

Title: Generating the Sine Curve in the First Quadrant Using the Calculator Based Laboratory/Motion Detector

A discovery/exploration lesson involving the sine curve.

Link to Outcomes:

- **Problem Solving** Students will develop a sine curve.
- **Communication** Students will interact and discuss mathematical concepts. They will record their findings and present their graphical interpretation.
- **Reasoning** Students will collect and graph data. Students will analyze the graphic results.
- **Connection** Students will associate degree measure with vertical distance.
- **Measurement** Students will use the TI-82 graphics calculator (TI-82)/Calculator Based Laboratory (CBL) connection with the Ultrasonic Motion Detector and the HIKER program.
- **Geometry** Students will plot points on the Cartesian plane.
- **Trigonometry** Students will generate through investigation the sine curve for $0^\circ \leq \theta \leq 90^\circ$.
- **Technology** Students will use the TI-82/CBL connection with the Ultrasonic Motion Detector and the HIKER program.
- **Cooperation** Students will work in groups of four to generate the sine curve through the existing properties of geometry and trigonometry.

Brief Overview:

The sine function is a continuous curve defined for all values of its domain. The coordinates of all points on the curve $y = \sin \theta$ are of the form (θ, y) . These points can be found on the circumference of a unit circle in the (x, y) plane where θ is in radian measure and y is the y -coordinate of the point. The students will select values for θ and determine the corresponding y values through a distance relation.

Grade/Level:

Grades 10 - 11: Trigonometry

Duration:

The lesson should take two to three days depending on student interaction and expertise with the TI-82.

Prerequisite Knowledge:

Students must:

- be familiar with the geometric ratio of the sides of a 30-60-90 right triangle and a 45-45-90 isosceles right triangle.
- have knowledge of the sine ratio.
- be able to manipulate data utilizing the TI-82 and transfer this data to a graphical system.

Objectives:

Students will:

- use the TI-82/CBL connection with the Ultrasonic Motion Detector to determine distance.
- reinforce their understanding of the geometric ratio of the sides of a 30-60-90 right triangle and 45-45-90 isosceles right triangle.
- reinforce their knowledge of the sine ratio in the 30-60-90 right triangle and 45-45-90 isosceles right triangle.
- observe the shape of the sine curve for $0^\circ \leq \theta \leq 90^\circ$.
- find various angles through the application of the motion detector.

Materials/Resources/Printed Materials:

- Meter stick
- Masking tape
- Protractor
- TI-82
- CBL unit
- Ultrasonic Motion Detector
- Student activity worksheets 1, 2, and 3

Development/Procedures:

Students will form groups of four and assemble equipment. Each group must have a designated recorder, reader, observer, and system's facilitator. The system's facilitator will rotate the meter stick θ degrees, allowing the reader to relay the distance measurement to the recorder. Upon completion of the experiment the group will process the data. Through use of the TI-82 <STAT> their results will be graphed. Students will then graph the actual sine values for 0° , 30° , 45° , 60° , and 90° .

Evaluation:

The teacher will observe the students performance in the successful completion of their activities. The student groups will submit a print out of their two graphs as well as the sine equation.

Extension/Follow Up:

The students could investigate the complete sine curve, if the motion detector is connected to a wheel.

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GENERATING THE SINE CURVE IN THE FIRST QUADRANT USING THE CBL AND MOTION DETECTOR

ACTIVITY SHEET #1

Discussion:

In a right triangle, the sine ratio can be used to determine the measure of the side opposite the acute angle under observation. If the hypotenuse of each right triangle takes on a value of 1, then the sine value of the angle of observation would be equal to the measure of the side opposite. By calculating this measurement not only can sine values of different angles be calculated, but the sine as a graphical function can be investigated. In this activity, students will investigate the changes in sine values as the angle under observation approaches 90° . This information will then be utilized to develop the sine curve.

Data will be collected using the Ultrasonic Motion Detector with the CBL. The gathered information will be entered into the TI-82, which will further assist in the manipulating, categorizing, and graphically analyzing your results.

Materials:

- Meter stick
- Masking tape
- Protractor
- TI-82 graphing calculator
- CBL unit
- Ultrasonic Motion Detector
- Table

Procedure:

A. Record table height.

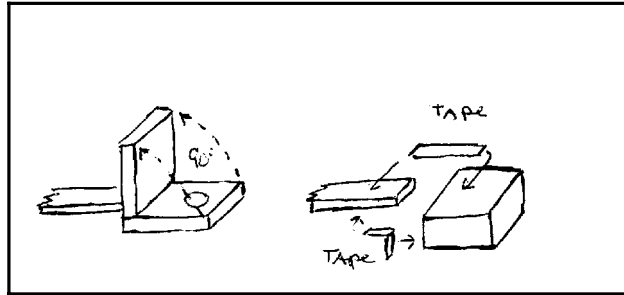
1. Insert motion detector cable into sonic port of the CBL unit.
2. Turn the CBL unit on by pressing the **<ON>**.
3. Press **<MODE>**, then **<CH VIEW>**.
4. To select the proper channel, press **<CH VIEW>** several times until SONIC M appears, and the detector starts clicking.
5. Place the motion detector face down on the edge of the table, and allow the unit to measure its height. (Note: Be careful to stand clear of the ultrasonic path.)
6. Record this height.

Height of table = _____

7. Turn the unit off by pressing **<2ND> <OFF>**.

B. Assemble motion detector and meter stick

1. Using masking tape affix the motion detector to the end of the meter stick. Allow 1/8 inch gap between detector and stick.
2. Check to make sure that the detector is able to rotate freely 90° without stick or tape interference. See diagram below.



3. Check to make sure the motion detector is still connected to the CBL.
4. Turn the CBL unit **<ON>**.
5. Press **<MODE>** followed by **<CH VIEW>** and continue to press until SONIC M is chosen. A clicking sound will be heard and the CBL/motion detector is now sonically reading distances.

C. Determine vertical measurement.

1. Support a protractor vertically on the edge of a table so that the center mark is at the edge.
2. Set the free end of the meter stick at the center mark and freely rotate the stick 90° for practice. Rotate from the vertical position to the horizontal position, and vice versa. At the same time support and keep the motion detector parallel to the floor.
3. When ready, rotate, stop, and record the vertical distance at 10° intervals starting at 0° and ending at 80°. (Be careful to keep the motion detector parallel to the floor and any feet clear from under the detector.)

	Distance
0°	_____
10°	_____
20°	_____
30°	_____
40°	_____
50°	_____

60° _____

70° _____

80° _____

D. Analyze with the TI-82.

1. To use the statistics mode in the TI-82, press **<ON>** and **<STAT>**; the edit menu will appear.
2. Press **<1>** for Edit, and a table will appear with the cursor on L₁, ready to receive information.
3. Enter your angle measures in the L₁ column, pressing **<ENTER>** after each entry.
4. Upon completion, press the right arrow key twice to select the L₃ column, and enter the corresponding measurements of distance.
5. Press the left arrow key to select the L₂ column and highlight L₂ by pressing the up arrow key.
6. Press the following **<2nd>**, **<L3>**, **<->**, and enter the reading for the height of the table from part A. Press **<ENTER>**.
7. L₂ now has a listing of values. What do these values represent?

E. Graph the collected data.

1. To graph the collected data, press **<2nd>** and **<STAT PLOT>**; the STAT PLOT screen will appear. Make sure all plots are off.
2. Press **<1>** for the Plot 1 screen, and turn the screen on by highlighting **On** and pressing **<ENTER>**.
3. Scroll down using the down arrow to **Type**, and select the line graph by pressing the right arrow key, then **<ENTER>**.
4. In the same fashion, select Xlist to be L₁ and Ylist to be L₂. Remember to press **<ENTER>** after selecting each.
5. Chose the **Mark** to be a dot by scrolling over and pressing **<ENTER>**.
6. Press **<ZOOM>**, scroll down to **9 Zoomstat**, and press **<ENTER>**.
7. A graph, from the collected data that you gathered is represented on the screen for all values of θ , where $0^\circ \leq \theta \leq 80^\circ$.

F. Repeat the activity using different angle measures

1.	Angle	Distance
	_____ °□	_____
	_____ °□	_____
	_____ °□	_____
	_____ °□	_____
	_____ °□	_____
	_____ °□	_____
	_____ °□	_____
	_____ °□	_____
	_____ °□	_____

2. Press <STAT> and <4> for Clrlist and <2nd>, <L3>, and <ENTER> to clear information in the L₃ column.
3. Press <STAT> and <1> for Edit, and enter angle measure in L₃.
4. Enter distance measure in L₅.
5. Highlight L₄ and press <2nd>, <L5>, and <->, and type in the table height recorded from **Activity A**. Press <ENTER>. L₄ is now complete.
6. To graph, push <2nd> and <STAT PLOT>. Leave Plot 1 on; select Plot 2.
7. Turn Plot 2 on; select Line graph, Xlist should be L₃; Ylist should be L₄; and the Mark should be +. Push <ENTER>.
8. Press <ZOOM>, scroll down to **9 Zoomstat**, and press <ENTER>.
9. Graphs representing the L₁/L₂ and L₃/L₄ tabulations will appear on the same graph system.

G. Using the two curves generated what conclusions can be drawn in regards to the sine curve over the interval $0^\circ \leq \theta \leq 90^\circ$?

GENERATING THE SINE CURVE THROUGH KNOWN VALUES OF SINE θ

ACTIVITY SHEET #2

A. Develop a table of known values.

1. On the TI-82, which should be disconnected from the CBL, press **<ON>**, **<STAT>**, **<CLR LIST>**, **<2nd>**, **<L5>**, and **<ENTER>** to clear column L_5 .
2. In column L_5 , enter five angle measures of 0° , 30° , 45° , 60° , 90° .
3. In column L_6 , enter the exact values which correspond to the angle measures in column L_5 .

B. Generate the actual sine curve through the know values

1. Press **<2nd>** and **<STAT PLOT>**, highlight **Plot 3** and press **<ENTER>**.
2. Turn **Plot 3** On; type is Line; Xlist is L_5 ; Ylist is L_6 ; and the Mark is the box. Press **<ENTER>**.
3. To graph, press **<GRAPH>**.
4. The sine curve, based on known values can now be compared to the first two generated curves.

GENERATION OF THE SINE CURVE IN THE FIRST QUADRANT THROUGH CONTINUOUS ROTATION

ACTIVITY SHEET #3

A. Preparation and set-up

1. Connect the TI-82 calculator to the CBL unit with a linking cable.
2. Make sure the Ultrasonic Motion Detector is still connected to the CBL unit through the sonic port.
3. Press <ON> to activate the TI-82 calculator, and then press <PRGM>.
4. Scroll down to the HIKER program, and push <ENTER> twice.
5. Turn on the CBL unit, and press <MODE> to set the channel.
6. Use the <CH VIEW> key to change the channel to SONIC M. If sampling is occurring, press <MODE> several times to end sampling.
7. The TI-82/CBL/Motion Detector is set and ready.

B. Calibration

1. Place the free end of the meter stick at one end of the table. The stick should be vertical to the table with one hand at the free end and the other holding the motion detector.
2. Holding the free end against the table and allowing it to become a pivot point, rotate the stick downwards until it is parallel to the floor. Do not twist the detector, and try to keep it parallel to the floor at all times.
3. When the stick and detector are parallel to the floor, press <ENTER>, and rotate position. Movement must be steady and the process will take approximately six seconds. Keep feet clear from the target floor. Do not go beyond 90° .

C. Graph Adjustment

1. Upon completion of the rotation, a graph should appear on the TI-82 screen.
2. Disconnect the TI-82 from the CBL unit.
3. Press <WINDOW> and enter the following values:
Xmin = 0
Xmax = 9
Xscl = 0
Ymin = 0
Ymax = 2
Yscl = 0
4. When completed, press <ENTER> and <GRAPH>.

5. Press <TRACE>. Find and record the following values:

Left endpoint $X =$ _____

$Y =$ _____

Coordinate of maximum value $X_1 =$ _____

$Y_1 =$ _____

6. Use these values to find the following:

Amplitude (a) = $Y_1 - Y =$ _____

Vertical shift (k) = $Y =$ _____

Period (p) = $4X_1 =$ _____

Frequency (b) = $2\pi/p =$ _____

7. By substituting a , b , and k into $y = a(\sin bx) + k$, determine the name of the sine curve that is defined by the above conditions.

D. Superimposing a generated sine curve over a manually generated curve and determine similarities

1. Press <ON> and <Y=>, then enter the curve.
2. Press <GRAPH> to place the sine curve on the same graph system as the curve.
3. Discuss the similarities and draw conclusions.
4. What reasons would cause discrepancies?