Supersymmetry Phenomenology (With a Broad Brush)

Michael Dine
Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064

These lectures provide an introduction to supersymmetry phenomenology. They include an overview of the Minimal Supersymmetric Standard Model. The notion of soft breaking is explained, constraints on the standard soft breaking parameters are reviewed, and the standard ansatz of universal soft masses is discussed. The rest of the lectures are devoted to understanding supersymmetry breaking more microscopically. Models of dynamical supersymmetry breaking are reviewed, after which we turn to the question of the scale of supersymmetry breaking. Both intermediate and low scales and their phenomenology are considered. Finally, we consider applications to string theory. The emphasis, throughout, is on general issues rather then extensive detail, but it is hoped that the listeners/readers are prepared after these lectures to delve into the details.

1 Introduction

The standard model is extraordinarily successful. While theorists (and sometimes even experimentalists) have become excited from time to time about small discrepancies between data and theory, at the moment of this writing, it is probably fair to say that wherever comparison of theory and experiment is possible, the agreement ranges between very good to extremely good. We are in the position, as we approach the twenty first century, that we know with great confidence all of the laws of nature which operate down to distances at least as small as 10^{-16} cm.

Yet we are firmly convinced that this structure is incomplete. The theory has too many parameters. It contains fundamental scalars, something difficult to reconcile with our current understanding of field theory. Finally, it does not incorporate gravity. It is tempting to speculate that a new, as yet undiscovered symmetry, supersymmetry, has something to do with the answers to these questions. Supersymmetry is the only framework in which we seem to be able to understand light fundamental scalars, i.e., scalars which are pointlike down to scales much smaller than their Compton wavelength. Supersymmetry addresses the question of parameters: first, unification of gauge couplings works much better with than without supersymmetry; second, it is easier to attack questions such as fermion masses in supersymmetric theories, in part simply due to the presence of fundamental scalars. Finally, supersymmetry seems to be intimately connected with gravity. String theory is the only theory we know – and quite possibly the only theory of any kind – which incorporates gravity and gauge interactions. But we have learned during the past two years that what we call string theory is really a code for some larger structure, whose precise nature we only partly understand. Supersymmetry seems to play a fundamental role in this structure. Moreover, it is hard to see how we will be able to make any sense of string theory if supersymmetry does not survive to comparatively low energies.

So there are a number of arguments that suggest that nature might be supersymmetric, and that supersymmetry might manifest itself at energies of order the weak interaction scale. There is also some weak but tantalizing experimental evidence, as I will discuss in these lectures. On the other hand, while many find these arguments for supersymmetry persuasive, I believe one should be skeptical. Perhaps some sort of technicolor theory can ultimately explain the presence of light scalars, or perhaps our ideas about hierarchy and fine tuning are not correct, there is no supersymmetry and yet there is a light fundamental scalar with mass less than a TeV. Experiment should decide this question over the next ten to fifteen years.

In these lectures, I will discuss several aspects of low energy supersymmetry. I will try, first, to make the case that supersymmetry may underlie the physics of weak interactions. I hope, also, to make clear that in the event that supersymmetry is discovered, the pattern of soft breaking – the masses of the partners of ordinary fields – should reveal a great deal. Indeed, one could imagine that the high energy physics of the next century will be devoted to unraveling this pattern, and deciphering its meaning, in much the same way that studying low energy weak interactions revealed the nature of the underlying gauge theory.

The rest of these lectures are organized as follows. In the next section, some features of $N = 1$ supersymmetric theories are briefly reviewed. Then I discuss the hierarchy problem, and how it may be resolved in a supersymmetric framework. I explain why $N = 1$ supersymmetry is special, and introduce a supersymmetric version of the standard model (the MSSM). In section 3, I introduce the notion of soft breaking and count the parameters of the model. A simple ansatz for these parameters is described, which is highly predictive, satisfies constraints from rare processes, and automatically yields $SU(2)\times U(1)$ breaking. This is followed by a critique of the ansatz, and a more careful examination of the various phenomenological constraints on the soft breaking parameters. These constraints arise from direct searches, from flavor changing neutral currents and other rare processes, and from the requirement of $SU(2)\times U(1)$ breaking. I also discuss coupling constant unification and the question of dark matter.

Sections 4-6 are devoted to the problem of supersymmetry breaking. Various mechanisms for dynamical supersymmetry breaking are introduced. I then turn to the question: what is the scale of supersymmetry breaking? Two possibilities are considered: a scale intermediate between M_Z and M_p, and a much lower energy scale, of order $10's$ to 100's of TeV. The virtues and difficulties of both approaches are discussed, as well as some dramatic experimental consequences. In particular, low energy supersymmetry seems a natural framework in which to understand the $e^+e^-\gamma\gamma$ event seen by the CDF detector at Fermilab.

Finally, section 7 is devoted to some questions in string theory. I examine how some of the issues raised earlier look in the context of strings. I also discuss how the problem of vacuum stability/instability – which appears to be one of the most fundamental questions of string dynamics – appears in a low energy framework. I consider this problem first from the perspective of weakly coupled strings, and then turn to the strongly coupled limit ("M theory"). M theory may well turn out to be more appropriate to the description of the real world then weakly coupled string theory.

2 N=1 Supersymmetry and "Low Energy" Physics

2.1 What is N=1 Supersymmetry?

In four dimensions, it is possible to have as many as eight supersymmetries. It is unlikely that theories with $N > 1$ play any role in low energy physics. First, such theories are nonchiral. Second, it is virtually impossible to break supersymmetry in theories with $N > 1$. The symmetries simply prevent one from writing any term in the effective lagrangian which could yield supersymmetry breaking.

Exercise: Check these statements, using results from Lykken's lectures.

So if nature is supersymmetric at scales comparable to the weak scale, there is almost certainly only one supersymmetry. The basic supersymmetry algebra is then

$$\{Q_\alpha, Q^*_{\dot\beta}\} = 2\sigma_\mu P^\mu. \tag{2.1}$$

There is a straightforward recipe for constructing theories with this symmetry. The construction has been described in Joe Lykken's lectures at this school, and an excellent introduction is provided by the text of Wess and Bagger [1]. We will first consider the case of global supersymmetry; later, we will consider the generalization to local supersymmetry. There are two irreducible representations of the supersymmetry algebra containing fields of spin less than or equal to one. These are the chiral and vector superfields. Chiral fields contain a Weyl spinor and a complex scalar; vector fields contain a Weyl spinor and a (massless) vector. In superspace (using the conventions of [1]), a chiral superfield may be written as

$$\Phi(x,\theta) = A(x) + \sqrt{2}\theta\psi(x) + \theta^2 F + \dots \tag{2.2}$$

Here A is the complex scalar, ψ the fermion, and F is an auxiliary field. Under a supersymmetry transformation with anticommuting parameter ζ, the component fields transform as

$$\delta A = \sqrt{2}\zeta\psi \tag{2.3}$$

$$\delta\psi = \sqrt{2}\zeta F + \sqrt{2}i\sigma^\mu\bar{\zeta}\partial_\mu A \qquad \delta F = -\sqrt{2}i\partial_\mu\psi\sigma^\mu\bar{\zeta}. \tag{2.4}$$

Exercise: For a chiral field, construct the supersymmetry *operators.* Verify that if F has an expectation value, supersymmetry is broken and that ψ is the Goldstone fermion (in particular, the supercurrent contains a term $\langle F\rangle\sigma^\mu\psi$).

For vector superfields, the physical content is most transparent in a particular gauge (really a class of gauges) know as Wess-Zumino gauge. This gauge is analogous to the Coulomb gauge in QED. In that case, the gauge choice breaks manifest Lorentz invariance, but Lorentz invariance is still a property of physical amplitudes. Similarly, the choice of Wess-Zumino gauge breaks supersymmetry, but physical quantities are still supersymmetric. In this gauge, the vector superfield may be written as

$$V = -\theta\sigma^\mu\bar{\lambda}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \tfrac{1}{2}\theta^2\bar{\theta}^2 D. \tag{2.5}$$

Here A_μ is the gauge field, λ_α is the gaugino, and D is an auxiliary field. The analog of the gauge invariant field strength is a chiral field:

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \tfrac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F_{\mu\nu} + \theta^2\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu\bar{\lambda}^{\dot{\beta}}. \tag{2.6}$$

To construct an action with $N = 1$ supersymmetry, one starts with a set of chiral superfields, Φ^i, transforming in various representations of some gauge group $\mathcal{G}$. For each gauge generator, there is a vector superfield, V^a. The most general renormalizable lagrangian, written in superspace, is

$$\mathcal{L} = \sum_i \int d^4\theta \Phi_i^\dagger d^V \Phi_i + \sum_a \frac{1}{4g_a^2}\int d^2\theta W_\alpha^2 + c.c. + \int d^2\theta W(\Phi_i) + c.c. \tag{2.7}$$

Here $W(\Phi)$ is a holomorphic function of chiral superfields known as the superpotential.

In terms of the component fields, the lagrangian takes the form (again in Wess-Zumino gauge):

$$\mathcal{L} = \sum_i \left(|D\phi_i|^2 + i\psi_i\sigma^\mu D_\mu \psi_i^* + |F_i|^2\right) \tag{2.8}$$

$$-\sum_a \frac{1}{4g_a^2}\left(F_{\mu\nu}^{a2} - i\lambda^a \not{D}\lambda^{a*} - \tfrac{1}{2}(D^a)^2\right)$$

$$+i\sqrt{2}\sum_{ia} g^a \psi_i T^a \lambda^a \phi^* + c.c.$$

$$+\sum_{ij} \frac{1}{2}\frac{\partial^2 W}{\partial\phi^i \partial\phi^j}\psi^i\psi^j .$$

Here I have already changed notation, and have used ϕ_i to denote the scalar component of Φ. This is a common practice, and I will often use it where it will not lead to (too much) confusion. I have also solved for the auxiliary fields F_i and D_a using their equations of motion:

$$F_i = \frac{\partial W}{\partial \phi_i} \qquad D^a = g^a \sum_i \phi_i^* T^a \phi_i . \tag{2.9}$$

The first two lines are just the gauge invariant kinetic terms for the various fields, as well as potential terms for the scalars. The third line corresponds to Yukawa interactions of the gaugino and matter fields, with strength controlled by the strength of the gauge interactions. The last line yields fermion mass terms and Yukawa couplings among the various chiral fields.

As you have already seen (for example in the lectures of Greene and Peskin at this school) it is often useful to consider effective lagrangians valid below some energy scale. In this case there is no restriction of renormalizability. On the other hand, one usually does want to expand the lagrangian in powers of momenta. It is a simple matter to generalize eq. (2.7) to include arbitrary terms with up to two derivatives:

$$\mathcal{L} = \sum_i \int d^4\theta K(\Phi^{i*}\Phi^i) + \sum_{ab}\int d^2\theta f_{ab}(\Phi) W_\alpha^a W^{\alpha b} + c.c. + \int d^2\theta W(\Phi_i) + c.c. \tag{2.10}$$

The functions W and f are holomorphic functions of the chiral fields (otherwise the last two terms are not supersymmetric); K is unrestricted. It is not hard to generalize the component lagrangian, and I will leave that as an exercise. Note that among the couplings are now terms:

$$\tfrac{1}{4}\mathrm{Re}\ \mathrm{f_{ab}}(\phi_\mathrm{i})\mathrm{F}^{\mathrm{a}}_{\mu\nu}\mathrm{F}^{\mu\nu\mathrm{b}} + \tfrac{1}{4}\mathrm{Im}\ \mathrm{f_{ab}}(\phi_\mathrm{i})\mathrm{F}^{\mathrm{a}}_{\mu\nu}\tilde{\mathrm{F}}^{\mu\nu\mathrm{b}} + \frac{\partial \mathrm{f_{ab}}}{\partial\phi_\mathrm{i}}\mathrm{F_i}\lambda^\mathrm{a}\lambda^\mathrm{b}. \tag{2.11}$$

Exercise: Work out the terms in the effective lagrangian. Be more careful about factors of two than I have been above.

2.2 Why should we be Interested in Low Energy Supersymmetry?

Figure 1: One loop corrections to the Higgs mass in the standard model.

It is quite possible that supersymmetry will be discovered in the near future. In the fall of 1996, for example, LEP will run at an energy of 172 GeV. During 1997, it will run at about 190 GeV. So it is conceivable that by the time these proceedings appear, superpartners of some ordinary particles might have been seen. Absent a discovery, however, there are three classes of reasons for thinking that supersymmetry might have something to do with nature, and that it might be broken at a scale comparable to the scale of weak interactions, rather than at some enormous energy such as the Planck scale. The first of these is the "hierarchy problem" [2], the fact that light fundamental scalars don't seem to make sense in quantum field theory. Here light means light compared to the largest interesting energy scale. One might imagine that this scale is the Planck mass or unification scale. To see the difficulty, consider the minimal standard model, with a single Higgs doublet. Then the diagrams of fig. 1 are quadratically divergent in the ultraviolet, so that the Higgs mass is given by a formula of the form

$$m_H^2 = (m_H^2)_o + \frac{\alpha_2}{4\pi}\Lambda^2. \tag{2.12}$$

If Λ is some extremely large scale, the Higgs mass can only be small if there is a delicate balance between classical and quantum efects. Such a conspiracy would represent a stupendous familure of dimensional analysis. In other words, the natural value of the Higgs mass would seem to be the largest scale in nature. That the Higgs boson is light (compared to, e.g., M_p) might indicate that it has struture on a scale comparable to the scale of weak interactions (technicolor). But failures of dimensional analysis often reflect the presence of symmetries. The only known symmetry which can suppress the quadratically divergent corrections is supersymmetry.

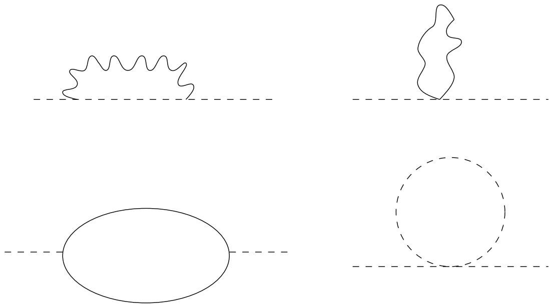

Figure 2: One loop gauge corrections to the scalar mass in a simple model.

To get some practice with supersymmetric theories, let's check that in a simple model, the quadratic divergences do indeed cancel. Take a $U(1)$ theory, with (massless) chiral fields ϕ^+ and ϕ^-. Before doing any computation, it is easy to see that provided we work in a way which preserves supersymmetry, there can be no quadratic divergence. In the limit that the mass term vanishes, the theory has a chiral symmetry under which ϕ^+ and ϕ^- rotate by the same phase,

$$\phi^{\pm} \rightarrow e^{i\alpha}\phi^{\pm}. \tag{2.13}$$

This symmetry forbids a mass term in the superpotential, $\Lambda\phi^+\phi^-$, the only way a supersymmetric mass term could appear. The actual diagrams we need to compute are shown in fig. 2. Since we are only interested in the mass, we can take the external momentum to be zero. It is convenient to choose Landau gauge for the gauge boson. In this gauge the gauge boson propagator is

$$D_{\mu\nu} = -i(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})\frac{1}{q^2} \tag{2.14}$$

so the first diagram vanishes. The second, third and fourth are straightforward to work out from the basic lagrangian. One finds:

$$I_b = g^2(i)(-i)\frac{3}{(2\pi)^4}\int \frac{d^4k}{k^2} \tag{2.15}$$

$$I_c = g^2(i)(-i)\frac{(\sqrt{2})^2}{(2\pi)^4}\int \frac{d^4k}{k^4}\mathrm{tr}(k_\mu\sigma^\mu k_\nu\bar{\sigma}^\nu) \tag{2.16}$$

$$= -\frac{4g^2}{(2\pi)^4}\int \frac{d^4k}{k^2} \tag{2.17}$$

$$I_c = g^2(i)(-i)\frac{1}{(2\pi)^4}\int \frac{d^4k}{k^2}. \tag{2.18}$$

It is easy to see that the sum, $I_a + I_b + I_c + I_d = 0$.

Clearly if nature is supersymmetric, supersymmetry is broken. We want, then, to ask, what is the likely scale of supersymmetry breaking? We can modify our computation above so as to address this question. Suppose that supersymmetry breaking induces a mass for the scalars, $\widetilde{m}^2$, but the fermions remain massless. Then only I_d changes;

$$I_d \rightarrow \frac{g^2}{(2\pi)^4}\int \frac{d^4k}{k^2 - \widetilde{m}^2} \tag{2.19}$$

$$= -i\frac{g^2}{(2\pi)^4}\int \frac{d^4k_E}{k_E^2 + \widetilde{m}^2}$$

$$= \quad \widetilde{\mathrm{m}}^2 \text{ independent} + \frac{\mathrm{i}g^2}{16\pi^2}\widetilde{\mathrm{m}}^2\ln(\Lambda^2/\widetilde{\mathrm{m}}^2).$$

We have worked here in Minkowski space, and I have indicated factors of i to assist the reader in obtaining the correct signs for the diagrams. In the second line, we have performed a Wick rotation. In the third, we have separated off a mass-independent part, since we know that this is cancelled by the other diagrams.

Exercise: Verify the expressions for $I_a - I_d$.

Summarizing, the one loop mass shift is

$$\delta\widetilde{m}^2 = -\frac{g^2}{16\pi^2}\widetilde{m}^2\ln(\Lambda^2/\widetilde{m}^2). \tag{2.20}$$

Note that the mass shift is proportional to $\widetilde{m}^2$, the supersymmetry breaking mass, as we would expect since supersymmetry is restored as $\widetilde{m}^2 \rightarrow 0$. In the context of the standard model, we see that the scale of supersymmetry breaking cannot be much larger than the the Higgs mass scale itself. Roughly speaking, it can't be much larger than this scale by factors of order $1/\sqrt{\alpha_W}$, i.e., factors of order 6.

A second reason to suspect that low energy supersymmetry might have something to do with nature comes from string theory. From the lectures at this school, you have surely gained the sense that supersymmetry is intrinsic to string theory. It is natural to suspect that any consistent fundamental theory must be supersymmetric.

By itself, this is not enough to argue that supersymmetry should survive to low energies. But it has been known for some time that there are a vast array of supersymmetric *solutions* of string theory. A general feature of these solutions is that if supersymmetry is unbroken in some lowest order approximation, the theory remains supersymmetric to all orders. Moreover, non-supersymmetric solutions are problematic: typically the vacuum is unstable already at one loop. While one cannot claim to have understood how string dynamics might break supersymmetry and choose some particular ground state, it is almost impossible to imagine how a sensible ground state could emerge in a non-supersymmetric vacuum. So if string theory describes nature, low energy supersymmetry is almost certainly a prediction.

Finally, there are some small experimental hints that supersymmetry might be true. The most dramatic of these is the unification of couplings [3]. To understand this, we first introduce a supersymmetric extension of the standard model, known as the "Minimal Supersymmetric Standard Model," or MSSM. In this model, the gauge symmetry is still taken to be $SU(3) \times SU(2) \times U(1)$. Each gauge generator is now associated with a vector multiplet, so there is a gaugino for each gauge boson. Similarly, all of the known quarks and leptons are promoted to chiral multiplets. In other words, each left-handed fermion of the standard model now has a complex scalar partner with the same quantum numbers. Finally, instead of one Higgs doublet, there must be two, each containing a boson and a fermion. Otherwise the model suffers from anomalies. We will denote these by H_U and H_D, with hypercharge ± 1, respectively.

Knowing the representation content of the theory, we can work out the β-functions of the different groups. The general expression for the one-loop β-functions is

$$b_o = \frac{11}{3} C_A - \frac{2}{3} \sum_{i=1}^{n_f} c_2^i - \frac{1}{3} \sum_{j=1}^{n_\phi} c_2^j. \tag{2.21}$$

Here the sum over i runs over all of the left-handed fermions of the theory, while that over j runs over the scalars. For the gauginos, $c_2 = C_A$, so for a supersymmetric theory with n_f chiral fields in the fundamental representation we obtain

$$b_o = 3C_A - \frac{1}{2} n_f. \tag{2.22}$$

For $SU(3)$, with three generations, this gives $b_o = 3$. Similarly, for $SU(2)$, one obtains $b_o = -1$ (remember to keep track of the two Higgs doublets). For the $U(1)$, some care is required with the normalization of the charge. Let us assume that the three gauge groups are unified in $SU(5)$. In that case, all of the generators must be normalized in the same way. In a singlet generation, one has a $\bar{5}$ and 10. The $\bar{5}$ contains the $\bar{d}$ quark and the lepton doublet. For this representation, the $SU(3)$ and $SU(2)$ generators satisfy $\mathrm{tr}T^2 = 1/2$. The corresponding $U(1)$ generator is then

$$\widetilde{Y} = \sqrt{\frac{3}{20}}\,\mathrm{diag}(2/3, 2/3, 2/3, -1, -1). \tag{2.23}$$

In other words, $\widetilde{Y}$ is related to the conventional hypercharge generator by:

$$\widetilde{Y} = \sqrt{3/20}Y \qquad g' = g_5\sqrt{3/5} \tag{2.24}$$

(remember that the hypercharge boson is taken to couple to $Y/2$). From this it follows that $sin^2(\theta_w) = 3/8$.

With this information, one can now run the gauge couplings to high energy. The simplest thing to do is to assume that all of the new particles predicted by supersymmetry have the same mass (say a few hundred GeV up to a TeV). At one loop, the couplings satisfy

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M_{GUT}) + \frac{b_o^{(i)}}{4\pi}\ln(M_{GUT}/M_Z). \tag{2.25}$$

Including also two loop corrections, one obtains quite good agreement. The couplings fail to unify in nonsupersymmetric theories. This might be a coincidence, but it is quite suggestive.[a]

2.3 More on the MSSM

Above, we have introduced the fields of the MSSM, and explained their gauge quantum numbers. Let us develop this model further. The lagrangian includes, first, the gauge invariant kinetic terms for all of the fields. In a supersymmetric theory, this includes Yukawa couplings of gauginos to matter fields and quartic

[a] Since the weak and electromagnetic couplings are far better known than α_s, one usually computes α_s, making some assumptions about thresholds both at the GUT scale and at the supersymmetric scale. One actually finds that, with the simplest assumptions for these, that one predicts a value of α_s slightly too large. This problem, and possible solutions, have been discussed in [4].

scalar couplings from the D^2 terms. The usual Yukawa couplings of fermions arise from terms in the superpotential. These can be written in the form

$$W = H_U Q y_u \bar{u} + H_D Q y_D \bar{d} + H_D L y_L \bar{e}. \tag{2.26}$$

In this expression, the quark and lepton superfields are understood to carry a flavor index, and the y's are 3×3 matrices. If H_U and H_D have non-zero masses, this gives mass for quarks and squarks, leptons and sleptons.

Exercise: Check that their are a set of supersymmetric ground states with equal, non-zero expectation values for H_u and H_D i.e., that the energy vanishes if $H_U = H_D = \binom{0}{v}$. Check that one obtains equal masses for bosons and fermions, first for the quarks and leptons, then for the gauge bosons and gauginos.

There are other terms which can also be present in the superpotential. These include the "μ-term," $\mu H_U H_D$. This is a supersymmetric mass term for the Higgs fields. We will see later that we need $\mu \sim M_Z$ to have a viable phenomenology.

A set of dimension four terms which are permitted by the gauge symmetries raise much more serious issues. For example, one can have terms

$$\bar{u}_f \bar{d}_g \bar{d}_h \Gamma^{fgh} + Q_f L_g \bar{d}_H \lambda^{fgh}. \tag{2.27}$$

These couplings violate B and L! This is our first serious setback. In the standard model, there is no such problem. The leading operators permitted by gauge invariance are four fermi operators of dimension 6, and it is easy to imagine that they are suppressed by some very large mass scale.

If we are not going to simply give up, we need to suppress B and L violation at the level of dimension four terms. This presumably requires additional symmetries. There are various possibilities one can imagine.

1. Global continuous symmetries: It is hard to see how such symmetries could be preserved in any quantum theory of gravity, and indeed in string theory, there is a theorem which asserts that there are no global continuous symmetries [5].

2. Discrete symmetries: Discrete symmetries can be gauge symmetries, and indeed such symmetries are common in string theory. These symmetries are often "R symmetries," symmetries which do not commute with supersymmetry.

A simple (though not unique), solution to the problem of baryon and lepton number violation by dimension four operators is known as R-parity or "matter parity." Under this symmetry, all ordinary particles are even, while their superparters are odd. Imposing this symmetry immediately eliminates all of the dangerous operators. For example,

$$\int d^2\theta \bar{u}\bar{d}\bar{d} \sim \psi_{\bar{u}}\psi_{\bar{d}}\tilde{\bar{d}} \tag{2.28}$$

(we have changed notation again: the tilde here indicates the superpartner of the ordinary field, i.e., the squark). This operator is clearly odd under the symmetry.

More formally, we can define this symmetry as the transformation on superfields:

$$\theta_\alpha \rightarrow -\theta_\alpha \tag{2.29}$$

$$(Q_f, \bar{u}_f \bar{d}_f, L_f \bar{e}_f) \rightarrow -(Q_f, \bar{u}_f \bar{d}_f, L_f \bar{e}_f) \tag{2.30}$$

$$(H_U, H_D) \rightarrow (H_U H_D). \tag{2.31}$$

While imposing this symmetry solves the immediate problem, it also has a striking consequence: the lightest of the new particles predicted by supersymmetry (the LSP) is *stable*. In order to avoid various cosmological disasters, this particle must be electrically neutral. It is then, inevitably, very weakly interacting. This in turn means:

- The generic signature of R-parity conserving supersymmetric theories is events with missing energy.
- Supersymmetry is likely to produce an interesting dark matter candidate.

In most of what follows, we will assume a conserved R-parity.

2.4 LSP as the Dark Matter

A stable particle is not necessarily a good dark matter candidate. But we can make a crude calculation which indicates that the LSP density is in a suitable range to be the dark matter. Consider particles, X, with mass of order 100 GeV interacting with weak interaction strength. Their annihilation cross sections go as $G_F^2 E^2$. So, in the early universe, the corresponding interaction rate is of order

$$\Gamma \approx \rho_X G_F^2 E^2 \approx \rho_X G_F^2 T^2. \tag{2.32}$$

These interactions drop out of equilibrium when

$$\Gamma \sim H \sim T^2/M_p, \tag{2.33}$$

i.e., when

$$\rho_X \sim \frac{G_F^2}{M_p} \sim 10^{-9}. \tag{2.34}$$

T here is of order 1 GeV, so

$$\frac{\rho_X}{\rho_\gamma} \sim 10^{-9}. \tag{2.35}$$

This means that the X particles have a number density today similar to that of baryons. So if their masses are of order 100 GeV, their density can be of order the closure density. This estimate is quite crude, but more careful studies indicate that the LSP can be the dark matter for a broad range of parameters.

So while it is disturbing that we need to impose additional symmetries in order to avoid proton decay, it is also exciting that this leads to a possible solution of one of the most critical problems of cosmology: the identity of the dark matter.

3 A First Look at Supersymmetry Breaking

3.1 Explicit Soft Breaking of Supersymmetry

If supersymmetry is a symmetry of nature, it is almost certainly an exact, local symmetry. This is the case, for example, in string theory. Even if string theory is not the correct underlying theory, it is hard to imagine how supersymmetry might arise "by accident." So supersymmetry must be spontaneously broken. We will devote a great deal of attention in these lectures to the problem of spontaneous breaking of supersymmetry. However, it turns out that most schemes for spontaneous breaking yield an effective lagrangian, at low energies, which is supersymmetric except for explicit, soft supersymmetry violating terms. So we will begin by simply adding such terms to the effective lagrangian of the MSSM, without altering the dimensionless couplings.

The addition of such soft terms is compatible with our original hope that supersymmetry could solve the hierarchy problem. Indeed, precisely because these terms are soft, they induce at most logarithmic ultraviolet divergences. As an example, consider a theory with a single massless chiral superfield, Φ, with superpotential

$$W = \frac{\lambda}{3}\Phi^3. \tag{3.1}$$

To the supersymmetric lagrangian for this theory, we add an explicit mass term for the scalar component of ϕ, i.e., we take the lagrangian to be

$$\mathcal{L} = |\partial_\mu \phi|^2 + i\psi \not{\partial} \psi^* + \lambda\psi\psi\phi + c.c. - |\lambda|^2|\phi|^4 - m_{sb}^2|\phi|^2. \tag{3.2}$$

Figure 3: One loop corrections to scalar masses arising from Yukawa couplings.

At one loop, the scalar masses receive corrections from the diagrams of fig. 3. One has

$$I_a = 4\frac{\lambda^2}{(2\pi)^4}\int \frac{d^4k}{k^2 - m_{sb}^2} \tag{3.3}$$

$$I_b = (-)2\frac{\lambda^2}{(2\pi)^4}\int \frac{d^4k}{k^2}\mathrm{tr}1.$$

Here the trace is over the Weyl indices and gives a factor of two, so as expected the two diagrams cancel in the supersymmetric limit.

Exercise: Check the signs and combinatorics of these diagrams.

For large momenta,

$$\frac{1}{k^2 - m_{sb}^2} \approx \frac{1}{k^2} + \frac{m_{sb}^2}{k^4}. \tag{3.4}$$

The leading divergence cancels, and one obtains for the mass shift,

$$\delta m^2 \approx -\frac{|\lambda|^2}{4\pi^2} m_{sb}^2 \ln(\Lambda^2/m^2). \tag{3.5}$$

Note the negative sign. This will be important when we come to consider the breaking of $SU(2) \times U(1)$.

It is not difficult to make a list of the symmetry breaking interactions which are soft in this sense [6]. These include:

1. Masses for scalars (either of the form $\phi^*\phi$, as above, or $\phi\phi$ when permitted by symmetries).

2. Masses for gauginos.

3. Cubic couplings for scalars, of the form $\phi_i\phi_j\phi_k + c.c.$, but not $\phi_i^*\phi_j\phi_k$.

We can understand this list in a simple way, by considering *spontaneous* supersymmetry breaking. The auxiliary fields, F_i and D^a, which sit in the chiral and vector supermultiplets, are candidate order parameters for this breaking.

To see this, consider the commutation relations (dropping the indices i and a, for simplicity):

$$\{Q_\alpha, \psi_\beta\} = \sqrt{2} F \epsilon_{\alpha\beta} + \ldots \ . \tag{3.6}$$

$$\{Q_\alpha, \lambda_\beta\} = \sqrt{2} D \epsilon_{\alpha\beta} + \ldots \ . \tag{3.7}$$

So if F or D have expectaion values, the Q_α's do not annhilate the vacuum and supersymmetry is broken. In general, these fields could be elementary or composite, but for our present discussion, we will assume that they are elementary.

Now suppose we add to the MSSM some fields Φ and V with nonvanishing auxiliary components, and which are neutral under the standard model gauge group. Then the soft breaking terms a-c can arise through terms in the effective action such as:

$$\frac{1}{M^2} \int d^4\theta \Phi^\dagger \Phi Q^\dagger Q = \frac{|\langle F \rangle|^2}{M^2} \widetilde{Q}^\dagger \widetilde{Q} + (\text{derivative terms}). \tag{3.8}$$

$$\frac{1}{M} \int d^2\theta \Phi W_\alpha^2 W^{\alpha a} = \frac{\langle F \rangle}{M} \lambda^a \lambda^a + \ldots \tag{3.9}$$

$$\frac{1}{M} \int d^2\theta \Phi H Q \overline{U} = \frac{\langle F \rangle}{M} H \widetilde{Q} \tilde{\bar{u}} + \ldots \ . \tag{3.10}$$

Without a microscopic theory of supersymmetry breaking, all of the soft terms are independent. It is interesting to ask, in the MSSM, how many soft breaking parameters are there? More precisely, let's count the parameters of the model beyond those of the minimal standard model with a single Higgs doublet. Having imposed R parity, the number of Yukawa couplings is the same in both theories, as is the number of gauge couplings and θ parameters. The quartic couplings of the Higgs fields are completely determined in terms of the gauge couplings. So the "new" terms arise from the soft breaking terms, as well as the μ term for the Higgs fields. We will speak loosely of all of this as the "soft breaking" lagrangian. Suppressing flavor indices:

$$\begin{aligned} \mathcal{L}_{sb} = {} & \widetilde{Q}^* m_Q^2 \widetilde{Q} + \tilde{\bar{u}}^* m_{\bar{u}}^2 \tilde{\bar{u}} + \tilde{\bar{d}}^* m_{\bar{d}}^2 \tilde{\bar{d}} \\ & + \tilde{L}^* m_L^2 \tilde{L} + \tilde{\bar{e}}^* m_{\bar{e}}^2 \tilde{\bar{e}} \\ & + H_U \widetilde{Q} A_u \tilde{\bar{u}} + H_D \widetilde{Q} A_d \tilde{\bar{d}} + H_D L A_l \bar{e} + \text{c.c.} \\ & + m_i \lambda\lambda + c.c. + m_{H_U}^2 |H_U|^2 + m_{H_D}^2 |H_D|^2 + \mu B H_U H_D + \mu \psi_H \psi_H . \end{aligned} \tag{3.11}$$

The matrices m_Q^2, $m_{\bar{u}}^2$, and so on are 3×3 hermitian matrices, so they have nine independent entries. The matrices A_u, A_d, etc., are general 3×3 complex matrices, so they each possess 18 independent entries. Each of the gaugino masses is a complex number, so these introduce 6 additional parameters. The quantities μ and B are also complex; this is four more. In total, then, there are 111 new parameters. As in the standard model, not all of these parameters are real; we are free to make field redefinitions. The counting, however, is significantly simplified if we just ask how many parameters there are beyond the usual 17 of the minimal theory, since this counting uses up most of our freedom.

To understand what redefinitions are possible beyond the transformations on the quarks and leptons which go into defining the usual KM parameters, we need to ask what are the symmetries of the MSSM before introducing the soft breaking terms and the μ term (the μ term is more or less on the same footing as the soft breaking terms, since it is of the same order of magnitude; as we will discuss later, it might well arise from the physics of supersymmetry breaking). Apart from the usual baryon and lepton numbers, there are two more. The first is a Peccei-Quinn symmetry, under which which two Higgs superfields rotate by the same phase, while the right handed quarks and leptons rotate by the opposite phase. The second is perhaps more interesting. It is an "R" symmetry. By definition, and R symmetry is a symmetry of the Hamiltonian which does not commute with the supersymmetry generators. Such symmetries can be continuous or discrete. In the case of continuous R-symmetries, by convention, we can take the θ's to transform by a phase $e^{i\alpha}$. Then the general transformation law takes the form

$$\lambda_i \to e^{i\alpha}\lambda_i \tag{3.12}$$

for the gauginos, while for the elements of a chiral multiplet

$$\Phi_i(x,\theta) \to e^{ir_i\alpha}\Phi(x,\theta e^{i\alpha}), \tag{3.13}$$

or, in terms of the component fields,

$$\phi_i \to e^{ir_i\alpha}\phi_i \qquad \psi_i \to e^{i(r_i-1)\alpha}\psi_i \qquad F_i \to e^{i(r_i-2)\alpha}F_i\ . \tag{3.14}$$

In order that the lagrangian exhibit a continuous R symmetry, the total R charge of all terms in the superpotential must be two. In the MSSM, we can take $r_i = 2/3$ for all of the chiral fields.

The soft breaking terms, in general, break two of the three lepton number symmetries, the R symmetry and the Peccei-Quinn symmetry. So there are four non-trivial field redefinitions which we can perform. In addition, the minimal standard model has two Higgs parameters. So from our 111 parameters, we can subtract a total of six, leaving 105 as the number of *new* parameters in the MSSM.

Clearly we would like to have a theory which predicts these parameters. Later, we will study some candidates. To get started, however, it is helpful to make an ansatz. The simplest thing to do is suppose that all of the scalar masses are the same, all of the gaugino masses the same, and so on. It is necessary to specify also a scale at which this ansatz holds, since it will not be respected by renormalization to lower energies. Almost all investigations of superymmetry phenomenology assume such a degeneracy at a large energy scale, typically the reduced Planck mass, $M = M_p/\sqrt{8\pi}$. This assumption is sometimes considered part of the definition of the MSSM, but I will use MSSM simply to refer to the particle content of the model. It is often said that degeneracy is automatic in supergravity models, so this is frequently called the supergravity ("SUGRA") model, but as well will see, supergravity by itself makes *no* prediction of degeneracy. In any case, the ansatz consists of the statement that at the high energy scale:

1. All of the scalar masses are the same, $\widetilde{m}^2 = m_o^2$. This asssumption is called "universality" of scalar masses.

2. The gaugino masses are the same $M_i = M_o$. This is referred to as the "GUT" relation, since it holds in simple grand unified models.

3. The soft-breaking cubic terms are assumed to be given by

$$\mathcal{L}_{tri} = A(H_U Q y_u \bar{u} + H_D Q y_d \bar{d} + H_D L y_l \bar{e}). \tag{3.15}$$

   The matrices y_u,y_d, etc. are the same matrices which appear in the Yukawa couplings. This is the assumption of "proportionality."

Note that with this ansatz, if we ignore possible phases, five parameters are required to specify the model $(m_o^2, M_o, A, B\mu, \mu)$. One of these can be traded for M_Z, so this is quite an improvement in predictive power. In addition, this ansatz automatically satisfies all constraints from rare processes. With this assumption, these constraints are automatically satisfied. On the other hand, we will want to ask: just how plausible are these assumptions? We will try to address this question later in these lectures.

3.2 $SU(2) \times U(1)$ Breaking

In the MSSM, there are a number of general statements which can be made about the breaking of $SU(2) \times U(1)$. The only quartic couplings of the Higgs fields arise from the $SU(2)$ and $U(1)$ D^2 terms. The general form of the soft breaking mass terms has been described above. So, before worrying about any detailed ansatz for the soft breakings, the Higgs potential is given, quite generally, by

$$V_{Higgs} = m_{H_U}^2 |H_U|^2 + m_{H_D}^2 |H_D|^2 - m_3^2 (H_U H_D + h.c.)$$
$$+ \frac{1}{8}(g^2 + g'^2)(|H_U|^2 - |H_D|^2)^2 + \frac{1}{2} g^2 |H_U H_D|^2. \qquad (3.16)$$

This potential by itself conserves CP; a simple field redefinition removes any phase in m_{12}^2. (As we will discuss shortly, there are many other possible souces of CP violation in the MSSM.) The physical states in the Higgs sector are usually described by assuming that CP is a good symmetry. In that case, there are two CP-even scalars, H^o and h^o, where by convention, h^o is the lighter of the two. There is a CP-odd neutral scalar, A^o, and charged scalars, $H^\pm$. At tree level, one also defines a parameter,

$$\tan(\beta) = \frac{|\langle H_U \rangle|}{|\langle H_D \rangle|} \equiv v_1/v_2. \qquad (3.17)$$

Note that with this definition, as $\tan(\beta)$ grows, so does the Yukawa coupling of the b-quark.

To obtain a suitable vacuum, there are two constraints which the soft breakings must satisfy:

1. Without the soft breaking terms, $H_U = H_D$ ($v_1 = v_2 = v$) makes the $SU(2)$ and $U(1)$ D terms vanish, i.e., there is no quartic coupling in this direction. So the energy is unbounded below unless

$$m_{H_U}^2 + m_{H_D}^2 - 2|m_3|^2 > 0. \qquad (3.18)$$

2. In order to obtain symmetry breaking, the Higgs mass matrix must have a negative eigenvalue. This gives the requirement:

$$|m_3^2|^2 > m_{H_U}^2 m_{H_D}^2. \qquad (3.19)$$

When these conditions are satisfied, it is straightforward to minimize the potential and determine the spectrum. One finds that

$$m_A^2 = \frac{m_{12}^2}{\sin(\beta)\cos(\beta)}. \qquad (3.20)$$

It is conventional to take m_A^2 as one of the parameters. Then one finds that the charged Higgs masses are given by

$$m_{H^\pm}^2 = m_W^2 + m_A^2, \tag{3.21}$$

while the neutral Higgs masses are:

$$m_{H^o,h^o}^2 = \frac{1}{2}\left(m_A^2 + m_Z^2 \pm \sqrt{(m_A^2+m_Z^2)^2 - 4m_Z^2 m_A^2\cos(2\beta)}\right). \tag{3.22}$$

Exercise: Derive these formulae.

Note the inequalities:

$$\begin{aligned} m_{h^o} &\leq m_A \\ m_{h^o} &\leq m_Z \\ m_{H^\pm} &\geq m_W . \end{aligned} \tag{3.23}$$

The middle relation is particularly interesting since LEP II will have enough energy to find h^o if $m_{h^o} \leq 95$ GeV. Thus it would appear that it might be able to rule out (or discover) the MSSM at this machine. The basic process through which one hopes to discover the higgs uses the $Z-Z-h$ vertex, as in fig. 4. From observations on the Z pole, one already has a limit of about 60 GeV. However, these are tree level relations. We will turn shortly to the issue of radiative corrections, and will see that these can be quite substantial – LEP II will not be able to rule out the MSSM [7].

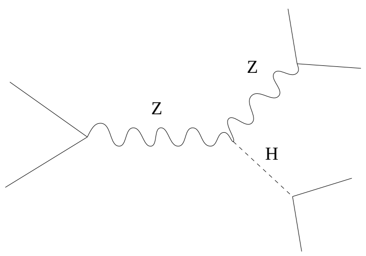

Figure 4: Leading contribution to Higgs production in $e^+ \ e^-$ annihilation.

There are other states in the MSSM which are likely discovery channels for supersymmetry. Particularly important among these are the "charginos," linear combinations of the partners of the $W^{\pm}$ and $H^{\pm}$. The mass matrix for these states, denoted $w^{\pm}$ and $\tilde{h}^{\pm}$ is given by

$$\mathcal{M}_{\chi^{\pm}} = \begin{pmatrix} M_2 & gv_1 \\ gv_2 & \mu \end{pmatrix}. \tag{3.24}$$

There are also four neutral fermions, referred to as the neutralinos, w^o, b, $\tilde{h}_U^o$, $\tilde{h}_D^o$. The lightest of these states is a natural dark matter candidate.

3.3 Why is one Higgs mass negative?

Within the simple ansatz, there is a natural way to understand why $m_{H_U}^2 < 0$ while $m_{H_D}^2 > 0$ [8,9]. What is special about H_U is that it has an $\mathcal{O}(1)$ coupling to the top quark. (If $\tan(\beta)$ is very large, of order 40–50, H_D has a comparable coupling to the b quark). We saw earlier in the Wess Zumino model that at one loop, there is a negative renormalization of the soft breaking scalar masses. This calculation can be translated to the MSSM, with a modification for the color and $SU(2)$ factors. One obtains:

$$m_{H_U}^2 = (m_{H_U})_o^2 - \frac{6y_t^2}{16\pi^2}\ln(\Lambda^2/m^2)\widetilde{m}_t^2. \tag{3.25}$$

$$\widetilde{m}_t^2 = (\widetilde{m}_t)_o^2 - \frac{4y_t^2}{16\pi^2}\ln(\Lambda^2/m^2)\widetilde{m}_H^2. \tag{3.26}$$

So we see that loop corrections involving the top quark Yukawa coupling reduce both the Higgs and the stop masses, but the reduction is larger for the Higgs. If $\Lambda \sim M_p$, and the typical soft breakings are of order a TeV, these corrections are $\mathcal{O}(1)$, so one needs a full renormalization group analysis to determine if $SU(2) \times U(1)$ is broken. For this we need the full set of renormalization group equations. These can be derived along the lines of the calculations we have already presented for the gauge and Yukawa contributions to the soft mass renormalizations. If only y_t is large, they can be written rather compactly:

$$\mu\frac{\partial}{\partial\mu}m_{H_U}^2 = \frac{1}{8\pi^2}[3y_t^2(m_{H_U}^2+m_{\tilde t}^2+m_{Q_3}^2+|A_U^{33}|^2)]-\frac{1}{2\pi^2}\left[\tfrac{3}{4}|M_2|^2g_2^2+\tfrac{1}{4}|M_1|^2g_1^2\right] \tag{3.27}$$

$$\mu\frac{\partial}{\partial\mu}m_{\tilde t}^2 = \frac{1}{8\pi^2}[2y_t^2(m_{H_U}^2+m_{\tilde t}^2+m_{Q_3}^2+|A_U^{33}|^2)]-\frac{1}{2\pi^2}\left[\tfrac{4}{3}|M_3|^2g_3^2+\tfrac{4}{9}|M_1|^2g_1^2\right] \tag{3.28}$$

$$\begin{aligned}\mu\frac{\partial}{\partial\mu}m_{Q_3}^2 &= \frac{1}{8\pi^2}\left[2y_t^2(m_{H_U}^2+m_{\tilde t}^2+m_{Q_3}^2+|A_U^{33}|^2)\right]\\ &-\frac{1}{2\pi^2}\left[\tfrac{4}{3}|M_3|^2g_3^2+\tfrac{3}{4}|M_2|^2g_2^2+\tfrac{1}{36}|M_1|^2g_1^2\right].\end{aligned} \tag{3.29}$$

For scalars besides H_U and the third generation squarks one has only the contribution from diagrams involving intermediate gauginos:

$$\mu\frac{\partial}{\partial\mu}m_i^2 = -\frac{1}{2\pi^2}\sum_a g_a^2|M_a|^2c_{ai} \tag{3.30}$$

where c_{ai} denotes the appropriate Casimir (1/2 for particles in the fundamental representation) while gaugino masses satisfy:

$$\mu\frac{\partial}{\partial\mu}\left(\frac{M_a^2}{g_a^2}\right)=0. \tag{3.31}$$

It is straightforward to integrate these equations numerically. For a significant range of parameters, one does obtain suitable breaking of $SU(2)\times U(1)$.

3.4 Radiative Corrections to the Higgs Mass Limit

At tree level, the form of the Higgs potential is highly constrained. The quartic terms are exactly known. Once supersymmetry is broken, however, there can be corrections to the quartic terms from radiative corrections.[b] These corrections are soft, in that the susy-violating four point functions vanish rapidly at momenta above the supersymmetry breaking scale. Still, they are important in determining the low energy properties of the theory, such as the Higgs vev's and the spectrum.

The largest effect of this kind comes from loops involving top quarks [7]. It is not hard to get a rough estimate of the effect. Consider the diagrams of fig. 5, and suppose that $\widetilde{m}_t \gg m_t$. In this limit, we can omit the top squark

[b] In sufficiently complicated models, there can be tree level corrections to the quartic couplings. This does not occur in the MSSM, but it can occur in models with singlets.

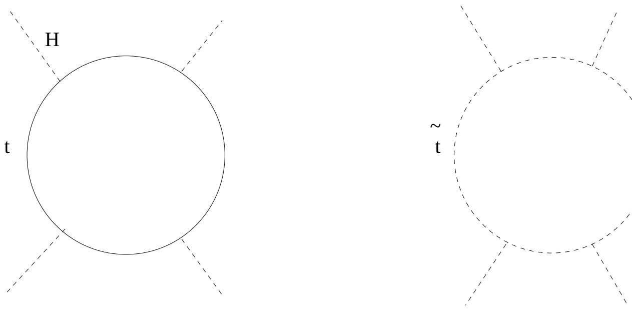

Figure 5: Corrections to quartic Higgs couplings from top loops.

from the computation. The result will be logarithmically divergent, and we can take the cutoff to be $\widetilde{m}_t$. So we have

$$\delta\lambda = (-1)y_t^4 \times 3 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr} \frac{1}{(\not{k} - m_t)^4} \tag{3.32}$$

$$= -\frac{12iy_t^4}{16\pi^2} \ln(\widetilde{m}_t^2/m_t^2). \tag{3.33}$$

The implications of this result are left for the following exercise.

Exercise: Verify these formulae. Evaluate y_t in terms of m_t and $\sin(\beta)$. Show that to this level of accuracy,

$$m_h^2 < m_Z^2 \cos(2\beta) + \frac{12g^2}{16\pi^2} \frac{m_t^4}{m_W^2} \ln(\widetilde{m}^2 m_t^2).$$

Check that you are in rough agreement with the numerical results in ref. [7].

3.5 Constraints on Soft Breakings

While we have stressed that there is a large number of soft breaking parameters, there are also many experimental constraints. These come from the failure of direct searches to see superpartners of ordinary fields, and also from indirect effects.

The direct searches are easy to describe, and production and decay rates can be computed given knowledge of the spectrum, since the couplings of the fields are known. If R parity is conserved, the LSP is stable and weakly interacting, so the characteristic signal for supersymmetry is *missing energy.* For example, in e^+e^- colliders, one can produce slepton pairs, if they are light

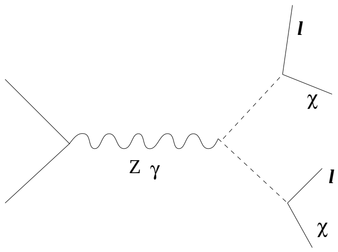

Figure 6: Slepton production in $e^+ \; e^-$ annihilation.

enough, through the diagram of fig. 6. These then decay, typically, to a lepton and a neutralino, as indicated. So the final state contains a pair of acoplanar leptons, and missing energy. From such processes, one has limits of order 45 GeV for all of the charged states of the MSSM. LEPII will eventually raise most of these limits to a range of order 95 GeV. Chargino masses are limited in this way to about 65 GeV; early reports from this winter's run indicate that the chargino limits are approaching 85 GeV.

One can obtain stronger limits – and in many cases greater discovery potential – from hadron machines. For example, because they are strongly coupled and they are octets of color, gluinos have very substantial production cross sections in hadron collisions. They can be produced both by $q\bar{q}$ and gg annihilation. Gluinos can decay to a large number of channels, and many of these are used in setting limits on the gluino mass. These limits range from 150 to 225 GeV, depending on the model assumptions (e.g., R-parity violating or not, masses of squarks) used in the analysis. Similar limits apply to squarks. Limits on charginos are similar to those currently set by LEP.

Rare processes provide another set of strong constraints on the soft breaking parameters. In the simple ansatz all of the scalar masses are the same at some very high energy scale. However, if this is true at one scale, it is not true at all scales, i.e., these relations are *renormalized.* Indeed, all 105 parameters are truly parameters, and it is not obvious that the assumptions of universality and proportionality are *natural.* On the other hand, there are strong experimental constraints which suggest some degree of degeneracy.

As one example, there is no reason, a priori, why the mass matrix for the $\tilde{L}$'s (the partners of the lepton doublets) should be diagonal in the same basis as the charged leptons. If it is not, there is no conservation of separate lepton numbers, and the decay $\mu \rightarrow e\gamma$ will occur (fig. 7). To see that we are potentially in serious trouble, we can make a crude estimate. If we suppose that the characteristic scale of the supersymmetric particles is of order m_W,

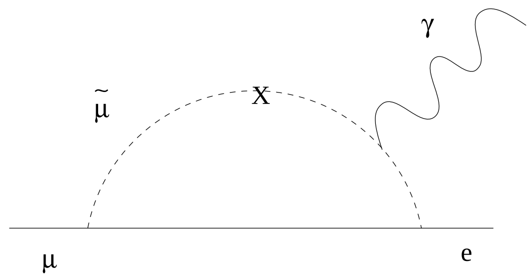

Figure 7: Contribution to $\mu \to e\gamma$.

then the branching ratio goes as

$$BR \sim \left(\frac{\alpha_2}{4\pi}\right)^2 \left(\frac{\delta \widetilde{m}_L^2}{m_{\rm susy}^2}\right)^2 \tag{3.34}$$

where $m_{\rm susy}^2$ denotes a typical susy-breaking mass scale, and $\delta \widetilde{m}_L^2$ denotes some off-diagonal term in the mass matrix. This branching ratio is about 10^{-4} if the off diagonal terms are large, so we are off by about 6 orders of magnitude. in this case. Things get better as the susy-breaking scale grows as $m_{\rm susy}^2$, but even if this scale is 500 GeV, we have to explain a dimensionless number of order 10^{-2}. With the assumption that m_L^2 is proportional to the unit matrix at some large scale, the separate lepton numbers are exact at that scale, and, of course, there is no problem.

Another troublesome constraint arises from the neutron electric dipole moment, d_n. Any non-zero value of this quantity signifies CP-violation. Currently, one has $d_n \leq 10^{-25} e$ cm. The soft breaking terms in the MSSM contain many new sources of CP-violation. Even with the assumptions of universality and proportionality, the gaugino mass and the A, μ and B parameters all are complex, and can violate CP. At the quark level, the issue is that one loop diagrams can generate a quark dipole moment, as in fig. 8. Note that this particular diagram is proportional to the phases of the gluino and the A parameter. It is easy to see that even if $m_{\rm susy} \sim 500$GeV, these phases must be smaller than about 10^{-2}. More detailed estimates can be found in [10].

Exercise: Make this estimate.

CP is violated in the real world, so it is puzzling that all of the soft supersymmetry violating terms should preserve CP to such a high degree. In fact, it is usually said that one of the triumphs of the minimal standard model is that it explains the observed CP violation with a CP-violating phase of order

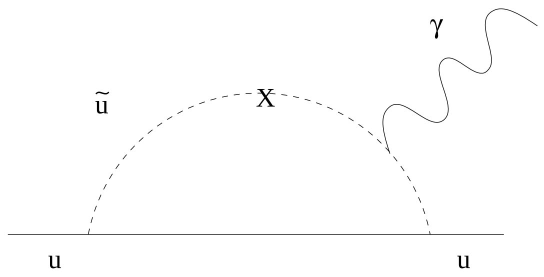

Figure 8: Contribution to d_n in supersymmetric theories.

one. It is thus a serious challenge to understand why CP should be such a good symmetry if nature is supersymmetric. Some possible explanations will be considered later in this lecture.

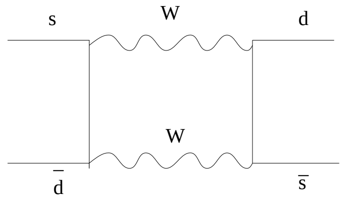

Figure 9: Contribution to $K \leftrightarrow \overline{K}$ in the standard model.

So far, we have discussed constraints on the slepton degeneracy and CP-violating phases. There are also constraints on the squark masses arising from various flavor violating processes. In the standard model, the most famous of these are strangeness changing processes, such as $K\overline{K}$ mixing. One of the early triumphs of the standard model was that it successfully explained why this mixing is so small. Indeed, the standard model gives a quite good estimate for the mixing. This was originally used to predict – amazingly accurately – the charmed quark mass [11].[c] The mixing recieves contributions from box diagrams such as the one shown in fig. 9. If we consider first, only the first two generations and ignore the quark masses (compared to M_W), we have that

$$\mathcal{M}(K^o \to \overline{K}^o) \propto (V_{di}V^\dagger_{is})(V^\dagger_{sj}V_{jd}) = 0. \tag{3.35}$$

[c] The computation of K–$\overline{K}$ mixing is discussed in many texts and reviews. See, for example, [12,13].

Including fermion masses leads to terms in $\mathcal{L}_{eff}$ of order

$$\frac{\alpha_W}{4\pi} \frac{m_c^2}{M_W^2} G_F \ln(m_c^2/m_u^2)(\bar{s}\gamma^\mu\gamma_5 d)(\bar{d}\gamma^\mu\gamma^5 s) + \ldots \tag{3.36}$$

The matrix element of the operator appearing here can be estimated in various ways, and one finds that this expression roughly saturates the observed value. Similarly, the CP-violating part (the "ϵ" parameter) is in rough accord with observation, for reasonable values of the KM parameter δ.

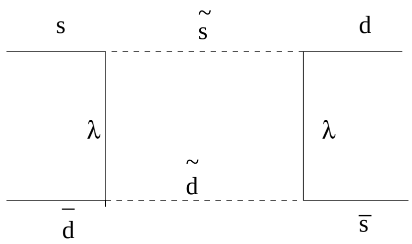

Figure 10: Gluino exchange contribution to kaon mixing in the MSSM.

In supersymmetric theories, if squarks are degenerate, there are similar cancellations. However, if they are not, there are new, very dangerous contributions. The most serious is that indicated in fig. 10, arising from exchange of gluinos and squarks. This is nominally larger than the standard model contribution by a factor of $(\alpha_s/\alpha_W)^2 \approx 10$. Also, the standard model contribution vanishes in the chiral limit, whereas the gluino exchange does not, and this leads to an addtional enhancement of nearly an order of magnitude. On the other hand, the diagram is highly suppressed in the limit of exact universality and proportionality. Proportionality means that the A terms in eq. (3.11) are suppressed by factors of light quark masses, while universality means that the squark propagator, $< \tilde{q}^* \tilde{q} >$, is proportional to the unit matrix in flavor space. So there are no appreciable off-diagonal terms which can contribute to the diagram. On the other hand, there is surely some degree of non-degeneracy. One finds that even if the characteristic susy scale is 500 GeV, one needs degeneracy in the down squark sector at the part in 10^2 level. More generally, it vanishes in the limit that the squark mass matrix is diagonal in the same basis as the quark mass matrix [14].

So K–$\overline{K}$ mixing tightly constrains the down squark mass matrix. The imaginary part provides further constraints. There are also strong limits on D–$\overline{D}$ mixing, which significantly restrict the mass matrix in the up squark sector. Other important constraints on soft breakings come from other rare processes. The current situation is carefully surveyed in ref. [15].

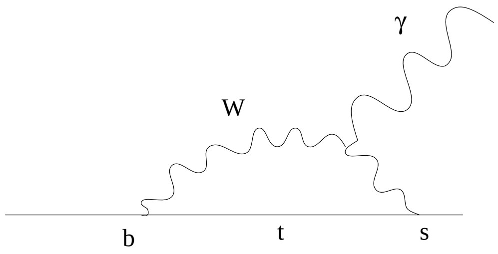

Figure 11: Standard model contribution to $b \to s + \gamma$.

Violations of universality might be expected to be largest in the third generation, so a quite powerful set of constraints comes from the process $b \to s\gamma$. This has been measured by CLEO (see Drell's talk at this school). One finds

$$BR(B \to s\gamma) = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4}. \tag{3.37}$$

This is quite consistent with the standard model prediction, which arises from the W loop in fig. 11:

$$BR(B \to s\gamma) = (3.25 \pm 0.30 \pm 0.40) \times 10^{-4}. \tag{3.38}$$

In the MSSM – and for that matter, in any theory with two Higgs doublets – there is at least one additional contribution, coming from a loop with the charged W replaced by a charged Higgs [16]. Unless there are cancellations, the mass of the charged Higgs must be greater than about 300 GeV. This is troubling. After all, we would like the Higgs mass parameters to be numbers comparable to M_Z. One might worry that such massive Higgs imply a fine tuning of parameters at the 10% level or worse. Whether this is acceptable or not, only time will tell. In many models, it turns out that the charged Higgs must be quite massive for other reasons as well.

Under some circumstances, there *are* additional, negative corrections to the rate which can ameliorate this problem. The principal new contributions in the MSSM are diagrams with stops and charginos in the loop (fig. 12.) [17]. These are unimportant unless these particles are relatively light. In models with exact degeneracy at the high scale, such light stops are not implausible in view of the renormalization group equations for the stop mass, which tend to lower the stop mass. So this is perhaps some support for the notion of high

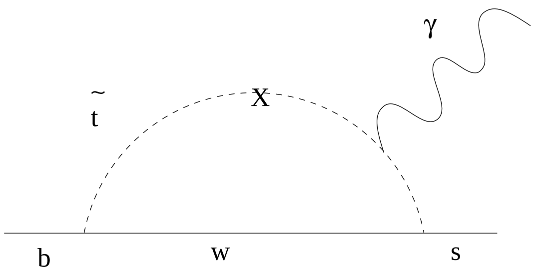

Figure 12: Additional contributions to $b \to s + \gamma$ in the MSSM.

scale universality (though as noted before, detailed phenomenological studies still often produce large values for the charged Higgs mass).

It has been suggested that light stops and charginos might be of interest for another reason. At the time that these lectures were presented, there was a substantial discrepancy between the standard model value of

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to X)}. \tag{3.39}$$

This discrepancy, averaging data from LEP and SLAC (SLD) was about 3.4σ. Since then, the ALEPH experiment has performed a further analysis, obtaining a result in good agreement with the standard model predictionThe world average now differs from the standard model by about 2σ. My own prejudice, which is widely shared, is that this discrepancy is not real. However, this is merely prejudice, and it is interesting to note that the MSSM with relatively light stops could account for most or all of this discrepancy [18]. This is a situation which bears further watching.

3.6 The Rarest Process: Proton Decay

We can summarize our discussion of rare processes up to now in the language of effective actions. One can think of the action of the standard model as an effective action, obtained by integrating out some as yet unknown physics at energy scales M above the W and Z masses. One of the virtues of this model is that several classes of rare processes are automatically suppressed by factors of $1/M$. So as long as M is large enough, the model is compatible with experiment. The problem in supersymmetry (and in any other framework, like technicolor, which seeks to explain the scale of weak interactions) is that M can't be much larger than M_Z. This means that other suppression mechanisms, such as approximate or exact symmetries, must be found. In the case of supersymmetry, reasonably simple mechanisms do exist in each case. Whether nature takes advantage of them is another matter, of course.

We have already mentioned the most dramatic of these constraints: proton decay. At the level of the supersymmetric effective action, the gauge symmetry permits dimension four terms in the superpotential which violate B and L. If one integrates out the squarks and sleptons, these generate various B and L violating terms in the standard model with coefficients of order $1/m_s^2$. So clearly these terms must be highly suppressed. Up to now, we have assumed that this suppression was achieved through R parity. However, it is not really necessary to eliminate every B and L violating operator in order to insure proton stability. For example, if L is violated, but not B, the proton will be stable. There are many other constraints which must be considered, such as $n - \bar{n}$ oscillations, and flavor changing processes [19]. Still, it is possible that some of these couplings are not too small. This would significantly alter the problem of susy detection. Most importantly, the LSP would not be stable, so missing energy would not necessarily be an important signal. One can easily imagine that in string theory one might have some more intricate discrete symmetry than the conventional R parity, which forbids some but not all of the R-parity violating operators.

Exercise: Show how various B-violating four fermi operators are generated by squark and slepton exchange, starting with the general set of B and L violating terms in the superpotential.

Finally even in models with R parity, the MSSM possesses B and L-violating dimension five operators which are permitted by all symmetries [20]. For example, R-parity doesn't forbid such operators as

$$\mathcal{O}_5^a = \frac{1}{M}\int d^2\theta \bar{u}\bar{u}\bar{d}e^+ \qquad \mathcal{O}_5^b = \frac{1}{M}\int d^2\theta QQQL. \tag{3.40}$$

These are still potentially very dangerous. When one integrates out the squarks and gauginos, they will lead to dimension six B and L-violating operators in the standard model with coefficients (optimistically) of order

$$\frac{\alpha}{4\pi}\frac{1}{M m_{\rm susy}}. \tag{3.41}$$

Comparing with the usual minimal $SU(5)$ prediction, and supposing that $M \sim 10^{16}$ GeV, one sees that one needs a suppression of order 10^9 or so.

Fortunately, such a suppression is quite plausible, at least in the framework of supersymmetric guts [21]. In a simple $SU(5)$ model, for example, the operators of eq. (3.40) will be generated by exchange of the color triplet partners of ordinary Higgs fields, and thus one gets two factors of Yukawa couplings. Also, in order that the operators be $SU(3)$-invariant, the color indices must be completely antisymmetrized, so more than one generation must be involved. This suggests further suppression by factors of order Cabibo angles. These numbers are typically of order $10^{-9} - 10^{-11}$. More detailed studies of this question can be found in the literature, and proton decay can be used to restrict the parameter space of particular models. But what is quite striking is that we are automatically in the right range to be compatible with experimental constraints, and perhaps even to see something. It is not obvious that things had to be this way.

So far we have phrased this discussion in terms of baryon-violating physics at M_{GUT}. But whatever the underlying theory at M_p may be, there is no reason to think that it should preserve baryon number. So one expects that already at scales just below M_p, these terms are present. This would certainly be the case in string theory. If their coefficients were simply of order $1/M_p$, the proton decay rate would be enormous. Is there any reason to expect further suppression? I believe the answer is yes. After all, the rate from Higgs exchange in GUT's is so small because the Yukawa couplings are small. We do not really know why Yukawa couplings are small, but it is natural to suspect that this is a consequence of (approximate) symmetries. These same symmetries, if present would also suppress dimension five operators from Planck scale sources, presumably by a comparable amount.

4 The Origin of Supersymmetry Breaking

4.1 Simple Models of Supersymmetry Breakdown

So far, we have treated supersymmetry as if it is explicitly broken. However, we have argued earlier that supersymmetry must be an exact symmetry which is spontaneously broken. Fortunately, this is not incompatible with the phenomenology we have done up to now. We have seen that soft breakings of the desired type can arise in a theory with spontaneous breaking, through operators in the effective lagrangian like

$$\frac{1}{M^2}\int d^4\theta Z^\dagger Z Q^\dagger Q \approx \frac{|\langle F_Z\rangle|^2}{M^2}\tilde{Q}^\dagger\tilde{Q} + \dots . \tag{4.1}$$

There are clearly a number of reasons to investigate the question of supersymmetry breaking. First, we have seen that without ad hoc assumptions, the MSSM has a huge number of free parameters. A theory of supersymmetry breaking might make predictions for these quantities. Another reason concerns the hierarchy problem. We have discussed the fact that supersymmetry can eliminate the problem of quadratic divergences. But supersymmetry also has the potential to *explain* the hierarchy [22]. In particular, if supersymmetry is unbroken to lowest order in perturbation theory in some theory, it is unbroken to all orders (with one possible exception, which we will discuss later). This means that supersymmetry breaking, if it does occur, must be smaller than any power of the coupling, e.g.,

$$m_{\mathrm{susy}} = e^{-\frac{8\pi^2}{Ng^2}}. \tag{4.2}$$

It has been known for some time that dynamical breaking can occur in four dimensions [23].

Before turning to dynamical supersymmetry breaking, it is instructive to consider models with supersymmetry breaking at tree level. We have seen that supersymmetry breaking is signalled by a non-zero expectation value of an F component of a chiral or D component of a vector superfield. Models involving only chiral fields with no supersymmetric ground state are called to as "O'Raifeartaigh" models. A simple example has three singlet fields, A, B, and X, with superspotential:

$$W = \lambda_1 A(X^2 - \mu^2) + \lambda_2 B X^2. \tag{4.3}$$

With this superpotential, the equations

$$F_A = \frac{\partial W}{\partial A} = 0 \qquad F_B = \frac{\partial W}{\partial B} = 0 \tag{4.4}$$

are incompatible. To actually determine the expectation values and the vacuum energy, it is necessary to minimize the potential. I'll leave this as an exercise. Note, however, that at this level not all of the fields are fully determined, since the equation

$$\frac{\partial W}{\partial X} = 0 \tag{4.5}$$

can be satisfied provided

$$\lambda_1 A + \lambda_2 B = 0. \tag{4.6}$$

This vacuum degeneracy is accidental, and as we will later see, is lifted by quantum corrections.

It is also possible to generate an expectation value for a D term. In the case of a $U(1)$ gauge symmetry a term

$$\mu^2 \int d^4\theta \; V = \mu^2 D \tag{4.7}$$

is gauge invariant. This is known as a "Fayet-Iliopoulos D term" and can lead to supersymmetry breaking. For example, if one has two charged fields, $\Phi^{\pm}$, with charges ± 1, and superpotential $m\Phi^+\Phi^-$, one cannot simultaneously make the two auxiliary F fields and the auxiliary D field vanish.

Exercise: Study the potentials in both models and verify these statements.

One important feature of both types of models is that at tree level, in the context of global supersymmetry, the spectra are never realistic. These spectra satisfy a sum rule,

$$\sum(-1)^F m^2 = 0. \tag{4.8}$$

Here $(-1)^F = 1$ for bosons and -1 for fermions. This guarantees that there are always light states, and often color and/or electromagnetism are broken. These statements are not true of radiative corrections, and of supergravity, as we will explain later.

It is instructive to prove this sum rule. Consider a theory of chiral fields only (no gauge interactions). The potential is given by

$$V = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2. \tag{4.9}$$

The boson mass matrix has terms of the form $\phi_i^* \phi_j$ and $\phi_i \phi_j$ + c.c. The latter terms, as we will now see, are connected with supersymmetry breaking. The various terms in the mass matrix can be obtained by differentiating the potential:

$$m^2_{i\bar{j}} = \frac{\partial^2 V}{\partial \phi_i \partial \phi^*_{\bar{j}}} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_k} \frac{\partial^2 W^*}{\partial \phi^*_{\bar{k}} \partial \phi^*_{\bar{j}}}, \tag{4.10}$$

$$m^2_{ij} = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} = \frac{\partial W}{\partial \phi^*_k} \frac{\partial^3 W}{\partial \phi_k \partial \phi_i \partial \phi_j}. \tag{4.11}$$

The first of theses terms has just the structure of the square of the fermion mass matrix,

$$\mathcal{M}_{\mathcal{F}ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}. \tag{4.12}$$

So writing the boson mass, $\mathcal{M}^2_B$ matrix on the basis $(\phi_i \;\; \phi^*_{\bar{j}})$, we see that eq. (4.8) holds.

The theorem is true whenever a theory can be described by a renormaliable effective action. We have seen that various non-renormalizable terms in the effective action can give additional contributions to the mass, and with a little thought it is clear that these will violate the tree level sum rule. Such terms arise in renormalizable theories when one integrates out heavy fields to obtain an effective action at some scale. In the context of supergravity, such terms are present already at tree level. This is perhaps not suprising, given that these theories are non-renormalizable and must be viewed as effective theories from the very beginning (perhaps the effective low energy description of string theory). Shortly, we will discuss the construction of realistic models. First, however, we turn to the issue of non-renormalization theorems and dynamical supersymmetry breaking.

4.2 Non-Renormalization Theorems

Non-supersymmetric theories have the property that they tend to be generic; any term permitted by symmetries in the theory will appear in the effective action, with an order of magnitude determined by dimensional analysis.[d] Supersymmetric theories are special in that this is not the case. This figures heavily in the duality story, as you have heard at this school. In $N = 1$ theories, there are non-renormalization theorems governing the superpotential and the gauge coupling functions, f, of eq. (2.10). These theorems assert that the superpotential is not renormalized beyond its tree level value, while f is at most renormalized at one loop [24].[e]

Originally, these theorems were proven by detailed study of Feynman diagrams [24]. Seiberg has pointed out that they can be understood in a much simpler way [26]. Both the superpotential and the functions f are holomorphic functions of the chiral fields, i.e., they are functions of the ϕ_i's and not the ϕ_i^*'s. This is evident from their construction. But the coupling constants of a theory may also be thought of as *expectation values* of chiral fields. For example, consider a theory of a single chiral field, Φ, with superpotential

$$W = \int d^2\theta(m\Phi^2 + \lambda\Phi^3). \tag{4.13}$$

We can think of λ and m as expectation values of chiral fields, $\lambda(x, \theta)$ and $m(x, \theta)$.

Exercise: One can make this more concrete, for example, by treating these as massive fields. Replace the term $m\Phi^2$ with $M(\frac{1}{2}S^2 - Sm + S\Phi^2)$. Check that

[d] Possibly up to a few powers of coupling.

[e] There is an important subtlety connected with these theorems. Both should be interpreted as applying only to a "Wilsonian" effective action, in which one integrates out physics above some scale, μ. If infrared physics is included, the theorems do not necessarily hold. This is particularly important for the gauge couplings. In ref. [25], the connection of the conventional β-function and the Wilsonian one is explained in some detail.

S has mass M while Φ has mass m. If M is large, it makes sense to think of the expectation value as frozen.

The lagrangian with λ and m has certain symmetries. In particular, if we first set λ to zero, it has an R symmetry under which Φ has R charge one. λ has R charge -1 under this symmetry. Now consider corrections to the effective action. For example, renormalizations of λ in the superpotential necessarily involve positive powers of λ. But such terms (apart from λ^1) have the wrong R charge to preserve the symmetry. So there can be no renormalization of this coupling. There can be wave function renoralization, since the symmetries allow the Kahler potential to depend on λ, $K = K(\lambda^\dagger \lambda)$.

There are many interesting generalizations of these ideas, and I won't survey them here, but I will mention two further examples. First, gauge couplings can clearly be thought of in the same way, i.e., we can think of g^{-2} as a chiral field. The real part of the scalar field in this multiplet couples to $F_{\mu\nu}^2$, but the imaginary part, a, couples to $F\tilde{F}$. $F\tilde{F}$ is a total derivative, and in perturbation theory there is a symmetry under constant shifts of a. But this means that the effective action should respect this symmetry. Because the gauge coupling function, f, is holomorphic, this implies that

$$f(g^2) = \tfrac{1}{4} g^{-2} + \text{const} \ . \tag{4.14}$$

The first term is just the tree level term. The constant term corresponds to one loop corrections. There are no higher order corrections in perturbation theory! This is quite a striking result. It is also paradoxical, since the two loop β-functions for supersymmetric Yang-Mills theories have been computed long ago, and are in general non-zero.

Before explaining the resolution of this paradox, there is one more non-renormalization theorem which we can prove rather trivially here [27]. This is the statement that if there is no Fayet-Iliopoulos D-term at tree level, this term can be generated at most at one loop. To prove this, write the D term as

$$\int d^4\theta d(g, \lambda) V. \tag{4.15}$$

Here $d(g, \lambda)$ is some unknown function of the gauge and other couplings in the theory. But if we think of g and λ as chiral fields, this expression is only gauge invariant if d is a constant, corresponding to a possible one loop contribution. Such contributions do arise in string theory.

In string theory, all of the parameters *are* expectation values of chiral fields. Indeed, non-renormalization theorems in string theory, both for world sheet[28] and string perturbation theory[29], were proved by the sort of reasoning we have used above, long ago.

Returning to the paradox we raised above, there is an important subtlety which must be discussed here. In textbooks, one often sees discussion of something called the effective action, which is defined to be the sum of one particle irreducible graphs. The effective action we are describing here is something different – and significantly more meaningful – called the Wilsonian action. This is defined as the action obtained by integrating out physics above some energy scale. So, for example, it includes one particle reducible diagrams containing massive fields, but it does not include the low energy parts of loop graphs containing light fields. Diagrams of the first type give effective, local interactions – an example is W and Z exchange at low energies in the standard model, which give rise to the four fermi interaction. Diagrams of the second type give non-local interactions. Since the non-renormalization theorems certainly rely on locality, they need only hold for the Wilsonian action. The paradox of the two loop renormalization of the gauge coupling is resolved in just this way; there *is no* renormalization beyond one loop for the Wilsonian action [25]. Unfortunately, we do not have a regulator for supersymmetric theories analogous to lattice regulators in (non-chiral) gauge theories, so it is sometimes difficult to make this discussion concrete. This problem will arise later, when we will want to discuss the problem of unification in theories which are strongly coupled.

4.3 Examples of Dynamical Supersymmetry Breaking

In speaking of dynamical supersymmetry breaking, there are two classes of models which one must consider.

1. Models with moduli (e.g., generic string vacuua). In such theories, there is a continuous set of degenerate vacuum states, at the classical level, and the question is: what effects may lift the degeneracy? We will see that the degeneracy *is* often lifted, but that there is usually no vacuum state at weak coupling.

2. Models without flat directions. In such theories, the generic behavior is that supersymmetry is unbroken, but under special circumstances, supersymmetry is broken.

These points can be illustrated by a theory known as supersymmetric QCD. By this I mean a theory with $SU(N)$ gauge group, with N_f flavors of "quarks." If the quarks are all massless, there is a large classical vacuum degeneracy. There are three distinct cases to consider:

1. $N_f > N_c$: The moduli space is exact quantum mechically.

2. $N_f = N_c$: There is still an exact moduli space, but the quantum moduli space is different than the classical moduli space.

3. $N_f < N_c$: Non-perturbatively a superpotential is generated in the effective theory describing the moduli, which lifts the flat directions. However, in the regime where one can do reliable calculations, the system has no ground state.

These statements can be understood, almost completely, on symmetry grounds. If there is no (classical) superpotential for the quark and antiquark fields, Q and $\overline{Q}$, the symmetry of the model at the classical level is: $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_A \times U(1)'_R$. $U(1)_B$ is just baryon number; $U(1)_A$ is the analog of the familiar axial $U(1)$ of QCD. Quantum mechanically, a linear combination of $U(1)_A$ and $U(1)'_R$ is unbroken by anomalies. If we think of the gauge coupling as a parameter, the anomaly in $U(1)'_R$ can be cancelled by a shift of the field a. Denote the quarks by Q and $\overline{Q}$. Then under $SU(N_f)_L \times SU(N_f)_R \times U(1) \times U(1)_R \times U(1)'_R$, the charges of these fields are:

$$Q = \left(N, 1, \frac{N - N_f}{N_f}, 0\right) \qquad \overline{Q} = \left(N, -1, \frac{N - N_f}{N_f}, 0\right). \tag{4.16}$$

It is convenient to describe the rotation of the gauge coupling in terms of the Λ parameter of the theory, $\Lambda = e^{-\frac{8\pi^2}{b_o g^2}}$. Λ has charges

$$\Lambda = (0, 0, 0, 0, \frac{2(N - N_f)}{3N - N_f}). \tag{4.17}$$

Exercise: Verify that symmetries $U(1)_R$ and $U(1)'_R$ are non-anomalous. It is necessary to carefully work out the coupling of a to $F\tilde{F}$, and to check that under suitable shifts of a, the shift in this term precisely cancels the usual anomaly from fermion loops.

The classical moduli spaces of these models are easily understood. The scalar potential arises just from the D^2 terms. We can define matrix-valued fields,

$$D_i^{\ j} = Q_i^* Q^j - \overline{Q}_i \overline{Q}^{*j} - \text{tr terms}. \tag{4.18}$$

These vanish for

$$Q = \overline{Q} = \text{diag}(a_1, \ldots, a_{N_f}). \tag{4.19}$$

For $N_f < N_c$, this is the most general solution up to symmetry transformations. For $N_f \geq N_c$, there are more general solutions. For example, one can have

$$Q = \text{diag}(v, v, v \ldots, v) \qquad \widetilde{Q} = \text{diag}(v', v'v' \ldots v'). \tag{4.20}$$

It is natural to try to parameterize these flat directions in a gauge-invariant way. We can define a set of "meson" operators,

$$M_a^b = \overline{Q}_a Q^b, \tag{4.21}$$

for any value of N_f; a and b are flavor indices. For $N_f \geq N_c$, we can also define baryon and antibaryon operators,

$$B = Q^{a_1} \ldots Q^{a_N} \qquad \bar{B} = \overline{Q}^{a_1} \ldots \overline{Q}^{a_N}. \tag{4.22}$$

Note that the number of such operators depends on N_f. If $N_f = N_c$, there is one baryon and one antibaryon, if $N_f = N_c + 1$, there are N_f baryons and N_f antibaryons, and so on.

For the case $N_f = N_c - 1$, let's check that the M's account for all of the light degrees of freedom. For a generic point in the flat directions, the gauge group is broken to $SU(N - N_f)$ (where $SU(1)$ is understood to be trivial). So the $N^2 - 1$ gauge fields gain mass. When a gauge field gains mass, it "eats" an entire chiral multiplet. So of the original $2 \times N \times N_f = N^2 - N$ chiral fields, N_f^2 remain massless. There is precisely the number of M's. You can check that this counting works for the other cases.

Exercise:

1. Verify that the D term has the form of eq. (4.18), and that for $N_f < N_c$, this is the most general solution.

2. Check that the meson and baryon superfields account for all of the light fields in the general flat direction for $N_f \geq N$.

Let's continue to focus on the case that $N_f \geq N - 1$. If the expectation values are large compared to the scale of the theory, Λ, all of the gauge bosons have masses large compared to Λ. The theory is then weakly coupled, and a semiclassical analysis should be valid. The low energy theory consists only of the fields M and B. The effective action which describes these light fields should be supersymmetric. If supersymmetry is broken, this should be understood as *spontaneous* breaking in the low energy theory. This can only occur if the effective theory contains a superpotential or a Fayet-Iliopoulos term, but the latter possibility is ruled out by our non-renormalization theorem (note that this theorem was non-perturbative). The form of any possible superpotential, however, is greatly restricted by the symmetries. In order that the action be invariant under $SU(N_f) \times SU(N_f)$, the superpotential must be a function of $\det M$. This determinant, however, vanishes in the flat direction if

$N_f > N$, and supersymmetry cannot be broken. If $N_f < N$, a superpotential is allowed, and the symmetries uniquely determine its form. In particular, noting that W must have R charge two, and the R charges of the fields in eqs. (4.16) and (4.17), the superpotential must be

$$W = \left(\det \overline{Q} Q\right)^{\frac{-1}{N-N_f}} \Lambda^{\frac{3N-N_f}{N-N_f}}. \tag{4.23}$$

This superpotential, in fact, respects all of the symmetries for any value of $N_f < N$. Note that it also respects $U(1)'_R$.

For $N_f = N - 1$, we have noted that the theory is weakly coupled, and that a semiclassical analysis should be valid. The superpotential cannot be generated in perturbation theory. In perturbation theory, there is an additional $U(1)$ symmetry which forbids a superpotential all together – this is the content of the standard non-renormalization theorem. Alternatively (and equivalently) we note that in perturbation theory only logs of Λ appear, not powers. In weakly coupled theories, the only non-perturbative effects which we understand are instantons. Without actually doing a computation, one can see that the instanton amplitude potentially has the correct form. Instanton effects go as

$$e^{-\frac{8\pi^2}{g^2(v)}} \tag{4.24}$$

where $g^2(v)$ is the coupling at the scale v. In the present case, noting that the leading term in the β function goes as $2N + 1$,

$$e^{-\frac{8\pi^2}{g^2(v)}} = (\Lambda/v)^{2N-1} \tag{4.25}$$

which gives precisely the expected dependence on Λ. I won't go through the details of the instanton computation here. They are reasonably straightforward [30]. Suffice it to say that in the end, one obtains a contribution to the effective action of the expected form, with a non-zero coefficient.

For $N_f < N_c - 1$, the gauge group is not completely broken, even in the most general flat direction. Instead, one is left with the mesons, M, and an unbroken gauge group, $SU(N - N_f)$, with no matter fields (i.e., a "pure supersymmetric gauge theory"). The gauge theory by itself presumably confines and has a mass gap, so our goal, again, is to obtain the effective theory of the light fields M. The superpotential expected from eq. (4.23) is again generated, now as a result of gluino condensation in the $SU(N - N_f)$ theory. To understand how this works, it is simplest to consider first the simplest case, $SU(3)$ with a single flavor. Then in the general flat direction,

$$Q = \overline{Q} = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}. \tag{4.26}$$

The low energy theory is an $SU(2)$ gauge theory with an additional, neutral singlet field, $M = \overline{Q}Q$. The effective action can be organized into terms with higher and higher powers of $1/v$. In leading order, one has a free, decoupled chiral multiplet and a pure $SU(2)$ gauge theory. General arguments indicate that the $SU(2)$ theory does not break supersymmetry, and posesses a mass gap [22,31].

The lowest order term which couples the two sectors involves the f function of eq. (2.10). In the present case, the form of this function can be determined by symmetry considerations. The microscopic theory possesses an R symmetry, under which

$$\lambda \rightarrow e^{i\alpha\lambda} \qquad Q, \overline{Q} \rightarrow e^{-2i\alpha} Q\overline{Q}. \tag{4.27}$$

The low energy theory contains only the gluinos and this symmetry now seems to be anomalous. The anomaly, however, will be cancelled if the theory includes a coupling

$$\frac{1}{16\pi^2}\int d^2\theta \ln(\overline{Q}Q) W_\alpha^2 = \frac{1}{16\pi^2}\int d^2\theta \ln(M) W_\alpha^2. \tag{4.28}$$

A simple one loop calculation verifies that this coupling is indeed present.

In the pure gauge theory, there is a non-zero gaugino condensate,

$$\langle\lambda\lambda\rangle = \Lambda_{SU(2)}^3. \tag{4.29}$$

Here, $\Lambda_{SU(2)}$ is the scale of the low energy $SU(2)$ theory; it is related to the original $SU(3)$ scale by

$$\Lambda_{\mathrm{su}(2)}^3 = v^3 e^{-\frac{8\pi^2}{b_o' g^2(v)}}, \tag{4.30}$$

with $b_o' = 6$, the low energy β function, and

$$\frac{8\pi^2}{g^2(v)} = \frac{8\pi^2}{g^2(M)} + b_o \ln(v/M), \tag{4.31}$$

where $b_o = 5$ is the microscopic β function.

Examining eq. (4.28), it is clear that the gluino condensate gives rise to a superpotential for M. It is not hard to check that this has precisely the correct form (it is easiest to do this in terms of the component fields, because of the dependence of the condensate on the fields).

So we have seen two possible behaviors for theories which classically have moduli. Either no potential is generated, and the moduli are exact even quantum mechanically. Alternatively, a potential is generated, but it falls to zero in the region where the coupling constants tend to zero. These results are not suprising. Later, we will discuss an example in which a flat direction is lifted and there is a stable ground state. This occurs because the coupling grows as the field becomes large.

We turn now to theories without classical flat directions. In such cases, the generic behavior is that supersymmetry is unbroken. This was already implicit in our discussion of the pure $SU(N)$ gauge theory. Consider, now, supersymmetric QCD with masses for the quarks. In particular, suppose that the mass is very small. Then there are still approximate flat directions, and we expect that the superpotential is simply a sum of the tree level (mass) term and the non-perturbative term. One can, in fact, prove that this form is exact, using symmetries and holomorphy. So, for example, for $N_f = N - 1$,

$$W = \frac{\Lambda^{2N+1}}{\det \overline{Q}Q} + m\overline{Q}Q. \tag{4.32}$$

Now there are no flat directions, classically or quantum mechanically. To make things simple, we have taken all of the quark masses equal. We can look for a supersymmetric minimum by assuming that Q has the form

$$Q = \begin{pmatrix} v_1 & 0 & \dots & 0 \\ 0 & v_2 & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & v_{N_f} \\ 0 & 0 & \dots & 0 \end{pmatrix}. \tag{4.33}$$

Then the equation $\frac{\partial W}{\partial Q} = 0$ gives that

$$v^{2N} = \frac{\Lambda^{2N+1}}{m}, \tag{4.34}$$

i.e., there are N supersymmetry preserving roots.[f]

Exercise: Prove the existence of a gluino condensate in pure $SU(N)$ gauge theory by the following indirect argument [26]. Start with the case $N_f = N - 1$, in which a direct calculation leads to the superpotential expected from eq. (4.23). Add a mass term for one of the quarks, Q_{N_f}, and integrate it out, by solving the equation

$$\frac{\partial W}{\partial Q_{N_f}} = 0.$$

Substitute this solution back into the superpotential to obtain an effective superpotential for the remaining fields. By holomorphy, this expression is

[f] This is compatible with the computation of the Witten index [31].

guaranteed to be correct for large m_{N_f}. Verify that the result is that expected from eq. (4.23). Note, in addition to the correct dependence on fields, this also has the correct dependence on the scale Λ. This demonstrates the existence of the gluino condensate, since we have seen that such a condensate is required to explain directly the existence of the non-perturbative superpotential.

A model in which supersymmetry turns out to be broken is the "3–2 model." This theory has gauge symmetry $SU(3) \times SU(2)$, and matter content:

$$Q(3,2) \qquad \bar{u}(\bar{3},1) \qquad L(1,2) \qquad \bar{d}(\bar{3},1). \tag{4.35}$$

This is similar to the field content of a single generation of the standard model, without the extra $U(1)$ and the positron. The most general renormalizable superpotential consistent with the symmetries is

$$W = \lambda Q L \bar{u}. \tag{4.36}$$

This model admits an R symmetry which is free of anomalies. There is also a conventional $U(1)$ symmetry, under which the charges of the various fields are the same as in the standard model (one can gauge this symmetry if one also adds an e^+ field).

While this model has global symmetries, it is different from supersymmetric QCD in that it does not have classical flat directions. To see this, note that by $SU(3) \times SU(2)$ transformations, one can bring Q to the form

$$Q = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \\ 0 & 0 \end{pmatrix}. \tag{4.37}$$

Suppose v_1 and/or v_2 is non-vanishing. Then if $v_1 \neq v_2$, vanishing of the $SU(2)$ D-term requires that L is non-zero. This, however, implies that $\frac{\partial W}{\partial \bar{u}}$ is non-zero. So v_1 must equal v_2, and L must vanish. But now one cannot make the $SU(3)$ D-term vanish unless both $\bar{u}$ and $\bar{d}$ are non-vanishing, in which case $\frac{\partial W}{\partial L}$ is non-vanishing. So there are no flat directions in this model.

To analyze the dynamics of this theory, consider first the case that $\Lambda_3 \gg \Lambda_2$. Ignoring, at first, the superpotential term, this is just $SU(3)$ with two flavors. In the flat direction of the D terms, there is a non-perturbative superpotential,

$$W_{np} = \frac{\Lambda^5}{\det \overline{Q} Q} \sim \frac{1}{v^4}. \tag{4.38}$$

The full superpotential in the low energy theory is a sum of this term and the perturbative term. It is straightforward to minimize the potential, and establish that supersymmetry is broken. The case that $\Lambda_2 \gg \Lambda_3$ has been analyzed more recently by Intriligator and Thomas. In this case, before including the classical superpotential, the theory is $SU(2)$ with two flavors. This is an example of a model with a "quantum moduli space" (see Peskin's lectures at this school). Studying the effects of the classical superpotential on this space, one again finds that supersymmetry is broken.

Even without detailed dynamical analysis, we could have anticipated that supersymmetry would be broken in this model from the following argument. This model has no flat directions. It also possesses a non-anomalous global symmetry. One expects that this global symmetry is spontaneously broken. In the limit of large Λ_3, since the superpotential blows up at the origin, the minimum of the potential must lie away from the origin. In the limit of large Λ_2, the point of unbroken symmetry does not lie in the quantum moduli space. This means that the R symmetry is spontaneously broken. Correspondingly, there must be a Goldstone boson. If supersymmetry were *unbroken* at the minimum, this field would necessarily be part of a chiral multiplet. The other scalar in this multiplet, like the Goldstone particle, would have no potential, and thus the full non-perturbative theory would have a flat direction. But this is essentially impossible, given that all of the couplings are weak and the classical theory has no flat directions.

This argument is often useful in considering models which are inherently strongly coupled, in which it is difficult to determine whether supersymmetry is broken by direct calculation. An example of this kind is a model with gauge group $SU(5)$ and chiral fields in the 10 and $\bar{5}$ representations. I will leave it as an exercise to check that there are no flat directions, and that there are two non-anomalous $U(1)$ symmetries. While it is difficult to prove, one can at least advance good arguments that one or both of these chiral symmetries are broken, so again we expect that supersymmetry is broken. This model has an infinite set of generalizations: theories with gauge group $SU(N)$, a single antisymmetric tensor, and $(N-4)\overline{N}$'s. All of these models are believed to break supersymmetry.

Exercise: Check that these models have no flat directions, and that they have a non-anomalous $U(1)$ symmetry.

During the past year, many generalizations of these models have been constructed [32,34]. In addition, new mechanisms of supersymmetry breaking have been discovered [33,35,36]. As an example, consider a model with gauge group $SU(2)$ and four doublets, $Q_I, I = 1 \ldots 4$ (two "flavors") [33]. Classically, this model has a moduli space labeled by the expectation values of the fields $M_{IJ} = Q_I Q_J$. These satisfy $\mathrm{Pf}\langle M_{IJ}\rangle = 0$, but, as we noted in our discussion of the 3–2 model when the same structure arose, the quantum moduli space is different, and satisfies:

$$\mathrm{Pf}\langle \mathrm{M_{IJ}} \rangle = \Lambda^4. \tag{4.39}$$

Now add a set of singlets to the model, S_{IJ}, with superpotential couplings

$$W = \lambda_{IJ} S_{IJ} Q_I Q_J. \tag{4.40}$$

Unbroken supersymmetry now requires

$$\frac{\partial W}{\partial S_{IJ}} = Q_I Q_J = 0. \tag{4.41}$$

However, this is incompatible with the quantum constraint. So it would appear that supersymmetry is broken.

On the other hand, the model, classically, has flat directions in which $S_{IJ} = s_{IJ}$, and all of the other fields vanish. So one might worry that there is runaway behavior in these directions, similar to that we saw in supersymmetric QCD. However, for large s, it turns out that the energy grows at infinity [37]. This can be established as follows. Suppose all of the components of S are large, $S \sim s \gg \Lambda_2$. In this limit, the low energy theory is a pure $SU(2)$ gauge theory. In this theory, gluinos condense,

$$\langle \lambda\lambda \rangle = \Lambda_{LE}^3 = \lambda s \Lambda_2^2. \tag{4.42}$$

Here, Λ_{LE} is the Λ parameter of the low energy theory.

At this level, then, the superpotential of the model behaves as

$$W_{\mathit{eff}} \sim \lambda S \Lambda_w^2, \tag{4.43}$$

and the potential is a constant,

$$V = |\lambda_2|^4 |\lambda|^2. \tag{4.44}$$

However, with a little thought, it is clear that one should think of λ in this expression as the effective λ at the scale s.The behavior of λ with s depends on the values of the couplings, but for a range of parameters λ grows with s. In some cases, it has a minimum at large s, where the theory is weakly coupled. Then one can determine the location of the minimum and the pattern of symmetry breakdown. In other cases, the minimum occurs in the region of small s, where the theory is strongly coupled and difficult to analyze.

We have seen, in this section, that dynamical breaking of supersymmetry is common. Flat directions are often lifted, and in many instances, supersymmetry is broken with a stable ground state. So we are ready to address the question: how might supersymmetry be broken in the real world?

5 Where is the Scale of Supersymmetry Breaking?

If supersymmetry has something to do with the hierarchy problem, it must, in some sense, be broken at a scale of order M_Z. More precisely, the soft breakings among the ordinary quarks, leptons and gauge particles must be of this order. However, the fundamental scale of supersymmetry breaking – the scale of the F or D fields which break supersymmetry – can be much larger. Virtually all existing models of supersymmetry breaking – whether based on dynamical breaking, or on tree level breaking as in the O'Raifeartaigh model – assume that some new set of fields and interactions are responsible for symmetry breakdown. The most popular approach has been to assume that supersymmetry is broken in a "hidden sector," i.e., by fields which have only very tiny couplings to ordinary matter, and that the breaking of supersymmetry is fed down to ordinary fields by gravitational strength interactions. In this case, the scale of breaking is intermediate between the Planck scale and the weak scale, of order 10^{11} GeV. Such an approach has a certain degree of elegance, and is suggested by string theory. As we will see, however, it is not easy to understand how the problems raised by the rare processes we have discussed earlier are resolved in this framework. An alternative possibility is that the breaking occurs at much lower scales, and is mediated by gauge interactions. This approach is remarkably predictive, and automatically avoids the problems of flavor changing neutral currents. In this section, we will review both of these possibilities.

5.1 Hidden Sector N=1 Supergravity

It would take many lectures (and a more expert lecturer) to give a proper exposition of $N = 1$ supergravity. Fortunately, there are only a few facts we will need to know. First, the terms in the effective action with at most two derivatives or four fermions are completely specified by three functions:

1. The Kahler potential, $K(\phi, \phi^\dagger)$, a function of the chiral fields
2. The superpotential, $W(\phi)$, a holomorphic function of the chiral fields.
3. The gauge coupling functions, $f^a(\phi)$, which are also holomorphic functions of the chiral fields.

The lagrangian which follows from these can be found, for example, in [38,39]. Let us focus, first, on the scalar potential. This is given by

$$V = e^K \left[\left(\frac{\partial W}{\partial \phi_i} + \frac{\partial K}{\partial \phi_i} W \right) g^{i\bar{j}} \left(\frac{\partial W^*}{\partial \phi_j^*} + \frac{\partial K}{\partial \phi_j^*} W \right) - 3|W|^2 \right], \qquad (5.1)$$

where

$$g_{i\bar{j}} = \frac{\partial^2 K}{\partial \phi_i \partial \phi_{\bar{j}}} \tag{5.2}$$

is the Kahler metric associated with the Kahler potential. In this equation, we have adopted units in which

$$G_N = \frac{1}{8\pi M^2}. \tag{5.3}$$

where $M \approx 2 \times 10^{18}$ GeV is the reduced Planck mass we encountered earlier.

There is a standard strategy for building supergravity models. One introduces two sets of fields, the "hidden sector fields," which will be denoted by Z_i, and the "visible sector fields," denoted y_a. The Z_i's are assumed to be connected with supersymmetry breaking, and to have only very small couplings to the ordinary fields, y_a. In other words, one assumes that the superpotential, W, has the form

$$W = W_z(Z) + W_y(y), \tag{5.4}$$

at least up to terms suppressed by $1/M$.

One also usually assumes that the Kahler potential has a "minimal" form,

$$K = \sum z_i^\dagger z_i + \sum y_a^\dagger y_a. \tag{5.5}$$

One chooses (tunes) the parameters of W_Z so that

$$\langle F_Z \rangle \approx M_w M \tag{5.6}$$

and

$$\langle V \rangle = 0. \tag{5.7}$$

Note that this means that

$$\langle W \rangle \approx M_W M_p^2. \tag{5.8}$$

To understand the structure of the low energy theory in such a model, suppose first that $W(y) = 0$, and that K is of the "minimal" form, eq. (5.5). Then

$$V(y) = e^{K(z)} |\langle W \rangle|^2 \sum |y_a|^2. \tag{5.9}$$

In other words, the scalars all have a common mass,

$$m_o^2 = e^{\langle K \rangle} \frac{|\langle W \rangle|^2}{M^4} \approx m_{3/2}^2. \tag{5.10}$$

Here $m_{3/2}$ is the mass of the gravitino. Note that with $F \sim M M_Z$, $m_{3/2} \sim M_Z$.

If we now allow for a non-trivial W_y, we find also A and $B\mu$ terms. For example, the terms

$$\frac{\partial W}{\partial y_a} y_a W_y = 3W_y \tag{5.11}$$

if W is homogeneous of degree three. Additional contributions arise from

$$\left\langle \frac{\partial W}{\partial z_i} \right\rangle y_j^* W^* + \text{c.c.} \tag{5.12}$$

Exercise: The simplest model of the hidden sector is known as the "Polonyi model." In this model,

$$W = m^2(z+\beta) \tag{5.13}$$

$$\beta = (2+\sqrt{3}M)\ . \tag{5.14}$$

Verify that the minium of the potential for Z lies at

$$Z = (\sqrt{3}-1)M \tag{5.15}$$

and that

$$m_{3/2} = (m^2/m)e^{(\sqrt{3}-1)^2/2} \quad m_o^2 = 2\sqrt{3}m_{3/2}^2 \quad A = (3-\sqrt{3})m_{3/2}. \tag{5.16}$$

So far, we have not addressed the question of gaugino mass. This can arise from a non-trivial gauge coupling function,

$$f^a = c\frac{Z}{M} \tag{5.17}$$

which gives a gluino mass, just as it would in the global case:

$$m_\lambda = \frac{cF_z}{M}. \tag{5.18}$$

So these models have just the correct structure. They have soft breakings of the correct order of magnitude, and they exhibit, with our assumption of minimal kinetic term, the properties of universality and proportionality. Indeed, this is a highly predictive framework, with only (assuming MSSM particle content) 5 parameters. A large amount of work has been done on these models, including investigations of[g]

[g] More detail on all of these points is provided in many excellent review articles. See, for example, Jon Bagger's 1995 TASI lectures, and references therein [42].

1. Renormalization group evolution: one finds that there is a substantial region of the parameter space in which $SU(2) \times U(1)$ is broken in the correct fashion.
2. Unification: The predictions for unification are far better than in the minimal standard model. However, the prediction for α_s tends to be somewhat larger than observed. This can be "fixed" by adding new thresholds at the high scale [4].
3. b-τ unification: One can also consider the possibility that the b and τ Yukawa's unify. This is certainly not a general requirement of unification; it depends, in grand unified models, on the Higgs content, and in string theory on the details of compactification. Still, this idea seems viable (and interesting).
4. Proton decay: with detailed assumptions about the susy spectrum and the structure of grand unification, it is possible to compute the proton lifetime and compare with experimental limits.
5. Dark matter: Again, in a detailed model, one can identify the LSP and compute its couplings to matter. This permits a precise calculation of the abundance.
6. $b \rightarrow s\gamma$: As we have already remarked, one can compute the rate for this process in particular models. One often finds that other constraints are stronger.

5.2 Beyond the Minimal Model

Almost all work on supersymmetry phenomenology has been based on the idea that supergravity is the messenger of supersymmetry breaking, and the assumption that, at the high scale, the theory is described by an effective lagrangian with a Kahler potential of the form of eq. (5.5). Yet this assumption is hard to justify. It is often said that it is plausible, since gravity is "flavor blind." But the flavor-blindness of gravity is a consequence of general covariance; no such principle forbids a Kahler potential of the form:

$$K = f(Z, Z^\dagger) + \sum y_a^\dagger y_a + \sum_{ab} h_{ab}(Z, Z^\dagger)\phi_a^\dagger \phi_b + \dots \, . \tag{5.19}$$

This last term is similar to terms we described in the context of global supersymmetry, and has *exactly* the same effect: it spoils universality and proportionality. These terms are not forbidden by any symmetry, and they are generated by radiative corrections, so they are surely present at some level.

$N = 1$ supergravity models are not renormalizable, and thus can't be viewed as in any sense complete, predictive theories. Without some understanding of the underlying microscopic theory, there is little more one can say. One possibility is that the underlying theory possesses some flavor symmetry. This is presumably a gauge symmetry (continuous or discrete), since it is widely believed that global symmetries do not make sense in the framework of quantum gravity. These symmetries must be broken, and it is a challenge to simultaneously obtain sufficient squark and slepton degeneracy while accomadating the strong violation of flavor observed in the fermion sector [43].

Ultimately, however, the only candidate we have for a microscopic theory of supergravity is string theory. In principle, it should be possible to calculate the Kahler potential in string theory and determine whether it yields sufficient degeneracy. Some tentative steps in this direction have been taken, and the results are mixed. There are limits in which string theory does yield the required structure. On the other hand, it is hard to see how these limits could have anything to do with the real world. We will return to this question later, in the chapter on string theory.

5.3 Incorporating Dynamical Supersymmetry Breaking

Models like the Polonyi model are extremely ad hoc. They introduce a small parameter by hand, subject to strong constraints. It would be much more satisfying if one had a dynamical understanding of this new scale. We have seen that dynamical supersymmetry breaking (DSB) occurs in many theories, and in such theories the appearance of a small scale is easily explained as an effect of order e^{-c/g^2}, for some weak coupling g.

There are two approaches which have been tried to incorporating DSB into the framework of supergravity models. The first is to replace the Polonyi sector with a model, such as the 3–2 model, in which supersymmetry is broken and there is a stable ground state. Attempts to do this, however, run into difficulty producing a phenomenologically acceptable gluino mass. The problem is that in the simplest theories, there are no gauge singlet superfields in this hidden sector, so gluino masses can only arise through operators of high dimension, such as

$$\int d^2\theta \frac{\Phi^3}{M_p^3} W_\alpha^2. \tag{5.20}$$

If the typical scale of the hidden sector is of order 10^{11}GeV, this generates a gluino mass of order 10^{-7} eV or so (actually, larger contributions will be generated by loops in the very low energy, non-supersymmetric theory, but these will not be nearly large enough).

It is possible to write DSB models with gauge singlets; the model of Intriligator and Thomas which we discussed earlier is an example. In the construction of these models, however, it is crucial that there are no terms of order S^2 or S^3. One might try to explain the absence of such terms by discrete symmetries. But such symmetries inevitably forbid the desired linear term in S in the gauge coupling function. Theses models also offer no further insight into the question of universality. No symmetry will forbid the dangerous terms in the Kahler potential of eq. (5.19).

The second framework in which to consider this question is string theory. Since all known string models possess moduli, it is necessary to understand how the degeneracy among the associated vacuum states is lifted. As we will discuss in the chapter on string theory, there are always directions in which the potential for the moduli tends to zero. In fact, it is not hard to argue that the potential for the moduli can only have local minima in regions where a perturbative or semiclassical analysis breaks down. If one assumes the moduli are stabilized, then the gaugino and scalar masses are comparable. However, one still has difficulty understanding degeneracy and proportionality. We will discuss these issues in some detail in section 7.

6 Low Energy Dynamical Supersymmetry Breaking

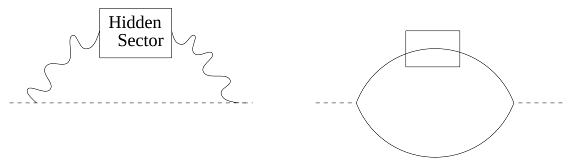

Figure 13: Schematic of gauge mediated supersymmetry breaking.

An alternative to the conventional supergravity approach is to suppose that supersymmetry is broken at some much lower energy, with gauge interactions serving as the messengers of supersymmetry breaking [40,41,32]. The basic idea is indicated in fig. 13. One again supposes that one has some set of new fields and interactions which break supersymmetry. Some of these fields are taken to carry ordinary standard model quantum numbers, so that "ordinary" squarks, sleptons and gauginos can couple to them through gauge loops. This approach, which is referred to as "gauge mediated supersymmetry breaking" (GMSB) has a number of virtues:

1. It is highly predictive: as few as 2 parameters describe all soft breakings.

2. The degeneracies required to suppress flavor changing neutral currents are automatic.

3. GMSB easily incorporates DSB, and so can readily explain the hierarchy.

4. GMSB makes dramatic and distinctive experimental predictions.

The approach, however, also has drawbacks. Perhaps most serious is related to the "μ problem," the question: why is the μ-parameter of the MSSM of order M_Z rather than, say M_p or any scale in between. I have not alluded to this problem earlier, because in the supergravity framework it is not a really a problem at all.[h] In string theory, one can give the following answer. It is quite common to find states which are massless at tree level, even though there is no symmetry which explains the absence of a mass term. In a non-supersymmetric theory, the mass would be corrected in loops, but now the non-renormalization theorems forbid a mass to any order. So the fact that μ is very small is not suprising. Assuming that supersymmetry is broken at an intermediate scale in the desired fashion, supersymmetry breaking effects tend to generate a μ term of precisely the correct order, through couplings like

$$\int d^4\theta \frac{Z^\dagger}{M} H_U H_D. \tag{6.1}$$

This phenomenon actually occurs in the simplest supergravity models [44,45].

The μ problem, however, finds a home in the framework of low energy breaking. The difficulty is that, if one is trying to explain the weak scale dynamically, one does not want to introduce the μ term by hand. However, operators like those of eq. (6.1) now do not generate a μ term of the correct order of magnitude. Various solutions have been offered for this problem [32,46], but none is yet compelling. Perhaps the most likely possibility is that there will be some structure beyond that of the MSSM at relatively low energies, such as singlets coupled to Higgs fields. In most of our discussion, we will simply assume that a μ term has been generated in the effective theory, and not worry about its origin.

6.1 Minimal Gauge Mediation (MGM)

The simplest model of gauge mediation contains, as messengers, a vectorlike set of quarks and leptons, q, $\bar{q}$, ℓ and $\bar{\ell}$. These have the quantum numbers of a 5 and $\bar{5}$ of $SU(5)$. The superpotential is taken to be

$$W_{mgm} = \lambda_1 q\bar{q} + \lambda_2 S\ell\bar{\ell}. \tag{6.2}$$

[h] I should note that this statement is not completely non-controversial.

We suppose that some dynamics gives rise to non-zero expectation values for S and F_S. We will not, in these lectures, explore detailed proposals for this dynamics; for this see [32]. Instead, we will go ahead and immediately compute the superparticle spectrum for such a model. Ordinary squarks and sleptons gain mass through the two-loop diagrams shown in fig. 14. While the prospect of computing a set of two loop diagrams may seem intimidating, the computation is actually quite easy. If one treats F_S/S as small, there is only one scale in the integrals. It is a straightforward matter to write down the diagrams, introduce Feynman parameters, and perform the calculation. There are also various non-trivial checks. For example, the sum of the diagrams must vanish in the supersymmetric limit.

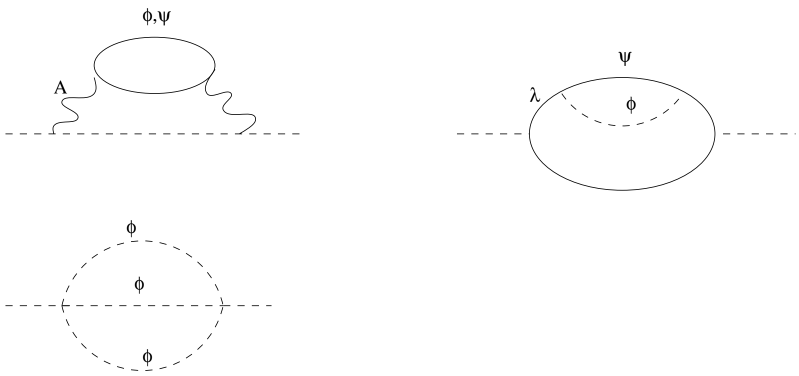

Figure 14: Two loop diagrams contributing to squark masses in a simple model of gauge mediation.

One obtains the following expressions for the scalar masses [41,32]:

$$\widetilde{m}^2 = 2\Lambda^2 \left[C_3 \left(\frac{\alpha_3}{4\pi}\right)^2 + C_2 \left(\frac{\alpha_2}{4\pi}\right)^2 + \frac{5}{3}\left(\frac{Y}{2}\right)^2 \left(\frac{\alpha_1}{4\pi}\right)^2 \right], \tag{6.3}$$

where $\Lambda = F_S/S$, and $C_3 = 4/3$ for color triplets and zero for singlets, $C_2 = 3/4$ for weak doublets and zero for singlets. For the gaugino masses one obtains:

$$m_{\lambda_i} = c_i \, \frac{\alpha_i}{4\pi} \, \Lambda \ . \tag{6.4}$$

This expression is valid only to lowest order in Λ. Higher order corrections have been computed in [47].

All of these masses are positive, and they are described in terms of a single new parameter, Λ. The lightest new particles are the partners of the

$SU(3) \times SU(2)$ singlet leptons. If their masses are of order 100 GeV, we have that $\Lambda \sim 30$ TeV. The spectrum has a high degree of degeneracy. In this approximation, the masses of the squarks and sleptons are functions only of their gauge quantum numbers, so flavor changing processes are suppressed. Flavor violation only arises through Yukawa couplings, and these can only appear in graphs at high loop order. It is further suppressed because all but the top Yukawa coupling is small.

Apart from the parameter Λ, one has the μ and $B\mu$ parameters (both complex), for a total of five. This is three beyond the minimal standard model. If the underlying susy-breaking theory conserves CP, this can eliminate the phases, reducing the number of parameters by 2.

6.2 $SU(2) \times U(1)$ Breaking

At lowest order, all of the squark and slepton masses are positive. The large top quark Yukawa coupling leads to large corrections to $m^2_{H_U}$, however, which drive $SU(2) \times U(1)$ breaking. The calculation is just a repeat of one we have done earlier. We can evaluate the diagram of fig. 3, treating the mass of $\tilde{t}$ as independent of momentum, provided we cut the integral off at a scale of order Λ (at this scale, the calculation leading to eq. (6.3) breaks down, and the propagator falls rapidly with momentum) and we have

$$m^2_{H_U} = (m^2_{H_U})_o - \frac{6y_t^2}{16\pi^2} \ln(\Lambda^2/\widetilde{m}_t^2)(\widetilde{m}_t^2)_o. \tag{6.5}$$

While the loop correction is nominally a three loop effect, because the stop mass arises from gluon loops while the Higgs mass arises at lowest order from W loops, we have

$$\left(\frac{\widetilde{m}_t^2}{m^2_{H_U}}\right)_o = \frac{16}{9}(\frac{\alpha_3}{\alpha_2})^2 \sim 20 \tag{6.6}$$

and the Higgs mass-squared is negative. This calculation was in fact done many years ago in [9]. In some sense, the situation here is more striking than in the supergravity case. There, the soft breakings were all parameters anyway, so the fact that there are negative radiative corrections to some, while suggestive, does not permit a "prediction" of weak interaction symmetry breaking. In the present case, this is a prediction of the theory.

6.3 *Light Gravitino Phenomenology*

There are other striking features of these models. One of the more interesting is that the LSP is the gravitino. Its mass is

$$m_{3/2} = 2.5 \left(\frac{F}{(100 \text{ TeV})^2} \right) \text{eV}. \tag{6.7}$$

The next to lightest supersymmetric particle, or NLSP, can be a neutralino, or a charged right handed slepton. The NLSP will decay to its superpartner plus a gravitino in a time long compared to typical microscopic times, but still quite short. The lifetime can be determined from low energy theorems, in a manner reminiscent of the calculation of the pion lifetime. Just as the chiral currents are linear in the (nearly massless)pion field,

$$j^{\mu 5} = f_\pi \partial^\mu \pi \quad \partial_\mu j^{\mu 5} = \partial^2 \pi \approx 0 \tag{6.8}$$

so the supersymmetry current is linear in the Goldstino, G:

$$j^\mu_\alpha = F \gamma^\mu G + \sigma^{\mu\nu} \lambda F_{\mu\nu} + \ldots \tag{6.9}$$

F, here, is the goldstino decay constant. From this, if one assumes that the LSP is mostly photino, one can calculate the amplitude for $\tilde{\gamma} \to G + \gamma$ in much the same way one considers processes in current algebra. From eq. (6.9), one sees that $\partial_\mu j^\mu_\alpha$ is an interpolating field for G, so:

$$\langle G\gamma | \tilde{\gamma} \rangle = \tfrac{1}{F} \langle \gamma | \partial_\mu j^\mu_\alpha | \tilde{\gamma} \rangle. \tag{6.10}$$

The matrix element can be evaluated by examining the second term in the current, eq. (6.9), and noting that $\not{\partial} \lambda = m_\lambda \lambda$.

Given the matrix element, the calculation of the NLSP lifetime is straightforward, and yields

$$\Gamma(\tilde{\gamma} \to G\gamma) = \frac{\cos^2 \theta_W \, m^5_{\tilde{\gamma}}}{16 \pi F^2}. \tag{6.11}$$

This yields a decay *length*:

$$c\tau = 130 \left(\frac{100 \text{ GeV}}{\text{m}_{\tilde{\gamma}}} \right)^5$$

$$\left(\frac{\sqrt{F}}{100 \text{ TeV}} \right)^4 \mu m. \tag{6.12}$$

In other words, if F is not too large, the NLSP may decay in the detector.

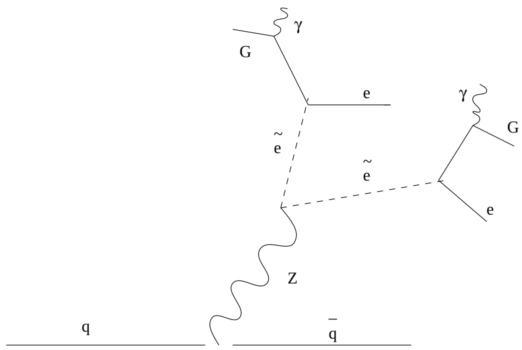

Figure 15: Decay leading to $e^+e^-\gamma\gamma$ events.

One even has the possibility of measurable displaced vertices [48,49]. The signatures of such low decay constants would be quite spectacular. Assuming the photino (bino) is the NLSP, one has processes such as $e^+e^- \rightarrow \gamma\gamma + \not{E}_t$ and $p\bar{b} \rightarrow e^+e^-\gamma\gamma + \not{E}_t$, as indicated in fig. 15. The first process has already been used to set limits on the parameter space of these models. The second process may, conceivably, already have been seen at the tevatron. One event has been seen with two electrons, two photons, and substantial missing energy [50]. Known standard model backgrounds for this process are negligible. One event is roughly consistent with the expected cross section if the selectron mass is in the 100 GeV range [51]. If the event is real, and this is the correct explanation, one would expect to see many such events at the upgraded tevatron. The collider phenomenology of these models has been studied in some detail in [52].

It is natural to ask how general is the MGM. Various modifications are possible, and these have been discussed in [48,53]. Among the possible modifications are changes in the particle content of the messenger sector, and mixings of messenger and "matter" fields.

So far, while we have assumed that supersymmetry is dynamically broken, we have not examined the details of the supersymmetry-breaking dynamics which might underlie the MGM. Indeed, perturbative models with the features of MGM were constructed long ago, using "O'Raifeartaigh" type breaking [54,55,9]. Realistic models in which supersymmetry is dynamically broken *have* been constructed [40,41,32]. All of the constructions so far involve a supersymmetry breaking sector separated from the fields of the MSSM. One might imagine that we could do better, using the many new things we have learned over the past three years. One could speculate on a theory with a very compact, tight structure, in which, perhaps, some of the gauge bosons we view as fundamental are in fact composite, and the scales of supersymmetry and weak interaction symmetry breaking are intimately tied together [56]. Ref. [53] makes arguments that many features of this ultimate theory are likely to be like those of the MGM. Still, one worries that we are not being clever enough. One possible alternative viewpoint, which starts by insisting on quite stringent – and plausible – notions of naturalness has been developed in [57].

7 String Phenomenology and Phenomenology of Strings

Supersymmetry seems to be an essential part of string theory. Vast numbers of string vacua are known with varying amounts of supersymmetry. Much – it is probably fair to say all – of the recent progress in string duality relies on supersymmetry. In fact, it is quite hard to make sense of string vacua which don't possess at least some degree of supersymmetry. In particular, there seems invarariably to be a cosmological constant at one loop. In addition to the fact that "constant" is far too large, what one is really determining in these computations is a potential for the dilaton. This potential always tends to zero for very weak coupling, which means that the vacuum is unstable [58].

The methods of the previous sections can be applied to understand many issues in string phenomenology. But they can also be used to give insights into the basic dynamics of string theory. In this section, we will content ourselves with a brief survey of these ideas.

At the classical level, moduli are ubiquitous in string theory. They are problematic for any string phenomenology. One wants to understand how a particular vacuum state is selected, and how the moduli gain mass. At the same time, they provide tools to understanding many aspects of string dynamics. Indeed, this is a large part of the duality story.

A simple example of this bad/good aspect of moduli is provided by toroidal compactification of the heterotic string to four dimensions [59]. These possess $N = 4$ supersymmetry. There is a large moduli space. At weak coupling and low energies, one can write an effective action for the light degrees of freedom – the moduli, the gauge fields and the graviton supermultiplet. This effective action must respect the full supersymmetry. But $N = 4$ supersymmetry is so restrictive that it forbids any terms in $\mathcal{L}_{eff}$ which might lift the degeneracy. This statement holds perturbatively and non perturbatively! In other words, the moduli in these theories are exact. There are also lots of $5, 6 \ldots 10$ dimensional vacua which are exact. To date, we have no idea what might provide a vacuum selection principle, which would explain why we live in an approximately $N = 1$ supersymmetric vacuum. Indeed, it is not easy to understand why there should even exist a vacuum with $N = 1$ supersymmetry broken to

$N = 0$, never mind explaining how the universe finds itself in such a state.

We will focus on a particular class of potential string vacua: perturbative ground states of the heterotic string theory with $N = 1$ supersymmetry. All known states of this type possess moduli. One of these controls the size of the four dimensional gauge couplings at the classical level; it is known as the "model-independent dilaton." It is convenient to write the (complex) scalar component of this multiplet as

$$S = \frac{8\pi^2}{g^2} + ia. \tag{7.1}$$

$S \to \infty$ corresponds to weak string coupling if the other moduli are held fixed. Non-perturbative effects inevitably fall to zero at weak coupling. Analyticity of the superpotential and the gauge coupling as a function of S, along with certain symmetry properties, allow us to make a number of strong statements about the non-perturbative structure of the theory.

First, we can prove a non-renormalization theorem for perturbation theory similar to that we encountered in field theory, and in almost the same way. In perturbation theory, string theory has a symmetry under the shift

$$a \to a + i\delta. \tag{7.2}$$

This symmetry can be understood by examining string vertex operators, or simply by noting that the axion is related by duality to the antisymmetric tensor field,

$$\partial_\mu a = \epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma}. \tag{7.3}$$

In perturbation theory, the effective action can be written in terms of H, as a consequence of the antisymmetric tensor gauge invariance, so only derivates of a appear. Non-perturbatively, this Peccei-Quinn symmetry is broken. This is familiar from field theory, where instantons break such symmetries. However, as in field theory, a discrete subgroup of the full group survives. This is a consequence of a quantization condition for the antisymmetric tensor field [60]. The coupling of a string to $B_{\mu\nu}$ to a string has the form:

$$I_B = \frac{1}{2} \int d^2\sigma B_{MN} (\partial X^M \partial X^N - M \leftrightarrow N). \tag{7.4}$$

In order that string amplitudes be single-valued, this action must be a multiple of $2\pi n$. If Σ is a closed three surface, this leads to

$$\int_\Sigma d^3\ \Sigma H = n. \tag{7.5}$$

The Peccei-Quinn charge is $j_\mu^{PQ} = \partial_\mu a$, so taking the divergence, and using the duality relation between a and H leads to a condition for the change of the Peccei-Quinn charge in any process [61]:

$$\Delta Q^{PQ} = \int d^4x \partial^2 a = n. \tag{7.6}$$

This means that, in the effective action, only operators like e^{ina} can appear. So the effective action is invariant under 2π shifts of a.

This periodicity is a subgroup of "S-duality." Clearly, however, it holds whether or not the particular string vacuum is S-dual. Since the superpotential is analytic in S, and must respect this periodicity property, it must be of the form:

$$W = w_o(M) + w_1(M)e^{-S} + w_2(M)e^{-2S} + \ldots \tag{7.7}$$

at least if the coupling is small. As a consequence, V goes rapidly to zero as $S \to \infty$. *Inevitably*, any good vacuum of string theory lies at strong coupling [62].

A little thought indicates that the true vacuum is unlikely to reside in any region where duality is very useful. The problem is that if the strong coupling region is mapped into some dual, weakly coupled theory, then one will still have the problem of vacuum instability. Unfortunately, we seem to be stuck with what David Gross has dubbed "The Principle of Minimal Calculability." In other words, the true vacuum of string theory must lie in a regime where no weak coupling analysis is applicable. One can imagine loopholes in this argument, so I offer the following exercise:

Exercise: Find the loophole in the above argument, find string vacua with broken supersymmetry and vanishing cosmological constant, and make first class reservations to Stockholm.

Actually, the situation is perhaps not so hopeless; holomorphy of W and f, and axion periodicity, might still permit us to make definite predictions about the low energy theory. Let us suppose that S is as large as typical computations of the unified coupling suggest,

$$\langle S \rangle = \frac{8\pi^2}{g^2} \sim 150. \tag{7.8}$$

We will give some arguments for this later. In that case, e^{-S} is an extremely small number, and can be ignored. In other words, the superpotential is unchanged from its tree level form, and the gauge couplings receive no appreciable corrections beyond one loop. However, the Kahler potential is not restricted by holomorphy, and thus one expects it to receive significant corrections at strong coupling.

One can conceive of corrections to K such that, even though the superpotential is of the form of eq. (7.7), the potential has a local minimum at $\langle S \rangle$. From the perspective of field theory, one might have expected perturbation theory to be good for such small values of the coupling. But there are two arguments against this. First, string perturbation theory is not as convergent as field theory perturbation theory, so perhaps the expansion of the Kahler potential has already broken down for such couplings. Second, S is not really the string expansion parameter in any case. The actual expansion parameter is the expectation value of the 10-dimensional dilaton, and as we will discuss shortly, this can be quite large even if the effective four dimensional coupling is small. In the following discussion, we will assume that e^{-S} is very small, but that corrections to the Kahler potential are large, and are responsible for stabilizing the potential for the dilaton and other moduli. Within our present understanding of string theory, such a picture seems almost inevitable, if the theory is to have anything to do with nature.

This assumption, plus the periodicity we discussed earlier, has significant implications for the effective action. The effective action we are considering here is the Wilsonian effective action, at a scale just below the string scale. In this action

$$W = W_{tree} + \mathcal{O}(e^{-S}) \qquad f = f_{tree} + f_{1-loop} + \mathcal{O}(e^{-S}) \ . \tag{7.9}$$

This means that

- The spectrum of light states is the same at weak coupling as at strong coupling.
- Yukawa terms in W are the same as at weak coupling, and so can be calculated at weak coupling. Note that these are not the same as the physical Yukawa couplings, which include terms in the Kahler potential, which are renormalized. In specific models, ratios of Yukawa couplings, for example, may not be renormalized.
- The gauge couplings are unified. Assuming that the low energy theory contains only three generations, without extra vector-like matter, then one obtains, as we have discussed, reasonable agreement with experiment. Note, however, that the unification scale computed in this way is not necessarily associated with any physical threshold. As one passes to strong coupling, there can be large "renormalizations" of the masses of the massive states. (In the framework of weakly coupled field theories, the issues described here have been considered in [25,63].)

In a picture like this, anything having to do with the Kahler potential cannot be calculated. This means that the soft breakings, which we have seen depend crucially on the Kahler potential, are not calculable unless one actually solves the strong coupling problem. This may seem quite disappointing, but with what we currently understand about string theory, this is probably the best we can do. However, I claim that if we were successful (e.g., found a model and made a few successful quantitative predictions) this would be an incredible triumph for string theory.

Despite these negative comments about computing the Kahler potential, I would like to mention a weak-coupling scenario which, if it were somehow realized, would be quite predictive and would give the sort of universality usually assumed for supergravity theories [64,65]. In this picture, it is assumed that the dilaton dominates supersymmetry breaking, i.e., that $\langle F_S \rangle$ is by far the largest non-vanishing F-component. It is also assumed that the weak coupling approximation is valid. In this case, the Kahler potential for S is known[66]

$$K = -\ln(S + S^\dagger) + K(\mathcal{M}, \mathcal{M}^\dagger). \tag{7.10}$$

The assumption that the dilaton dominates supersymmetry breaking means that $W = W(S)$. There are no terms in K at this order which couple matter fields to S. As a result, the squark and slepton masses are universal. It is straightforward to calculate the soft breaking terms. One has, for the scalar masses, gaugino masses, and A terms:

$$m_o^2 = m_{3/2}^2 \qquad M = \frac{\sqrt{3}}{2} m_{3/2} \qquad A = -\sqrt{3} m_{3/2}. \tag{7.11}$$

So, while we have argued that there is no reason to think that a supergravity theory should yield universality and proportionality, we see that there is a limiting case of string theory which yields just that! On the other hand, in the limit that the coupling is small enough that one can do perturbation theory, so that the expression above is a good approximation to the Kahler potential, we have argued one cannot expect to find a stable ground state of string theory.[i] One has to hope that, accidentally, some Kahler corrections are large and some are small. Even if one assumes a good vacuum, degeneracy may not hold to the required level of accuracy [68]. This model makes strong predictions for the soft susy parameters, and it is not clear that these are compatible with

[i] A possible loophole is provided by racetrack models [67]. However, efforts to construct such models have not been particularly successful, either in generating models with reasonable supersymmetry breaking or in generating a phenomenologically acceptable value of S.

experimental constraints, at least if the low energy theory has MSSM particle content [69].

Returning to our picture of Kahler stabilization of the dilaton, we have seen that at the high scale, supersymmetry breaking effects are of order e^{-S}, i.e., far too small to be of interest. Effects in the low energy theory, however, can be much larger. As an example, consider a Calabi-Yau theory with the standard embedding of he gauge group, and perhaps some Wilson lines. Suppose that there is a hidden sector with some gauge group and associated one-loop β-function b_o. Gluinos condense in this theory:

$$\langle\lambda\lambda\rangle = e^{(-3\frac{8\pi^2}{b_o g^2}-3i\frac{a}{b_o})}. \tag{7.12}$$

This leads to an effective superpotential for S,

$$W_{eff}(S) = e^{-\frac{3S}{b_o}}. \tag{7.13}$$

It is possible to invent Kahler potentials which lead to a minimum for the potential at S_o with vanishing cosmological constant, and we will assume that this is what occurs. For suitable b_o, this can lead to a value of $m_{3/2}$ of order the weak scale.

7.1 The View from 11 Dimensions

It is usually said that the compactification scale, in string theory, must be comparable to the Planck mass [70,71]. The argument is quite simple. In the heterotic string, the ten dimensional gravitational and gauge constants are given in terms of the dimensionless coupling of the theory, which for now I will denote by λ, by

$$\kappa_{10}^2 = \tfrac{1}{4}\lambda^2(2\alpha')^4 \qquad g_{10}^2 = \lambda^2(2\alpha')^3. \tag{7.14}$$

Thus the string tension, $T = (2\alpha')^{-1}$ is

$$T = \frac{g_{10}^2}{4\kappa_{10}^2} = \frac{g_4^2}{4\kappa_4^2}. \tag{7.15}$$

or

$$T \approx 5 \times 10^{17}\mathrm{GeV}. \tag{7.16}$$

On the other hand, $g_4^2 = \lambda^2/VT^3$ where V is the volume of the internal space. So if we require *weak string coupling*, $\lambda^2 \leq 1$, $g_4^2 \approx 1$, we have that $VT'^3 \approx 1$. On the other hand, if $R^{-1} \approx M_{GUT} \approx 2\times 10^{16}$, this gives $\lambda^2 \sim 10^7$!

In the past, people have insisted that the coupling should be less than or of order one, and have therefore assumed that R^{-1} couldn't be much different than M_p. However, we have just argued that string theory should be strongly coupled. Moreover, in light of the recent developments in duality, we are not frightened by the limit $\lambda \to \infty$: this should just be M theory on $X \times S_1/Z_2$, where X is a six dimensional manifold such as a Calabi-Yau manifold [72]. This argument suggests that M theory might well give a qualitatively better description of the real world than weakly coupled string theory [73].

Assuming that the M-theory description is valid, one can determine the 11-dimensional Planck mass, M_{11} and the size of the 11'th dimension, R_{11}:

$$M_{11} = R^{-1} \left(2(4\pi)^{-2/3}\alpha_{GUT}\right)^{-1/6}. \tag{7.17}$$

$$R_{11}^2 = \frac{\alpha_{GUT}^3 V}{512\pi^4 G_N^2}, \tag{7.18}$$

Substituting $M_{GUT} = R^{-1} = 10^{16}$ GeV, $\alpha_{GUT} = \frac{1}{25}$ and the correct value for Newton's constant, one finds that

$$R \sim 2M_{11}^{-1} \tag{7.19}$$

$$M_{11}R_{11} \sim 72 \ . \tag{7.20}$$

These numbers are quite striking. M_{11} is not much different than the unification scale. The radius of the 11'th dimension is much larger than the others, so there is an approximation in which the universe is five dimensional. The important scales of new physics are not set by the usual Planck scale, but in fact lie at significantly smaller scales. This observation has implications for proton decay, since it means that dangerous dimension five operators are not suppressed by M_p but rather only by M_{GUT}. Presumably, one would will need approximate symmetries to account for the smallness of the proton decay amplitude [74].

In the framework of M theory, we can revisit the question of the stabilization of the moduli. In weakly coupled string theories, in addition to the modulus S, one typically speaks of a set of moduli, which can be thought of as describing the size and shape of the internal manifold. To simplify the discussion, we will speak of one overall size, usually denoted by T (not to be confused with the string tension) $T = R^2$, measured in units of the string tension. The moduli S and T are now, up to constants:

$$S = V_{11}M_{11}^6 \qquad T = R_{11}M_{11}^3R^2. \tag{7.21}$$

This can be verified by using the relations between 11-dimensional and string theory quantities. Roughly speaking, large S and fixed T now corresponds to large Calabi-Yau space, while large T and fixed S corresponds to large radius for the 11'th dimension. It is not hard to work out the Kahler potential for these fields. One can do this by first compactifying on the orbifold, yielding a 10 dimensional supergravity Yang-Mills theory, and then reducing on the Calabi-Yau. This yields a result identical to that for the ordinary compactification of string theory at large radius on a Calabi-Yau space. Now, however, one has an interesting new possibility. If one calculates the f function at one loop, one finds that it has the form, for the hidden sector fields [75,76,73],

$$f_8 = S - cT \ . \tag{7.22}$$

This has been calculated both at strong and at weak coupling; the two computations must – and do – agree, by holomorphy and the periodicity arguments we have used repeatedly [74]. This result means that the coupling blows up for $S = cT$ [73]. Since in the weakly coupled theory, $\lambda \rightarrow S/T^3$, this inevitably implies strong coupling; this is presumably why this possibility was ignored in the past. Witten has suggested that this point, where the coupling blows up, is special and might be the location of the true vacuum. This assumption yields a prediction for the ratio of the unification scale and the Planck scale which is not unreasonable.

While this result is intriguing, it is not easy to go further. First, if one examines gluino condensation in the hidden sector, this tends to drive one away from the strong coupling point. Of course, one expects large corrections as one approaches this point, so the potential still might have a minimum there. Second, while this observation might explain how one linear combination of the moduli is fixed, it is not easy to see how the other would be. Indeed, given that the fifth (eleventh) dimension is so large, there are important restrictions on the leading terms in the Kahler potential coming from five dimensional supersymmetry. These make it hard to write any sort of effective theory which would stabilize the other modulus. As we did earlier for the dilaton, we need to suppose that there are corrections to the Kahler potential for large R_{11} which are surprisingly large [74]. All of this clearly bears further study [77].

8 Conclusions

Supersymmetry may well be the next layer of structure. If it is there, and has something to do with the hierarchy problem, we should see it at the LHC, if not before. Supersymmetry is a predictive framework, in that the quantum numbers and interactions of the array of new particles expected are well determined. Much can be said about its phenomenology, even without a detailed model of the superparticle spectrum. Strong constraints can be placed on the spectrum by rare processes. Unlike the case of, say, technicolor models, it is not hard to provide models in which all of these constraints are well satisfied. Still, one would like to predict the soft breakings. This leads to the crucial question of supersymmetry breaking.

We have explored two quite distinct ways in which supersymmetry might be broken. The first, high energy breaking, raises many puzzles: why is their squark degeneracy, why is CP-violation so small, what fixes the numerous parameters of the model? The second, low energy breaking, is a much tighter structure, with few parameters, automatic flavor conservation, and other desirable features. Still, it is not at all clear which structure might ultimately be correct. No beautiful and compelling model of low energy breaking yet exists, and the μ problem is particularly troubling in this framework. Intermediate scale breaking looks like a much more likely outcome of string theory, though we can hardly be said to understand string dynamics well enough to make any definite statements.

Skeptics often ask why one is so interested in supersymmetry, and ask at one point one will give up on it. I hope these lectures have made clear that supersymmetry is of interest from many points of view: the hierarchy problem, dark matter, string theory, and more. It is hard to imagine that nature does not take advantage of such a rich and beautiful structure. On the other hand, as compelling as this set of ideas may sometimes seem, we should all be skeptics. The experimental support for supersymmetry is, at best, extremely slender. As the mass scales associated with supersymmetry are gradually pushed higher, one worries that the original argument for low energy supersymmetry may soon no longer make sense. Hopefully, time will tell. In the meantime, there is much for theorists to do. In particular, we would like to know what string theory predicts for the soft breakings. If we could make progress on this question, we might someday be in a position similar to that of gauge theories in the 1970's, exploring and testing the theory through purely low energy measurements. Had we never built the CERN SPS and higher energy machines, we would still be convinced of the validity of the standard model. If we could understand the predictions of string theory for supersymmetry breaking, we could similarly establish the theory without having to wait for a Planck-scale machine. Perhaps, on that highly optimistic note, it is time to conclude these lectures.

References

1. J. Wess and J. Bagger, *Supersymmetry and Supergravity*, (Princeton University Press, Princeton, 1983).

2. L. Susskind, Phys. Rev. **D20** (1979) 2619
3. U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. **B260** (1991) 447; C. Giunti, C.W. Kim and U.W. Lee, Mod. Phys. Lett. **16** (1991) 1745; J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. **B249** (1990) 441; **B260** (1991) 131; P. Langacker and M.-X. Luo, Phys. Rev. **D44** (1991) 817.
4. D. Pierce, J.A. Bagger, K. Matchev and R.-J. Zhang, hep-ph/9606211 and references therein.
5. T. Banks and L. Dixon, Nucl. Phys. **B299** (1988) 613.
6. M. Grisaru, Nucl. Phys. **B194** (1982) 65.
7. H.E. Haber and R. Hempfling, Phys. Rev. Lett. **66** (1991) 1815; J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. **B262** (1991) 477.
8. L. Ibanez and G.G. Ross, Phys. Lett. **110B** (1982) 215.
9. L. Alvarez-Gaume, M. Claudson and M.B. Wise, Nucl.Phys. **B207** (1982) 96.
10. J. Polchinski and M.B. Wise, Phys. Lett. **125B** (1983) 393.
11. M.K. Gaillard and B.W. Lee, Phys. Rev. **D10** (897) 1974; M.K. Gaillard, B.W. Lee and J.L. Rosner, Rev. Mod. Phys. **47** 1975) 277.
12. H. Georgi, *Weak Interactions and Modern Particle Physics*, (Benjamin-Cummings, 1984).
13. Y. Nir, in Proceedings of the SLAC Summer Institute, 1992, SLAC-PUB-5847.
14. M. Dine and A. Nelson, *Phys. Rev.* **D48** (1993) 1277.
15. F. Gabbiani, E. Gabrielli, A. Masiero, L. Silvestrini, "A Complete Analysis of FCNC and CP Constraints in General SUSY Extensions of the Standard Model," Nucl. Phys. **B477** (1996) 321, hep-ph/9604387.
16. For a recent discussion and references, see J. Hewett and J.D. Well, "Searching for Supersymmetry in Rare B Decays," hep-ph/9610323, and references therein.
17. R. Barbieri and G.F. Giudice, "$b \to s\gamma$ Decay and Supersymmetry," hep-ph/9303270, Phys. Lett. **B309** (1993) 86.
18. The theoretical literature on R_b is very large. See, for example, P.H. Chankowski and S. Pikorski, "Chargino Mass and R_b Anomaly, hep-ph/9603310, Nucl. Phys. **B475** (1996) 3 and references therein.
19. S. Dimopoulos and L.J. Hall, Phys. Lett. **B207** (1987) 210.
20. S. Weinberg, Phys. Rev. **D26** (1982) 287.
21. S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. **112B** (1982) 133.
22. E. Witten, *Nucl. Phys.* **B188** (1981) 513.
23. I. Affleck, M. Dine and N. Seiberg, *Nucl. Phys.* **B241**, (1984) 493; *Nucl. Phys.* **B256** (1986) 557.

24. M.T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. **B159** (1979) 429.
25. M.A. Shifman and A.I. Vainshtein, Nucl. Phys. **B277** (1986) 456; Nucl. Phys. **B359** (1991) 571; L. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. **B355** (1991) 649; H.S. Li and K.T. Mahanthappa, Phys. Lett. **B319** (1993) 152; M. Dine and Y. Shirman, "Some Explorations in Holomorphy," hep-th/9405155, Phys. Rev. **D50** (1994) 5389.
26. N. Seiberg, "Naturalness Versus Supersymmetric Nonrenormalization Theorems," hep-ph/9309335, Phys. Lett. **B318** (1993) 469; K. Intriligator, R.G. Leigh and N. Seiberg, "Exact Superpotentials in Four Dimensions," hep-th/9403198, Phys. Rev. **D50** (1994) 1092.
27. W. Fischler, H.P. Nilles, J. Polchinski, S. Raby and L. Susskind, Phys. Rev. Lett. **47** (1981) 757.
28. E. Witten, Nucl. Phys. **B268** (1986) 79.
29. M. Dine and N. Seiberg, Phys. Rev. Lett. **57** (1986) 2625.
30. I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. **B241** (1984) 493; Phys. Rev. Lett. **B51** (1026) 1983.
31. E. Witten, Nucl. Phys. **B202** (1982) 253.
32. M. Dine, A. Nelson, Y. Nir and Y. Shirman, New Tools for Low Energy Dynamical Supersymmetry Breaking, hep-ph/9507378, *Phys. Rev.* **D53** (1996) 2658.
33. K. Intriligator and S. Thomas, "Dynamical Supersymmetry Breaking on Quantum Moduli Spaces," hep-th/9603158, Nucl. Phys. **B473** (1996) 121.
34. P. Pouliot, Phys. Lett. **B367** (1996) 131, hep-th/9510148; P. Pouliot and M.J. Strassler, Phys. Lett. **B375** (1996) 175, hep-th/9602031; T. Kawano, YITP-96-5, hep-th/9602035; E. Poppitz, Y. Shadmi and S.P. Trivedi, EFI-96-15, hep-th/9605113; EFI-96-24, hep-th/9606184; K-I. Izawa and T. Yangida, Prog. Theor. Phys. **95** (1996) 829, hep-th/9602180; C. Csaki, L. Randall and W. Skiba, MIT-CTP-2532; hep-th/9605108; C. Csaki, L. Randall, W. Skiba and R.G. Leigh, MIT-CTP-2543, hep-th/9607021.
35. K. Intriligator, N. Seiberg and S.H. Shenker, "Proposal for a Simple Model of Dynamical SUSY Breaking," hep-ph/9510203, Phys. Lett. **B342** (1995) 152.
36. A.E. Nelson, "A Viable Model of Dynamical Supersymmetry Breaking in a Hidden Sector," hep-ph/9508367, Phys. Lett. **B369** (1996) 277.
37. Y. Shirman, "Dynamical Supersymmetry Breaking Versus Runaway Behavior in Supersymmetry Gauge Theories," hep-th/9608147.
38. E. Cremmer, B. Julia, J. Scherk, S. Ferrar, L. Girardello and P. von Nieuvenhuizen, Nuclear Physics **B147** (1979) 105.

39. H.P. Nilles, Phys. Rept. **110** (1984) 1.
40. M. Dine and A.E. Nelson, "Dynamical Supersymmetry Breaking at Low Energies," hep-ph/9303230, Phys. Rev. **D48** (1993) 1277.
41. M. Dine and A.E. Nelson, "Low Energy Dynamical Supersymmetry Breaking Simplified," hep-ph/9408384, Phys. Rev. **D51** (1995) 1362.
42. J. Bagger, "Weak Scale Supersymmetry: Theory and Practice," hep-ph/9604232.
43. This subject has been widely studied. Some relevant papers are: M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. **B420** (1994) 468; M. Dine, R. Leigh, and N. Kagan, Phys. Rev. **D48** (1993) 4269; D.B. Kaplan and M. Schmaltz, Phys. Rev. **D49** (1994) 3741; L.J. Hall and M. Murayama, Phys. Rev. Lett. **D48** (1993) 4269.
44. A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. **49** (1982) 970.
45. L. Alvarez-Gaume, J. Polchinski, M.B. Wise, Nucl. Phys. **B221** (1983) 495.
46. G. Dvali, G.F. Giudice and A. Pomarol, "The Mu Problem in Theories with Gauge Mediated Supersymmetry Breaking," hep-ph/9603238.
47. S. Dimopoulos, G.F. Giudice and A. Pomarol, "Dark Matter in Theories of Gauge Mediated Supersymmetry Breaking," hep-ph/9607225.
48. S. Dimopoulos, M. Dine, S. Raby and Thomas,"Experimental Signatures of Low-Energy Gauge Mediated Supersymmetry Breaking," hep-ph/9601367, Phys. Rev. Lett. **76** (1996) 3494.
49. D.R. Stump, M. Wiest and C.P. Yuan, "Detecting a Light Gravitino at Linear Collider to Probe the SUSY Breaking Scale, hep-ph/9601362.
50. D. Toback (for the CDF Collaboration), "The Diphoton Missing E_t Distribution at CDF," FERMILAB-CONF-96-240-E.
51. H. Baer, C.-h Chen, F. Paige and X. Tata, Phys. Rev. **D49** (1994) 3283.
52. S. Dimopoulos, S. Thomas and J.D. Wells, "Sparticle Spectroscopy and Electroweak Symmetry Breaking with Gauge Mediated Supersymmetry Breaking," hep-ph/9609434; S. Dimopoulos, S. Thomas and J.D. Wells, "Implications of Low-Energy Supersymmetry Breaking at the Tevatron," hep-ph/9604452, Phys. Rev. **D54** (1996) 3283.
53. M. Dine, Y. Nir and Y. Shirman, "Variations on Minimal Gauge Mediated Supersymmetry Breaking," hep-ph/9607397.
54. M. Dine and W. Fischler, Phys. Lett. **B110** (1982) 227.
55. C.R. Nappi and B.A. Ovrut, Phys. Lett. **113B** (175) 1982.
56. N. Seiberg, "Electric-Magnetic Duality in Supersymmetric Nonabelian Gauge Theories," hep-th/941149, Nucl. Phys. **B435** (1995) 129.
57. A.G. Cohen, D.B. Kaplan and A.E. Nelson, "The More Minimal Super-

symmetric Standard Model," hep-ph/9607394.
58. R. Rohm, Nucl. Phys. **B237** (1984) 553.
59. K.S. Narain, Phys. Lett. **169B** (1986) 41; K.S. Narain, M.H. Sarmadi and E. Witten, Nucl. Phys. **B279** (1987) 369.
60. R. Rohm and E. Witten, Ann. Phys. **170** (1986) 454.
61. T. Banks and M. Dine, "Coping with Strongly Coupled String Theory," hep-th/9406132, Phys. Rev. **D50** (1994) 7454.
62. M. Dine and N. Seiberg, Phys. Lett. **162B**, 299 (1985), and in *Unified String Theories*, M. Green and D. Gross, Eds. (World Scientific, 1986).
63. M. Shifman, "Little Miracles of Supersymmetric Evolution of Gauge Couplings," hep-ph/96006281.
64. I.E. Ibanez and D. Lust, Nucl. Phys. **B382** (1992) 305;
65. V. Kaplunovsky and J. Louis, Phys. Lett. B **306** (1993) 269.
66. E. Witten, Phys. Lett. **155B** (1985) 151.
67. V. Krasnikov, Phys. Lett. **193B** (1987) 37; L. Dixon, in the *Proceedings of the DPF Meeting, Houston, 1990*; J.A. Casas, Z. Lalak, C. Munoz and G.G. Ross, Nucl. Phys. **B347** (1990) 243; T. Taylor, Phys. Lett. **B252** (1990) 59.
68. J. Louis and Y. Nir, "Some Phenomenological Implications of String Loop Effects," hep-ph/9411429, Nucl. Phys. **B447** (1995) 18.
69. R. Barbieri, J. Louis and M. Moretti, "Phenomenological Implications of Supersymmetry Breaking by the Dilaton," hep-ph/9305262, Phys. Lett. **B312** (1993) 451, Erratum – ibid **B316** (1993) 632.
70. V. Kaplunovsky, Mass Scales of String Unification, *Phys. Rev. Lett.* **55** (1985) 1036.
71. M. Dine and N. Seiberg, Couplings and Scales in Superstring Models, *Phys. Rev. Lett.* **55** (1985) 366.
72. P. Horava and E. Witten, "Heterotic and Type I String Dynamics From Eleven Dimensions", *Nucl. Phys.* **B460**, (1996) 506, hep-th/9510209.
73. E. Witten, "Strong Coupling Expansion of Calabi-Yau Compactification," hep-th/9602070, Nucl. Phys. **B471** (1996) 135.
74. T. Banks and M. Dine, "Couplings and Scales in Strongly Coupled Heterotic String Theory", hep-th/9605136.
75. K. Choi and J.E. Kim, *Phys. Lett.* **165B**, (1985), 71.
76. L. Ibanez and H.P. Nilles, Phys. Lett. **180B** (1986), 354.
77. P. Horava, "Gluino Condensation in Strongly Coupled Heterotic String Theory," hep-th/9608019.