DEPARTMENT OF COMMERCE
FREDERICK H. MUELLER, Acting Secretary

WEATHER BUREAU
F. W. REICHELDERFER, Chief

MONTHLY WEATHER REVIEW

JAMES E. CASKEY, JR., Editor

Volume 87
Number 5

MAY 1959

Closed July 15, 1959
Issued August 15, 1959

SOME STATISTICS ON THE MAGNITUDE OF THE AGEOSTROPHIC WIND OBTAINED FROM CONSTANT LEVEL BALLOON DATA*

J. K. ANGELL
U.S. Weather Bureau, Washington, D.C.

[Manuscript received February 11, 1959; revised March 11, 1959]

ABSTRACT

On the basis of wind speeds and accelerations derived from U.S. Navy sponsored 300-mb. constant level balloon or transosonde flights made during 1953, 1955, and 1956, statistics are presented on the magnitude of the ageostrophic wind and its variation with latitude and wind speed. These statistics indicate that at 300 mb. the average angle between wind and geostrophic wind is 11 degrees and the mean magnitude of the vector deviation between wind and geostrophic wind is 12 m. sec.$^{-1}$. The data also show that, through the use of the geostrophic and gradient wind approximations, half the time errors greater than 29 percent and 11 percent, respectively, are introduced into the derived results.

1. INTRODUCTION

Previous evaluations of the magnitude of the ageostrophic wind have in large part been based on comparisons of wind data with geostrophic wind data obtained from synoptic weather maps, as illustrated by the works of Neiburger et al. [1] and Machta [2]. A few evaluations of the ageostrophic wind have been based on estimations of the partial wind derivatives in the equations of motion, as illustrated by the work of Godson [3]. Durst and Gilbert [4] and Neiburger and Angell [5] evaluated the ageostrophic wind by determining the value of the individual wind derivatives from constant level balloon flights and introducing this value into the equation of motion. Since limited constant level balloon data were available for analysis in the latter two papers, it seemed desirable to carry through further analysis upon the receipt of more constant level balloon or transosonde data. This paper presents statistics on the magnitude of the ageostrophic wind, the variation of this magnitude with latitude and speed, and indicates the errors made in assuming the wind to be geostrophic or gradient, all based on 635 evaluations of the ageostrophic wind from 28 transosonde flights made at 300 mb. Of these 28 transosonde flights, 8 were launched from Minneapolis, Minn. in 1953, 4 were launched from Vernalis, Calif. in 1955, and 16 were launched from Oppama, Japan in 1956. The average duration of these flights was 65 hours and the average trajectory length was 4,700 nautical miles. The flights were positioned at 1-hourly or 2-hourly intervals by means of the excellent Federal Communications Commission (FCC) radio-direction-finding network. Anderson [6] found, by comparison of FCC positions with transosonde positions determined by photographs of the underlying terrain, that within the United States two-thirds of the time the FCC positions are in error by less than 20 nautical miles. Over the Atlantic and Pacific Oceans one would expect this error to be larger owing to the greater distances between tracking stations and balloons.

---

*This work was performed under Office of Naval Research Contract Nonr-233(21) for the Naval Research Laboratory in connection with BuAer Problem No. TED-NRL-MA505.

2. DETERMINATION OF THE WIND AND AGEOSTROPHIC WIND

With minor approximations, the vector equation of motion may be written:

$$\frac{d\mathbf{V}}{dt}=f(\mathbf{V}-\mathbf{V}_g)\times\mathbf{k}+\mathbf{F} \qquad (1)$$

where $\mathbf{V}$ is the horizontal component of the velocity, $\mathbf{V}_g$ is the geostrophic velocity, f is the Coriolis parameter, $\mathbf{F}$ is the frictional force per unit mass, and $\mathbf{k}$ is the unit vertical vector. The term on the left hand side of (1) represents the change of horizontal velocity following the three-dimensional path of an air parcel and may be expanded into three terms involving partial derivatives,

$$\frac{d\mathbf{V}}{dt}=\frac{\partial\mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}+w\frac{\partial\mathbf{V}}{\partial z} \qquad (2)$$

where w is the vertical velocity and ∇ is the horizontal differential operator. The constant level balloon data determine the sum-value of the first two terms on the right hand side of (2). The sum of these two terms (local change of velocity plus horizontal advection of velocity) is hereafter called the partial acceleration. The value of the third term on the right hand side of (2), the vertical advection of velocity, may be approximated along a constant level balloon trajectory by determining the vertical motion by the adiabatic technique (vertical motion proportional to the change of temperature following the balloon and inversely proportional to the deviation of the lapse rate from the process lapse rate along the trajectory), and by determining the vertical wind shear from rawin data. Based on a limited number of computations from the 1953 flights and reported in reference [5], the magnitude of the vertical advection of velocity averages about 25 percent of the magnitude of the partial acceleration, and thus is certainly not negligible. Nevertheless, in this paper it is assumed that the partial acceleration data derived from the transosondes represents a reasonable approximation to the total acceleration since the bulk of the transosonde positions are over oceans where it is impractical to estimate the vertical advection of velocity by the techniques described above.

In order to pass from the value of the acceleration to the value of the geostrophic deviation it is necessary, as seen from (1), to neglect friction. While attempts have been made to estimate the magnitude of the frictional force from a combination of radiosonde and transosonde data, the evaluations, particularly over the oceans, are far too crude for inclusion here. Hence, an additional degree of uncertainty (in addition to the neglect of the vertical advection of velocity) enters into the values of the geostrophic deviations presented below.

With the neglect of friction and the vertical advection of velocity we find, after taking the dot product of (1) with the unit vector along and normal to the wind, that, respectively,

$$dV/dt=fV_g\sin i \qquad (3)$$

and

$$Vd\theta/dt=-f(V-V_g\cos i) \qquad (4)$$

where $d\theta/dt$ is the angular velocity of an air parcel and i is the angle of indraft (the angle between wind and geostrophic wind). Taking into account the findings of Neiburger and Angell with respect to the time intervals for which the velocity and acceleration should be computed in order to avoid large errors in these parameters due to radio-direction-finding errors in the positioning of the transosondes, the speed was determined from a 4-hour transosonde displacement; the tangential acceleration (dV/dt) was determined from the change in 4-hour average transosonde speed in 8 hours; and the normal acceleration ($Vd\theta/dt$) was determined as the product of the mean transosonde speed during an 8-hour time interval and the change of 4-hour average transosonde direction in 8 hours.

Upon eliminating the geostrophic wind speed between (3) and (4), the angle of indraft can be expressed in terms of three known variables; namely, the tangential acceleration, the normal acceleration, and the Coriolis acceleration (fV). From knowledge of the angle of indraft, the geostrophic wind can be evaluated from either (3) or (4). The cross-contour component of the ageostrophic wind ($V \sin i$) can then be determined as well as the along-contour component of the ageostrophic wind ($V \cos i-V_g$). In the diagrams in this paper $V-V_g$ is substituted for $V \cos i-V_g$ since, for the magnitudes of the angles of indraft obtained at 300 mb., the two quantities are practically identical.

3. VELOCITY AND AGEOSTROPHIC VELOCITY STATISTICS

In this section and in sections 4 and 5 we show by means of histograms and cumulative frequency curves the magnitudes of various velocity and ageostorophic velocity parameters obtained from the transosonde flights. Where comparison of two parameters is desirable, histograms and cumulative frequency curves are superimposed on the same diagram.

The magnitude of the 4-hour average wind speed at 300 mb. for this series of transosonde flights is indicated in the upper diagram of figure 1. Based upon 755 calculations, the mean wind speed is 38 m. sec.$^{-1}$ while the mode is 20–30 m. sec.$^{-1}$. Ten percent of the winds exceed 67 m. sec.$^{-1}$. The maximum wind speed determined from a 4-hour transosonde displacement is 150 m. sec.$^{-1}$. This value occurred on Flight 23 approximately 1,000 miles east-southeast of Tokyo in January 1956, and is undoubtedly one of the largest, reasonably reliable, values of wind speed ever recorded.

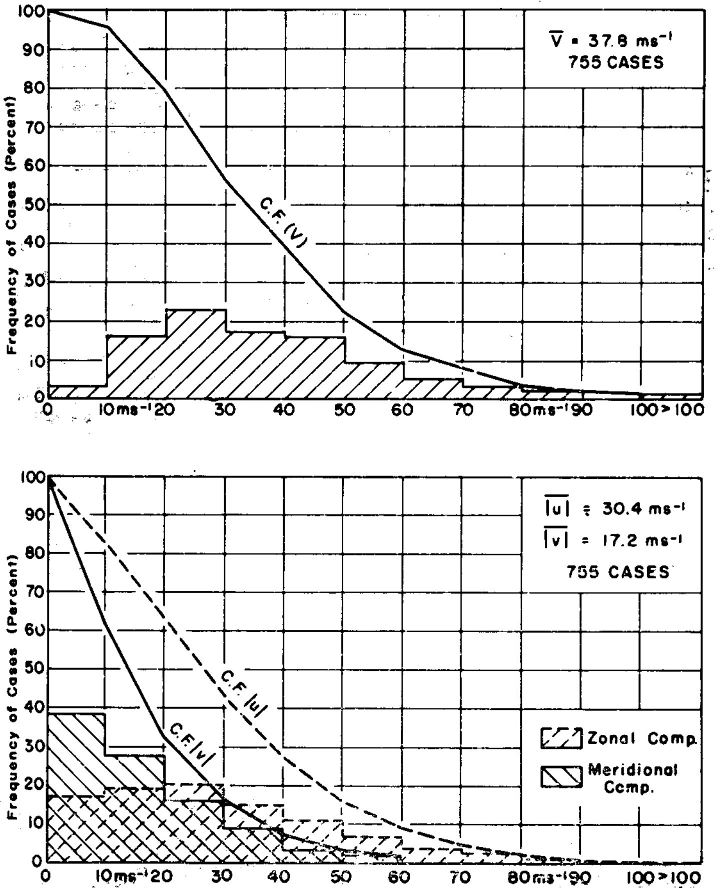

FIGURE 1.—Magnitude of the 300-mb. wind speed, V, (top) and its zonal, $|u|$, and meridional, $|v|$, components (bottom).

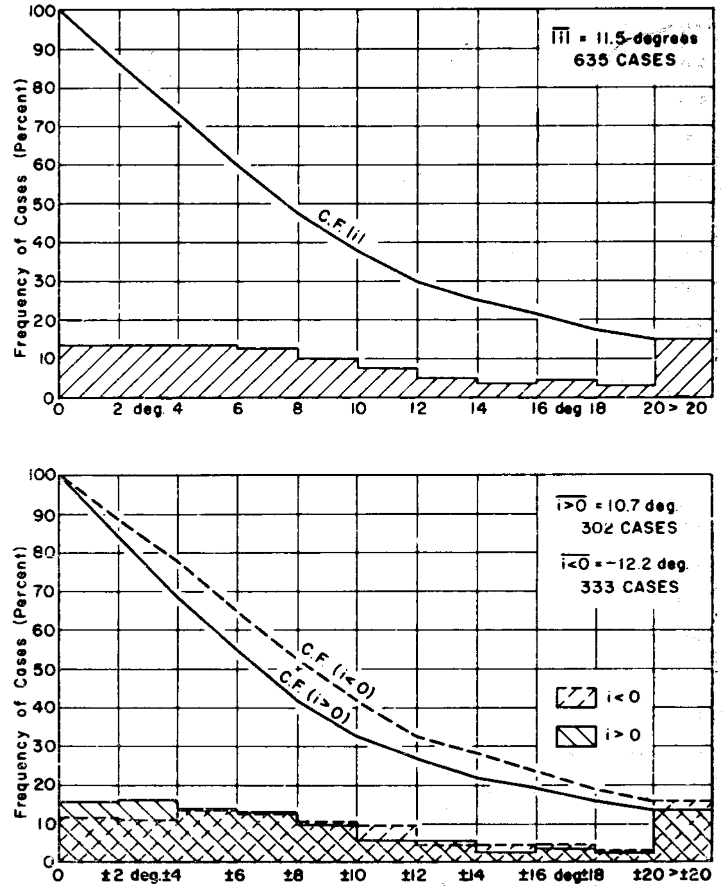

FIGURE 2.—Absolute magnitude (top) and algebraic magnitude (bottom) of angle of indraft, i, at 300 mb.

The magnitudes of the zonal and meridional wind components are indicated in the lower diagram of figure 1. These data demonstrate the well known fact that the mean value of the zonal wind component (here 30 m. sec.$^{-1}$) is considerably larger than the mean value of the meridional wind component (17 m. sec.$^{-1}$).

The absolute magnitude of the angle of indraft, indicated in the top diagram of figure 2, is based on 635 calculations. The mode is indistinct and lies in the range 0–6 degrees, while the median is 8 degrees and the mean is 11 degrees. In 15 percent of the cases the angle exceeds 20 degrees. The considerable difference between median and mean is attributable to a relatively few large values of the angle of indraft of doubtful validity. For comparison, Godson found by substituting synoptic wind data at 700 mb. in the equation of motion a mean value of 14 degrees for the angle of indraft. The larger mean mean value for his data would be expected since the magnitude of the angle of indraft is inversely proportional to the wind speed, as shown in section 6 of this paper.

In the lower diagram of figure 2 is shown the algebraic magnitude of the angle of indraft, considered positive when the flow is toward low pressure and negative when the flow is toward high pressure. It is evident from the cumulative frequency curves in this diagram that in the mean the angle of indraft is of greater magnitude when the flow is toward high pressure than when the flow is toward low pressure. It is believed that this tendency is at least partially connected with the occasional loss of geostrophic control in regions of inertial instability in the deceleration region downstream from sharply curved ridges.

The magnitude of the vector geostrophic deviation $|\mathbf{V}-\mathbf{V}_g|$ is indicated in the upper diagram of figure 3. The mode is 2–4 m. sec.$^{-1}$, the median is 10 m. sec.$^{-1}$ and the mean is 12 m. sec.$^{-1}$. In 15 percent of the cases the deviation is greater than 20 m. sec.$^{-1}$. For comparison, Machta found an average value of 13 m. sec.$^{-1}$ for the vector geostrophic deviation at 300 mb. during the winter months. His data were obtained by comparing the wind and pressure gradient (geostrophic wind) on analyzed synoptic maps.

The absolute magnitudes of the cross-contour ($V \sin i$) and along-contour ($V-V_g$) components of the vector geostrophic deviation are given in the lower diagram of figure 3. In the case of the cross-contour component, the

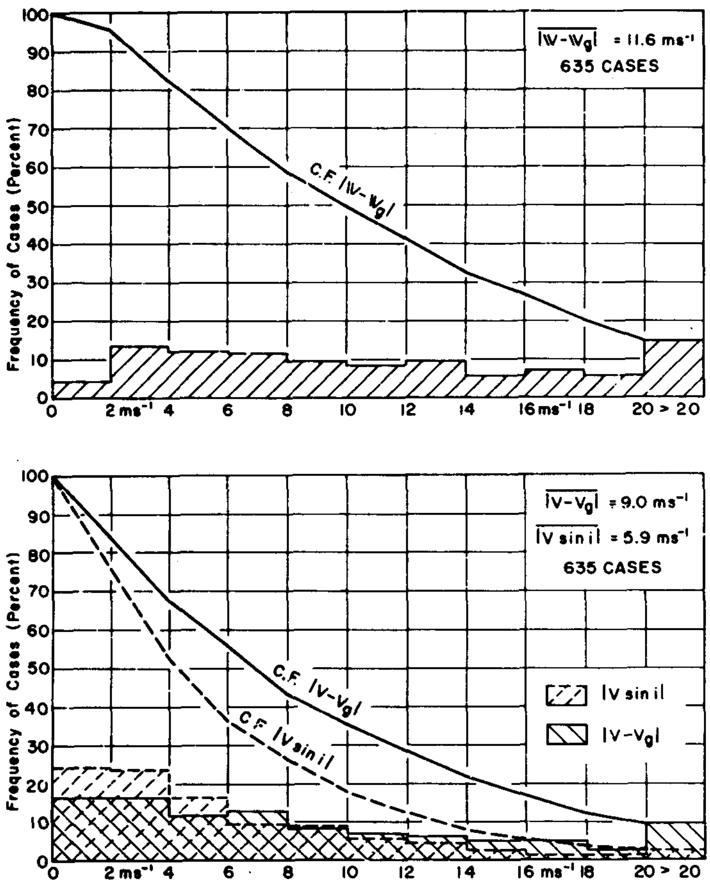

FIGURE 3.—Magnitude of the 300-mb. vector geostrophic deviation, $|\mathbf{V}-\mathbf{V}_g|$, (top) and its cross-contour, $|V \sin i|$, and along-contour, $|V-V_g|$, components (bottom).

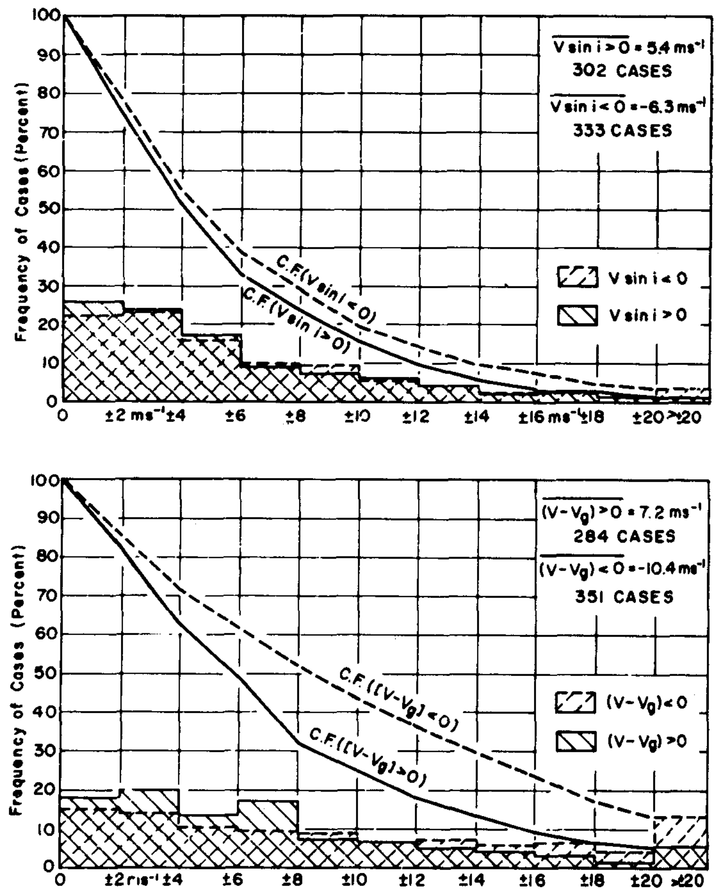

FIGURE 4.—Algebraic magnitude of the 300-mb. cross-contour flow, $V \sin i$, (top) and along-contour component of the vector geostrophic deviation, $V-V_g$, (bottom).

mode, the median, and the mean are respectively 0–4 m. sec.$^{-1}$, 4 m. sec.$^{-1}$, and 6 m. sec.$^{-1}$ whereas for the along-contour component they are, respectively, 0–4, m. sec.$^{-1}$, 7 m. sec.$^{-1}$, and 9 m. sec.$^{-1}$. In the case of the cross-contour component, 10 percent of the values exceed 13 m. sec.$^{-1}$ whereas in the case of the along-contour component, 10 percent of the values exceed 19 m. sec.$^{-1}$. For comparison, Neiburger et al. found at 700 mb. a mean value of 3 m. sec.$^{-1}$ for the along-contour component of the vector geostrophic deviation. This smaller value would be expected since the magnitude of this component is a function of the wind speed, as will be shown below.

The algebraic magnitude of the cross-contour flow is shown in the upper diagram of figure 4. In agreement with the findings for the angle of indraft, the cumulative frequency curves indicate that the magnitude of the cross-contour flow is greater when the flow is toward high pressure than when the flow is toward low pressure.

The algebraic magnitude of the along-contour component of the vector geostrophic deviation is given in the lower diagram of figure 4. In this diagram the difference between the cumulative frequency curves is striking indeed, with the difference between the wind speed and geostrophic wind speed being much larger, in the mean, when the flow is cyclonic than when the flow is anticyclonic. The respective median values are -8 m. sec.$^{-1}$ and 6 m. sec.$^{-1}$. It may be that this asymmetry is due to the existence of a type of upper bound on the degree to which the wind may be supergeostrophic on ridges (twice the geostrophic value), whereas there is no such bound in troughs.

4. ERRORS RESULTING FROM THE USE OF GEOSTROPHIC AND GRADIENT WIND EQUATIONS

The magnitude of the partial acceleration $(\partial\mathbf{V}/\partial t+\mathbf{V}\cdot\nabla\mathbf{V})$ in comparison with the Coriolis acceleration (fV) is of interest since the geostrophic assumption states that the partial acceleration is negligible in comparison with the Coriolis acceleration. The magnitude of the ratio of partial acceleration and Coriolis acceleration is given in the upper diagram of figure 5. The mode of the ratio is roughly 0.10–0.20 while the median is 0.29 and in 10 percent of the cases the ratio exceeds 0.83. In other words these data suggest that,

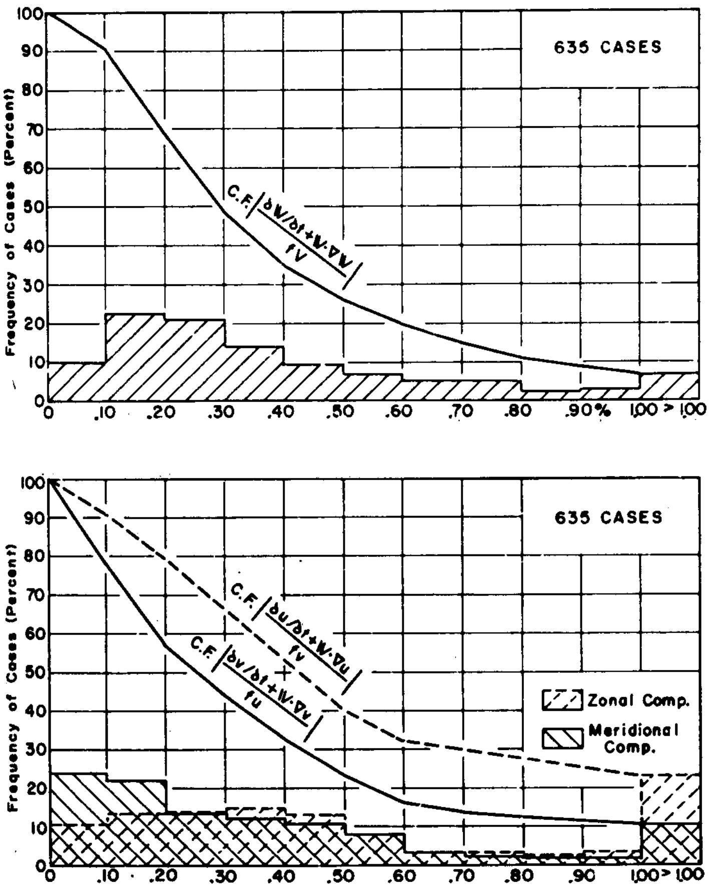

FIGURE 5.—Ratio of partial and Coriolis acceleration, $\left|\frac{\partial \mathbf{V}/\partial t+\mathbf{V}\cdot\nabla \mathbf{V}}{fV}\right|$, at 300 mb. (top) and the ratio of the zonal, $\left|\frac{\partial u/\partial t+\mathbf{V}\cdot\nabla u}{fv}\right|$, and meridional, $\left|\frac{\partial v/\partial t+\mathbf{V}\cdot\nabla v}{fu}\right|$, components (bottom).

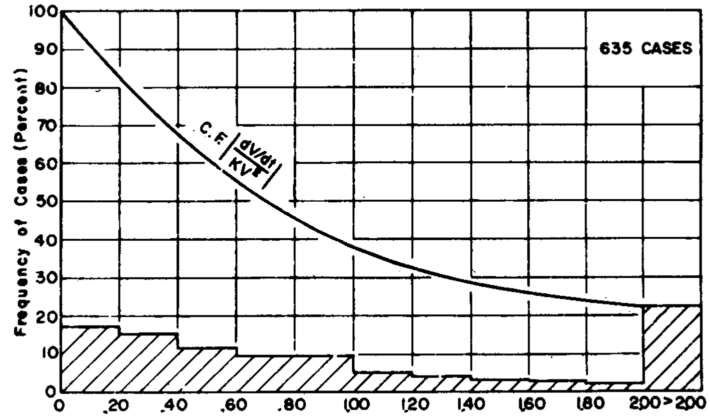

FIGURE 6.—Ratio of tangential and normal components of partial acceleration, $\left|\frac{dV/dt}{KV^2}\right|$, at 300 mb.

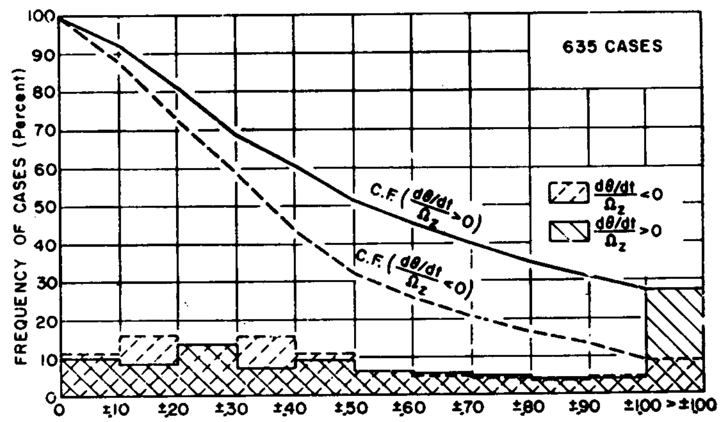

FIGURE 7.—Algebraic ratio of the angular velocity of the wind and the angular velocity of the earth about the local vertical $(d\theta/dt)/\Omega_z$ at 300 mb.

through the introduction of the geostrophic approximation, half the time an error exceeding 29 percent is introduced into the derived results.

The magnitudes of the ratios of zonal and meridional components of partial acceleration and Coriolis acceleration are given in the lower diagram of figure 5. These data suggest that the neglect of the acceleration in the zonal component of the equation of motion results in an untenable approximation, since for this component the mode of the ratio is 0.30–0.40 and the median is 0.41. In the case of the meridional component of the equation of motion, however, the ratio of partial acceleration and Coriolis acceleration has a mode of only 0–0.10 and a median value of only 0.25. In this diagram the relatively large percentage of cases where the ratio exceeds 1.00 is due mainly to the existence of "cusp" points in the trajectories. At such points the velocity component may go toward zero at a time when the component of the partial acceleration is far from zero.

The magnitude of the ratio of tangential and normal components of partial acceleration is of interest since the gradient wind approximation assumes the tangential component to be negligible in comparison with the normal component. It is seen from figure 6 that the mode of this ratio is 0–0.20 while the median is 0.67. In 38 percent of the cases the tangential component of the partial acceleration is larger than the normal component. From consideration of the median ratio of the tangential and normal components of partial acceleration and the median ratio of partial acceleration and Coriolis acceleration we estimate that through the use of the gradient wind approximation half the time an error exceeding 11 percent is introduced into the derived results.

5. FREQUENCY OF OCCURRENCE OF "ABNORMAL" FLOW

In figure 7 is indicated the ratio of the algebraic angular velocity of the air parcel or transosonde and the angular velocity of the earth about the local vertical. It is seen that in 27 percent of the cases the angular velocity of the air parcel exceeds that of the earth about the local vertical

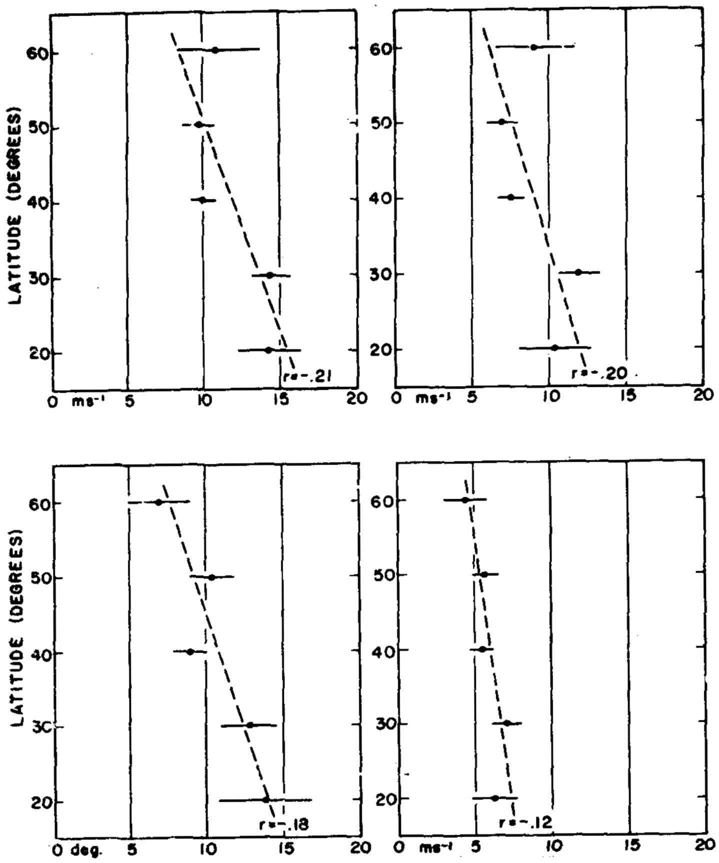

FIGURE 8.—Variation with latitude of angle of indraft, $|i|$, (lower left), and of the vector geostrophic deviation, $|\mathbf{V}-\mathbf{V}_g|$, (upper left) and its along-contour, $|V-V_g|$, (upper right) and cross-contour, $|V \sin i|$, (lower right) components at 300 mb.

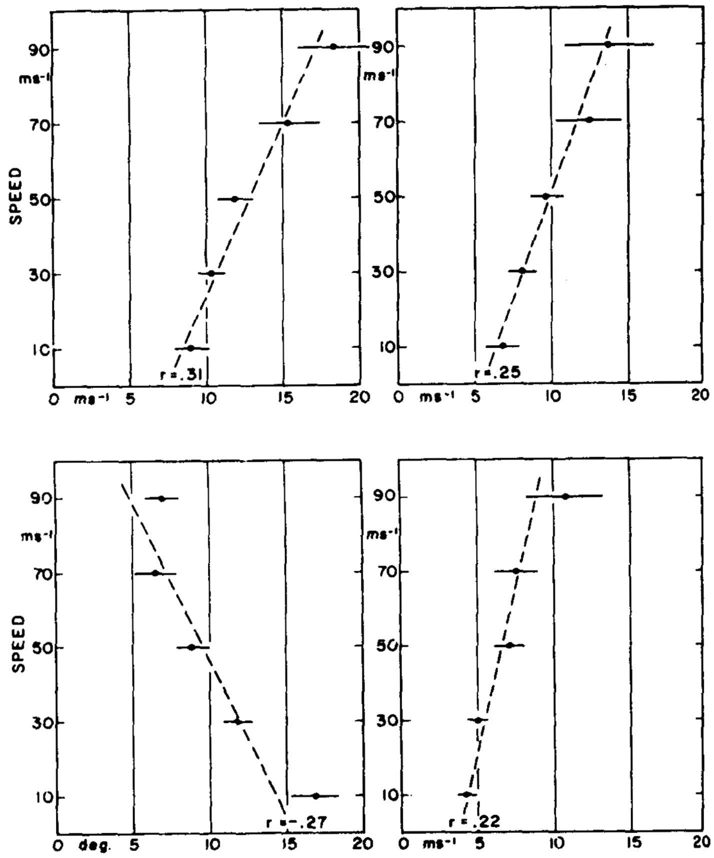

FIGURE 9.—Variation with speed of angle of indraft, $|i|$, (lower left), and of the vector geostrophic deviation, $|\mathbf{V}-\mathbf{V}_g|$, (upper left) and its along-contour, $|V-V_g|$, (upper right) and cross-contour, $|V \sin i|$, (lower right) components at 300 mb.

when the angular velocity is positive (cyclonic flow), but that this criterion is satisfied in only 9 percent of the 398 cases when the angular velocity of the air parcel is negative (anticyclonic flow). Thus in 5 percent of the total 635 cases the flow allegedly possesses anticyclonic rotation in space. Flow which satisfies this criterion is called "abnormal" or "anomalous" flow by Holmboe et al. [7] and Gustafson [8].

6. VARIATION OF AGEOSTROPHIC PARAMETERS WITH LATITUDE AND SPEED

In this section and in section 7 the data are presented in the form of group means, regression lines, and correlation coefficients (figs. 8–11). On either side of the group means, lines have been drawn extending a distance equal to two standard errors from the mean. If the data are randomly drawn from a normal population there is only a 5 percent chance that the true group mean lies outside the extent of these lines. An estimate of significance of variability at the 5 percent level can then be obtained by noting whether a straight vertical line can be drawn so as to intersect all these lines extending either side of the group mean. The regression lines and correlation coefficients serve to yield an indication of the overall trend but it is not meant to imply thereby that these trends are actually linear in nature. It is also possible to determine the correlation coefficient which would indicate significance at the 5 percent level. Based on the z transformation presented in Hoel [9], it is found that for the number of cases available here a correlation greater than ± 0.08 is significant at the 5 percent level.

With regard to the variation of the parameters with respect to latitude it must be emphasized that the results are biased since the transosondes sample chiefly cyclonic flow patterns at southerly latitudes and anticyclonic flow patterns at northerly latitudes. If transosondes were released from stations along a given meridian but at different latitudes, it is probable that different mean values would be found from those presented below.

Figure 8 gives the variation with latitude of the absolute magnitude of the angle of indraft and the vector geostrophic deviation and its along-contour and cross-contour components. All four parameters decrease in magnitude with increasing latitude, with the smallest decrease being found for $V \sin i$ ($r=-0.12$). The group means show that the variability is ragged. The magnitude of the vector geostrophic deviation varies from a mean of 10 m. sec.$^{-1}$ at northerly latitudes to 14 m. sec.$^{-1}$ at southerly latitudes with most of the change occurring abruptly at latitude 35°. The absolute magnitude of the angle of indraft varies from a mean value of about 8 degrees at northerly latitudes to a mean value of 13 degrees at southerly latitudes, while the absolute magnitude of $V \sin i$ varies from a mean value of 4–5 m. sec.$^{-1}$ in northerly latitudes to a mean value of 7–8 m. sec.$^{-1}$ in southerly latitudes.

Figure 9 gives the variation with speed of the absolute magnitude of the angle of indraft and the vector geostrophic deviation and its along-contour and cross-contour components. While the angle of indraft shows a significant decrease in magnitude with increase in speed ($r=-0.27$) the ageostrophic parameters show significant increases in magnitude with increase in speed. It is of interest to note that the magnitude of the cross-contour flow increases with increase in speed despite the counteracting tendency of the angle of indraft. The vector geostrophic deviation varies in average magnitude from 8 m. sec.$^{-1}$ at low speeds to 18 m. sec.$^{-1}$ at high speeds. The absolute magnitude of the angle of indraft varies from 17 degrees at low speeds to 7 degrees at high speeds, while the absolute magnitude of the cross-contour flow varies from 4 m. sec.$^{-1}$ at low speeds to 11 m. sec.$^{-1}$ at high speeds.

7. VARIATION WITH LATITUDE AND SPEED OF ERRORS RESULTING FROM THE USE OF GEOSTROPHIC AND GRADIENT WIND EQUATIONS

The two diagrams in figure 10 give the variations with latitude of the ratio of partial acceleration and Coriolis acceleration and the ratio of tangential and normal components of partial acceleration. As would be expected, the ratio of partial acceleration and Coriolis acceleration varies greatly with latitude ($r=-0.32$), ranging from a mean value of 0.30 at northerly latitudes to a mean value of 0.70 at southerly latitudes. The group means indicate that the ratio is almost constant north of latitude 40°, with a rapid, nearly linear increase in the value of the ratio as one progresses southward from latitude 40°. The ratio of the tangential and normal components of the partial acceleration shows little variation with latitude ($r=-0.03$). The group means also indicate no significant variations except at latitude 60° where the large values of the normal components of partial acceleration on the crests of trajectories overbalance the tangential components.

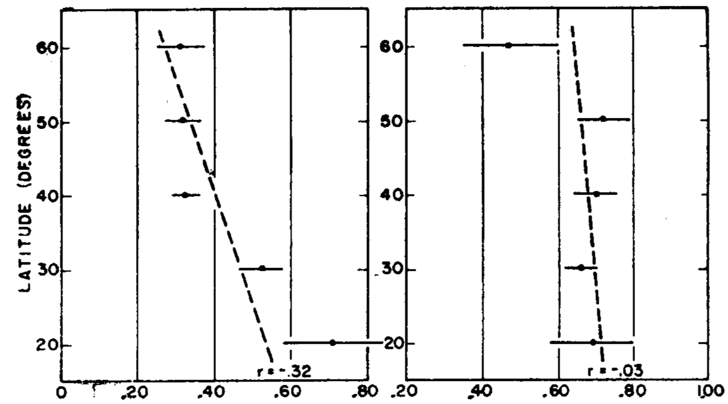

FIGURE 10.—Variation with latitude of ratio of partial and Coriolis acceleration, $\left|\dfrac{\partial \mathbf{V}/\partial t+\mathbf{V}\cdot\nabla\mathbf{V}}{fV}\right|$, (left) and ratio of tangential and normal components of partial acceleration, $\left|\dfrac{dV/dt}{KV^2}\right|$, (right) at 300 mb.

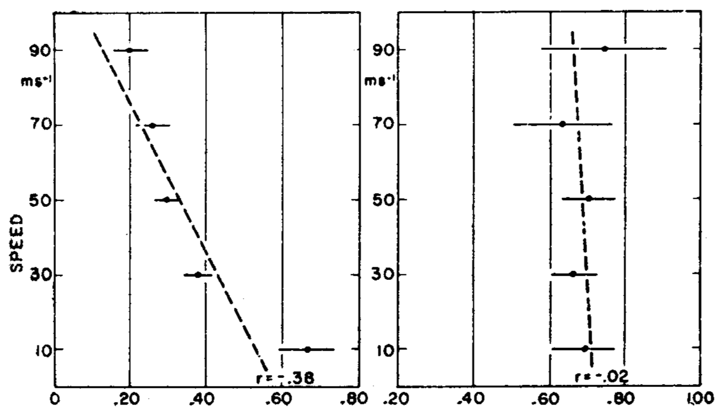

FIGURE 11.—Variation with speed of ratio of partial and Coriolis acceleration, $\left|\dfrac{\partial \mathbf{V}/\partial t+\mathbf{V}\cdot\nabla\mathbf{V}}{fV}\right|$, (left) and ratio of tangential and normal components of partial acceleration, $\left|\dfrac{dV/dt}{KV^2}\right|$, (right) at 300 mb.

The two diagrams in figure 11 give the variations with speed of the ratio of partial acceleration and Coriolis acceleration and the ratio of tangential and normal components of partial acceleration. As would be expected, the ratio of partial acceleration and Coriolis acceleration shows a significant decrease in magnitude with increase in speed ($r=-0.38$) with the mean value of the ratio varying from 0.66 at low speeds to 0.20 at high speeds. The ratio of tangential and normal components of partial acceleration shows no significant variation with speed ($r=-0.02$), the group means at all speeds resting close to 0.70.

8. CONCLUSION

Statistics based on 635 evaluations from 300-mb. constant level balloon data indicate that through the use of the geostrophic wind approximation half the time an error greater than 29 percent will be introduced into the derived results, whereas through the use of the gradient wind approximation half the time an error greater than 11 percent will be introduced into the derived results. On a percentage basis both the geostrophic and gradient wind approximations are about twice as bad at latitude 20° as at latitude 60° and about twice as bad at speeds of 10 m. sec.$^{-1}$ as at speeds of 50 m. sec.$^{-1}$. The magnitudes of angles of indraft and ageostrophic parameters are in fair agreement with results obtained by other techniques, with the transosonde data yielding a mean angle of indraft of 11 degrees, a mean cross-contour flow of 6 m. sec.$^{-1}$, a mean deviation between wind and geostrophic wind speed of 9 m. sec.$^{-1}$, and a mean vector geostrophic deviation of 12 m. sec.$^{-1}$.

REFERENCES

1. M. Neiburger et al., "On the Computation of Wind from Pressure Data," *Journal of Meteorology*, vol. 5, No. 3, June 1948, pp. 87–92.
2. L. Machta, A Study of the Observed Deviations from the Geostrophic Wind, ScD thesis, Dept. of Meteorology, Massachusetts Institute of Technology, 1948. (Unpublished).
3. W. L. Godson, "A Study of the Deviations of Wind Speeds and Directions from Geostrophic Values," *Quarterly Journal of the Royal Meteorological Society*, vol. 76, No. 327, Jan. 1950, pp. 1–15.
4. C. L. Durst and G. H. Gilbert, "Constant-Height Balloons—Calculation of Geostrophic Departures," *Quarterly Journal of the Royal Meteorological Society*, vol. 76, No. 327, Jan. 1950, pp. 75–86.
5. M. Neiburger and J. K. Angell, "Meteorological Applications of Constant-Pressure Balloon Trajectories," *Journal of Meteorology*, vol. 13, No. 2, Apr. 1956, pp. 166–194.
6. A. D. Anderson, "A Study of the Accuracy of Winds Derived from Transosonde Data," (Memo. Rep. 498), Naval Research Laboratory, Washington, D.C., 1955, 10 pp.
7. J. Holmboe, G. E. Forsythe, and W. Gustin, *Dynamic Meteorology*, New York, John Wiley and Sons, 1945, 378 pp.
8. A. F. Gustafson, "On Anomalous Winds in the Free Atmosphere," *Bulletin of the American Meteorological Society*, vol. 34, No. 5, May 1953, pp. 196–201.
9. P. G. Hoel, *Introduction to Mathematical Statistics*, New York, John Wiley and Sons, 1947, 258 pp.