The "Interior" of a QCD Traveling Wave

Robi Peschanski[a]
(SPhT, Saclay)
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- Geometric Scaling from Non-Linear QCD:

  Traveling Waves

- Exact Scaling Solutions:

  The "Interior" of a wave

- Outlook:

  Running Coupling

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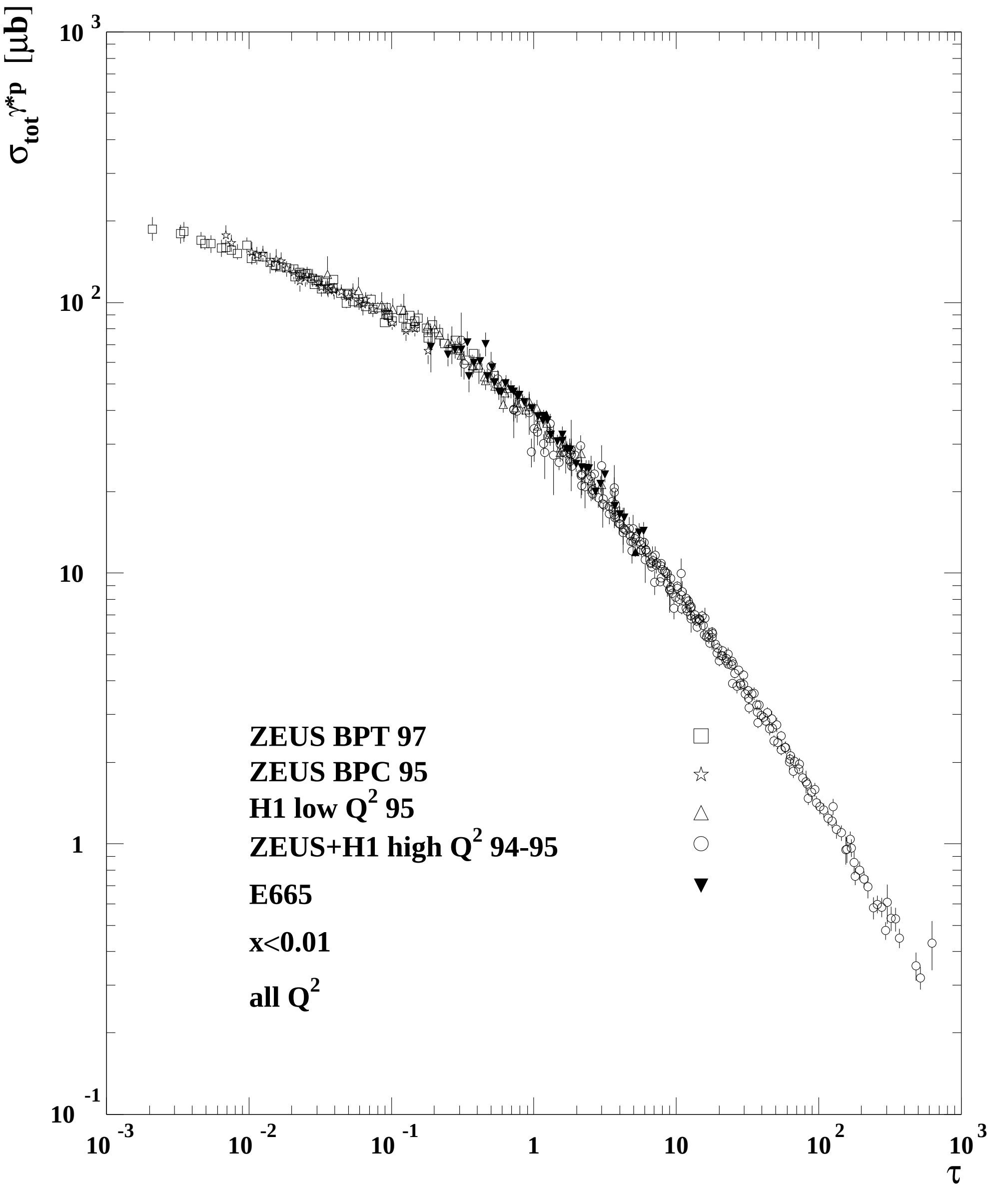
[a] R.P., hep-ph/050523; C.Marquet,R.P.,G.Soyez, to appear.

Geometric Scaling

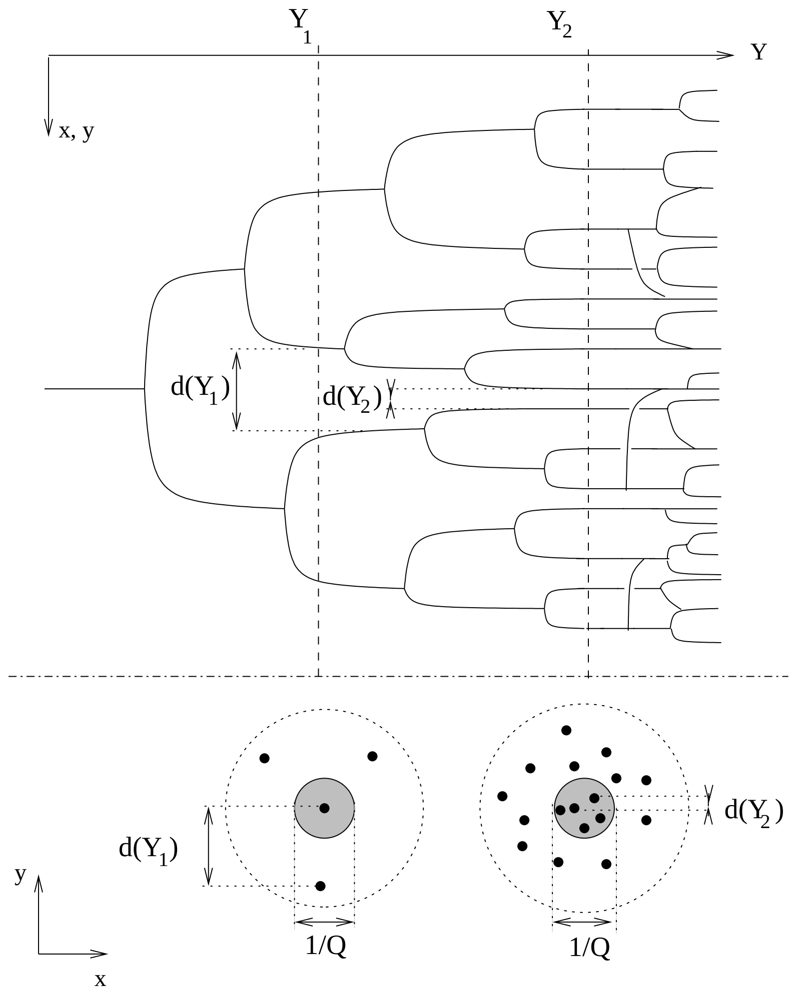
K.Golec-Biernat, J.Kwiecinski, A.Stasto (2000)



$$\tau = Q^2/Q_s(Y = \log 1/x)$$

Saturation and Non-Linear Equations

The Tree of Partons/Dipoles



$d(Y) \to 0 =$ Non-Linear Density effects

$Y \sim Y_1$: Exponential regime: BFKL

$Y \sim Y_2$: Transition to Saturation: BK

$Y > Y_2$: JIMWLK,fluctuations,CGC

The Balitskii-Kovchegov Equation

- **The Dipole Tree Observed in DIS:**

$$\sigma^{\gamma^*}(Y,Q) = \int_0^\infty x_{01}^3\, dx_{01}\, |\psi(x_{01}Q)|^2 \int k dk J_0(kx_{01})\, \mathcal{N}(Y,k)$$

x_{01} : Dipôle Size

$\psi(x_{01}Q)$: $q\bar{q}$ Dipole Wave Function

$\mathcal{N}(Y,k)$: $\sim$ Unintegrated Gluon k-Distribution

- **The Non-Linear BK Equation for $\mathcal{N}$:**

$$\boxed{\partial_Y \mathcal{N} = \bar{\alpha}\chi\left(-\partial_L\right)\mathcal{N} - \bar{\alpha}\,\mathcal{N}^2}$$

- BFKL kernel

$$\chi\left(-\partial_L\right) = 2\psi(1) - \psi\left(-\partial_L\right) - \psi\left(1+\partial_L\right)\ ;\ L \equiv \log\frac{k^2}{\Lambda^2}$$

- QCD Coupling (fixed or running)

$$\bar{\alpha}\ = cste.\ or\ \bar{\alpha}\ = \frac{1}{bL}$$

- Equation valid for uncorrelated *effective probes*

Mathematical Problem

1^{rst} step: $\rightarrow$ **Non-Linear Diffusion**

- **Diffusive Approximation of BK ($\bar{\alpha} = cst.$)**

$$\bar{\chi}\,(-\partial_L) \sim \chi\,(\tfrac{1}{2}) + \frac{D}{2} \times \left(\partial_L + \frac{1}{2}\right)^2$$

- **Equation BK $\Rightarrow$ F-KPP**

S.Munier, R.P., 2003,2004

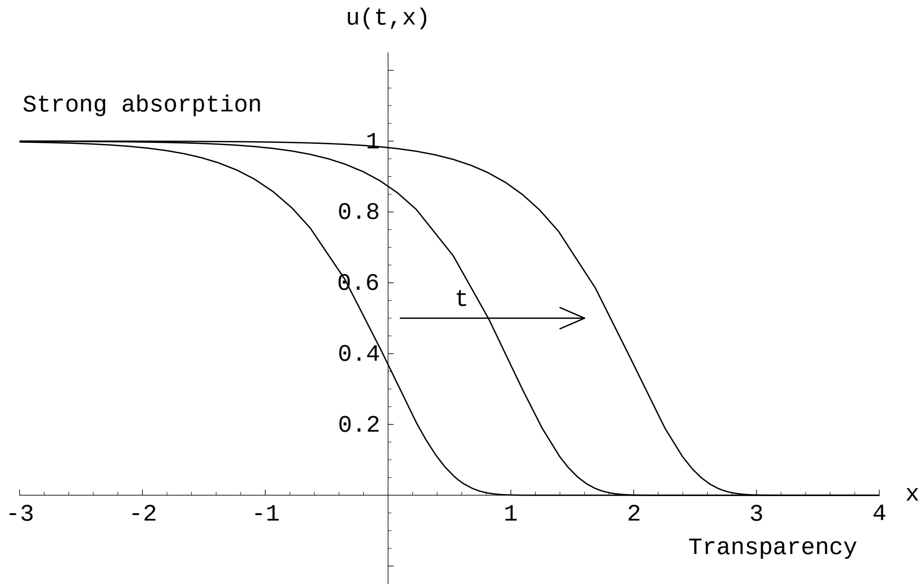
$$\boxed{\partial_t u(t,x) = \partial_x^2 u(t,x) + u(t,x)(1 - u(t,x))}$$

- **"Dictionnary"**

$$\begin{aligned} Time &= t \propto Y \\ Space &= x \propto L + \frac{\bar{\alpha}D}{2}\,Y \\ Wave\ Front &= u(t,x)\ \propto\ \mathcal{N}(Y,k) \end{aligned}$$

Traveling wave Solutions

Bramson (1983)



- $\rightarrow$ Geometric Scaling

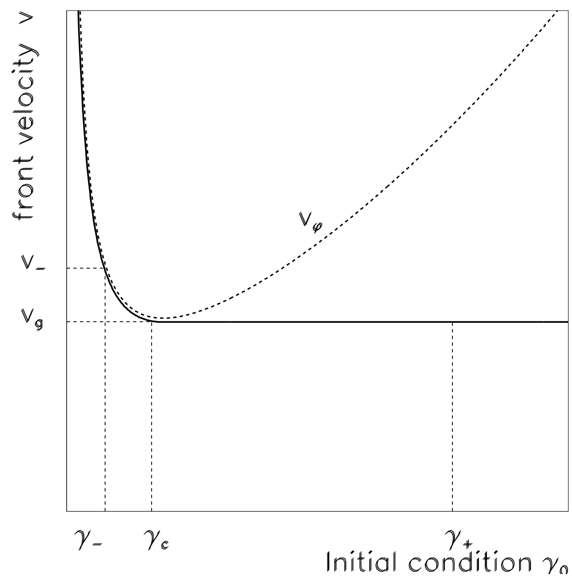
$$u(t,x) \xrightarrow[t\to\infty]{} w(x - m_{\bar\gamma}(t)) \Rightarrow \boxed{\mathcal{N}(Y,k) = \mathcal{N}\left(\frac{k}{Q_s(Y)}\right)}$$

- $\rightarrow$ Saturation Scale: "Universal terms"

$$\boxed{\log Q_s^2(Y) = \bar\alpha \frac{\chi(\bar\gamma)}{\bar\gamma} Y - \frac{3}{2\bar\gamma}\log Y - \frac{3}{(\bar\gamma)^2}\sqrt{\frac{2\pi}{\bar\alpha\chi''(\bar\gamma)}}\frac{1}{\sqrt{Y}} + \mathcal{O}(1/Y)}$$

Generic Wave Solutions for BK

2^{nd} step: **Beyond the Diffusive Approximation**



- Sub-critical regime: phase velocity

$$\gamma_0 = \gamma_- \Rightarrow v \equiv v_\varphi(\gamma) = \bar{\alpha}\chi(\gamma)/\gamma$$

- Critical regime: phase $\equiv$ group

$$\gamma_0 = \bar{\gamma} = .6275... \Rightarrow v_\varphi(\bar{\gamma}) \equiv v_g(\bar{\gamma}) = \bar{\alpha}\chi'(\bar{\gamma}) \sim 4.883\bar{\alpha}$$

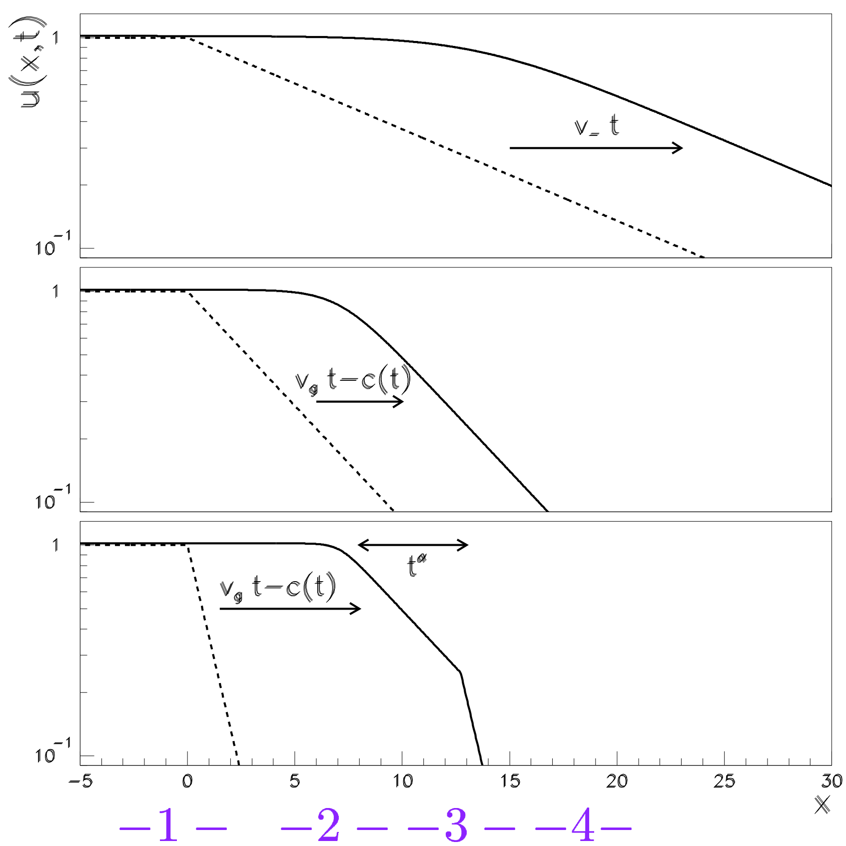
- Super-critical regime (cf. QCD at $\gamma_+ = 1$)

$$\gamma_0 = \gamma_+ \Rightarrow \bar{v} \equiv v_g(\bar{\gamma}) < v_\varphi(\gamma)$$

- NB: for F-KPP: $v \geq v_g \rightarrow c \geq c_g = 2$

The Wave Front Structure

Derrida, Brunet, Van Saarlos: “Pulled vs. Pushed fronts”



Supercritical “Pulled” Fronts: 4 Regions

- -1- “Absorptive”: Deep Saturation
- -2- “Interior”: Geometric Scaling
- -3- “Leading Edge”: Transition to Saturation
- -4- “Pulling Tail”: Transparency limit

Exact Scaling Solutions

Logan 1984, R.P. 2005

$\rightarrow$ **Insert** $u(x,t) \rightarrow U(s = \frac{x}{c} - t)$

- **Diffusive Approximation**

$$\boxed{U(1-U) + \frac{dU}{ds} + \frac{1}{c^2}\frac{d^2U}{(ds)^2} = 0}$$

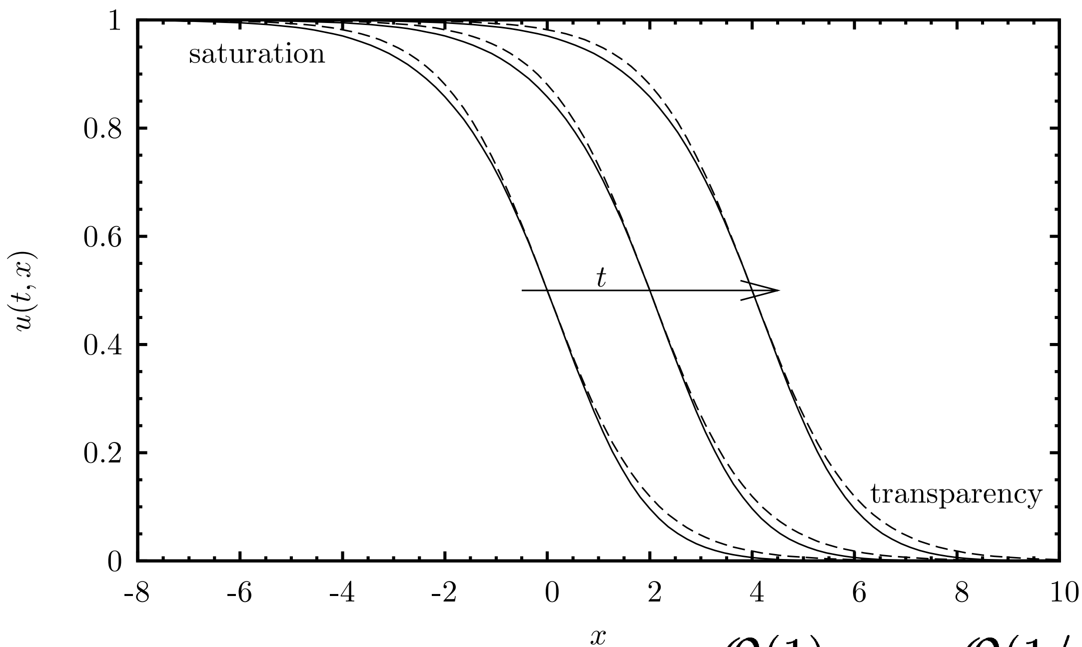
- **Expansion in** $1/c^2 \leq 1/4 = 1/c_g^2$

$$h(s) = h_0 + \frac{1}{c^2}h_2 + \sum_{p\geq 2}\frac{1}{c^{2p}}h_{2p} \equiv \frac{1}{2} - U(s)$$

- **Hierarchy of Linear Equations Except First**

$$\begin{aligned} h_0' + h_0^2 - 1/4 &= 0 \\ h_2' + 2h_0 h_2 + h_0'' &= 0 \\ h_4' + h_2^2 + 2h_0 h_4 + h_2'' &= 0 \cdots \end{aligned}$$

Universal Parametric Wave



- - - $\mathcal{O}(1)$; —- $\mathcal{O}(1/c^2)$

- Result up to $\mathcal{O}(1/c^2)$

$$U(s) = \frac{1}{1+e^s} - \frac{1}{c^2}\ \frac{e^s}{(1+e^s)^2}\ \log \frac{(1+e^s)^2}{4e^s}$$

- Geometric Scaling

$$\mathcal{N} \propto \frac{1}{1+\left[\frac{k^2}{Q_s^2(Y)}\right]^{\mu_1}} - \frac{1}{c^2}\ \frac{\left[\frac{k^2}{Q_s^2(Y)}\right]^{\mu_1}}{\left(1+\left[\frac{k^2}{Q_s^2(Y)}\right]^{\mu_1}\right)^2}\ \log \frac{\left(1+\left[\frac{k^2}{Q_s^2(Y)}\right]^{\mu_1}\right)^2}{4\left[\frac{k^2}{Q_s^2(Y)}\right]^{\mu_1}}$$

Effective saturation scale $Q_s^2(Y) = \exp\left(\mu_2 Y\right)$

Outlook

- **Running Coupling**

$$\boxed{bL\ \partial_Y \mathcal{N}(L,Y) = \chi\,(-\partial_L)\,\mathcal{N}(L,Y) - \mathcal{N}^2(L,Y)}$$

- **Scaling Solution**

$$\boxed{\mathcal{N}(L,Y) \equiv \mathcal{N}\left\{L\ \varphi(\frac{Y}{L^2})\right\}}$$

- **New scaling variable**

$$\boxed{s \propto L - \kappa\sqrt{L^2 + \frac{v_g^2}{\kappa^2}Y} \sim L - v_g\sqrt{Y}}$$

- **Same Universal Parametric Wave**

$$\boxed{U(s) = \frac{1}{1+e^s} - \frac{1}{c^2}\ \frac{e^s}{(1+e^s)^2}\ \log\frac{(1+e^s)^2}{4e^s}}$$

Conclusions

- Balitskii-Kovchegov Equation
  Previous: Universal Asymptotic "Leading Edge","Pulled Fronts" (2005: $\rightarrow$ "weakly pushed"...)
- New: Exact Scaling Solutions:
  Global description of the wave "interior" $\equiv$ Geometric Scaling
- Results:
  Universal Parametric Form of the Front Profile
- Outlook:
  Running Coupling, Phenomenology, Stochasticity