RBRC Memo 3/11/99

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Re: New CP variable: Twist Tensor and $p - \bar{p}$ asymmetry in AA

1 Intro

At the 3/11/99 RBRC meeting several CP violation observables were discussed. In particular, Dima and Jack presented estimates for the magnitude of parity violation measured through the pseudoscalar variable

$$p = (\mathbf{k} \times \bar{\mathbf{k}}) \cdot \hat{z} \quad . \tag{1}$$

Here it was assumed that k referes to the momentum of a π^+ and $\bar{k}$ refers to momentum of a π^-, The z direction is fixed externally. This is a good variable for looking for P violation, but TD stressed the importance of constructing observables sensitive to CP. He proposed a four pion CP observable

$$cp = (\mathbf{k} \times \bar{\mathbf{k}}) \cdot (\mathbf{k}' - \bar{\mathbf{k}}') \tag{2}$$

where $(\mathbf{k}' - \bar{\mathbf{k}}')$ is an axis formed from a second independent $\pi^\pm$ pair. The estimate of p was made considering the correlated deflection suffered by a back-to-back quark and anti-quark jet in a "TPC" type chromo field with parallel $\mathbf{E}(x)$ and $\mathbf{B}(x)$ field that could arise if a CP domain were formed.

I questioned whether the assumption of homogeneity of the fields is a good assumption and considered whether that assumption could be relaxed and a simpler CP observable could be constructed. I believe that any CP violation in AA must incorporate the very inhomogenious random field configurations produced. Initial chromo-electric and chromo-magnetic field fluctuations are induced by copious minijet production in AA or the classical Yang-Mills evolution. This *random* background field configuration has zero local mean

$$\langle E_i^a(x) \rangle = \langle B_i^a(x) \rangle = 0 \tag{3}$$

but is characterized by large local rms fluctuations

$$\langle E_i^a(x) E_j^b(y) \rangle = \langle B_i^a(x) B_j^b(y) \rangle = \frac{\delta_{ab}}{N_c^2 - 1} \frac{\delta_{ij}}{3} \Delta(x-y) \epsilon(\tau) \tag{4}$$

The scale of the fluctuations is set by the proper energy density $\epsilon(\tau)$, which under RHIC conditions for central $Au+Au$ has been estimated to on the order of $\epsilon(\tau) \sim (10 \text{ GeV/fm}^3)(\tau_0/\tau)$ with $\tau_0 \sim 0.2-0.5$ fm/c. Here $\Delta(x-y) \sim \exp(-|x-y|/r_D)$ is a Euclidean correlator with range on the order of the Debye screening length $r_D \sim gT(\tau)$. More precicely, These correlators are related in equilibium to the gauge variant retarded propagator $\theta(x_0-y_0)\text{Tr}(\rho[A^\mu(x)A^\nu(y)])$ through the fluctuation dissipation theorem as used for example in ref. [4].

The propagation of any jet through this random background field leads to large deflections due to collisional and radiation energy loss that cannot be neglected in estimating parton deflections or equivalently the acoplanarity of initial back-to-back jets.

The novel distinguishing feature of CP violating domains is the existence of nonvanishing longitudinal correlations between the **E** and **B** fields. To model those correlations I assume that in such a domain

$$\langle E_i^a(x)B_j^b(y)\rangle = \frac{\delta_{ab}}{N_c^2-1}Q_{ij}\Delta_{CP}(x-y)\langle\mathbf{E}\cdot\mathbf{B}\rangle f_{CP}(\tau) \tag{5}$$

where $\Delta_{CP}(x-y) \sim \exp(-|x-y|/r_{CP})$ controls the spatial extent of such correlations. I consider the possibility that characteristic correlation scale r_{CP} may in fact be small compared to the CP violating domain size R. In this picture, E and B are only locally parallel on a scale τ_{CP} but the sign of that correlation may extend over a larger domain of size perhaps comparable to the nuclear radius. There is some unknown proper time dependence $f_{CP}(\tau)$ that depends on the transient dynamics of the formation and decay of metastable CP violating domains. Presumable f_{CP} is only nonvashing in some time interval, (τ_1, τ_2), after the deconfinement transition has been reached ($\epsilon(\tau_1) \sim \epsilon_Q \sim 1$ GeV/fm^3 and before the system freezes out ($\epsilon(\tau_2) \sim \epsilon_H \sim 0.1$ GeV/fm^3. In practice, for a high energy pair of jets the upper time is cut off by the jet exit time into the vacuum, R.

The strength of the **E** and **B** correlation is given by $\langle\mathbf{E}\cdot\mathbf{B}\rangle$ which we take from estimates [1]

$$\langle\alpha_s G\tilde{G}\rangle \sim 0.04 \text{ GeV}^4 \tag{6}$$

I have also included a diagonal tensor, Q_{ij}, to allow for quadrapole deformation of this E B correlation relevant to the CP twist tensor considered below.

2 Twist or Torsion Tensor as CP Observable

Due to the assumed parallel correlated **E** and **B** background fields a high energy quark q with initial momentum k_0 in direction n_0 acquires a momentum kick δk characteristic of helical motion

$$\delta k = \delta e + n_0 \times \delta b \tag{7}$$

where the electric and magnetic kicks along the trajectory $x(\tau) = n_0\tau + x_0$ is

$$\begin{aligned} \delta e_i &= \int_{\tau_1}^{\tau_2} d\tau g_a E_i^a(x(\tau),\tau) \\ \delta b_i &= \int_{\tau_1}^{\tau_2} d\tau g_a B_i^a(x(\tau),\tau) \end{aligned} \tag{8}$$

Note that the nonabelian charge g_a also precesses as the quark moves through the background field[4] while keeping $g_a g_a = g^2 C_F$ fixed.

In contrast, an antiquark, $\bar{q}$, with initial momentum $\bar{k}_0$ in direction $\bar{n}_0$, acquires a kick

$$\delta\bar{k} = \delta\bar{e} + \bar{n}_0 \times \delta\bar{b} \tag{9}$$

along another independent trajectory $\bar{x}(\tau) = \bar{n}_0\tau + \bar{x}_0$ is

$$\begin{aligned} \delta\bar{e}_i &= -\int_{\tau_1}^{\tau_2} d\tau g_a E_i^a(\bar{x}(\tau),\tau) \\ \delta\bar{b}_i &= -\int_{\tau_1}^{\tau_2} d\tau g_a B_i^a(\bar{x}(\tau),\tau) \end{aligned} \tag{10}$$

I allow for the possibility that two jets could in principle have been produced in independent hard processes in causally disconnected regions.$(x_0 - \bar{x}_0)^2 < 0$. Also the E and B fields along the two independent trajectories need not be correlated at all! I only assume a local E B correlation.

The unique feature of parallel E and B fields that we want to exploit is that there is a natural handedness associated with the helical motion in such fields. Consider for example Dima's "TPC" type chromo field with B pointing in direction n with E field parallel. As the quark drifts "north" along E as it spirals around the B field, the antiquark drifts "south" as it spirals around the B field. The handednes of the helical motion of both q and $barq$ is the same though they move in opposite directions. The trajectoriy of both have the same twist or torsion. This suggests that we should consider the correlation between the transverse and longitudinal momenta of the pair.

A possible (tensorial) measure of such helical correlation between the trajectories of the q and $\bar{q}$ is provided by the following two particle twist or torsion tensor observable

$$t_{ij} = (k \times \bar{k}) \cdot n_i n_j \cdot (k - \bar{k}) \tag{11}$$

For $n = z$, the beam twist $t_{zz} \propto (y - \bar{y}) \sin(\phi - \bar{\phi})$. Unlike the pseudoscalar observable (1) the twist is C even by construction while P odd. Hence it provides a two particle CP violation observable in addition to the four particle CP observable (2). Note that

$$\mathrm{Tr} t = t_{xx} + t_{yy} + t_{zz} = 0 \tag{12}$$

Therefore, if there is azimuthal axial symmetry about the beam axis, $t_{xx} = -t_{zz}/2$.

As an aside we note that in ref.[5] another parity violation variable called "screwiness" was proposed to test for helical strings configurations in jet fragmentation. The "screwiness" is defined there as a measure of the collective coorelation between rapidities and azimuthal angle of particles:

$$\mathcal{S}(\omega) = \sum_e P_e \left| \sum_j \exp(i(\omega y_j - \phi_j)) \right|^2 . \tag{13}$$

The first sum is over all the configurations e found in the phase space and the second goes over the gluons in the configuration. For ω-values close to zero, screwiness must be small if the gluons are emitted isotropically in the azimuthal angle. For large values of ω the phases should be close to chaotic and then screwiness only depends on the mean number of emitted gluons. Parity violation is signalled by $S(\omega) \neq S(-\omega)$. They proposed this in what i think is a wrong context: parity conserving pQCD radiation. However, the variable is useful for checking parity violation in strong interaction due to novel nonperturbative phenomena.

One could imagine many variants of the above that would be appropriate to CP hunting. For example the CP screwiness

$$s(\omega) = \sum_{\pi^+} \cos(\omega y_j - \phi_j)) + \sum_{\pi^-} \cos(\omega \bar{y}_j - \bar{\phi}_j)) \tag{14}$$

nder P, $y \to -y$ $\phi \to \phi + \pi$,and $s(\omega) \to -s(-\omega)$. Under C it is even.

3 Beam twist in CP violation

If the initial distribution of q and $\bar{q}$ momenta are symmetric

$$\langle (k_0 \times \bar{k}_0) \cdot n_i n_j \cdot (k_0 - \bar{k}_0) \rangle = 0 \tag{15}$$

Many reality factors could "screw" this zero order assumption up. The C non-eigenstate of the nuclear beams and the Coulomb final state interactions are obvious spoilers. In the end, there is another spoiler associated with the fact that a final π^+ has equal probability of arisng from the fragmentation of a u quark or a $\bar{d}$ quark! That is why I will reinterpret the twist observable at the end to be really more relevant as a baryon antibaryon asymmetry observable.

We proceed here following the idealized world q and $\bar{q}$ trajectories in random "TPC" type chromo fields. The *random* medium scenario implies no net deflection on the average:

$$\langle \delta k \rangle = \langle \delta \bar{k} \rangle = 0 \tag{16}$$

Therefore, the first order corrections to the twist vanish:

$$\begin{aligned} \langle (k_0 \times \delta \bar{k}) \Delta (k_0 - \bar{k}_0) \rangle &= 0 \\ \langle (\delta k \times \bar{k}_0) \Delta (k_0 - \bar{k}_0) \rangle &= 0 \\ \langle (k_0 \times \bar{k}_0) \Delta (\delta k - \delta \bar{k}) \rangle &= 0 \end{aligned} \tag{17}$$

Here we use $\Delta_{ij} = n_i n_j$ as shorthand for the projection tensor.

Also since the fields along the two different trajectories are uncorrelated in this random medium, $\langle \delta k \times \delta \bar{k} \rangle = 0$ by assumption. The twist is thus determined by the following two quadratic corrections to the trajectories:

$$\langle t \rangle = \langle (k_0 \times \delta \bar{k}) \Delta (-\delta \bar{k}) \rangle + \langle (\delta k \times \bar{k}_0) \Delta \delta k \rangle \tag{18}$$

We can rewrite this as

$$\langle t_{ij} \rangle = \langle \delta \bar{k} \cdot ((k \times n_i) \otimes n_j) \cdot \delta \bar{k} \rangle + \langle \delta k \cdot \Big((\bar{k} \times n_i) \otimes n_j \Big) \cdot \delta k \rangle \tag{19}$$

where we used (16) to replace k_0 by k.

Thus t_{ij} simply measures the correlation between orthogonal components of the momentum kicks suffered during the passage of the jets through the medium.

If the off-diagonal ($i \neq j$) EE and BB correlations vanish, $\langle E_i E_j \rangle = \langle B_i B_j \rangle = 0$, as assumed, then only the cross terms involving the product of electric and magnetic impulses survive in the average.

Consider the beam twist, t_{zz} by setting $n_i = n_j = z$, and look at opposite side jets with $\vec{k}_\perp = -k_\perp \hat{x}$ and $\vec{\bar{k}}_\perp = +k_\perp \hat{x}$. Then $k \times z = k_\perp \hat{y}$ while $\bar{k} \times z = -k_\perp \hat{y}$. In this case,

$$\langle t_{zz} \rangle = k_\perp (\langle \delta \bar{k}_y \delta \bar{k}_z \rangle - \langle \delta k_y \delta k_z \rangle) \tag{20}$$

Noting that

$$\begin{aligned} \delta k_y &= \delta e_y + \delta b_z \\ \delta k_z &= \delta e_z - \delta b_y \\ \delta \bar{k}_y &= \delta \bar{e}_y - \delta \bar{b}_z \\ \delta \bar{k}_z &= \delta \bar{e}_z + \delta \bar{b}_y \end{aligned} \tag{21}$$

$$\begin{aligned} \langle \delta k_y \delta k_z \rangle &= \langle \delta e_z \delta b_z \rangle - \langle \delta e_y \delta b_y \rangle \\ \langle \delta \bar{k}_y \delta \bar{k}_z \rangle &= -\rangle \delta \bar{e}_z \delta \bar{b}_z \langle + \rangle \delta \bar{e}_y \delta \bar{b}_y \langle \end{aligned} \tag{22}$$

Now we evaluate the $\delta e_i \delta b_i$ expectation value assuming eq.(5):

$$\begin{aligned} \langle \delta e_i \delta b_i \rangle &= \int d\tau_1 d\tau_2 f_{CP}(\tau_1) f_{CP}(\tau_2) \Delta_{CP}(x(\tau_1) - x(\tau_2)) g^2 C_F \langle \mathbf{E} \cdot \mathbf{B} \rangle \\ &\approx r_{CP} \Delta\tau \; g^2 C_F \langle \mathbf{E} \cdot \mathbf{B} \rangle Q_{ii} \end{aligned} \tag{23}$$

Here $\Delta\tau$ is the time duration of the jet propagating in the CP violating domain. The final twist about the beam axis is

$$t_{zz} \propto r_{CP} (\Delta\tau + \Delta\bar{\tau}) \; \langle \mathbf{E} \cdot \mathbf{B} \rangle (Q_{yy} - Q_{zz}) \tag{24}$$

Note that if rotation invariance is not broken, $Q_{yy} = Q_{zz}$ then there is no average twist and we must look for an enhanced width in the twist distribution. However, rotational invariance is already broken by the beam axis in AA and it may be that $Q_{yy} \neq Q_{zz}$. In this case there is an average twist in spite of the random nature of the field fluctuations as assumed here. In particular, the twist survives if the alignment of the E and B fields prefers the beam beam axis in a quadrapole sense.

Of course if the sign of the alignment $\langle \mathbf{E} \cdot \mathbf{B} \rangle$ varies from event to event, then all average twists vanish and we are forced back again into considering enhanced widths relative to large fluctuations induced by the background fields. Should CP violating domains have a unique sign of $\langle G\tilde{G} \rangle$? What physics would drive an asymmetry in the sign of that condensate? Could it be T violation?

4 Final remarks

In the above we simply considered the motion of quarks and anti-quarks in correlated random fields. Unfortunately neither quarks nor antiquarks are observable. The proposed mesonic observable using pions are the least sensitive to the difference between q and qbar propagation. As noted before, the $\pi^{\pm}$ have equal fragmentation probabilities from light quarks or anti-quarks. One way out is to consider heavy flavor mesons. $K^{\pm}$ preferentially produced by direct $\bar{s}$ and s fragmentation rather than the strangeness suppressed non-strange quark fragmentation. Unfortunately, most light mesons including the K at moderate high transverse mometum are produced in any case through gluon fragmentation in the mid rapidity region. On the other hand, $D^{\pm}$ with charmed quarks and antiquarks may be much cleaner messengers of the evolution of quarks and antiquarks in dense matter. This would require PHENIX type open charm correlation measurements perhaps though correlated leptons of high invariant mass.

Alternatively, we could turn to baryons and hyperons and their anti matter in STAR. The twist of quarks could translate directly into the twist baryon observables. This is perhaps one of the most interesting observables in any case, since CP violation is presumably responsible for the baryon asymmetry of the universe. The suggestion therefore is that one should look for baryon antibaryon twist asymmetry in AA to probe possible CP violation in strong interactions.

References

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