Twisted $\mathcal{N} = 2$ exact SUSY on the lattice for BF and Wess-Zumino

D'Adda, Alessandro	(INFN, Torino)
Kanamori, Issaku	(Hokkaido Univ.)
Kawamoto, Noboru	(Hokkaido Univ.)
Nagata, Kazuhiro	(Hokkaido Univ)

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Introduction

Supersymmetry on lattice:

Nonperturbative formulation of SUSY

Need *exact* SUSY

...but

Difficult due to *breakdown of the Leibniz rule*

Our approach

- Discretization of SUSY algebra itself
- Twisted SUSY (S.Catterall et al.; F.Sugino)
- Superspace with "mild noncommutativity"

$\Downarrow$

BF; Wess-Zumino (2 dim.)

Twisted Supersymmetry

(E. Witten '88)

$\mathcal{N} = 2$ SUSY algebra(Majorana):

$$\boxed{\{Q_{\alpha i},\ Q_{\beta j}\} = i2\delta_{ij}(\gamma_\mu)_{\alpha\beta}\partial_\mu}$$

$$\Downarrow$$

Twist: Extended SUSY $\rightarrow$ spinor

$$Q_{\alpha\beta} = \left(\mathbb{1}Q + \gamma_\mu Q_\mu + \gamma_5 \tilde{Q}\right)_{\alpha\beta}$$

$$\boxed{\begin{array}{c} \{Q, Q_\mu\} = i\partial_\mu \qquad \{\tilde{Q}, Q_\mu\} = -i\epsilon_{\mu\nu}\partial^\nu \\ Q^2 = \tilde{Q}^2 = \{Q, \tilde{Q}\} = \{Q_\mu, Q_\nu\} = 0 \end{array}}$$

Q : scalar
Q_μ : vector
$\tilde{Q}$: pseudo scalar

Easier to treat on a lattice than spinor!

Noncommutativity

Discretized algebra:

$$[\theta Q, \theta_\mu Q_\mu] = i\theta\theta_\mu \partial_{+\mu}$$

forward diff. $\partial_{+\mu} f(x) = f(x + 2\hat{n}_\mu) - f(x)$

$$\begin{aligned}
&\theta\theta_\mu \partial_{+\mu}(f(x)g(x)) \\
&= \theta\theta_\mu [f(x+2\hat{n}_\mu)g(x+2\hat{n}_\mu) - f(x)g(x)] \\
&= \Big(\theta\theta_\mu \partial_{+\mu} f(x)\Big) g(x) + \theta\theta_\mu f(x+2\hat{n}_\mu)\partial_{+\mu} g(x)
\end{aligned}$$

Noncommutativity $\boxed{\theta\theta_\mu f(x+2\hat{n}_\mu) = f(x)\theta\theta_\mu}$

$$= \Big(\theta\theta_\mu \partial_{+\mu} f(x)\Big) g(x) + f(x)\theta\theta_\mu \partial_{+\mu} g(x)$$

To compensate breakdown of Leibniz rule, we need the noncomm. Grassmann parameter

$$\Downarrow$$

Superspace formalism

cf. Noncomm. differential form
(A.Dimakis et al.;P.Aschieri et al.; ...)

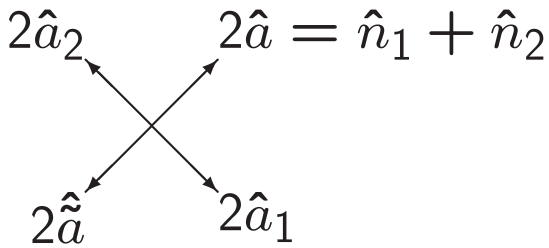
Superspace

- Supercharges(Q_A) as derivative of θ_A
- Use both of forward and backward diff.
- Chiral and anti-Chiral superfield

Supercharges

$$Q = \frac{\partial}{\partial\theta} + \frac{i}{2}\theta\partial_{+\mu} \qquad \tilde{Q} = \frac{\partial}{\partial\tilde{\theta}} - \frac{i}{2}\theta^{\mu}\epsilon_{\mu\nu}\partial^{-\nu}$$

$$Q_\mu = \frac{\partial}{\partial\theta^\mu} + \frac{i}{2}\theta\partial_{+\mu} - \frac{i}{2}\tilde{\theta}\epsilon_{\mu\nu}\partial_{-\nu}$$



Chiral superfield

$$\Psi(x) = ic(x) + \theta^\mu\omega_\mu(x + a_\mu) + i\theta^1\theta^2\lambda(x) + \cdots$$

$$\bar{\Psi}(x) = i\bar{c}(x) + \theta b(x + a) + \tilde{\theta}\phi(x - a) - i\theta\tilde{\theta}\rho(x) + \cdots$$

Transformations

$$Q_A\Psi(x) = s_A\Psi(x)$$

Op. on superfield   Op. on *component fields*

$$s_A f(x) = f(x + 2\hat{a}_A)s_A$$

$\boxed{\text{Action(Abelian BF)}}$

$$
\begin{aligned}
S &= \sum_x \bar{\Psi}(x)\Psi(x)\Big|_{\theta^4} \\
&= \sum_x i s_1 s_2 \tilde{s} s\Big(\overline{c}(x)c(x)\Big) \\
&= \sum_x \Big[\phi(x+a)\Big\{\partial_{+1}\omega_2(x+a_2) - \partial_{+2}\omega_1(x+a_1)\Big]\Big\} \\
&\quad + b(x-a)[\partial_{-1}\omega_1(x+a_1) + \partial_{-2}\omega_2(x+a_2)] \\
&\quad - i\overline{c}(x)\partial_{+\mu}\partial^{-\mu}c(x) + i\rho(x)\lambda(x)\Big]
\end{aligned}
$$

$s_A^2 = 0 \implies \boxed{S \text{ is SUSY inv.}}$

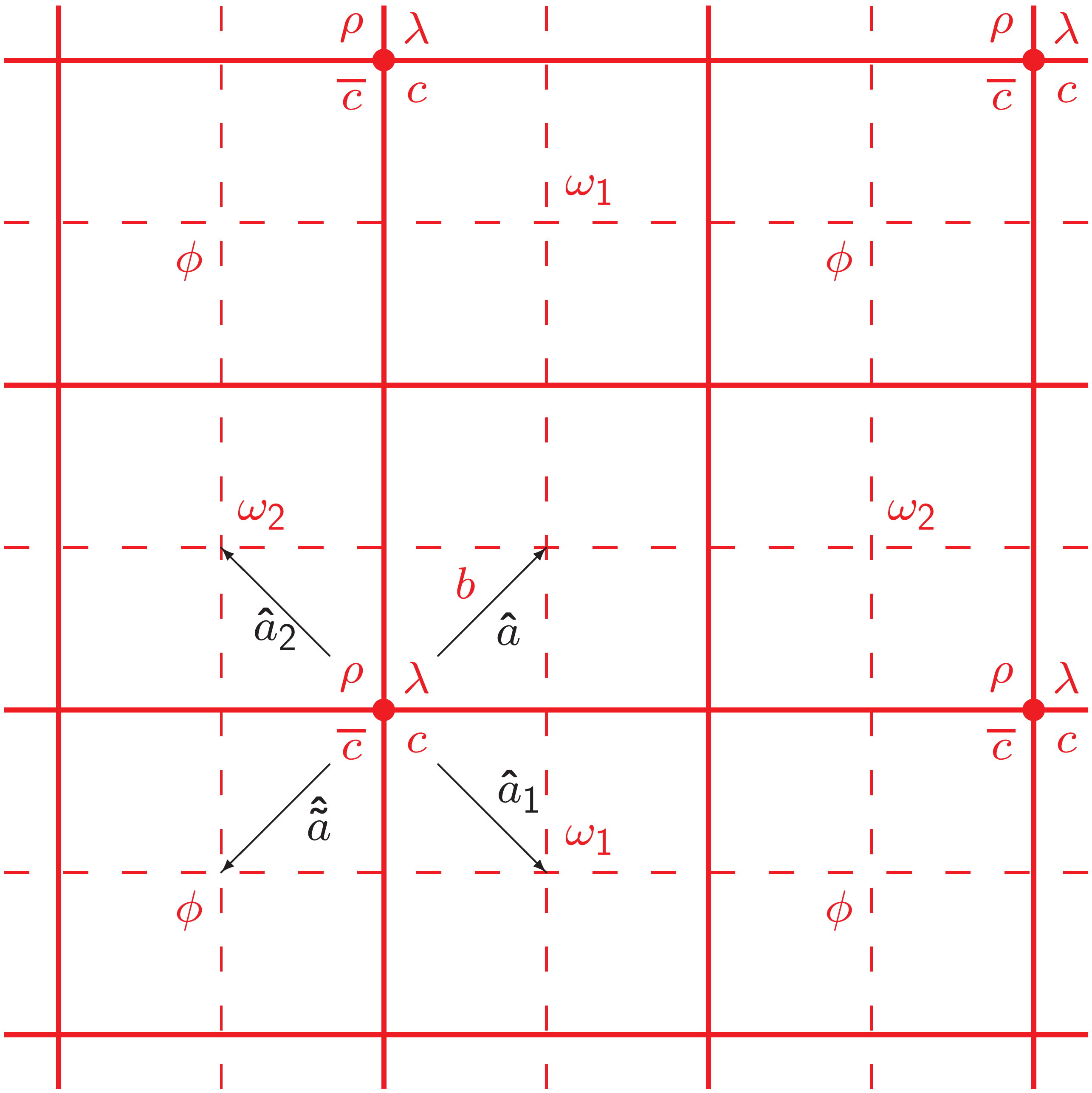
$$
\left[\begin{array}{c} \text{continuum: BF + gauge fixing terms} \\ \int d^2x\Big(\phi\epsilon_{\mu\nu}\partial^\mu\omega^\nu + b\partial_\mu\omega^\mu - i\overline{c}\partial_\mu\partial^\mu c + i\rho\lambda\Big) \end{array}\right]
$$

Summation: $x =$ (even, even), (odd, odd)

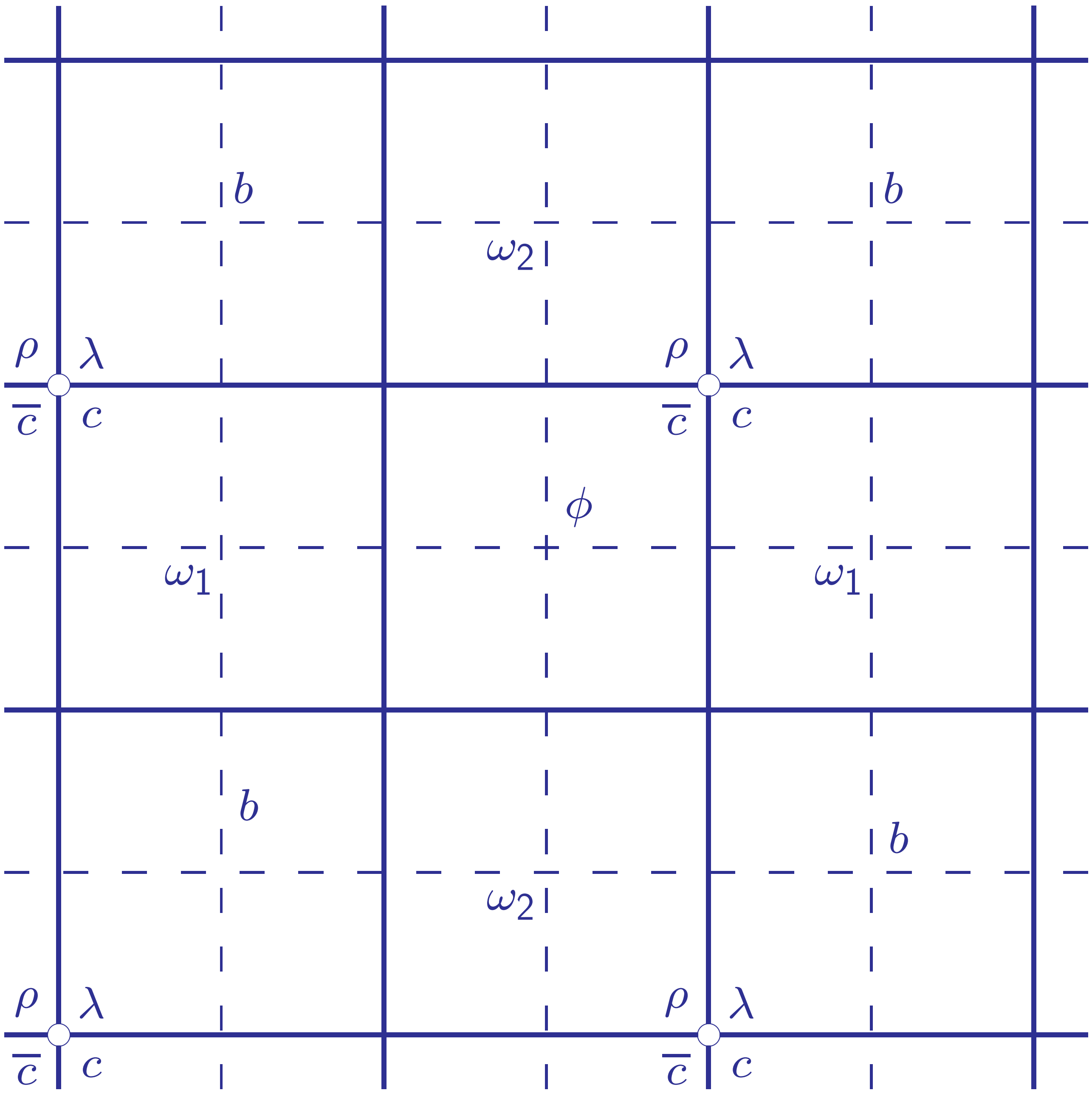
cf. $s(\overline{c}(x)c(x)) = (s\overline{c}(x))c(x) - \overline{c}(x+\underbrace{2a}_{n_1+n_2})sc(x)$

$$\partial_{\pm\mu}f(x) = \pm\Big(f(x \pm 2n_\mu) - f(x)\Big)$$

$$
\begin{aligned}
\{s, s_\mu\} &= -i\partial_{+\mu}, \\
\{\tilde{s}, s_\mu\} &= i\epsilon_{\mu\nu}\partial_{-\nu}, \\
\{s, \tilde{s}\} = \{s_\mu, s_\nu\} &= 0 \\
s^2 = \tilde{s}^2 = s_\mu^2 &= 0
\end{aligned}
$$



● (even, even)



b
b
ω_2
ρ
λ
ρ
λ
$\bar{c}$
c
$\bar{c}$
c
ϕ
ω_1
ω_1
b
b
ω_2
ρ
λ
ρ
λ
$\bar{c}$
c
$\bar{c}$
c

○ (odd, odd)

Action(non-Abelian BF)

BF with *interactions* is also SUSY invariant

- $\Phi(x+\hat{a}) = \Psi(x+\hat{a}) + i\theta\Psi(x+3\hat{a})\Psi(x+\hat{a})$
- $\overline{\Psi}(x+\hat{a})$

$$
\begin{aligned}
&S^{\text{lat}}_{\text{NABF}} \\
&= \sum_x \operatorname{tr} i\overline{\Psi}(x+\hat{a})\Phi(x+\hat{a})\Big|_{\theta^4} \\
&= \sum_x \operatorname{tr} s\tilde{s}s_1 s_2\Big(-i\overline{c}(x+\hat{a})c(x+\hat{a})\Big) \\
&= \sum_x \operatorname{tr}\Big[\phi(x+\hat{a}-\hat{\tilde{a}})\Big[\epsilon_{\mu\nu}\partial_{+\mu}\omega_\nu(x-\hat{a}_\mu-\hat{\tilde{a}}) \\
&\quad + (\epsilon_{\mu\nu}\omega_\mu\omega_\nu)(x+\hat{a}-\hat{\tilde{a}}) - \{c,\lambda\}(x+\hat{a}-\hat{\tilde{a}})\Big] \\
&\quad + b(x)\partial_{-\mu}\omega_\mu(x+\hat{n}_\mu) \\
&\quad - i\overline{c}(x+\hat{a})\partial_{-\mu}D_{+\mu}c(x+\hat{a}) + i\rho(x-\hat{\tilde{a}})\lambda(x-\hat{\tilde{a}})\Big]
\end{aligned}
$$

$$
\begin{aligned}
[\omega_\mu, c]\,(x) &\equiv \omega_\mu(x+\hat{a})c(x-\hat{n}_\mu) - c(x+\hat{n}_\mu)\omega_\mu(x-\hat{a}) \\
(\epsilon_{\mu\nu}\omega_\mu\omega_\nu)(x) &\equiv \epsilon_{\mu\nu}\omega_\mu(x+\hat{n}_\nu)\omega_\nu(x-\hat{n}_\mu) \\
\{c,\lambda\}(x) &\equiv c(x+\hat{n}_1+\hat{a}_2)\lambda(x-\hat{a}) + \lambda(x+\hat{a})c(x-\hat{n}_1-\hat{a}_2) \\
D_{+\mu}c(x) &\equiv \partial_{+\mu}c(x) + [\omega_\mu, c](x+\hat{n}_\mu)
\end{aligned}
$$

Action(Wess-Zumino)

Superfields: Fermionic $\Longrightarrow$ Bosonic

$$S=\sum_x\Big[\varphi(x)\partial^{+\mu}\partial_{-\mu}\phi(x)+\tilde{\varphi}(x)\tilde{\phi}(x)$$
$$-i\Big(\partial_{+\mu}\chi(x-a)-\epsilon_{\mu\nu}\partial^{-\nu}\tilde{\chi}(x+a)\Big)\psi^{\mu}(x+a_\mu)\Big]$$

$\Downarrow$

untwist

$$\xi_{\alpha i}(x) = \frac{1}{2}\Big(\mathbb{1}\chi(x-a)+\gamma_\mu\psi^\mu(x+a_\mu)+\gamma_5\tilde{\chi}(x+a)\Big)_{\alpha i}$$

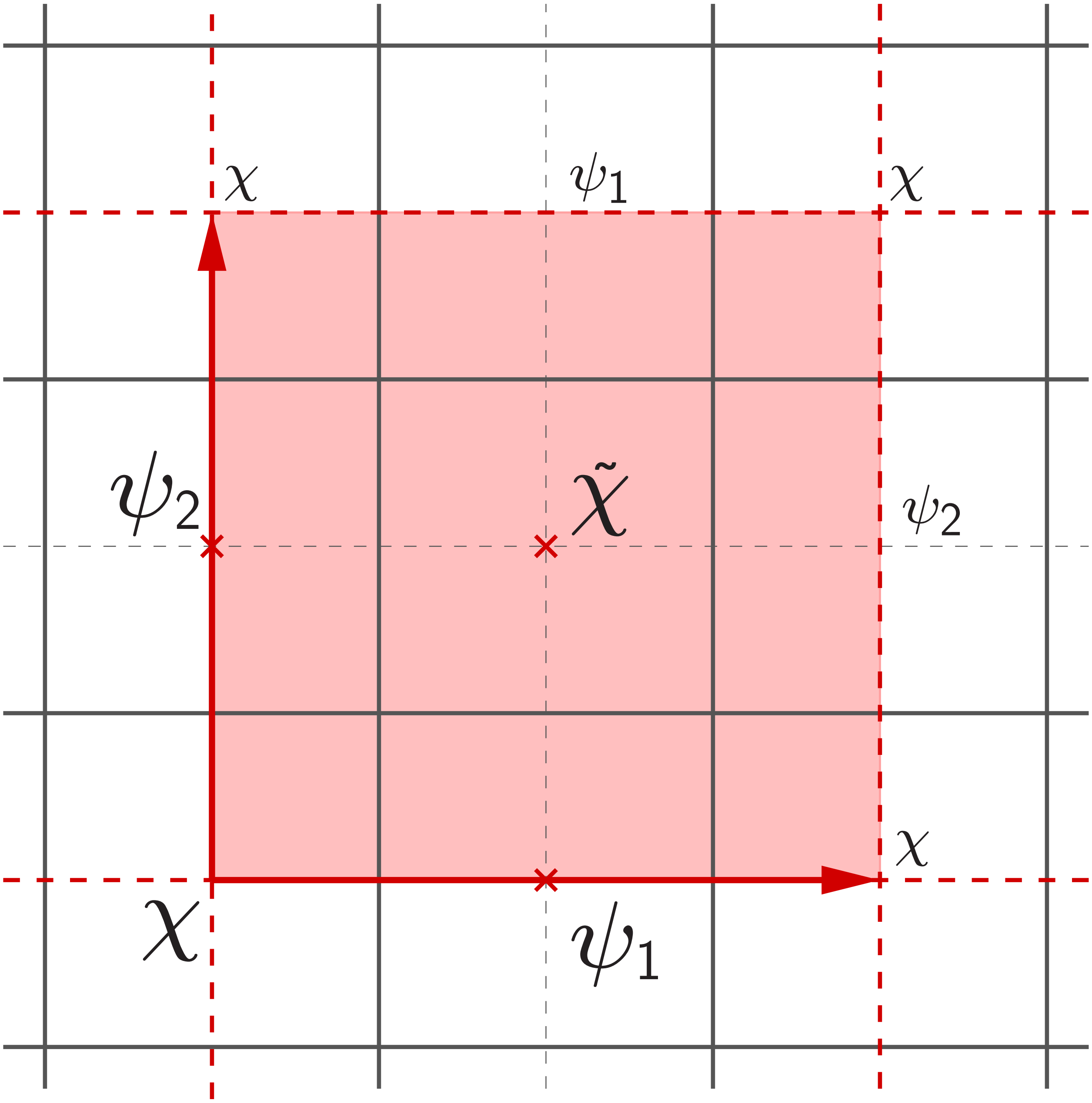
Dirac-Kähler mechanism(K-S fermion)

$\mathcal{N}=2$ Wess-Zumino model

2-flavor $\Longrightarrow$ $\mathcal{N}=2$

$$S=\sum_x\Big[-\phi_i(x)\partial^{+\mu}\partial_{-\mu}\phi_i(x)+F_i(x)F_i(x)$$
$$+\frac{i}{2}\bar{\xi}_{i\alpha}(x)(\gamma_\mu)_{\alpha\beta}(\partial_{+\mu}+\partial_{-\mu})\xi_{\beta i}(x)$$
$$-\frac{i}{2}\bar{\xi}_{i\alpha}(x)(\gamma_5)_{\alpha\beta}(\partial_{+\mu}-\partial_{-\mu})\xi_{\beta j}(x)(\gamma_5\gamma_\mu)_{ji}\Big]$$

$$\xi_{\alpha i}(x) = \frac{1}{2}\Big(\mathbb{1}\chi(x-a) + \gamma_\mu\psi^\mu(x+a_\mu) + \gamma_5\tilde{\chi}(x+a)\Big)_{\alpha i}$$



$$\Psi = \chi + \psi_\mu\, dx^\mu + \tilde{\chi}\, dx^1\, dx^2$$

$\boxed{\text{Conclusion}}$

- $D = \mathcal{N} = 2$ *exact* twisted supersymmetric model

- Twist can be understood as Dirac-Kähler mechanism

- *Mild Noncommutativity*:
  Discretization of superspace & algebra
  New definition of SUSY?
  — G-W relation modified the chiral sym.

Future Work

- Supersymmetric Yang-Mills
- 4 dim.($\mathcal{N} = 4$?)
- Numerical analysis

Transformation Table for BF Model

	s	$\tilde{s}$	s_μ
$c(x)$	0	0	$-i\omega_\mu(x+\hat{a}_\mu)$
$\omega_\nu(x+\hat{a}_\nu)$	$\partial_{+\nu}c(x)$	$-\epsilon_{\nu\rho}\partial^{-\rho}c(x)$	$-i\epsilon_{\mu\nu}\lambda(x)$
$\lambda(x)$	$\epsilon_{\mu\nu}\partial^{+\mu}\omega^\nu(x+\hat{a}_\nu)$	$-\partial_{-\mu}\omega^\mu(x+\hat{a}_\mu)$	0
$\overline{c}(x)$	$-ib(x+\hat{a})$	$-i\phi(c+\hat{\hat{a}})$	0
$b(x+\hat{a})$	0	$-i\rho(x)$	$\partial_{+\mu}\overline{c}(x)$
$\phi(x+\hat{\hat{a}})$	$i\rho(x)$	0	$-\epsilon_{\mu\nu}\partial^{-\nu}\overline{c}(x)$
$\rho(x)$	0	0	$-\partial_{+\mu}\phi(x+\hat{\hat{a}}) - \epsilon_{\mu\nu}\partial^{-\nu}b(x+\hat{a})$