ON THE INFLUENCE OF THE MEAN MERIDIONAL CIRCULATION ON THE ZONAL FLOW[1,2]

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ABSTRACT

The mean meridional circulation calculated earlier from observed values of transports of momentum and sensible heat by solving the zonally averaged form of the quasi-geostrophic omega equation is used to investigate the influence of this secondary flow on the zonally averaged values of the wind and temperature in the atmosphere. The contributions from the horizontal transport processes and the mean meridional circulations are computed separately in order to estimate their relative importance. It is found that the mean meridional circulation counterbalances the horizontal transport of momentum in the upper troposphere, while the two effects work in the same direction in the lower part of the atmosphere. With respect to changes in the zonally averaged temperature field, it is found that the effect of the mean meridional circulation opposes the effect of the horizontal transport of sensible heat almost everywhere.

The recent results of calculations of the mean meridional circulation are also used to discuss the role of zonal heating and friction in quasi-geostrophic models.

1. INTRODUCTION

It has been known for several years (Bristor [1], Thompson [8]) that significant errors appear in the mean zonal wind as a function of latitude in barotropic forecasts. The main error seems to be a shift of the wind maximum to the north and a general increase of the mean zonal winds in the middle latitude and a decrease in the low and high latitudes. The mean error patterns are explainable in terms of the observed horizontal momentum transport in the average as shown by Thompson [8] who also points out that as far as the vertical mean flow is concerned the compensating factor is the vertical transport of momentum near the earth's surface.

When we restrict our attention to a specific isobaric level there are several other factors which play an important role in explaining changes in the mean zonal winds. In addition to the convergences of horizontal and vertical momentum transports, it is also possible that the mean meridional circulation and frictional effects may influence the mean zonal winds. The latter effects are very difficult to estimate from observational studies because of our lack of knowledge of frictional processes in the free atmosphere and the ageostrophic nature of the mean meridional circulations. They are, however, readily obtained in numerical simulations of the general circulation as seen from the descriptions, given by Phillips [6] and Smagorinsky [7], of the results of their numerical experiments. It is also possible to estimate the influence of the mean meridional circulation from the results of short-range forecasts with a quasi-geostrophic model because such a model indirectly takes account of the mean meridional circulation. Such an attempt was made by one of the authors (Wiin-Nielsen [10]) calculating the mean meridional circulation inherent in a two-level quasi-geostrophic model using data from 85 and 50 cb.

The changes in the zonal average of the temperature at a specific isobaric level is influenced by the convergence of the horizontal transport of heat, the mean meridional circulation, and the diabatic heating. While the effect of the horizontal northward transport of sensible heat has been investigated by several investigators, as for example Wiin-Nielsen, Brown, and Drake [13, 14], we are not aware of any observational studies of the role of the mean meridional circulation on the temperature field. This effect can be evaluated easily in numerical studies of the general circulation.

A method to calculate the mean meridional circulation from observed values of the horizontal transports of momentum and heat has recently been developed. It consists of solving the quasi-geostrophic ω-equation after the equation has been averaged with respect to longitude. When this is done it is possible to express the forcing function in terms of the momentum and heat transports in addition to the terms depending on the diabatic heating and friction. The results of such a study using

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momentum and heat transports computed by Wiin-Nielsen, Brown, and Drake [14] for the 5 months of January, April, July, and October 1962 and January 1963 has recently been given by Vernekar [9] who calculated the mean meridional circulation forced by the eddy transports of momentum and heat. It is the purpose of this study to make use of the mean meridional wind component, v_Z, obtained in Vernekar's study to calculate the influence of the mean meridional circulation on changes in the mean zonal winds, and to make use of the mean zonal vertical velocity, ω_Z, to estimate the changes in the zonal mean of the temperature. The results of such calculations will be described in the following sections.

From other investigations of the mean meridional circulation by Holopainen [3] and Kung [4] it has been concluded that the effects of friction in the free atmosphere can not be neglected near the level of the jet stream. Such frictional effects are not included in the present study, but they could conceivably change the intensity of the mean meridional circulation. However, it should be pointed out that the conclusions obtained by the investigators mentioned above are based on investigations using observed winds. Errors in the wind observations near the jet stream level may lead to fictitious values of the zonal average of the meridional wind components. The question of the importance of internal friction for the mean meridional circulation is therefore not resolved, although it warrants further investigation.

Most of the investigations in this study will be based on the annual mean data for 1962. The values of v_Z and ω_Z are reproduced in tables 1 and 2, respectively, as a function of latitude and pressure.

2. CHANGES IN THE MEAN ZONAL WINDS

Although the calculations of the mean meridional circulation were carried out for 4 months during 1962, we shall in this study restrict ourselves to a consideration of the annual average for the year 1962. The average was obtained by taking a simple mean value of the 4 months mentioned above.

By averaging the first equation of motion with respect to longitude, we obtain:

$$\frac{\partial u_Z}{\partial t}=-\frac{1}{a\cos^2\varphi}\frac{\partial[(u_E v_E)_Z\cos^2\varphi]}{\partial\varphi}+fv_Z$$
$$-\frac{1}{a\cos^2\varphi}\frac{\partial[u_Z v_Z\cos^2\varphi]}{\partial\varphi}-\frac{\partial u_Z\omega_Z}{\partial p}$$
$$-\frac{\partial(u_E\omega_E)_Z}{\partial p}+F_{x,Z}\qquad(2.1)$$

in which a subscript Z indicates a zonal average, i.e.

$$(\quad)_Z=\frac{1}{2\pi}\int_0^{2\pi}(\quad)d\lambda,\qquad(2.2)$$

TABLE 1.—*Annual average of v_Z based on data from January, April, July, and October 1962. Unit: cm. sec.$^{-1}$*

Lat.	88.75 cb.	68.75 cb.	50.00 cb.	32.50 cb.	12.50 cb.
87.5°	−4.7	−0.7	1.9	7.9	−1.5
85.0	−6.7	−1.1	2.5	11.1	−1.9
82.5	−8.1	−1.5	2.9	13.5	−2.0
80.0	−8.9	−1.8	2.9	14.7	−1.9
77.5	−8.6	−1.9	2.5	14.5	−1.7
75.0	−7.3	−1.5	2.0	12.8	−1.6
72.5	−4.7	−0.7	1.3	9.7	−2.2
70.0	−0.9	1.0	0.6	5.3	−3.6
67.5	4.0	3.3	0.2	−0.2	−5.9
65.0	9.6	6.3	0.1	−6.2	−9.4
62.5	15.6	9.7	0.3	−12.3	−13.7
60.0	21.5	13.2	0.8	−17.9	−18.5
57.5	26.5	16.6	1.7	−22.4	−23.3
55.0	30.2	19.2	2.6	−25.3	−27.5
52.5	32.0	20.9	3.6	−26.4	−30.4
50.0	31.8	21.1	4.3	−25.5	−31.6
47.5	29.3	20.0	4.8	−22.5	−30.8
45.0	24.9	17.5	4.8	−17.9	−27.7
42.5	18.8	13.7	4.4	−12.0	−22.8
40.0	4.7	9.1	3.7	−5.7	−16.4
37.5	1.8	4.2	2.6	0.2	−9.3
35.0	−2.4	−0.6	1.3	6.1	−2.1
32.5	−8.1	−4.6	0.0	10.2	4.4
30.0	−12.1	−7.6	−1.1	12.3	9.6
27.5	−14.3	−9.4	−2.0	13.2	13.1
25.0	−14.5	−9.8	−2.5	12.1	14.6
22.5	−13.1	−9.0	−2.7	9.6	14.4
20.0	−10.5	−7.3	−2.6	6.4	12.8

TABLE 2.—*Annual average of ω_Z based on the same data as table 1. Unit: 10^{-5} mb. sec.$^{-1}$*

Lat.	77.5 cb.	60.0 cb.	40.0 cb.	25.0 cb.
87.5°	3.8	4.3	3.0	−1.4
85.0	3.5	4.0	2.8	−1.1
82.5	3.0	3.4	2.6	−0.7
80.0	2.2	2.6	2.1	−0.3
77.5	1.3	1.5	1.3	−0.2
75.0	0.1	0.0	0.1	−0.3
72.5	−1.3	−1.7	−1.4	−0.7
70.0	−2.6	−3.6	−3.2	−1.5
62.5	−3.9	−5.4	−5.1	−2.5
65.0	−4.9	−6.9	−6.8	−3.6
62.5	−5.5	−8.0	−8.2	−4.6
60.0	−5.7	−8.4	−8.8	−5.3
57.5	−5.3	−7.9	−8.6	−5.5
55.0	−4.4	−6.7	−7.5	−5.1
52.5	−2.9	−4.7	−5.5	−4.1
50.0	−1.2	−2.1	−2.7	−2.5
47.5	0.8	0.8	0.4	−0.6
45.0	2.3	3.6	3.4	1.5
42.5	4.1	6.0	6.2	3.4
40.0	5.2	7.7	8.2	5.0
37.5	5.6	8.5	9.2	5.9
35.0	5.4	8.3	9.2	6.2
32.5	4.7	7.2	8.1	5.8
30.0	3.4	5.4	6.2	4.8
27.5	2.0	3.2	3.8	3.3
25.0	0.4	0.8	1.3	1.7
22.5	−0.9	−1.3	−1.1	0.1
20.0	−1.9	−2.9	−2.9	−1.2

while a subscript E indicates a deviation from the zonal average, i.e.

$$(\quad)_E=(\quad)-(\quad)_Z.\qquad(2.3)$$

The other notations in (2.1) follow the standards. It should be pointed out that the effects of the first two terms on the right hand side of (2.1) are included in a quasi-geostrophic model because the first term is part of the advection of relative vorticity, and the second term is part of the divergence term in the vorticity equation. Since our calculations of the mean meridional circulation are based on a quasi-geostrophic, adiabatic, and frictionless model, we shall be concerned mainly with these two terms while the remaining terms are assumed to be small in a quasi-geostrophic formulation. It should be noted that the third and fourth term in (2.1) can be computed knowing the mean meridional circulation. Such a calculation will be described later in this section. In an earlier investigation by Wiin-Nielsen [2], it was shown that the contribution from the vertical advection of momentum was one order of magnitude smaller than the contribution from either the first or the second term in (2.1) when the calculations were based on a two-parameter model. We shall also make some comments on the frictional term $F_{x,Z}$.

It is convenient to measure the influence of the first two terms in (2.1) by introducing the following notations:

$$\Delta u_{Z,H} = -\frac{1}{a\cos^2\varphi}\frac{\partial[(u_E v_E)_Z\cos^2\varphi]}{\partial\varphi}\cdot\Delta t \tag{2.4}$$

and

$$\Delta u_{Z,M} = f v_Z \cdot \Delta t, \tag{2.5}$$

where $\Delta u_{Z,H}$ and $\Delta u_{Z,M}$ are the changes in the mean zonal winds which would occur if the processes expressed on the right hand sides of (2.4) and (2.5) acted alone through the time interval Δt.

The quantities (2.4) and (2.5) were computed for each of the 4 months during 1962. Figure 1 shows $\Delta u_{Z,H}$ obtained as an average for the months expressed in the unit: m. sec.$^{-1}$ day^{-1}. We notice that the main effect of the horizontal eddy momentum transport is to increase the zonal winds in a latitude interval from about 35°N. to about 65°N. The maximum increase occurs at 30 cb. with a value of almost 5 m. sec.$^{-1}$ day^{-1}. There is a decrease of the zonal winds due to this effect of almost equal magnitude south of 35°N. with the maximum decrease occurring between 30 and 20 cb. We note finally that there is also a decrease in the zonal winds north of 65°N., but this decrease does not exceed 2 m. sec.$^{-1}$ day^{-1}. Figure 1 may be compared with similar figures constructed by Wiin-Nielsen, Brown, and Drake [13] for the month of January 1962.

Figure 2 shows the distribution of $\Delta u_{Z,M}$ as a function of latitude and pressure in the unit m. sec.$^{-1}$ day^{-1} using a similar procedure as in the computations leading to figure 1. Since the calculations of v_Z made in Vernekar's study [9] resulted in a very pronounced and regular three-cell circulation, we naturally find the same distribution here. A comparison between figure 1 and figure 2 shows that $\Delta u_{Z,H}$ and $\Delta u_{Z,M}$ oppose each other in the upper troposphere in almost all latitudes with $\Delta u_{Z,M}$ being numerically smaller everywhere. On the other hand, $\Delta u_{Z,H}$ and $\Delta u_{Z,M}$ have in general the same sign in the lower part of the troposphere with the numerically larger values obtained from $\Delta u_{Z,M}$ especially in the middle latitudes.

The sum of the two contributions is shown in figure 3 which shows a general increase of the zonal wind speeds in the middle latitudes and a decrease in the low and high latitudes. We may consider figure 3 as showing the typical errors which would be obtained in numerical predictions based on a multi-level quasi-geostrophic model which does not include friction. Such forecasts will still have an error distribution qualitatively of the same kind as the simple barotropic model with an increase in the jet stream in the middle latitudes, especially at the lower levels, and a decrease of the mean zonal winds at low latitudes, in particular at the higher levels. There is also a decrease in the high latitudes, but it is smaller than the decrease found in the low latitudes. The fact that error patterns such as those displayed in figure 3 occur is naturally due to the importance of the remaining terms in (2.1). Staying with the quasi-geostrophic theory, we neglect the third, fourth, and fifth terms in (2.1). Expressing the frictional term in the form $g\partial\tau_{x,Z}/\partial p$ it is then possible to calculate the surface stress from (2.1) by integrating with respect to pressure and assuming a steady state. The surface stress, $\tau_{x,Z}$ calculated in this way, is shown in figure 4 using the unit: dynes cm.$^{-2}$ The distribution of the stress with respect to latitude agrees fairly well in the form of the curve with values computed in a similar way by Mintz [5], but our values are about 40

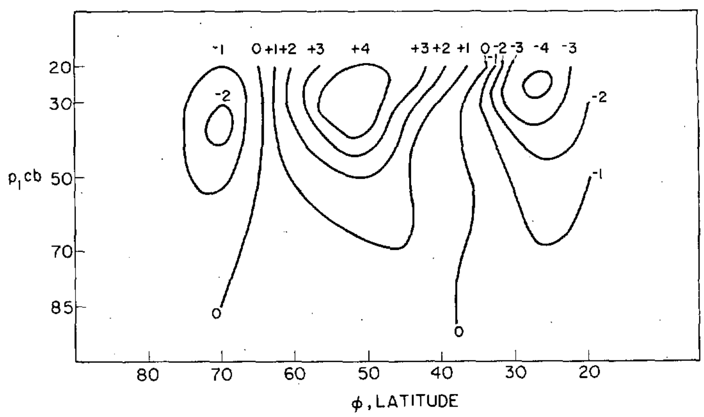

FIGURE 1.—Changes in the mean zonal wind caused by the convergence of the horizontal eddy transport of momentum. Unit: m. sec.$^{-1}$ day^{-1}. Average for the year 1962.

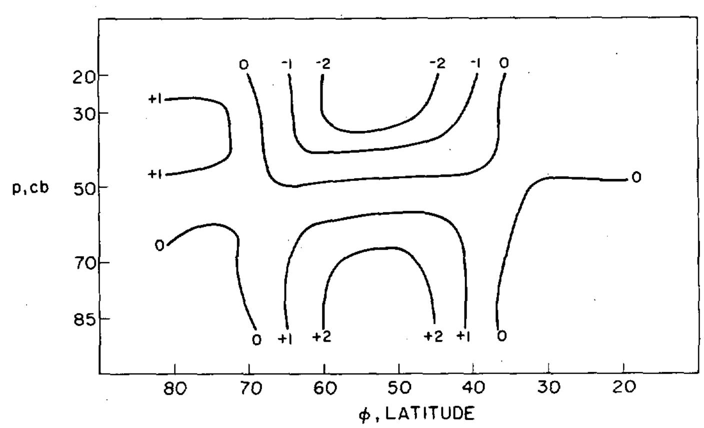

FIGURE 2.—Changes in the mean zonal wind caused by the Coriolis effect. Unit: m. sec.$^{-1}$ day^{-1}. Average for the year 1962.

percent larger than his values in the middle latitudes. It is believed that the reason for this discrepancy is the larger values of the momentum transport which is obtained on the basis of the objective analyses produced by the National Meteorological Center (NMC). Such a discrepancy has been noted earlier, but it is beyond the scope of this paper to investigate the apparent systematic errors in the NMC analysis procedures. However, figure 4 shows the stress which must be included in a multi-level quasi-geostrophic model in order to maintain an annual steady state in the zonal winds as a function of latitude. Figure 4 may furthermore be compared with the stress values obtained by Smagorinsky [7] in his general circulation experiment. The qualitative agreement is good, but we find again a discrepancy in magnitude (see fig. 10.11 in Smagorinsky's paper).

It is furthermore of interest to compare the contribution Δu_{ZM}, found in this study with the values obtained in numerical experiments. Since the published numerical experiments have been carried out using two-level models, we have found it necessary to average our results for the layers above and below 50 cb. Since the vertical average of v_Z is zero and v_Z is almost zero at 50 cb. in these calculations it is only necessary to consider the upper layer. Figure 5 shows Δu_{ZM} in m. sec.$^{-1}$ day^{-1} averaged for the layer from 50 to 20 cb. as the solid curve. The dashed curve is the corresponding quantity obtained by Smagorinsky [7], while the dashed-dotted curve is taken from the study by Phillips [6]. We notice that there is general agreement in the sign of Δu_{ZM} realizing that the dashed curve artificially goes to zero at 64°N. due to the design of the numerical experiments. It has been pointed out earlier by Wiin-Nielsen [1] that the eddy available potential energy and the eddy kinetic energy are much smaller in Smagorinsky's experiment than the observed mean values. Since the northward transport of sensible heat and momentum also are smaller in the numerical experiment than in the atmosphere it is understandable that the intensity of the mean meridional circulation is weaker in Smagorinsky's experiment than in our study. The reason that the intensity of the mean meridional circulation is stronger in Phillips' experiment than in our calculation is most likely that the period over which his averages are formed does not represent a steady state, but a period during which there is a building up of a zonal wind maximum.

In spite of the fact that our calculation of the mean meridional circulation is based on a quasi-geostrophic formulation, it is interesting to investigate the contributions from the third and fourth term on the right hand side of equation (2.1). The period which we have selected for this part of the study is January 1964. The mean zonal wind as a function of latitude and pressure is given in figure 6 which shows a maximum (the subtropical jet stream) of slightly more than 40 m. sec.$^{-1}$ at 20 cb. and 30°N. A secondary maximum (the polar jet stream) of about 20 m. sec.$^{-1}$ occurs at 10 cb. and about 65°N.

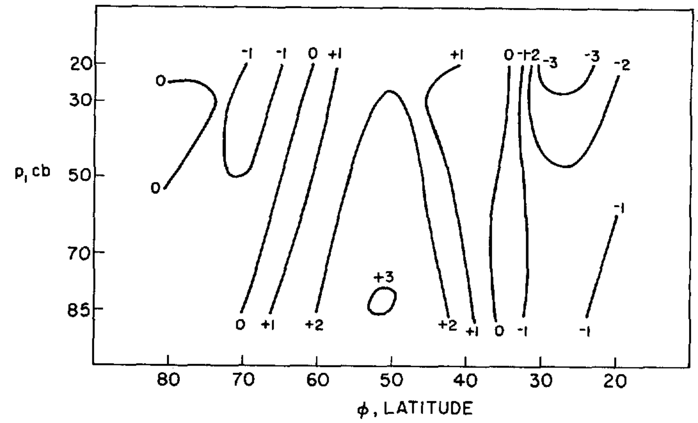

FIGURE 3.—Changes in the mean zonal wind caused by the combined effect of horizontal eddy momentum transport and the Coriolis effect. Unit: m. sec.$^{-1}$ day^{-1}. Average for the year 1962.

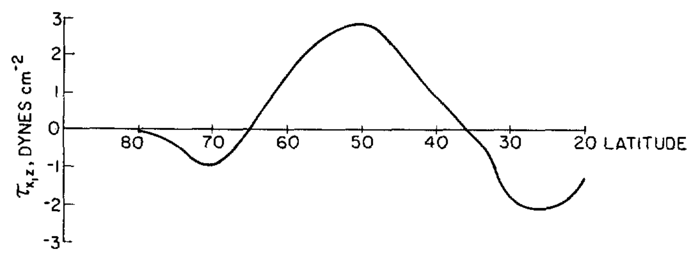

FIGURE 4.—Computed surface stress as a function of latitude. Unit: dynes cm.$^{-2}$ Average for 1962.

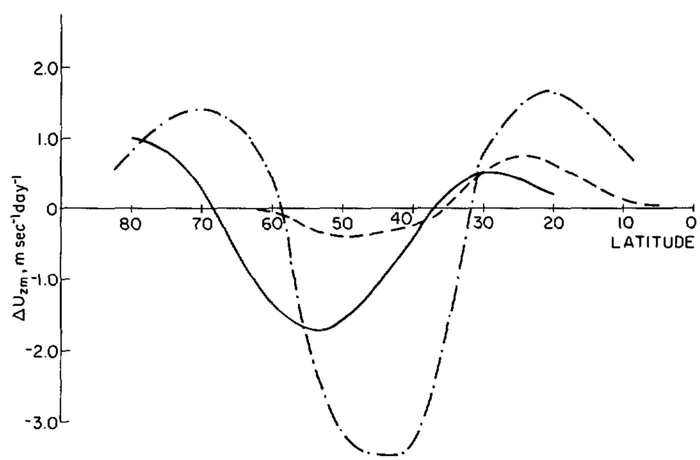

FIGURE 5.—Changes in the mean zonal wind caused by the Coriolis effect for the layer 20–50 cb. (solid curve), at the upper level in Smagorinsky's [7] model (dashed curve), and at the upper level in Phillips' [6] model (dashed-dotted curve). Unit: m. sec.$^{-1}$ day^{-1}.

It is the maintenance of this wind system which is governed by equation (2.1).

To get a preliminary idea of the importance of the mean meridional circulation as compared to the influence of the eddies, we have prepared figures 7 and 8 which show the momentum transports by the eddies $(u_E v_E)_Z$ and by the mean meriodional circulation $(u_Z v_Z)$, respectively. Figure

8 was prepared using the monthly mean values of u_Z and v_Z. However, an inspection of the mean meridional circulation for the individual days (Vernekar [9]) shows that the general nature of the mean meridional circulation was maintained throughout the month. We notice first of all that the momentum transport by the mean meridional circulation, $u_Z v_Z$, is about one order of magnitude smaller than the momentum transport by the eddies, $(u_E v_E)_Z$. The largest values of the former occur at 20 cb. and about 40°N. and amount to 10 m.^{2} sec.$^{-2}$, while the largest values of the latter occur at the same pressure, but somewhat farther to the south (35°N.). This maximum is about 100 m.^{2} sec.$^{-2}$ The momentum transport, $(u_E v_E)_Z$, displayed in figure 7 is quite similar to the annual mean value for the year 1962, although the values are larger in winter than for the annual mean. It should be noticed that since u_Z (fig. 6) is positive almost everywhere except at low elevations in the very high and very low latitudes, the sign of $u_Z v_Z$ is determined mainly by v_Z. Since v_Z turns out to indicate the familiar three-cell circulation we can immediately recognize these patterns in figure 8 with the largest values occurring in the high troposphere between 25°N. and 45°N. A comparison of figure 6 and 8 shows that the momentum transport, $u_Z v_Z$, is arranged in such a way around the subtropical jet stream that it will tend to increase the jet stream, because the transport is to the south, north of the jet stream, but to the north, south of the wind maximum. We shall investigate this question in detail later in this section.

The changes in u_Z, expressed in the unit m. sec.$^{-1}$ day^{-1}, due to the convergence of the horizontal eddy momentum transport for January 1964 is given in figure 9 showing the characteristic increase in middle latitudes with a decrease in the low and high latitudes. A comparison of figure 9 and figure 1 shows great similarity in patterns. The influence of the term fv_Z on changes in u_Z for January 1964 is shown in figure 10, which shows great similarity to figure 2 which applies to the annual average for the year 1962. The combined effect of the convergence of the horizontal momentum transport and the Coriolis term is given in figure 11 which shows patterns similar to figure 3.

We shall next proceed to investigate the changes in the zonal wind caused by those effects which are excluded in a quasi-geostrophic model. These effects are expressed by the third and fourth terms on the right hand side of equation (2.1). Our first evaluation is of the convergence of the horizontal momentum transport by the mean meridional circulation. The result of this calculation is given in figure 12 for January 1964 in the unit: m. sec.$^{-1}$ day^{-1}. As expected from an inspection of figure 8 we find the largest contribution at 20 cb. close to the wind maximum (30–35°N.) where the change in u_Z amounts to about 1.4 m. sec.$^{-1}$ day^{-1}. The major maximum in figure 12 occurs in a region in which the contributions expressed in figures 9 and 10 are small. We notice furthermore that the contributions from the convergence of the horizontal

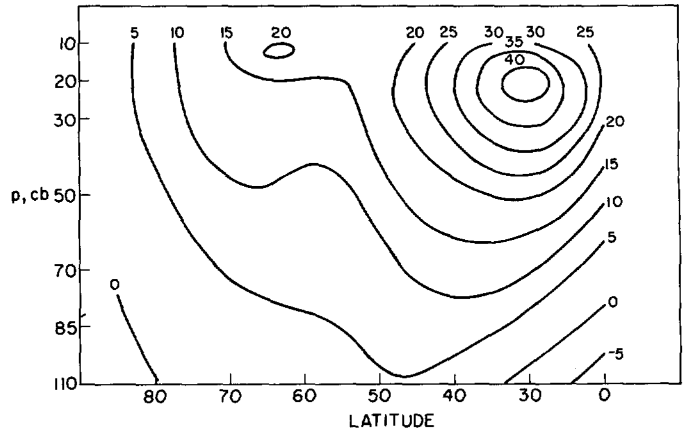

FIGURE 6.—The mean zonal wind as a function of latitude and pressure. Unit: m. sec.$^{-1}$ January 1964.

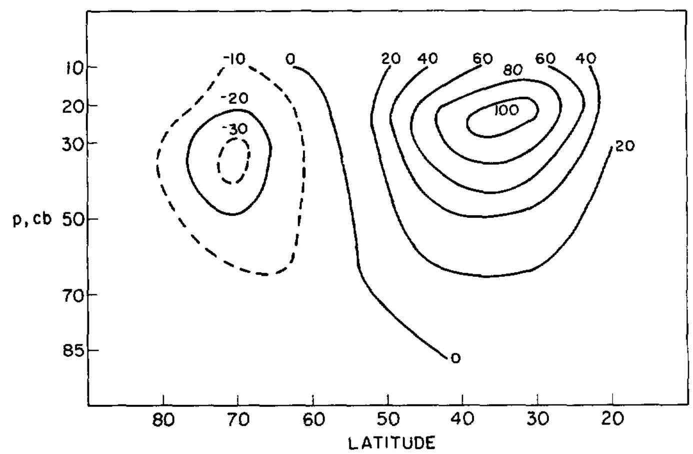

FIGURE 7.—The horizontal momentum transport by the eddies. Unit: m.^{2} sec.$^{-2}$ January 1964.

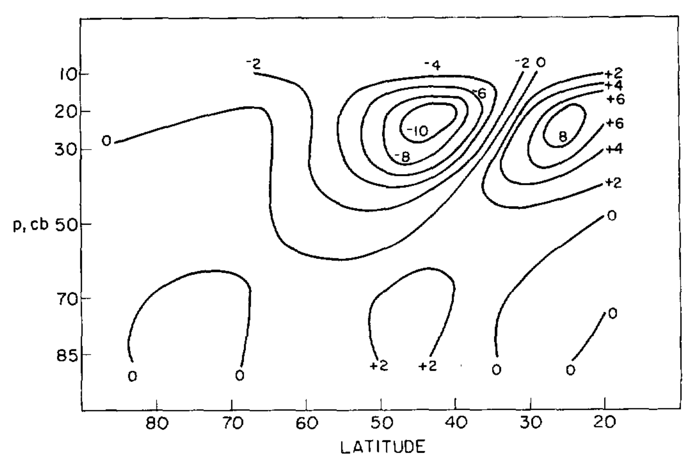

FIGURE 8.—The horizontal momentum transport by the mean meridional circulation. Unit: m.^{2} sec.$^{-2}$ January 1964.

momentum transport $u_Z v_Z$ at other places act in the same direction as the Coriolis effect fv_Z although the latter effect in general is much larger. The contributions displayed in figure 12 are small in the lower part of the troposphere. Based on figure 12 one is tempted to conclude that the momentum transport by the mean meridional circulation plays a minor role except close to the subtropical jet stream where the contributions are of a magnitude which is significant compared to other terms. However, such a conclusion is of smaller significance if the changes in u_Z displayed in figure 12 to a large extent are canceled by other contributions.

The changes in u_z caused by the convergence of the vertical transport of momentum, $u_z\omega_z$, are shown in figure 13. We note that the general order of magnitude of the changes shown in this figure is comparable to those shown in figure 12, but there is a strong tendency for the two contributions to be of opposite sign. Note, in particular, that the largest positive contribution in figure 12 at 20 cb. and 30–35°N. is opposed by an equally large negative contribution in the same region in figure 13. To illustrate this point we have prepared figure 14 which shows the changes in u_z at 20 cb. caused by the convergence of the horizontal transport of momentum by the eddies (solid curve), the convergence of the horizontal transport of momentum by the mean meridional circulation (dashed curve), and the convergence of vertical transport of momentum by the mean meridional circulation (dashed-dotted curve). Based on figures 12, 13, and 14, we conclude that the terms which are excluded in the quasi-geostrophic theory in general are small except in the region of the subtropical jet stream where they have a tendency to cancel. It is impossible to evaluate one of the terms, the convergence of the vertical transport of momentum by the eddies, from the available data. In view of the fact that our knowledge of friction in the free atmosphere is limited, we are not yet in a position to complete a study of the total momentum budget for the zonally averaged winds. Such a study could conceivably be made using an extended time period and a detailed calculation of all components of the vertical velocity.

3. CHANGES IN THE ZONALLY AVERAGED TEMPERATURE

A technique similar to the one which was applied in section 2 to investigate changes in the mean zonal winds can be applied to compute changes in the zonal average of temperature. The basic equation is the thermodynamic equation. In this paper we are mostly interested in the role played by the mean meridional circulation. Since our calculation of ω_z and v_z is based on a quasi-geostrophic model we should for consistency assume that the horizontal wind is non-divergent and that our measure of static stability is a function of pressure only. However, as in the case of the mean zonal wind it is interesting to calculate the additional terms which depend entirely on the mean meridional circulation.

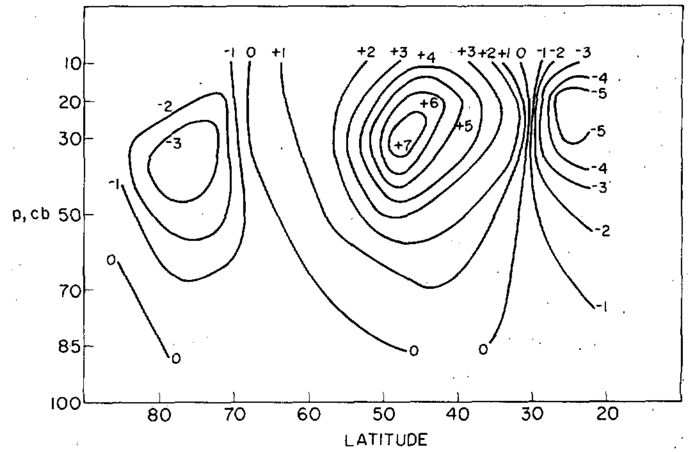

FIGURE 9.—Same as figure 1, but for January 1964.

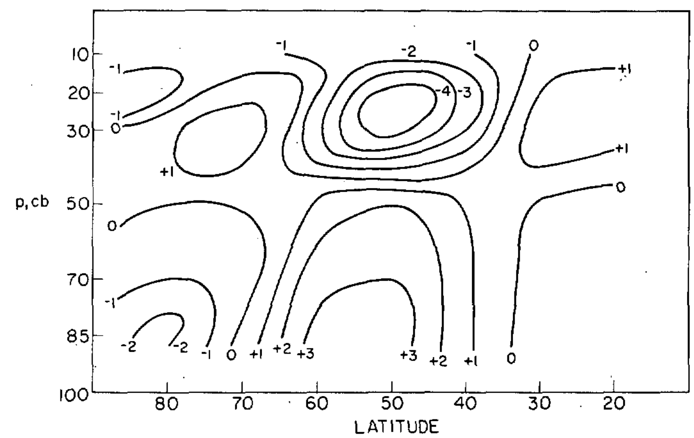

FIGURE 10.—Same as figure 2, but for January 1964.

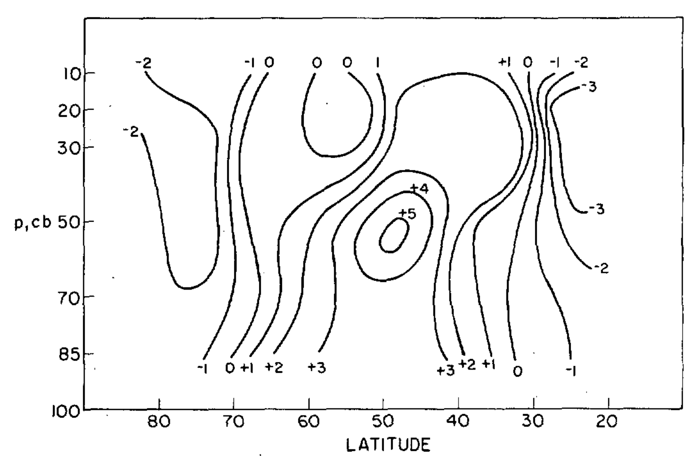

FIGURE 11.—Same as figure 3, but for January 1964.

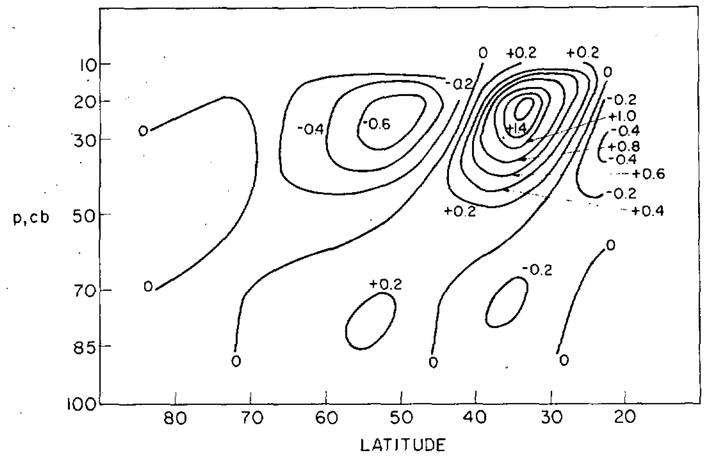

FIGURE 12.—Changes in the mean zonal wind caused by the convergence of the horizontal transport of momentum by the mean meridional circulation. Unit: m. sec.$^{-1}$ day^{-1}. January 1964.

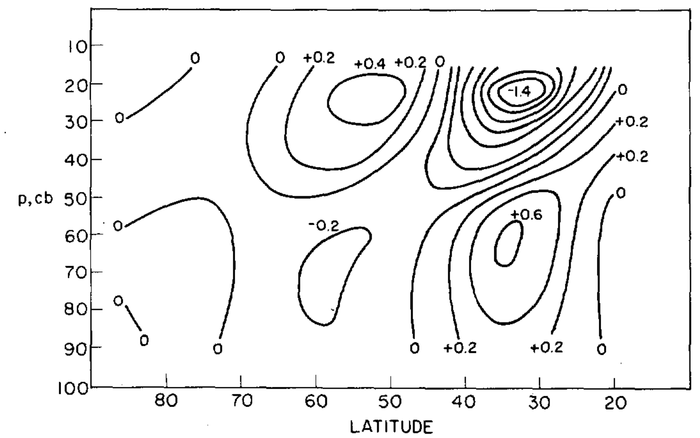

FIGURE 13.—Changes in the mean zonal wind caused by the convergence of the vertical transport of momentum by the mean meridional circulation. Unit: m. sec.$^{-1}$ day^{-1}. January 1964.

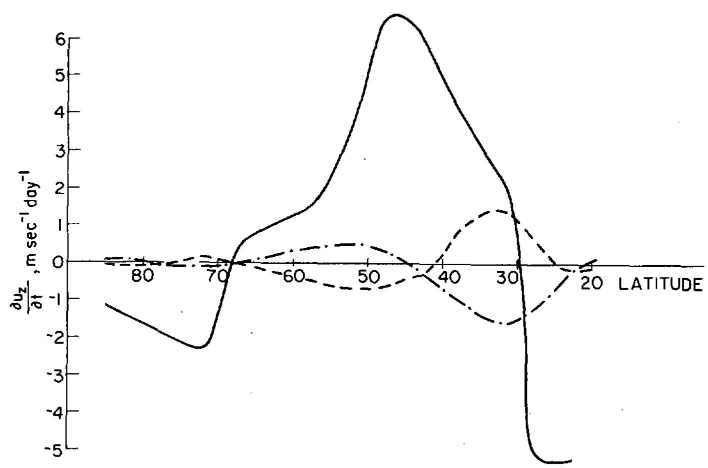

FIGURE 14.—Changes in the mean zonal wind as a function of latitude at 20 cb. due to the convergence of the horizontal eddy momentum transport (solid curve), the convergence of the horizontal momentum transport by the mean meridional circulation (dashed curve) and the converegence of the vertical transport of momentum by the mean meridional circulation (dashed-dotted curve). Unit: m. sec.$^{-1}$ day^{-1}. January 1964.

The thermodynamic equation may be written in the following form:

$$\frac{\partial \alpha}{\partial t}+\frac{u}{a\cos\varphi}\frac{\partial \alpha}{\partial \lambda}+\frac{v}{a}\frac{\partial \alpha}{\partial \varphi}-\sigma\omega=\frac{R}{c_p}\frac{1}{p}\frac{dQ}{dt}, \tag{3.1}$$

in which α is the specific volume, dQ/dt the diabatic heating per unit mass and unit time, and $\sigma=-\alpha\partial\ln\theta/\partial p$ a measure of static stability. When (3.1) is averaged with respect to longitude we obtain the following equation:

$$\begin{aligned}\frac{\partial \alpha_Z}{\partial t}=&-\frac{1}{a\cos\varphi}\frac{\partial(\alpha_E v_E)_Z\cos\varphi}{\partial\varphi}+\sigma_Z\omega_Z\\ &-\frac{1}{a\cos\varphi}\frac{\partial\alpha_Z v_Z\cos\varphi}{\partial\varphi}-\alpha_Z\frac{\partial\omega_Z}{\partial p}\\ &-\left(\alpha_E\frac{\partial\omega_E}{\partial p}\right)_Z+(\sigma_E\omega_E)_Z+\frac{R}{c_p}\frac{1}{p}\left(\frac{dQ}{dt}\right)_Z.\end{aligned} \tag{3.2}$$

The terms on the right hand side of (3.2) fall into several categories. The two terms in the first line are incorporated in a quasi-geostrophic theory. They can be computed from the available data and these calculations will be described below. The next two terms appearing in the second line are the result of incorporating the advection by the divergent part of the wind. They are not present in the quasi-geostrophic theory, but may still be computed from our data on the mean meridional circulation. The three terms in the last line of (3.2) can not be computed from our data either because a knowledge of ω_E is required or because the diabatic heating $(dQ/dt)_Z$ has to be known.

The contributions from the terms in the second line would be straightforward to compute knowing the zonally averaged thicknesses h_Z of the various layers, because α_Z can be related to h_Z through the hydrostatic equation. During our calculations we did not include a computation of h_Z. We have therefore found it necessary to calculate these terms by relating them to the vertical wind shear through the use of the geostrophic thermal wind equation. Using first the zonally averaged continuity equation we obtain

$$-\left[\frac{1}{a\cos\varphi}\frac{\partial\alpha_Z v_Z\cos\varphi}{\partial\varphi}+\alpha_Z\frac{\partial\omega_Z}{\partial p}\right]=-\frac{v_Z}{a}\frac{\partial\alpha_Z}{\partial\varphi}, \tag{3.3}$$

but the geostrophic thermal wind equation may be written in the form:

$$\frac{\partial u_Z}{\partial p}=\frac{1}{fa}\frac{\partial\alpha_Z}{\partial\varphi}. \tag{3.4}$$

Substituting (3.4) in (3.3), we therefore obtain:

$$-\frac{v_Z}{a}\frac{\partial\alpha_Z}{\partial\varphi}=-fv_Z\frac{\partial u_Z}{\partial p}. \tag{3.5}$$

Using (3.5) and $\alpha_Z=(RT_Z)/p$, we may finally write (3.2) in the form:

$$\frac{\partial T_Z}{\partial t} = -\frac{1}{a\cos\varphi}\frac{\partial (T_E v_E)_Z \cos\varphi}{\partial \varphi} + \frac{p}{R}\sigma_Z\omega_Z - \frac{p}{R} f v_Z \frac{\partial u_Z}{\partial p} - \left(T_E \frac{\partial \omega_E}{\partial p}\right)_Z + \frac{p}{R}(\sigma_E\omega_E)_Z + \frac{1}{c_p}\left(\frac{dQ}{dt}\right)_Z \quad (3.6)$$

The contributions from the first three terms on the right hand side of (3.6) to changes in T_Z will be described in the following sections. The available data on the horizontal transport of sensible heat can be used to calculate the first term in (3.6). We express this contribution in the form:

$$\Delta T_{ZH} = -\frac{\Delta t}{a\cos\varphi}\frac{\partial (T_E v_E)_Z \cos\varphi}{\partial\varphi}. \quad (3.7)$$

Figure 15 shows ΔT_{ZH} in the unit: deg. day^{-1} for the annual average 1962 as a function of latitude and pressure. This figure may be compared with a similar figure for January 1962 published by Wiin-Nielsen, Brown, and Drake [13]. We observe the well known result that the horizontal heat transport tends to increase the temperature in the high latitudes and decrease the temperature in the low latitudes. The largest contributions are found in the middle and lower part of the troposphere with the absolute magnitude amounting to 1 deg. day^{-1}.

The contribution from the second term in (3.6) is expressed in the form:

$$\Delta T_{ZM} = \Delta t \frac{p}{R}\sigma_Z\omega_Z \quad (3.8)$$

and is displayed in figure 16 using the same unit and arrangement as in figure 15. Since σ_Z in our formulation is everywhere positive it follows that ΔT_{ZM} will have the same sign as ω_Z. The effect of the three-cell meridional circulation can clearly be seen in figure 16. The numerically large values are found in the upper troposphere with more than 0.6 deg. day^{-1} at 35°N. and about −0.5 deg. day^{-1} at 60°N. One of the reasons that ΔT_{ZM} has a maximum at the higher elevations is naturally that the variable part of the coefficient to ω_Z in (3.8) increases with height. A comparison of figures 15 and 16 shows that the two terms incorporated in an adiabatic, quasi-geostrophic model, i.e. ΔT_{ZH} and ΔT_{ZM}, are of about equal importance in changing the zonally averaged temperature. It is, however, also obvious that the two effects do not cancel each other.

The sum of the terms, ΔT_{ZH} and ΔT_{ZM}, is shown in figure 17 indicating that positive temperature changes will be found in the high latitudes (essentially north of 50°N.), while negative temperature changes will be observed in the major part of the region south of 50°N. with the exception of a small region at about 35°N. in the high troposphere. The quantity displayed in figure 17 may with a change in sign be considered as the diabatic heating which has to be introduced in a quasi-geostrophic model in order to have vanishing changes in the zonally averaged temperature. Considered in this way we observe that cooling takes place in the higher latitudes with the numerically largest values at the lower elevations. A considerable amount of heating occurs in the lower latitudes at lower elevations, while cooling apparently is necessary in the upper troposphere in the same latitude band. The gross features of figure 17 agree reasonably well with other knowledge concerning diabatic heating of the troposphere. The vertical average of the pattern in figure 17 corrected in such a way that the area average of the diabatic heating vanishes is shown in figure 18. This figure may be compared with similar figures calculated by Brown [2].

Our interpretation of the sum of ΔT_{ZH} and ΔT_{ZM} as a diabatic heating in the meridional plane is only approximately correct provided the additional terms in (3.6) are small. In order to investigate part of this question we shall proceed to compute the third term in (3.6). This calculation is based on data from January 1964. The temperature changes due to this effect and expressed in the unit: deg. day^{-1} are shown in figure 19. Over most of the meridional plane we find temperature changes which are small compared with the changes due to the horizontal convergence of the transport of sensible heat by the eddies. This effect is shown in figure 20 based on data from the same time period. A comparison of figures 19 and 20 shows that the only region in which the transport by the mean meridional circulation is relatively large is at lower elevations in the middle latitudes. The relatively large temperature changes appearing in figure 19 in this region are connected with the strength of the Ferrel cell combined with a large horizontal temperature gradient at the lower altitudes. However, even in this region there will only be minor modifications. We may therefore conclude that our earlier interpretation of $\Delta T_{ZH}+\Delta T_{ZM}$ as a measure of the intensity of diabatic heating is approximately correct within the framework of the quasi-geostrophic theory. We mention finally that the remaining terms in equation (3.6) cannot be computed directly from our data.

4. CONCLUDING REMARKS

The main purpose of this investigation has been to study the changes in the mean zonal winds and the mean zonal temperature field using recent calculations of the mean meridional circulation based on observations. It has thus been possible to measure the relative importance of the mean meridional circulation as compared to the effects of the horizontal eddy transports. With respect to the changes in the mean zonal winds, it is found that the effects included in a quasi-geostrophic approximation without friction will result in an increase of the zonal winds in middle latitudes (35–65° N.) with a decrease in the low and high latitudes. Part of the errors in quasi-geostrophic forecasts may be removed by including fric-

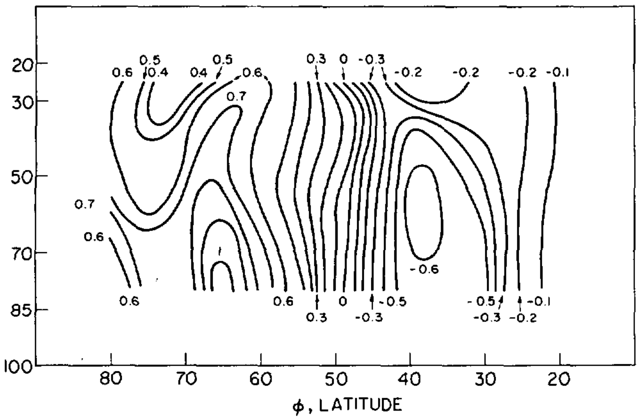

FIGURE 15.—Changes in the mean zonal temperature caused by the convergence of the horizontal eddy transport of sensible heat. Unit: deg. day^{-1}. Average for the year 1962.

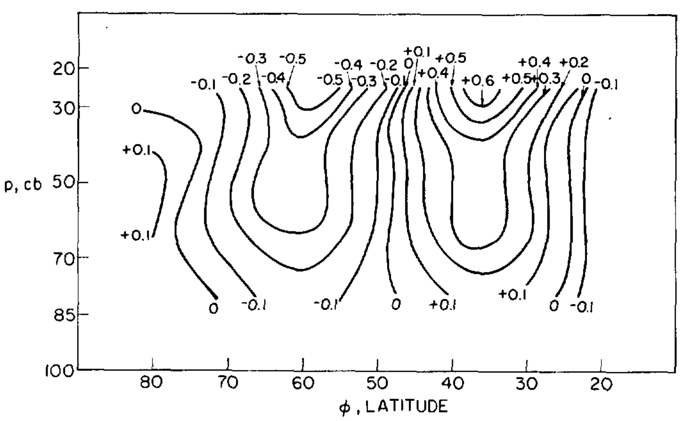

FIGURE 16.—Changes in the mean zonal temperature caused by the mean meridional circulation through adiabatic expansion. Unit: deg. day^{-1}. Average for the year 1962.

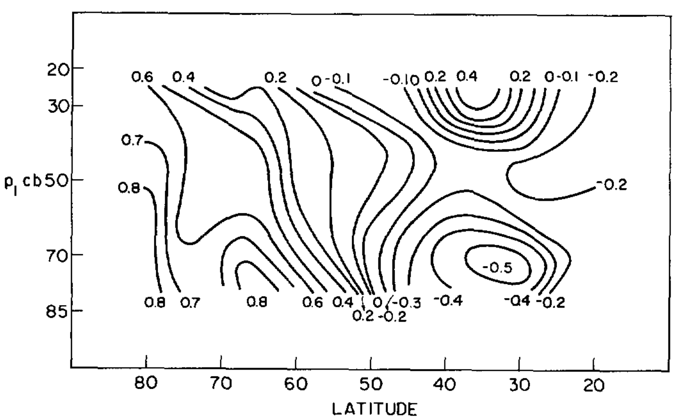

FIGURE 17.—Changes in the mean zonal temperature caused by the combined effect of horizontal eddy transport of sensible heat and the mean meridional circulation. Unit: deg. day^{-1}. Average for the year 1962.

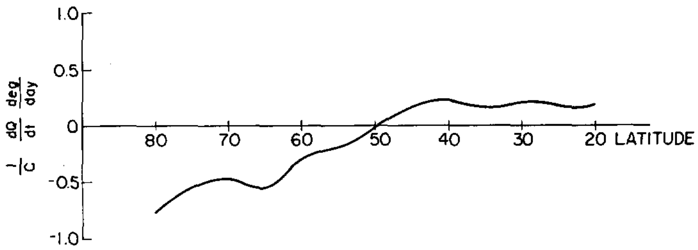

FIGURE 18.—Vertical average of diabatic heating as a function of latitude. Unit: deg. day^{-1}. Average for the year 1962.

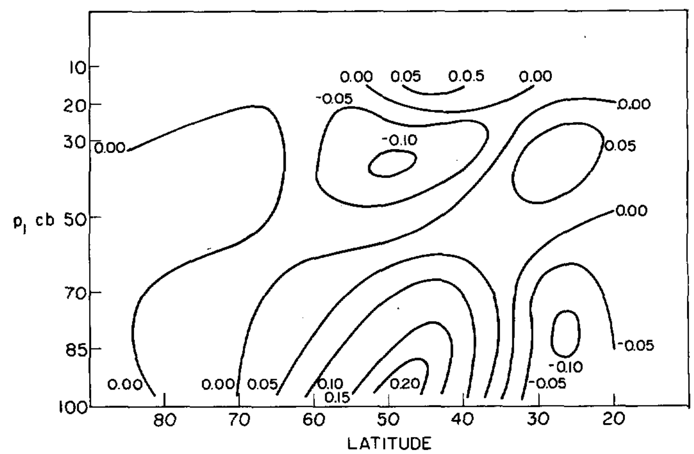

FIGURE 19.—Changes in the mean zonal temperature caused by the convergence of the horizontal transport of sensible heat by the mean meridional circulation. Unit: deg. day^{-1}. January 1964.

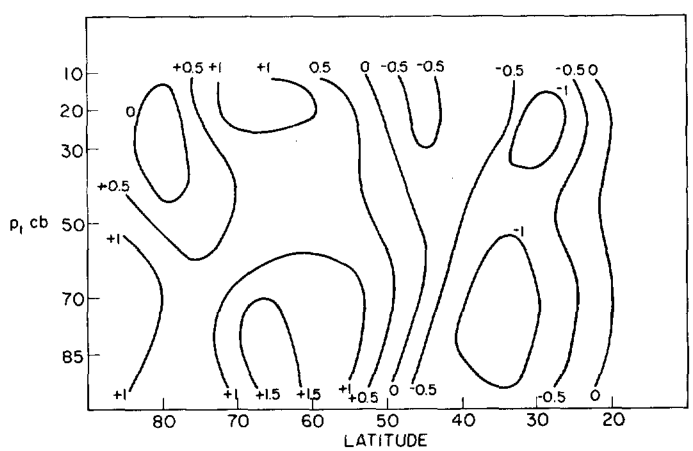

FIGURE 20.—Same as figure 15, but for January 1964.

tion. A study of the effects excluded in the quasi-geostrophic theory shows that they in general are small except in the region of the subtropical jet stream where meridional circulation and the vertical transport of momentum are of some importance although there is a tendency for cancellation between the two effects. The latter conclusions are based on data from January 1964 only.

The main error in forecasts based on an adiabatic, quasi-geostrophic model in the zonally averaged temperature field is a decrease of the temperature in the lower latitudes and an increase in the higher latitudes. These error patterns may in the first approximation be ascribed to the neglect of the zonal part of diabatic heating, because an evaluation of the temperature changes caused by the transport of sensible heat by the mean meridional circulation indicates that the temperature changes are small almost everywhere in the meridional plane with the exception of the lower elevations in the middle latitudes.

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