Determining the Spin of the χ in $B \to \chi(\to \psi\gamma)K_S$

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1 Angular distributions from cartesian tensors

Riccardo Faccini has asked whether we can actually demonstrate that the χ seen in $B \to \chi(\to \psi\gamma)K_S$ is, in fact, a spin-one object. Of course the χ states are p-wave $c\bar{c}$ states and thus have $J^{PC} = 0^{++}, 1^{++}, 2^{++}$. The angular distributions in the radiative decays are conveniently calculated using cartesian tensors, as advocated in R. N. Cahn and L. S. Brown, *Phys. Rev.* **D13**, 1195 (1976). This is particularly effective because we can treat the charmonium states as non-relativistic since there is little energy available in the radiative decays. None of the result depends on the $c\bar{c}$ structure itself, though.

2 Application to radiative charmonium decays

Invariant decay amplitudes are constructed from tensors (or vectors) representing the particles. Thus the photon can be represented by E_i or B_i, while we write for the polarization vector of the ψ just ψ_i. The ψ is observed through its $\ell^+\ell^-$ decays and we can show that summing over the polarizations of the ψ gives the replacement

$$\sum_{pol} \psi_i \psi_j = \delta_{ij} - l_i l_j \tag{1}$$

where l_i is unit vector giving the direction of the lepton Similarly, summing over photon polarizations gives

$$\sum_{pol} E_i E_j \quad = \quad \delta_{ij} - k_i k_j$$

$$\sum_{pol} B_i B_j = \delta_{ij} - k_i k_j \tag{2}$$

where k_i is the unit vector giving the direction of the photon. These last two equations are just the statement that the photon polarization is perpendicular to its momentum. Of course this works just the same whether you use B or E. This means, in particular, that the angular distributions for E1 transitions and for M1 transitions are the same as long as the polarization is not measured.

3 $J(\chi) = 0$

What about the χ itself? First, suppose the χ is a scalar. Then the decay amplitude for $\chi \to \psi\gamma$ must be simply

$$\mathcal{M} \propto E_i \psi_i \tag{3}$$

Because we sum over photon polarizations, we can treat them as real. The angular distribution is thus

$$\begin{aligned} \frac{dN}{d\Omega_\gamma d\Omega_l} &\propto (\delta_{ij} - k_i k_j)(\delta_{ij} - l_i l_j) \\ &\propto 3 - 1 - 1 + (l \cdot k)^2 \\ &\propto 1 + (l \cdot k)^2 \end{aligned} \tag{4}$$

Of course the distributions relative to the direction of the χ are flat when the χ is a scalar:

$$\begin{aligned} \frac{dN}{d\Omega_{l-\chi}} &\propto 1 \\ \frac{dN}{d\Omega_{\gamma-\chi}} &\propto 1 \end{aligned} \tag{5}$$

while

$$\frac{dN}{d\Omega_{\gamma-l}} \propto 1 + \cos^2 \theta_{\gamma-l} \tag{6}$$

4 $J(\chi) = 1$

Next suppose that χ is a vector. Then it is represented by, say, χ_i. What is χ_i? The χ is produced in $B \to \chi K_S$ which itself must have a decay amplitude

$$\mathcal{M} = \chi_i z_i \tag{7}$$

where z_i is the direction of the three-momentum of the χ (or K_S) in the B rest frame. (The decay $B \to \chi K_S$ does have a significant energy release, but our argument is still correct. Write the amplitude in the rest frame of the χ.) This is one of those "what else could it be" arguments. It is the only vector quantity in the problem! If we take three orthogonal polarization states for the χ with one of them longitudinal, i.e. z_i itself, we find, of course that only the longitudinal polarization contributes. That is, $\chi_i = z_i$.

What is the decay amplitude for $\chi \to \psi\gamma$? The E1 transition has no extra factors of momentum in it:

$$\mathcal{M} = E_i \chi_j \psi_k \epsilon_{ijk} \tag{8}$$

The angular distribution is thus

$$\begin{aligned}
\frac{dN}{d\Omega_\gamma d\Omega_l} &\propto E_i \chi_j \psi_k E_m \chi_n \psi_p \epsilon_{ijk} \epsilon_{mnp} \\
&\propto (\delta_{im} - k_i k_m)(\delta_{kp} - l_k l_p) z_j z_n \epsilon_{ijk} \epsilon_{mnp} \\
&\propto (\delta_{jn}\delta_{kp} - \delta_{jp}\delta_{kn})(\delta_{kp} - l_k l_p) z_j z_n - k_i k_m (\delta_{kp} - l_k l_p) z_j z_n \epsilon_{ijk} \epsilon_{mnp} \\
&\propto 3 - 1 - 1 + (z \cdot l)^2 - k_i k_m z_j z_n (\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}) + |k \cdot (l \times z)|^2 \\
&\propto (z \cdot l)^2 + (z \cdot k)^2 + |k \cdot (l \times z)|^2
\end{aligned} \tag{9}$$

If we integrate out the direction of the χ (z) we find

$$\frac{dN}{d\Omega_{l-\gamma}} \propto 1 - \tfrac{1}{3} \cos^2 \theta_{l-\gamma} \tag{10}$$

If, on the other hand we ignore the direction k of the γ, we find

$$\frac{dN}{d\Omega_{l-\chi}} \propto 1 + \cos^2 \theta_{l-\chi} \tag{11}$$

Finally, ignoring instead the direction l of the lepton,

$$\frac{dN}{d\Omega_{\gamma-\chi}} \propto 1 + \cos^2 \theta_{\gamma-\chi} \tag{12}$$

5 $J(\chi) = 2$

If χ has spin two, it is represented by a traceless symmetric tensor χ_{ij}. Because it is produced in the decay $B \to \chi K_S$, this tensor must be built entirely out of z_i, the direction of the χ. Thus

$$\chi_{ij} \propto z_i z_j - \tfrac{1}{3}\delta_{ij} \tag{13}$$

The E1 transition is described by

$$\mathcal{M} \propto E_i \chi_{ij} \psi_j \tag{14}$$

The angular distribution is therefore

$$\begin{aligned}
\frac{dN}{d\Omega_\gamma d\Omega_l} &\propto E_i \chi_{ij} \psi_j E_m \chi_{mn} \psi_n \\
&\propto (\delta_{im} - k_i k_m)(z_i z_j - \tfrac{1}{3}\delta_{ij})(\delta_{jn} - l_j l_n)(z_m z_n - \tfrac{1}{3}\delta_{mn}) \\
&\propto (z_m z_l - k_m z \cdot k z_l - \tfrac{1}{3}\delta_{ml} + \tfrac{1}{3}k_l k_m)(z_l z_m - l_l l \cdot z z_m - \tfrac{1}{3}\delta_{lm} + \tfrac{1}{3}l_l l_m) \\
&\propto \tfrac{4}{9} - \tfrac{1}{3}(z \cdot l)^2 - \tfrac{1}{3}(z \cdot k)^2 - \tfrac{2}{3}(z \cdot l)(l \cdot k)(k \cdot z) \\
&\qquad + (k \cdot z)^2 (l \cdot z)^2 + \tfrac{1}{9}(k \cdot l)^2
\end{aligned} \tag{15}$$

Again, we can consider the partially integrated versions:

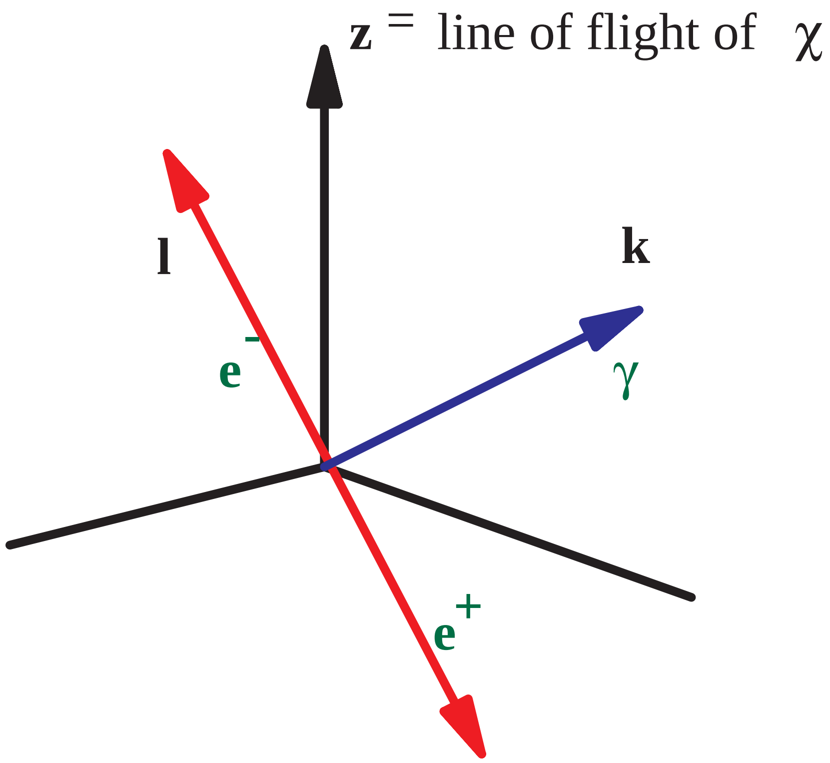
$$\frac{dN}{d\Omega_{l-\gamma}} \propto 13 + \cos^2\theta_{l-\gamma} \tag{16}$$

If, on the other hand we ignore the direction of the γ, we find

$$\frac{dN}{d\Omega_{l-\chi}} \propto 5 - 3\cos^2\theta_{l-\chi} \tag{17}$$

Finally, ignoring instead the direction of the lepton,

$$\frac{dN}{d\Omega_{\gamma-\chi}} \propto 5 - 3\cos^2\theta_{\gamma-\chi} \tag{18}$$


z = line of flight of χ
l
k
e⁻
γ
e⁺