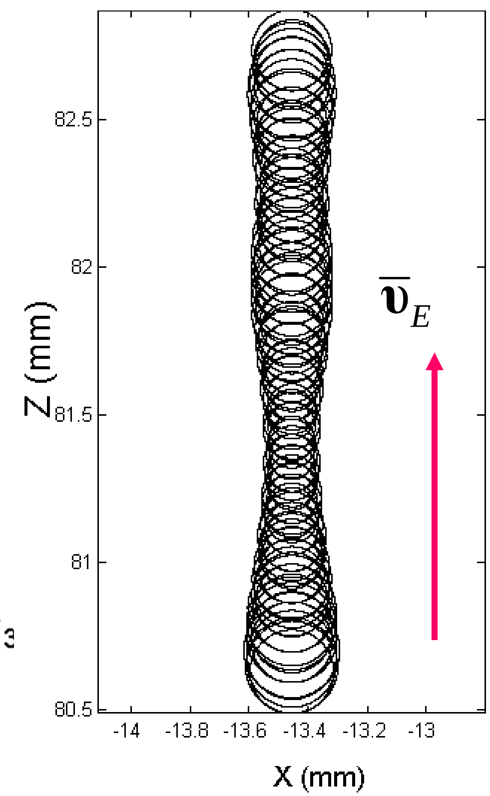
Typical motion of electron in a strong dipole

- **Gyration motion round B field line**
- **Interaction between e- and positron bunch (Receive energy from positron beam)**
- **E×B drift in beam direction**

$$\overline{\upsilon}_E = \frac{e\mathbf{E}\times\mathbf{B}}{eB^2} = \frac{E}{B}\hat{e}\times\hat{B}$$



B_y=0.1T



B_y=0.8484T

Electron energy gain in strong dipole magnet

$$\Delta E(X) = -c\beta\sqrt{\frac{me}{2\pi\varepsilon_0}\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}}\left(1-\frac{X^2}{a^2}+\ln\frac{b^2}{a^2}\right)\left(aG+\int_{\sqrt{a^2-X^2}}^{\sqrt{b^2-X^2}}\left(\ln\frac{b^2}{X^2+y^2}\right)^{-1/2}dy\right)$$

$$+\frac{1}{2}c\beta\sqrt{\frac{me}{2\pi\varepsilon_0}\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}}\int_0^{\sqrt{a^2-X^2}}\frac{y^2}{a^2}\left[\ln\left(1-\frac{X^2}{a^2}+\ln\frac{b^2}{a^2}+\frac{y^2}{a^2}\right)\right]^{-1/2}dy$$

$$+\frac{1}{2}c\beta\sqrt{\frac{me}{2\pi\varepsilon_0}\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}}\int_{\sqrt{a^2-X^2}}^{\sqrt{b^2-X^2}}\left(1-\frac{X^2}{a^2}+\ln\frac{X^2+y^2}{a^2}\right)\left[\ln\left(\frac{b^2}{X^2+y^2}\right)\right]^{-1/2}dy$$

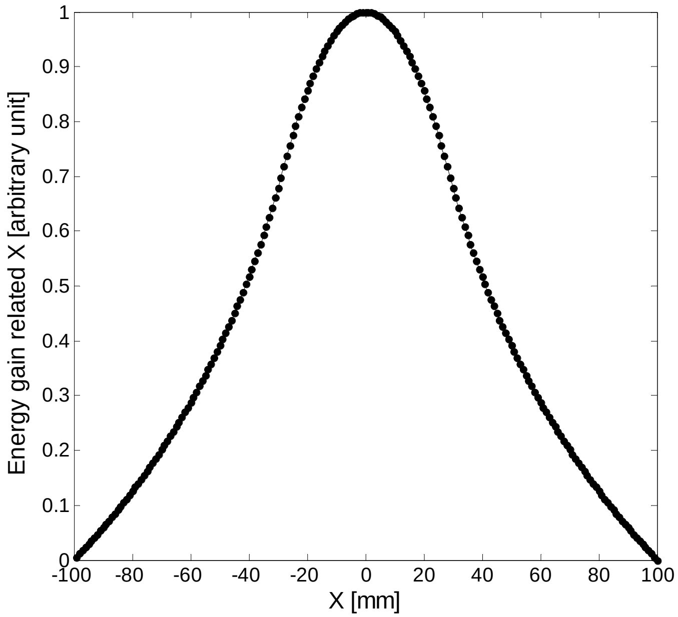
(|X|<a)

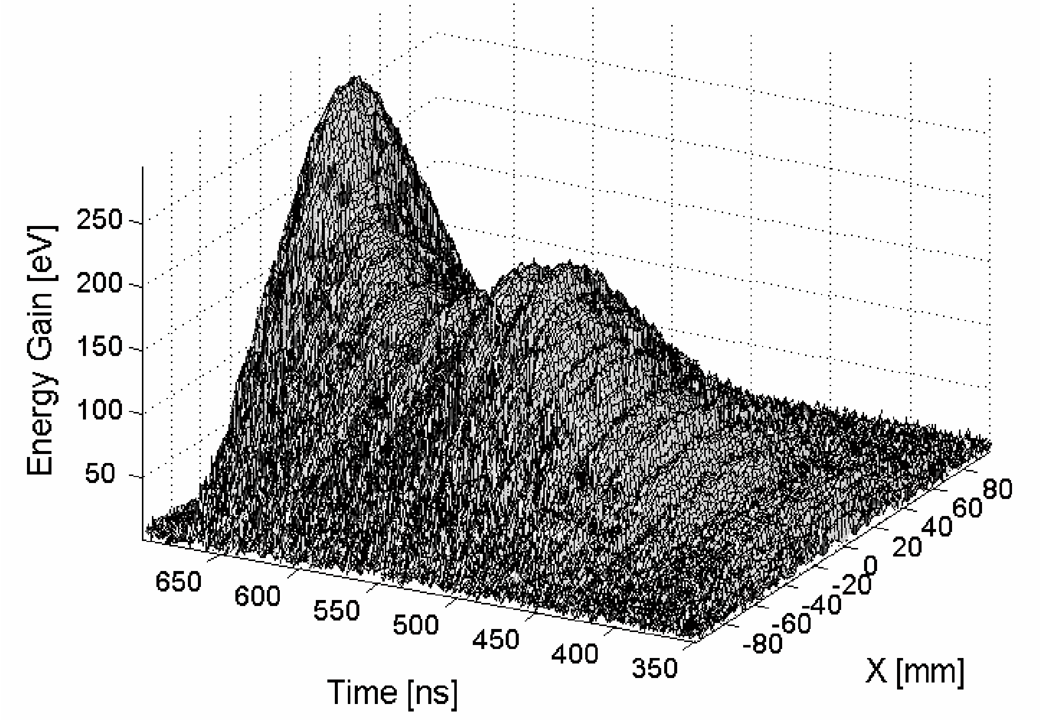
$$\Delta E(X) = -c\beta\sqrt{\frac{me}{2\pi\varepsilon_0}\frac{\partial\lambda}{\partial z}\frac{1}{\sqrt{\lambda}}}\left[\frac{b^2}{X^2}\int_0^{\sqrt{b^2-X^2}}\left(\ln\frac{b^2}{X^2+y^2}\right)^{-1/2}dy-\frac{1}{2}\int_0^{\sqrt{b^2-X^2}}\frac{X^2+y^2}{X^2}\left(\ln\frac{b^2}{X^2+y^2}\right)^{-1/2}dy\right]$$

$$G=\arcsin\left[\frac{\sqrt{a^2-X^2}}{a}\left(1-\frac{X^2}{a^2}+\ln\frac{b^2}{X^2+a^2}\right)^{-1/2}\right]$$

(|X|>a)

Electron energy gain in strong dipole magnet

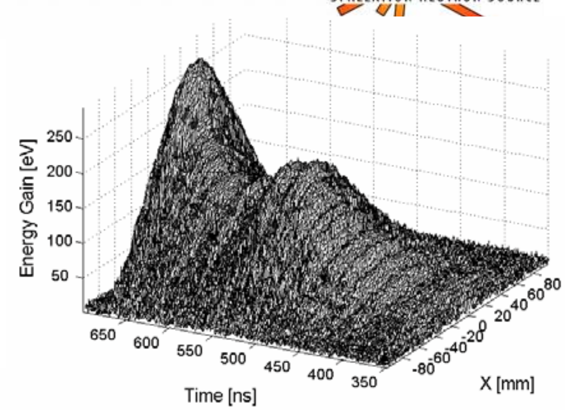


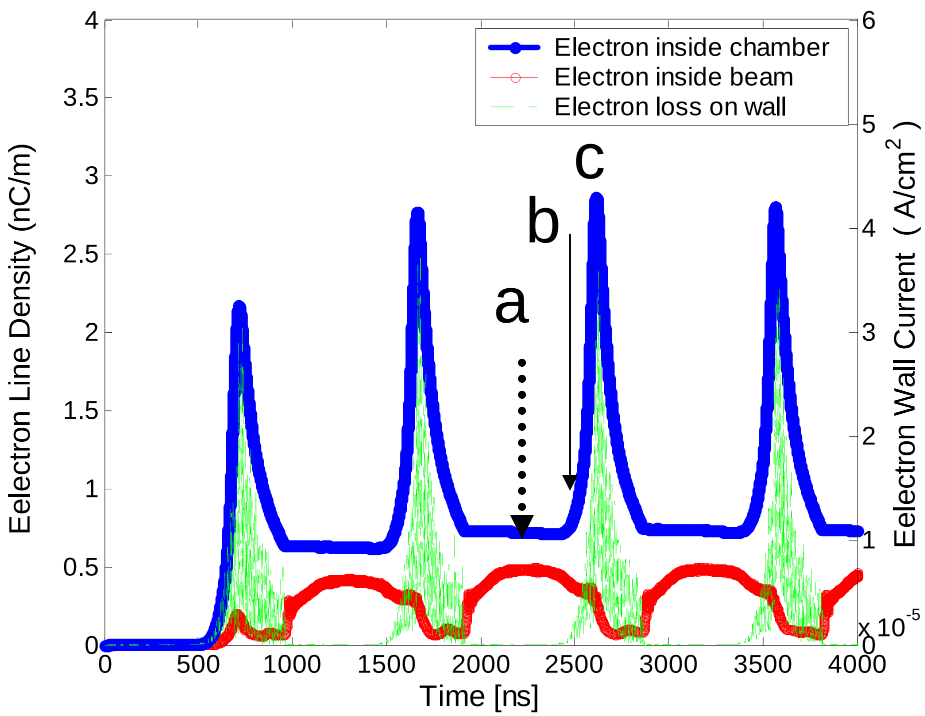


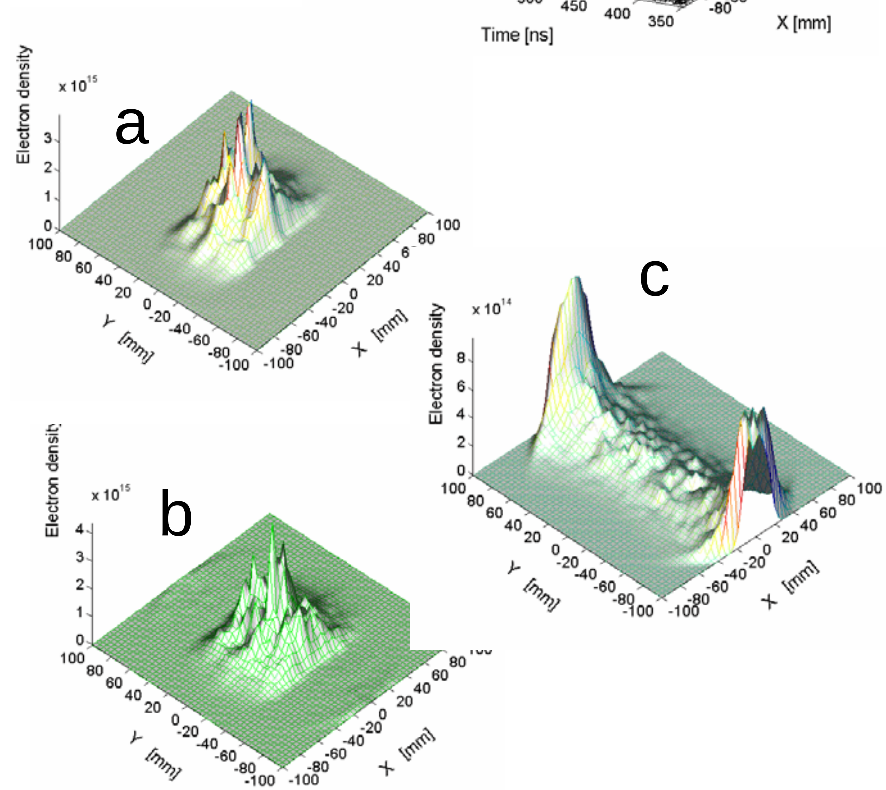
Energy gain at the wall surface for different X-coordinates. Left plot shows the electron energy gain as a function of horizontal coordinate. It is normalized by the peak energy gain at the chamber center X=0. Right plot shows the energy gain of direct drifting electrons in SNS dipole magnets with By=7935 Gauss.

Multipacting in Dipole magnets (By=0.79T)

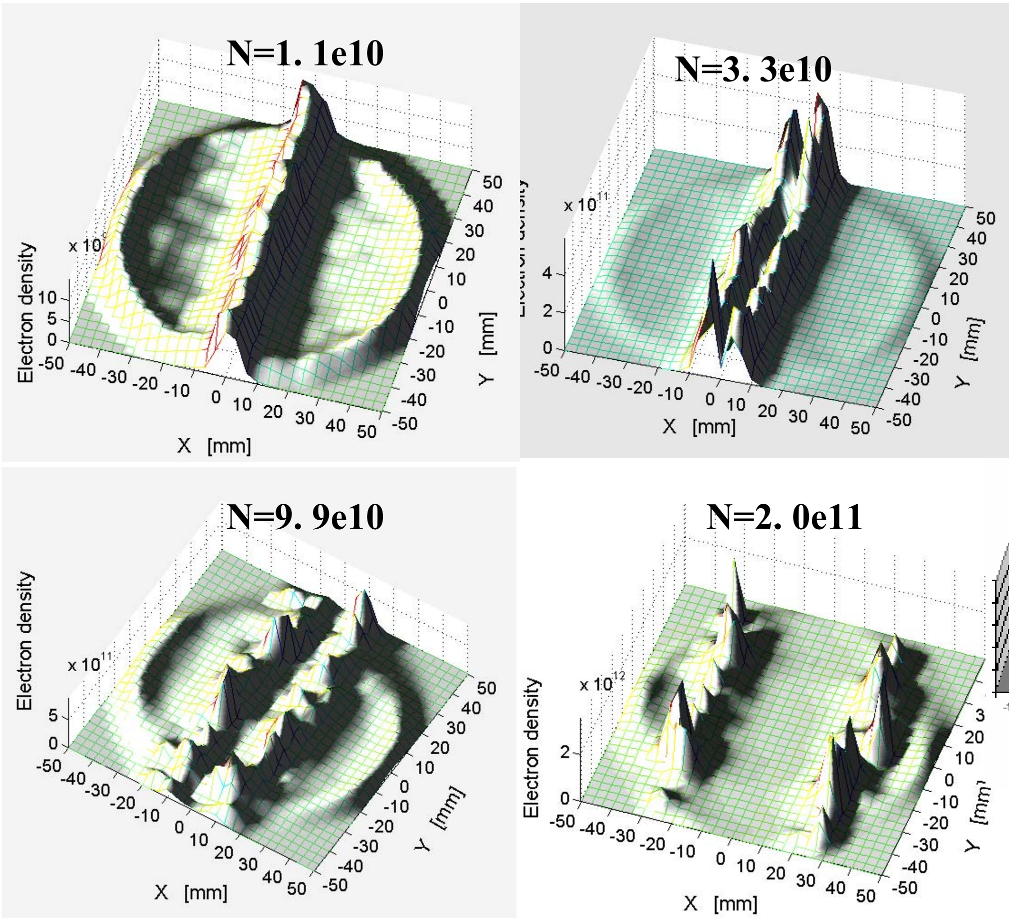
- Multipacting happen at the horizontal chamber center (**1 strip**, agree with estimation)
- E-cloud density is **about 2 times smaller** than the drift region



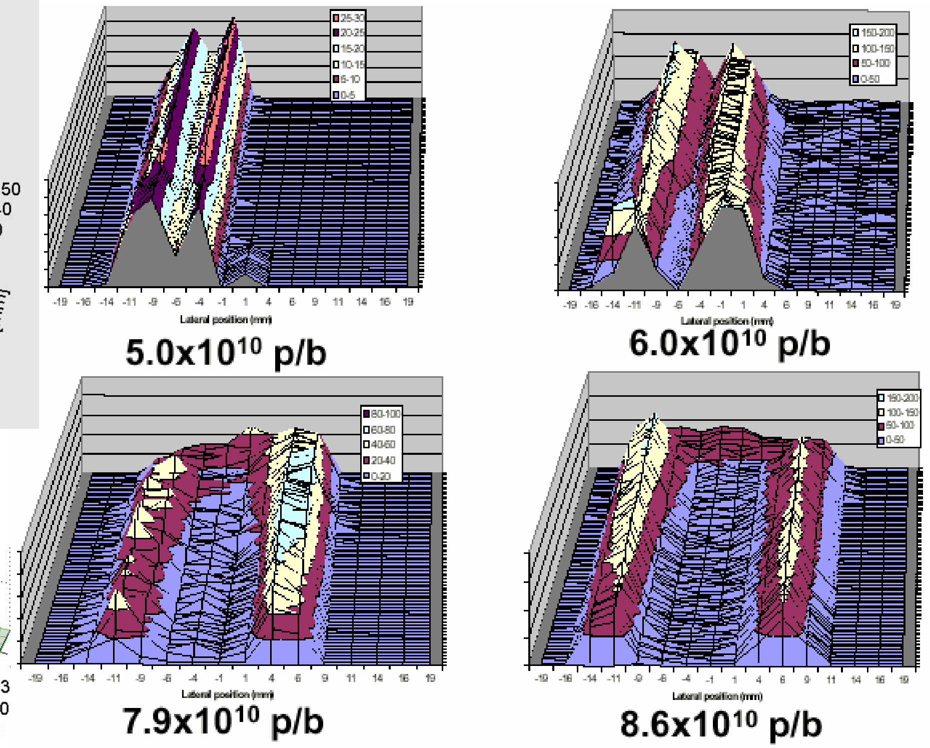




Bunch current effects on Multipacting in *dipole* *for* ***short bunch***---*strip position and lost charge density*



KEKB LER, simulation



Effect of a dipole field: 2 strips at different bunch intensities, CERN SPS experimental results, J. M.Jimenez, ECloud'02, CERN, 2002

Summary

- ***Electron motion under beam space charge field is investigated. (Adiabatic invariant, Nonlinear oscillation frequency, electron energy gain). Mechanism of trailing edge multipacting is clearly explained***
- ***Many factors related to the multipacting has been investigated one by one using 3D code. The results qualitatively agree with the our analysis and experiment studies. Beam intensity, Longitudinal beam profile, transverse beam size, beam in gap are important.***