PNNL-14106

Pacific Northwest National Laboratory
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Statistical Analyses of AY-101 Ultrasonic Measurements of Wall Thickness

D. R. Weier

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Prepared for the U.S. Department of Energy
under Contract DE-AC06-76RL01830

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Denny Weier
PNNL Statistics Group
10-31-02

Summary and Conclusions: Tank AY-101 wall thicknesses have been measured using ultrasonic (UT) images. Interest is in using the available data to estimate a worst-case minimum wall thickness for regions of the tank that remain unexamined with the UT approach. Each UT image can provide the wall thickness at a very large number of pixels, but only the minimum value for each image is used in this work. Since the data arise in this manner, one of several candidate statistical extreme value distributions should successfully fit these reported minimum values.

Given a set of multiple UT images and the resulting minimum measured wall thicknesses, extreme value distributions are fit to the data with subsequent extrapolations made to estimate the "minimum measured wall thickness" expected for the entire tank. Such a minimum estimate incorporates both the variability in wall thicknesses and the uncertainty associated with the measurement method. Uncertainties of the estimated parameters are also derived and used in propagation of variance methods to obtain confidence bounds on the estimated minimum measured wall thickness as well.

Three distinct populations are identified for investigation: the Liquid Air Interface (LAI) within Plate 2, Plate 3, and Plate 2 outside of the LAI. The body of this report establishes estimates for overall minimum measured wall thickness for these three Tank AY-101 regions. These minimums are respectively 0.4014, 0.4212, and 0.4251 inches. Ninety-five percent confidence bounds for these minimums that reflect the uncertainties in the estimated extreme value distribution parameters are respectively given as 0.3913, 0.4181, and 0.4173.

Note that several other tanks have similar UT data available. To confirm measurement system performance, Tank AP-108 was briefly considered as well. A display of the data for this tank is given in an Appendix.

Introduction and Background: Data in this report are obtained from the following three PNNL letter reports prepared by Gerald J. Posakony and Allan F. Pardini:

> *Ultrasonic Examination of Double-Shell Tank 241-AY-101*, August 21, 2001
>
> *Ultrasonic Examination of Double-Shell Tank 241-AY-101 – Riser 91*, June 4. 2002
>
> *Ultrasonic Examination of Double-Shell Tank 241-AY-101*, June 25, 2002

Each data value is the minimum wall thickness resulting from an ultrasonic (UT) scan. Most scans consist of a 3.5 by 12 inch image; such images consist of pixels measuring about 0.035 in. square. Therefore about 34,000 pixels are available in such an image, and the single reported value is then the minimum of the 34,000 wall thicknesses observed at these pixel locations. This measurement scenario suggests that the behavior of these minimum measured wall thicknesses should therefore be adequately modeled by using statistical extreme value distributional theory. This is discussed in more detail later.

The objective is to determine the potential minimum wall thickness for the entire tank (and thus the tank's fitness for use) based on what has been measured using UT. The circular tank wall consists of five plates as listed in the following. The given heights are at the top edges of the plates as measured from the bottom of the tank. Note by subtracting the successive heights, that the heights for each of Plate 2 and 3 cover just a bit less than a 10 foot range.

Plate	Height	Nominal Thickness
1	418.9 in.	0.375 in.
2	380.8 in.	0.500 in.
3	262.7 in.	0.500 in.
4	144.5 in.	0.750 in.
5	36.5 in.	0.875 in.

Preliminary investigation readily shows the plates of concern with respect to minimum wall thickness are Plates 2 and 3 with the 0.5-inch nominal thickness. Wall thickness for these two plates is the subject of this report.

Discussion: The data provided in the reports listed above for Plates 2 and 3 are illustrated in Figure 1 on the following page. Note that the figures and computational requirements in this report were done using JMP software (see JMP(2002)). The three reports are referred to in the figure and throughout the remainder of the report as being respectively for years 2001, 2001.5, and 2002, which indicate the order in which the measurement activities were conducted.

Vertical axes in the figure are remaining minimum measured wall thickness as determined by the ultrasonic (UT) measurement. Horizontal axes represent the heights on the tank walls of the UT images. The top of the tank is therefore to the right on the axis. The specific height reported for each image is along the top boundary of the image. Therefore a 12-inch vertical image reported at 258 inches actually scans the tank wall from height 246 to 258.

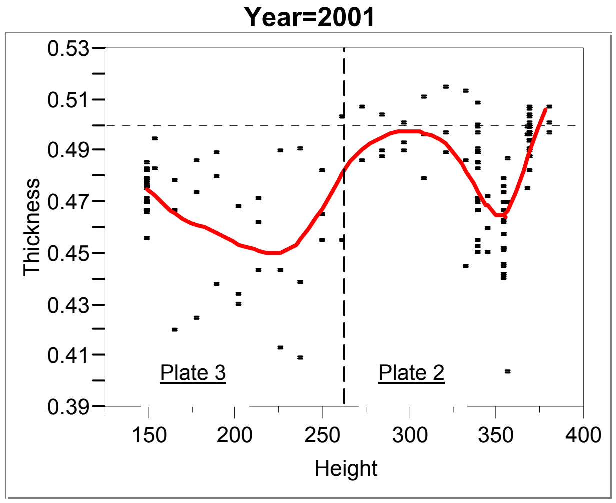
Note in the table above that the boundary between Plates 2 and 3 is at height 262.7. UT measurements recorded at height 262 are therefore at the top edge of Plate 3, but contained entirely within the plate. Note in Figure 1 the presence of the Liquid Air Interface (LAI) within Plate 2 from heights approximately 340 to 360. This is the level at which the top of the stored waste was located for many years. The interaction of waste, condensation, and air on the tank wall is thought to have accelerated corrosion at this level. Note also in Figure 1 that considerable differences in measured wall thickness can be observed between Plate 2 (outside of the LAI) and Plate 3, that is, on either side of height 262.

Note that the 2001 data consists of 15 by 12 UT images while the 2001.5 and 2002 consisted of 3.5 by 12 inch images. The exception is that most of the LAI measurements in 2002 were also 15 by 12, or 12 by 15, or even 12 by 30. This will be discussed in more detail in further discussion of the LAI.

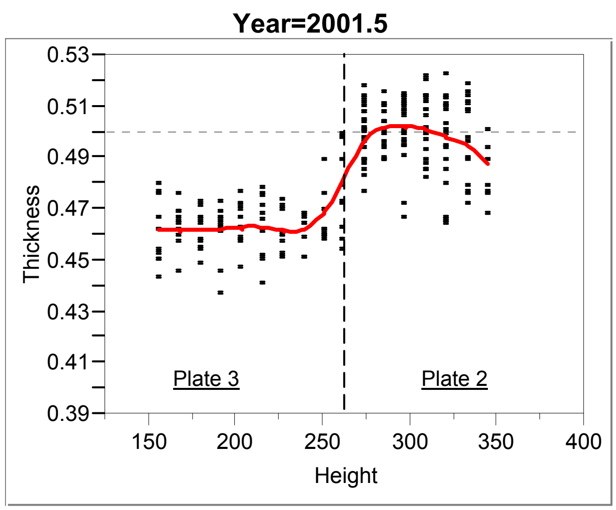
Each measurement in 2001 in some sense represents the minimum of four measurements in the 2001.5 and 2002 data. This is since four adjacent 3.5 by 12 inch images approximate a 15 by 12 inch image. By reporting the minimum wall thickness for each image, four measurements would be generated for the four 3.5 by 12 images, but only a single measurement, the minimum of these four, would have been generated for the 15 by 12 image. Therefore less variability generally would be expected in the 2001 data. Yet more variability is shown for 2001 in Figure 1.

The differing image size, and discussions with Cogema personnel involved in taking the UT measurements led to the decision to omit the 2001 data from the Plate 2 (outside the LAI) and Plate 3 analyses. For the 2001 data a grinding tool was drug up and down the exterior of the wall where the UT measurement device traveled. This wasn't very successful at removing the hard, accumulated material on the wall surface. This left a very rough surface on which the UT device rested. For the 2001.5 and 2002, a high-pressure water device was instead used to clean the wall and gave a much smoother surface that was described as being pretty clean. The greater variability in the 2001 data, when less variability would be expected due to the larger image size, is likely due to the rough surface. Since considerable data are available without including the 2001 data, it was omitted except for in the LAI region. The decision to include 2001 data for the LAI is discussed later.

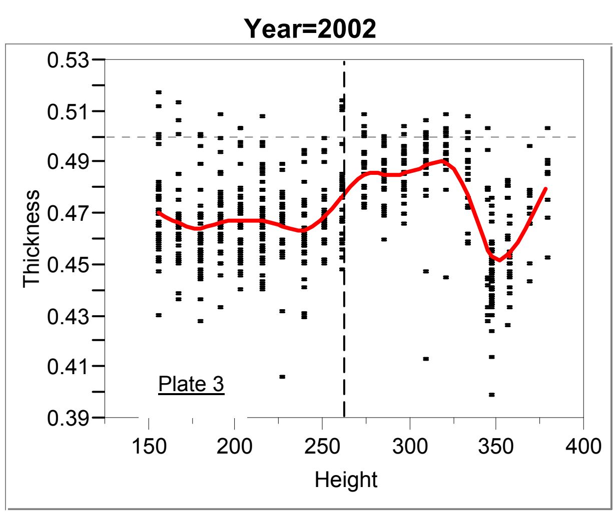
Figure 1: Wall Thickness By Height for Nominal 0.5 inch Plates



15 by 12 inch UT segments



3.5 by 12 inch UT segments



3.5 by 12 inch UT segments except in 343 Liquid Air Interface (LAI)

The 2001.5 and 2002 data for Plates 2 and 3 is shown in Figure 2, by riser in the top four figures, and combined over all four risers in the bottom figure. Note the consistent and substantial differences between Plate 2 (outside the LAI) and Plate 3. Plate 3 can be seen to average about 0.465 in. while Plate 2 outside the LAI can be seen to average about 0.490 in. Such consistent difference is likely due to inherent differences in the two plates. The simplest explanation would be that Plate 2 had nominal thickness generally about 0.025 inches greater than Plate 3 and that subsequent corrosion in fact affected the plates similarly (other than in the LAI).

The combined bottom figure suggests that the data should be divided as coming from three distinct populations: Plate 3, Plate 2 outside the LAI, and the LAI. That is the approach used in this report. Henceforth in this report, "Plate 2" will refer to Plate 2 outside of the LAI. Three cases then need to be examined; they are Plate 3, Plate 2, and the LAI. The initial step is to better define the boundaries of these three groups.

Figure 3 displays the 2001.5 and 2002 data a bit differently to help define the population boundaries. Each diamond represents the location of the mean measured minimum UT wall thickness (over all four risers) observed at the associated height. The vertical extent of the diamond indicates the uncertainty associated with the estimate of the mean at that height. The means at each height are listed in the adjacent table.

To better examine Plate 3, Figure 4 gives the corresponding illustration only for that plate. The only unusual feature is the greater mean at the plate edge at height 262. The mean at this height is statistically significantly greater than the two smallest means, which are located at heights 180 and 240. No other pairs of means would be considered significantly different. This raises the question as to whether height 262 should be included when the primary interest is predicting minimum wall thickness (in the direction towards the bottom of Figure 4). Analyses were in fact performed both with and without the measurements for height 262, and which approach was used made virtually no difference in the minimum wall thickness inference. Remaining discussion for Plate 3 in this report does not include the data for height 262.

Figure 5 shows Plate 2 results to facilitate identifying the extent of the LAI. Figures 6, 7, and 8 are used for this same purpose. Figure 5 contains no 2001 data for the LAI. Note from Figure 1 that the 2001.5 data have few LAI measurements. The 2002 LAI data are a mix of 3.5 by 12, 15 by 12, 12 by 15, and even 12 by 30 images. Significant additional amounts of LAI data are available in the 2001 data, all based on 15 by 12 images. Since we already have a mix of image sizes, and since a significant amount of 2001 LAI data are available relative to the amount of 2001.5 and 2002 LAI data, the 2001 LAI data are included in Figure 6. In spite of the potentially rougher wall surface, the 2001 LAI data behave much like the other LAI data. Perhaps the material buildup on the wall exterior was less of a problem higher on the tank wall.

This gives a mix for the LAI of several image sizes as just described. In particular, the 3.5 by 12 inch images, which constitute the measurements at the 334, 346, 358, 370 heights, are now in the minority. To get a more homogeneous mix of measurements based on more closely similar UT images, the groupings of the 3.5 by 12 images were examined and only the minimum of four adjacent values was used to emulate what would have been obtained with 15 by 12 inch images. This results in a reduced number of values considered at those four heights. Generally the larger values at these heights are deleted in this manner. In particular, twenty-two 3.5 by 12 images were omitted from what will subsequently be identified as the LAI. These twenty-two values would not have been reported with 15 by 12 images. (Discussion continues after Figure 8.)

Figure 2: Wall Thickness By Height
(2001 data not included)

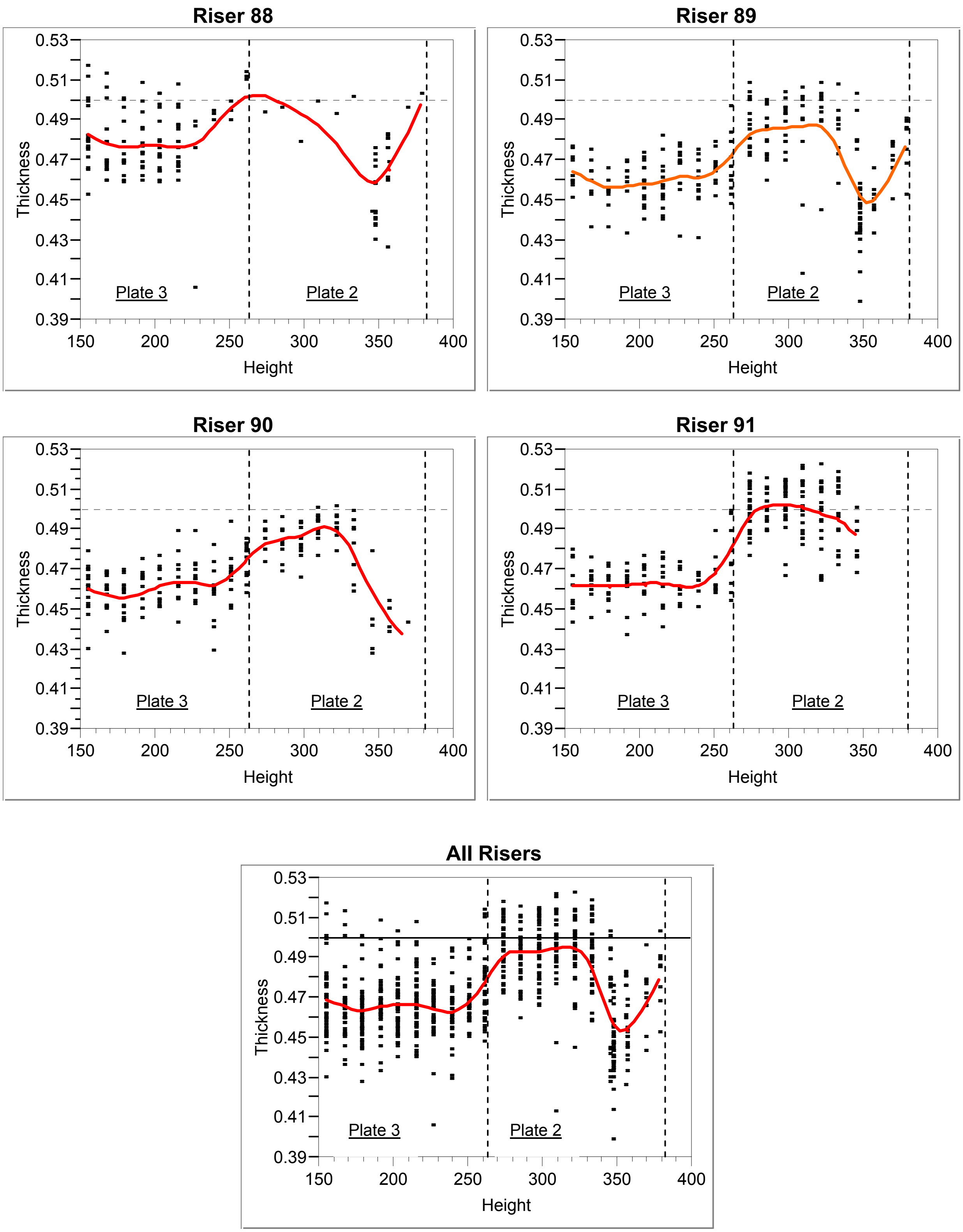
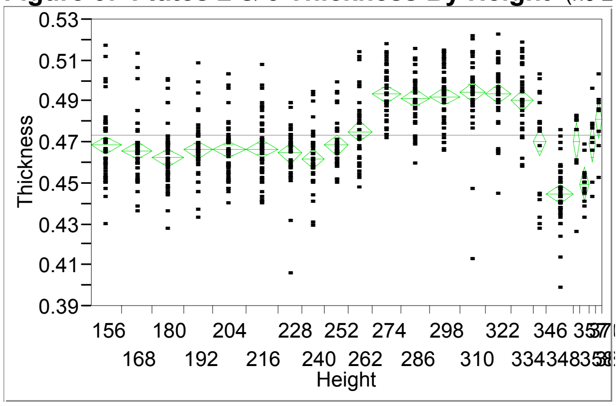


Figure 3: Plates 2 & 3 Thickness By Height (no 2001 data)

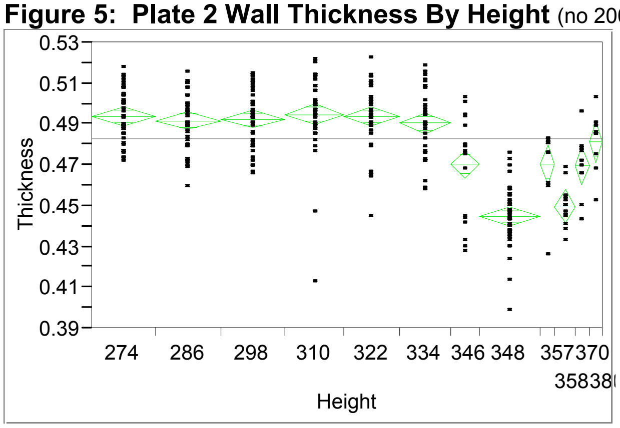


Heightl	Number	Mean
156	49	0.468735
168	49	0.465939
180	49	0.462673
192	45	0.466267
204	53	0.466434
216	52	0.466962
228	37	0.464811
240	36	0.462028
252	37	0.468703
262	37	0.475432
274	46	0.493674
286	46	0.491848
298	46	0.492565
310	41	0.494951
322	40	0.493750
334	36	0.490528
346	21	0.470333
348	43	0.444837
357	10	0.470200
358	14	0.449857
370	10	0.469700
380	9	0.481667

Figure 4: Plate 3 Thickness By Height (no 2001 data)

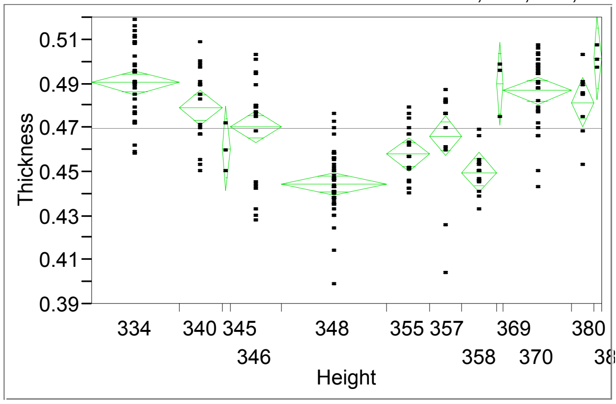
Note: Height 262 is statistically significantly different from heights 180 and 240.

Figure 5: Plate 2 Wall Thickness By Height (no 2001 data)



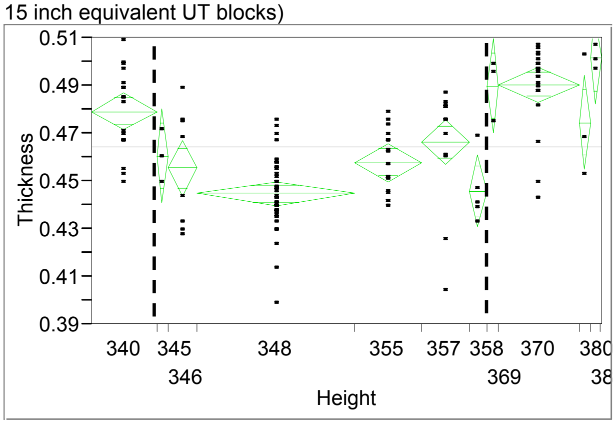
Height	Number	Mean
274	46	0.493674
286	46	0.491848
298	46	0.492565
310	41	0.494951
322	40	0.493750
334	36	0.490528
346	21	0.470333
348	43	0.444837
357	10	0.470200
358	14	0.449857
370	10	0.469700
380	9	0.481667

Figure 6: LAI Wall Thickness By Height (with 2001 data; mixes 15 inch and 3.5 inch UT blocks - 3.5 at 334, 346, 358, 370)



Height	Number	Mean
334	36	0.490528
340	18	0.479278
345	3	0.460667
346	21	0.470333
348	43	0.444837
355	18	0.457889
357	13	0.466385
358	14	0.449857
369	3	0.490000
370	28	0.486857
380	9	0.481667
381	3	0.501667

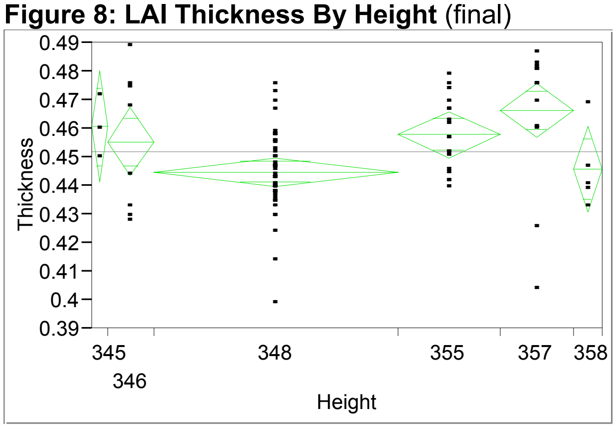
Figure 7: LAI Wall Thickness By Height (with 2001 data; 15 inch equivalent UT blocks)



Height	Number	Mean
340	18	0.479278
345	3	0.460667
346	8	0.455375
348	43	0.444837
355	18	0.457889
357	13	0.466385
358	5	0.445800
369	3	0.490000
370	22	0.490455
380	3	0.474667
381	3	0.501667

Means – LAI taken to be at heights 345 to 358

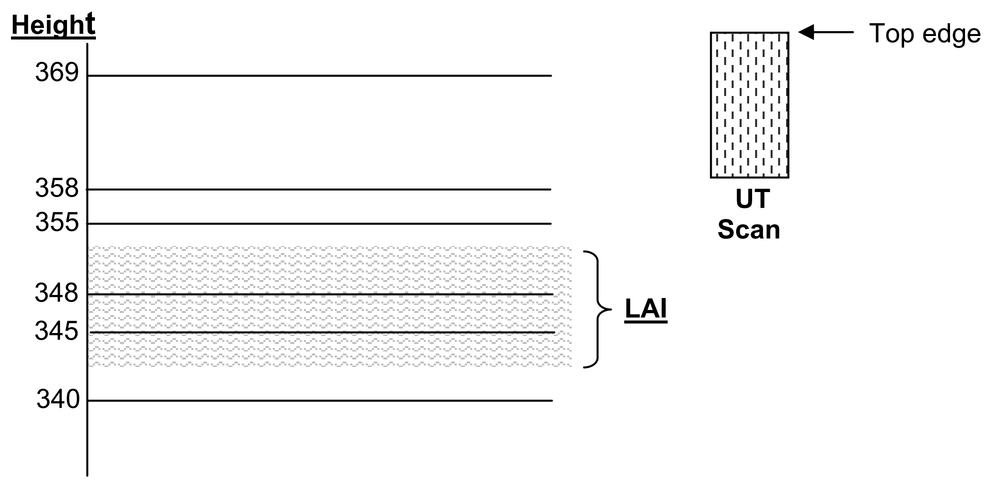
Figure 8: LAI Thickness By Height (final)



Height	Number	Mean
345	3	0.460667
346	8	0.455375
348	43	0.444837
355	18	0.457889
357	13	0.466385
358	5	0.445800

Figure 7 therefore contains results that represent more consistent image sizes. The vertical dotted lines are meant to delineate the proposed extent of the LAI. Height 340 is somewhat marginal with its mean of approximately 0.48. Other non-LAI Plate 2 averages are about 0.49. Within the LAI defined in Figure 7 and shown alone in Figure 8, the means are considerably less ranging from 0.444 to 0.466. This is why height 340 will not be considered part of the LAI. Note that since all data at height 340 is from the 2001 report data, and the Plate 2 analyses are only using the 2001.5 and 2002 3.5 by 12 image data, the height 340 data are not used in the Plate 2 analyses either.

Some thought was given to whether a v-shaped pattern within the LAI should be used in modeling LAI wall thickness, that is, with the thinnest wall towards the center of the V. Figure 8 with the relatively consistent means at the different heights somewhat discounts the need for doing this. The following diagram suggests why this is an appropriate approach.



Assume the shaded area represents the true extent of the LAI, and the upper right figure represents a UT image with at least a 12-inch vertical length. Then images at height 369 and 340 would not intersect the LAI as illustrated. All the other shown heights would result in an LAI intersection. It therefore may not specifically depend on whether the top edge of the image is actually anywhere from 345 to 358. The worst-case LAI minimums appear to be picked up on the UT images and reported as the minimum wall thickness somewhat independently of the height within this range. Therefore the LAI data is taken to include all images with top edge heights from 345 to 358, and a v-shaped pattern will not be used to model the LAI. Note that whether such images actually have vertical length 12, 15 or even 30 inches is not particularly important since the worst-case LAI thicknesses are apparently still being intersected (no 30 inch vertical images were positioned at height 369).

In summary, the previous logic has led to three populations to be considered. They are represented by Plate 3 (without including the top edge at height 262), Plate 2 without the LAI, and the LAI. Note that the respective sample sizes for these three groupings are 407, 274, and 90. These are the number of UT images, and thus minimum wall thicknesses, used to represent the three groups. All Plate 2 and 3 measurements come from 3.5 by 12 inch images. The LAI measurements have been “standardized” to come from images with widths varying from 12 to 14 (from 4 times 3.5) to 15 inches; they will be taken in subsequent discussion to be 12 inches wide which is in fact the case for most of them.

The resulting populations are displayed by the histograms in Figure 9. Note that to facilitate subsequent modeling, the negatives of the wall thicknesses are used. A sharper boundary is then expected on the left side of the distributions since the tails in this direction are bounded by the original nominal thicknesses. Interest is then in inference into the right hand tails of the distributions which then represent the negative of the minimum wall thicknesses.

Figure 9: Three Populations

Plate 2 Minus Wall Thickness

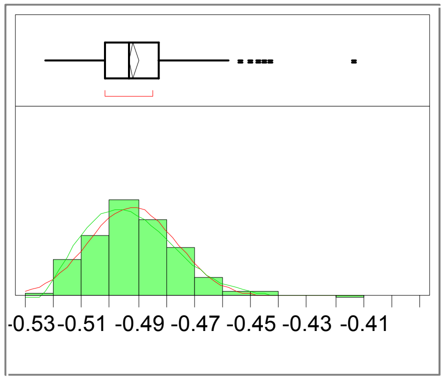
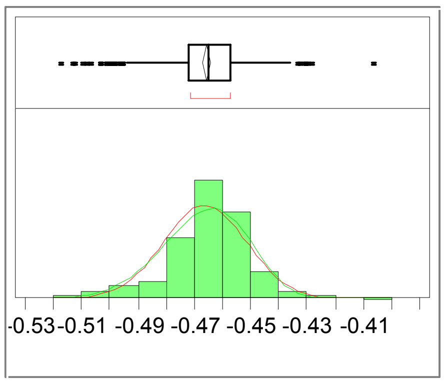
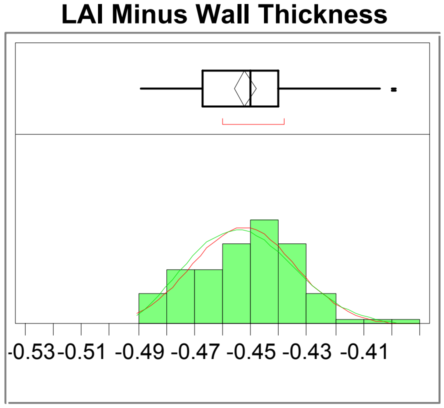


Plate 3 Minus Wall Thickness



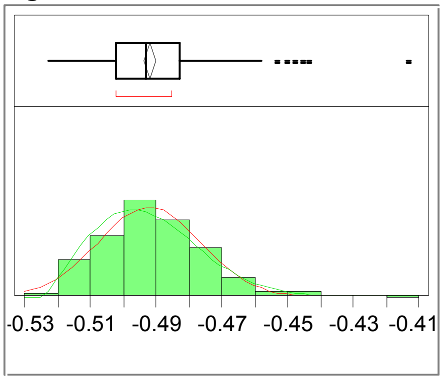
LAI Minus Wall Thickness



Note the means of these distributions are clearly different from each other, decreasing in magnitude from Plate 2 to Plate 3 to the LAI, but that is not the focus of these analyses. Rather inference out into the right hand tails of the distributions to estimate the worst-case minimum wall thickness in the uninspected areas of the tanks is the goal. Therefore the tail of the distribution, and in particular the parametric form of the model chosen to represent the associated population, is of prime importance.

Estimating out into the tail of a distribution is much more difficult than simply estimating where the distribution is centered (for example, the mean). Estimating the mean, even with only a modest amount of data, and even more so with large amounts of data, can be done relatively independent of the shape of the underlying population distribution. This is not the case for estimating tail values. Their estimates are instead very dependent on the chosen shape of the distribution, even for large amounts of data. Additional summary information on the three sets of data is given on the subsequent three pages in Figures 10, 11, and 12. (Discussion follows Figure 12.)

Figure 10: Plate 2 Minus Wall Thickness



Normal(-0.4917,0.01564)

Weibull with Threshold(2.17536,0.03662,-0.5242)

Smallest 5 values: .450 .447 .445 .443 .413

Quantiles

100.0%	maximum	-0.4130
99.5%		-0.4243
97.5%		-0.4574
90.0%		-0.4720
75.0%	quartile	-0.4827
50.0%	median	-0.4930
25.0%	quartile	-0.5020
10.0%		-0.5110
2.5%		-0.5160
0.5%		-0.5226
0.0%	minimum	-0.5230

Moments

Mean	-0.491701
Std Dev	0.0156413
Std Err Mean	0.0009449
upper 95% Mean	-0.48984
lower 95% Mean	-0.493561
N	274

Fitted Normal

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	Mu	-0.491701	-0.493561	-0.489840
Dispersion	Sigma	0.015641	0.014432	0.017073

Goodness-of-Fit Test

Shapiro-Wilk W Test

W	Prob<W
0.960194	<.0001

Fitted 3 parameter Weibull

Parameter Estimates

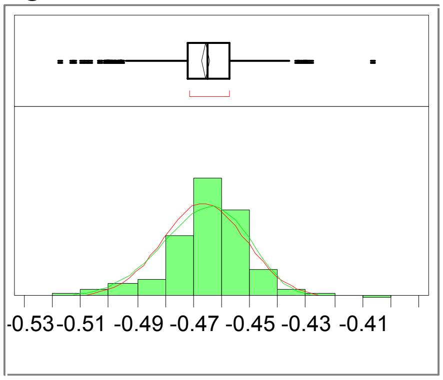
Type	Parameter	Estimate	Lower 95%	Upper 95%
Shape	Beta	2.17536	1.94925	2.47154
Scale	Alpha	0.03662	0.03393	0.04063
Threshold	Theta	-0.52417	-0.52733	-0.52309

Goodness-of-Fit Test

Cramer-von Mises W Test

W-Square	Prob>W^2
0.161297	0.0161

Figure 11: Plate 3 Minus Wall Thickness



Normal(-0.4659,0.0147)

Weibull with Threshold(5.11295,0.07331,-0.5336)

Smallest 5 values: .431 .430 .429 .428 .406

Quantiles

100.0%	maximum	-0.4060
99.5%		-0.4280
97.5%		-0.4374
90.0%		-0.4500
75.0%	quartile	-0.4570
50.0%	median	-0.4650
25.0%	quartile	-0.4720
10.0%		-0.4850
2.5%		-0.5010
0.5%		-0.5130
0.0%	minimum	-0.5170

Moments

Mean	-0.465916
Std Dev	0.0147031
Std Err Mean	0.0007288
upper 95% Mean	-0.464484
lower 95% Mean	-0.467349
N	407

Fitted Normal

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	Mu	-0.465916	-0.467349	-0.464484
Dispersion	Sigma	0.014703	0.013758	0.015789

Goodness-of-Fit Test

Shapiro-Wilk W Test

W	Prob<W
0.967382	<.0001

Fitted 3 parameter Weibull

Parameter Estimates

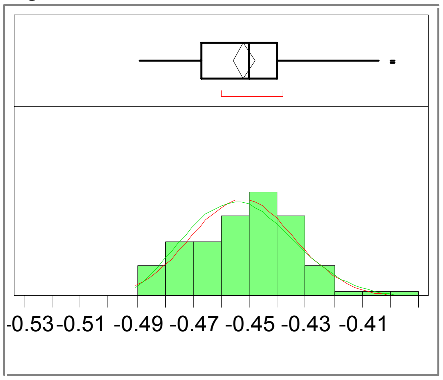
Type	Parameter	Estimate	Lower 95%	Upper 95%
Shape	Beta	5.11295	4.27046	6.50758
Scale	Alpha	0.07331	0.06273	0.09260
Threshold	Theta	-0.53359	-0.55252	-0.52344

Goodness-of-Fit Test

Cramer-von Mises W Test

W-Square		Prob>W^2
0.740589	<	0.0100

Figure 12: LAI Minus Wall Thickness



Normal(-0.4521,0.01833)

Weibull with Threshold(2.64407,0.05011,-0.4966)

Quantiles

100.0%	maximum	-0.3990
99.5%		-0.3990
97.5%		-0.4068
90.0%		-0.4303
75.0%	quartile	-0.4400
50.0%	median	-0.4500
25.0%	quartile	-0.4673
10.0%		-0.4760
2.5%		-0.4859
0.5%		-0.4890
0.0%	minimum	-0.4890

Smallest 5 values: .426 .424 .414 .404 .399

Moments

Mean	-0.452078
Std Dev	0.0183274
Std Err Mean	0.0019319
upper 95% Mean	-0.448239
lower 95% Mean	-0.455916
N	90

Fitted Normal

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	Mu	-0.452078	-0.455916	-0.448239
Dispersion	Sigma	0.018327	0.015985	0.021480

Goodness-of-Fit Test

Shapiro-Wilk W Test

W	Prob<W
0.968145	0.1276

Fitted 3 parameter Weibull

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Shape	Beta	2.64407	1.94274	4.36730
Scale	Alpha	0.05011	0.04030	0.07902
Threshold	Theta	-0.49662	-0.52395	-0.48962

Goodness-of-Fit Test

Cramer-von Mises W Test

W-Square	Prob>W^2
0.162074	0.0149

For Plate 2 data in Figure 10, note the extreme outlying wall thickness of 0.413. The next smallest values are 0.443, 0.445, 0.447, and 0.450. It will be very difficult for any parametric distribution with the other 273 observed values at 0.443 or above to suddenly account for a measurement at 0.413. It is a statistical outlier. It is similarly difficult to envision a physical phenomenon that would generate 273 values at or above 0.443 to also generate one at 0.413.

Two distribution curves are fit to the histogram data. The one slightly more to the left is a three-parameter Weibull, which is related to “extreme value distributions” as discussed later. The second curve is the normal. Goodness of fit information further down the page suggests the Weibull is the better fit although it is not particularly strong (the smaller the “Prob<W” or “Prob>W^2” values, the poorer the distributional fit). Note the rather abrupt left hand tail of the distribution ending within the ceiling of the nominal wall thickness. Interest is in projecting out into the longer right hand tail to predict a minimum tank wall thickness for Plate 2. This fairly abrupt tail at one end and extended tail at the other is what make the Weibull distribution preferred over the normal, which is a symmetric distribution.

Plate 3 in Figure 11 similarly has an outlying value at 0.406. Note again the smallest 5 measurements. The next smallest are now 0.428, 0.429, 0.430, and 0.431. The other 406 measurements were found to be at or above 0.428, yet one was found at 0.406. This will again be difficult to model with any parametric distribution.

Somewhat unusual for Plate 3 is the rather lengthy tail towards the greater wall thicknesses. While Plate 2 seemed reasonably bounded above by the nominal wall thickness, Plate 3 has a pretty lengthy tail reaching to the left towards the nominal thickness. This left-hand tail may actually be more extreme than the right hand tail of interest that extends towards thinner walls. This effect is so extreme here that in the figure the relative positions of the normal and Weibull distributions are reversed relative to those for Plate 2. Goodness of fit information again indicates the Weibull better fits the data, but also as before, it does not have a very strong fit.

The LAI data in Figure 12 show no extreme outlying values. The more abrupt tail towards the greater wall thicknesses is again evident, but note it is well below any expected nominal wall thickness. This indicates a relatively consistent and substantial amount of corrosion in the LAI. Goodness of fit information here actually rather surprisingly indicates that a normal distribution provides a better fit than a Weibull. This is because the data show the largest bulge to the right of where it would be expected for the Weibull, even more so than for the normal.

Discussion now turns towards the concept of extreme value distributions. If one were to take multiple samples of the same size n from an underlying distribution, the interest might be in how the largest (or smallest) result from each sample behaves. This behavior would depend on the sample size. And as the sample size n increases, interest might be in some type of limiting distribution of such maximum measured values.

This is a very similar scenario to the wall thickness measurement process for Tank AY-101. As discussed earlier, each 3.5 by 12 inch UT image could be considered a sample size 34,000 based on the number of pixels being considered. The interest is not actually in the distribution of these 34,000 wall thicknesses. Rather what is reported is only the extreme value, in this case the minimum, which becomes the maximum when the negatives of the wall thicknesses are considered. The actual distribution of the 34,000 values would determine what extreme value distribution the minimums fit, but the specific determination of that underlying distribution is not needed for the current task. This is fortunate since all 34,000 values are not readily available for even one image, much less the many images actually involved. Instead the extreme value fit of the minimums across many UT images is considered directly.

Given many UT images, how might the minimum measured wall thicknesses behave from one image to the next? In theory, such values should generate an extreme value distribution. To determine a worst-case minimum wall thickness for an entire tank, or say instead for Plate 3, an extreme value distribution could be fit to the 407 observed minimum wall thicknesses resulting from the 407 UT images. Then since much of Plate 3 remains uninspected, a worst-case minimum wall thickness could be extrapolated for all of Plate 3 by determining how many UT images would be needed to inspect the entire Plate. That is the approach used here. This approach was simlarly used earlier in Anantamula (2002) when five LAI wall thickness measurements were taken to have an extreme value distribution; his work was based on an approach used at Savannah River (SRP) on essentially the same application.

Before data investigation discussion continues, the different types of extreme value distributions are discussed. Statistical literature shows that depending on the particular underlying parametric distribution of a particular phenomenon (for example, the 34,000 wall thicknesses from a single UT image), the limiting (that is, for very large n) extreme value distribution will take on one of three forms. This discussion is from Johnson et al (1995, Vol. 2).

Typically three forms of extreme value distributions are considered. They are represented by the following cumulative distribution functions:

$$\text{Type 1:} \quad F(t) = \Pr[\, X \le t \,] = \exp\left\{ -e^{-(t-\alpha)/\beta} \right\}. \tag{1}$$

$$\text{Type 2:} \quad F(t) = \Pr[\, X \le t \,] = \exp\left\{ -\left(\frac{t-\alpha}{\beta} \right)^{-k} \right\}, \quad t \ge \alpha. \tag{2}$$

$$\text{Type 3:} \quad F(t) = \Pr[\, X \le t \,] = \exp\left\{ -\left(\frac{\alpha - t}{\beta} \right)^{k} \right\}, \quad t \le \alpha. \tag{3}$$

In the above, *X* is a random variable. The given expressions are the probabilities that *X* takes on a value less than or equal to *t* (note that the derivative of *F(t)* with respect to *t* is the probability density function of *X*). α, β, and *k* are parameters that can be estimated from the data, although the estimation schemes can get rather complex due to the nature of the expressions. The Type 1 distribution is the more commonly applied and was used in Anantamula (2002) and the SRP work after which it was modeled.

For a data set of size n, cumulative probabilities related to *F(t)* in the above can be represented as *i/(n+1)* for *i = 1, 2, . . ., n,* and then paired with the ordered *n* measurements in the data set. With the appropriate log transformations of the above expressions, each can be reduced to a straight line relationship in terms of the parameters α, β, and *k*, and the values *F(t).* Scatterplots of the resulting transformed values should fit a straight line if the particular distribution really does fit the data. This approach will be used in subsequent discussion.

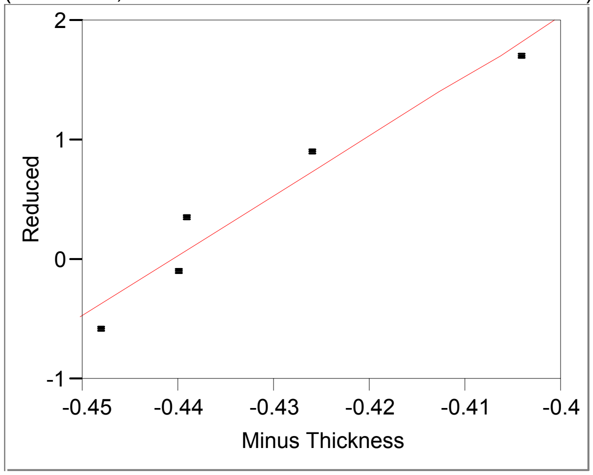
Note that for each of the distributions above, the distribution of –*X* is also considered an extreme value distribution. In fact for the Type 3 distribution, the distribution of –*X* is a three-parameter Weibull distribution. That will be of particular interest since, for the three groups Plate 2, Plate 3, and LAI, the minus wall thicknesses discussed earlier can reasonably be fit by a three-parameter Weibull. This is in fact then a Type 3 extreme value distribution. The Type 2 distribution was similarly investigated, but it provided much worse fits than either the Type 1 and Type 3 distributions in each case, so it is not included in this discussion.

To illustrate an initial example, consider the five LAI measurements from Anantamula (2002). They are 0.404, 0.426, 0.439, 0.440, 0.448. While five measurements will not give much information in terms of picking an appropriate model, consider Figure 13. Five measurements are not sufficient to estimate three parameters for a Type 3 distribution (three parameter Weibull); the required numerical methods do not converge to estimates. Instead α, a threshold parameter, must be fixed at some reasonable maximum wall thickness and then a two-parameter Weibull fit instead. The resulting straight-line fits in Figure 13 evaluate which of Type 1 or Type 3 might then be more appropriate.

Figure 13: Five Points Extreme Value Fits

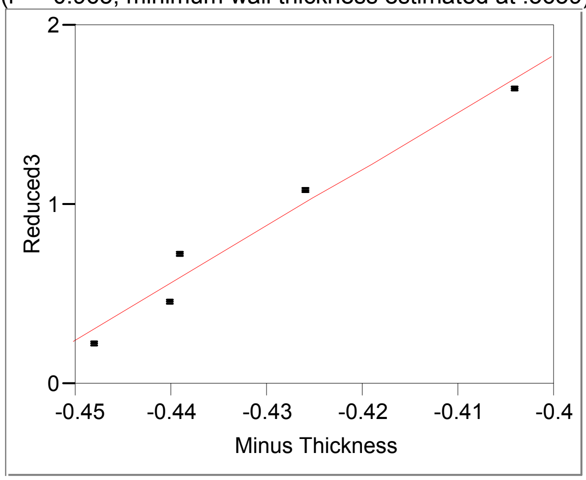
Type1

(r^2 = 0.940, minimum wall thickness estimated at .3311)



2 Parameter Weibull

(r^2 = 0.965, minimum wall thickness estimated at .3659)



With only the five points very little apparent difference exists in the fits. On the vertical axes in the two figures, “Reduced” and “Reduced3” are the appropriate transformed values respectively for Type 1 and Type 3 that should lie on a straight line if that extreme value distribution fits this minus wall thickness data. Note in the headings above the graphs the slightly larger R-square value (0.965 versus 0.940) for the Weibull fit. This means the “Reduced3” transformation converts the values $F(t) = i/(n+1)$ to a straight line fit slightly better than the “Reduced” transformation. This suggests the Weibull fits the data a bit better, but with only five points any differences certainly wouldn’t be statistically significant. The points can be seen to lie very slightly closer to the line for the Type 3 fit than for the Type 1 fit.

From these “Reduced” and “Reduced3” transformations, extrapolated values can be predicted to determine the minimum wall thickness projected for the entire LAI. Referring back to the LAI discussion on page 8, it can be argued that the extent of the LAI can simply be considered as a linear horizontal ring around the tank. Its vertical extent is not of particular importance since all the UT images intersected the LAI as discussed. The 75-foot diameter tank has total circumference of 75π or about 236 feet. Therefore if we were to exhaustively measure the LAI with adjacent 12-inch wide UT images, it would take 236 images. The minimum wall thicknesses reported for each would be taken to behave like these five measurements and thus taken to fit the corresponding distribution. Therefore to estimate the expected thinnest of the 236 minimum wall thicknesses, the extrapolation would go out to the 1/236 = .00423 percentile in the tail of the distribution. Given the respective estimated parameters for the Type 1 and Type 3 extreme value distributions shown, this process results in predicting minimum LAI wall thicknesses for the two distributions respectively as 0.331 and 0.366 inches (again shown in the graph headings in Figure 13). Little preference between these values can be established due to the minimal amount of data. The only indicator is the slightly higher R-square value for Type 3, so the 0.366 predicted value might be preferred.

Fortunately much more data are available for which preferences might be more readily apparent for Plate 2, Plate 3, and the LAI from the 2001 through 2002 data. Figure 14 compares the analogous Type 1 and Type 3 transformed fits for these data. Again, the closer to a straight line the better.

For Plate 2 the two extreme value fits in Figure 14 are comparable. Neither can adequately model the outlying value. It appears that the Type 1 is doing a better job accommodating the extreme value, but in reality these fits are determined more so by the 230 or so measurements found between 0.47 and 0.51.

Little preference is evident again for these Plate 2 data. The R-square value is larger for the Type 1, but that is primarily due to its coming considerably closer to the outlying point. Other than for that single point, the fit may actually be better for the Type 3 case. The predictions are rather different however. It is evident how the line for the Type I case will extrapolate out lower to the right than that for the Type 3 case. Thus the differing predicted values of 0.3888 for Type 1 and 0.4251 for Type 3 listed at the top of the respective graphs. Which is the more appropriate? Consider that without the outlying value, none of the other 273 points was less than 0.443. And recall the minimum five values 0.450, 0.447, 0.445, 0.443, and 0.413. The outlying value 0.413 can't be accommodated by the parametric distributions, and without it, the 0.3888 predicted value for the Type 1 case seems overly conservative. The Type 3 predicted value 0.4251 might then be considered the more sensible estimate.

Note that to perform the Plate 2 extrapolation, the total area of Plate 2 outside of the LAI was determined. The total area for each of Plate 2 and 3 is 9ft. 10in. times 75ft times π. The UT images associated with the LAI were taken to extend 27 inches vertically, so 27in. times 75ft times π was subtracted from the Plate 2 total to give the total area in Plate 2 outside the LAI. This total was then divided by the area in a 3.5 by 12 inch UT image. This results in approximately 6127 total UT images needed to cover all of Plate 2 outside the LAI. Thus the 1/6127 = 0.000163 (or 0.999837) percentiles in the tails of the two extreme value distributions were determined and provide the predicted values as given.

The Plate 3 data are clearly better fit by the Type 3 extreme value distribution than by the Type 1. Note again the lines are actually determined by the nearly 400 measurements between 0.44 and 0.50, and the Type 1 fit is very poor here. It coincidentally makes the line come quite close to the outlying value, which again cannot be reasonably modeled by these distributions since it is so dramatically different from the other measurements. The Type 3 fit is not all that great, but it is better than the Type 1.

To perform the Plate 3 extrapolation, the one-foot height associated with the omitted height 262 data was subtracted from the entire Plate 3 surface area. This gives a required 7150 UT scans of size 3.5 by 12 inches to cover the associated portion of Plate 3. Thus extrapolations were made out to the 1/7150 = 0.00014 percentile (or 0.99986).

The differing predictions behave again as for Plate 2. Without the outlying value, 406 measurements were at or greater than 0.428 with the minimum five being 0.431, 0.430, 0.429, 0.428, and 0.406. The Type 1 estimate 0.3606 is just not reasonable in this case, and it reflects the poor fit of the Type 1 data to the line. Note the R-square differences of 0.966 to 0.892. The Type 3 prediction 0.4213, but for the outlying value, again seems the more reasonable. Note that this predicted value is considerably closer to the "end of the data", that is, the four smallest values other than the outlier, for Plate 3 than for Plate 2.

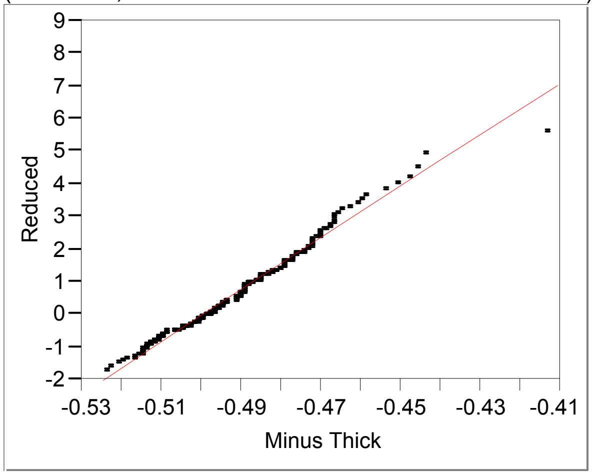
Note again the histograms in Figure 9. The longer tail for Plate 2 extends towards the thinner wall thicknesses while the longer tail for Plate 3 extends the other direction. The right hand tail towards

Figure 14: Extreme Value Fits

Type1 **Type3 (Weibull)**

Plate 2

(r^2 = 0.986, minimum wall thickness estimated at .3888)



(r^2 = 0.978, minimum wall thickness estimated at .4251)

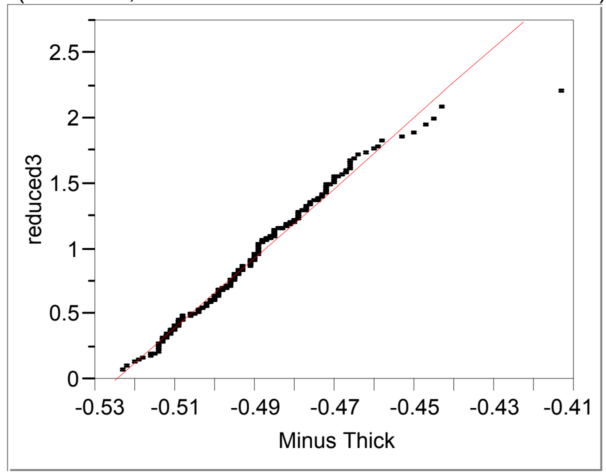
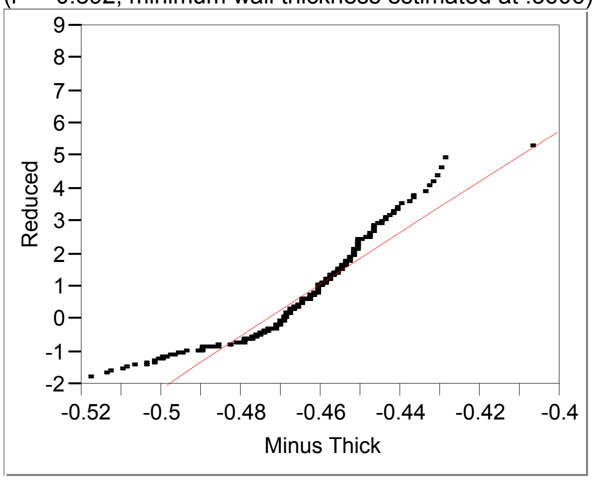
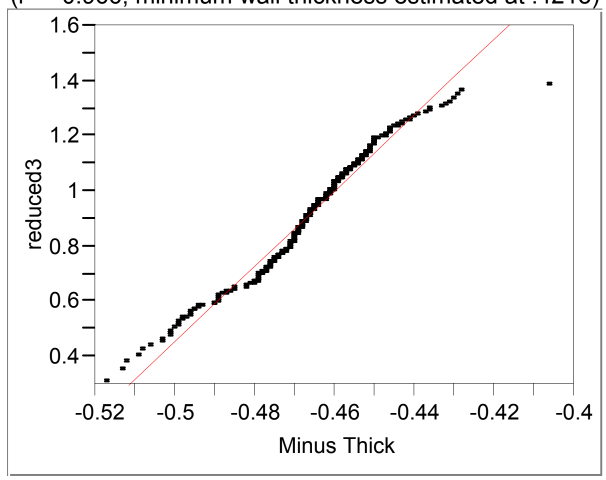


Plate 3

(r^2 = 0.892, minimum wall thickness estimated at .3606)

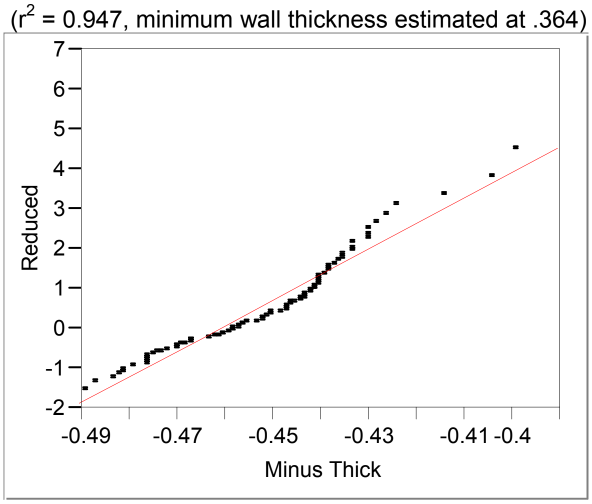


(r^2 = 0.966, minimum wall thickness estimated at .4213)

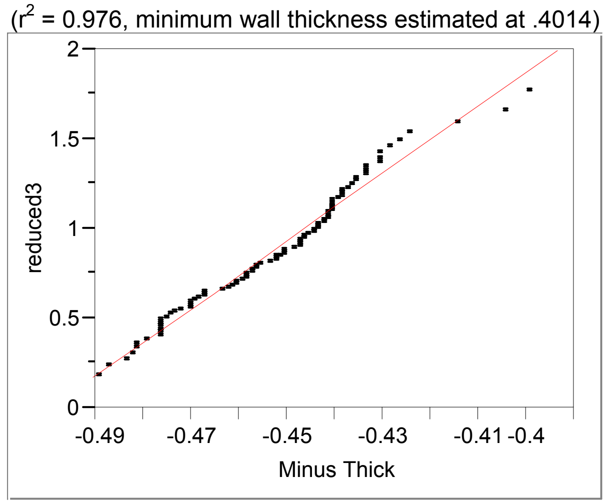


LAI

(r^2 = 0.947, minimum wall thickness estimated at .364)



(r^2 = 0.976, minimum wall thickness estimated at .4014)



thinner walls ends somewhat more abruptly for Plate 3 than for Plate 2, so the extrapolation goes relatively further out for Plate 2.

For the LAI, the Type 3 fit is again better visually and with R-square value of 0.976 compared to 0.947 for the Type 1. Again the Type 1 prediction might seem overly conservative at 0.3756 as compared to 0.4014 for the Type 3. Recall the minimum five values 0.426, 0.424, 0.414, 0.404, and 0.399. No outlying value is included as for Plates 2 and 3. The Type 3 estimate is 0.0014 in. short of including the thinnest observed wall thickness.

Recall 90 "equivalent" UT measurements were accumulated from the 2001, 2001.5, and 2002 data that are taken to have generally 12 inch widths. 236 such measurements would cover the entire LAI. This is means a dramatically greater proportion of the LAI has been measured than has Plates 2 or 3. As a result, the extrapolation is not nearly as severe.

To this point in the discussion, an argument can be made that the Type 3 extreme value distribution better fits the minimum measured wall thicknesses on UT images than does the Type 1 extreme value distribution. However, all the minimum wall thickness predictions made so far are simply point estimates with no associated uncertainties. Certainly the estimates based on 400 UT scans should be given more credibility than those based on only 5 UT scans. But nothing in the estimation process discussed so far makes that distinction. The distinction comes when the uncertainties of the parameter estimates in the distributions are considered, and in turn, their impact on the uncertainties in the predicted values.

For the two-parameter Weibull, closed form expressions are available for the uncertainties of the parameter estimates (see Johnson et al (1995 Vol. 1)). Such uncertainties would include the variances of the parameter estimates and their covariance as well since the parameter estimates are not independent. The covariance becomes important when the uncertainties are propagated through the functional form of the parameters that is needed to obtain the extreme value predicted estimate and its uncertainty. Standard Taylor series variance propagation methods are used to do this, and results will include the covariance terms as well as those involving the variances.

For the Type 3 extreme value (three-parameter Weibull), numerical estimation methods are required to obtain the parameter estimates as well as their estimated variances and covariances. Similar variance propagation can then be performed. The 2-parameter case is included primarily to see the performance of uncertainty bounds for the five-point case, which cannot use the 3-parameter approach. Both the 2-parameter and 3-parameter cases were computed for the Plate 2, Plate 3 and LAI data. The 2-parameter case was done for comparison to the five-point result; the 3-parameter case gives the recommended "final" answers.

Uncertainty results are given below, but first an important reminder is mentioned. Note in the following the term "Minimum Measured Wall Thickness Prediction". Refer again, for example, to the bottom illustration in Figure 2. The data there reflects the "measured wall thicknesses", not necessarily the "true wall thicknesses". The considerable variability in the data has two causes. The first is the true variability of the actual wall thicknesses. The second is in the inability to perfectly measure those wall thicknesses, that is, uncertainty due to measurement error. When extrapolation out into the tail of the resulting distribution is performed, the obtained estimate is of the minimum "measured wall thickness". This estimate takes into consideration the variability both in the actual minimum wall thicknesses and in the UT measurement process of those thicknesses.

The measurement error in this application is a nuisance parameter. Ideally extreme value predictions would only be based on the variability of the actual minimum wall thicknesses without incorporating the associated measurement uncertainty as well. But without somehow separating out measurement error, the values being given here are predictions of minimum "measured" wall thicknesses. This accommodates a "worst-case" combination of wall thinning and underestimation of that thinning due to measurement error. Realize therefore that actual wall thicknesses are likely not as thin as the estimates indicate. Rather they are "worst case" projections of how small a measured wall thickness can get relative to both the variability in the walls and the measurement error. If the measurement process were well understood, some attention could be given to "backing out" measurement error as a nuisance parameter to get a less severe prediction of actual wall thinning.

Recall once again the smallest five values in each case as shown:

	Smallest 5 values measured
LAI (5 measurements)	.448 .440 .439 .426 .404
LAI (90 measurements)	.426 .424 .414 .404 .399
Plate 3 (407 measurements without HT 262)	.431 .430 .429 .428 .406
Plate 2 (274 measurements without LAI)	.450 .447 .445 .443 .413

Now consider the following columns in order from left to right as discussed below.

Minimum "Measured Wall Thickness" Predictions

	Estimates			95% Confidence Bounds	
	Type 1 Extreme Value	Type 3 Weibull 3-parameter	Weibull 2-parameter	Weibull 2-parameter	Weibull 3-parameter
LAI (5 points)	0.3311	- - -	0.3660	0.2531	- - -
LAI	0.3756	0.4014	0.4014	0.3925	0.3913
Plate 3	0.3606	0.4213	0.4212	0.4182	0.4181
Plate 2	0.3888	0.4251	0.4251	0.4176	0.4173

The "Type 1 Extreme Values" in the first column are as given previously in Figures 13 and 14. They are the minimum measured wall thickness predictions based on the best fit possible from a Type 1 extreme value distribution. Generally they are very conservative, probably much too conservative due to the poor fits shown earlier. The "Type 3 Weibull 3-Parameter" column gives the corresponding extreme value results as given in Figure 14. Recall that the three parameter Weibull could not be used with only the five data points in the first case due to lack of convergence. These predicted values are proposed as being more reasonable and based on better fits.

To include a 5-point estimate based on Weibull distributions, a "Weibull 2-Parameter" case was included in the next column that used a "threshold parameter" for wall thickness of 0.45 for the five points. The estimated threshold parameters in each case were similarly used for the other 2-parameter Weibull cases for purpose of comparisons. It can be seen very similar estimates to the 3-parameter approach are thereby obtained for those cases with the 0.3660 estimate then available for the five point case.

Again to this point, these are all point estimates that in no way address uncertainties in the estimation process. This was the case in the SRP approach used in Anantamula (2002). The added feature here is the 95% confidence bounds given in the last two columns. The "Weibull 2-Parameter" bounds are generated from variance propagation as previously described. The impact of the considerable uncertainty resulting from only having five points can be seen in the bound estimate of 0.2531 that corresponds to the point estimate of 0.3660.

To understand the meaning of this lower bound, consider the bottom illustration in Figure 13. Say the process is repeated with five new points obtained, a new line or distribution fit to the new five, and a new minimum wall thickness prediction made. Then another five points and the process repeated - and on and on. How variable can the five points, the line, and ultimately the predicted minimum thickness be? If 100 such samples size 5 were used one at a time to generate 100 predicted minimums, 95 of them would be expected to be greater than 0.2531 based on the particular five data points that have actually been observed. The associated statement would be that with 95% confidence, the predicted minimum LAI wall thickness, based on the five measurements, is expected to be greater than 0.2531 inches. This shows the considerable uncertainty due to the small sample size involved in the 0.3660 point estimate.

The 2-parameter Weibull lower bound for the LAI case is not as extreme since now about 90 measurements are used instead. Analogously, repeated samples of size 90 would be expected to give predicted minimum measured wall thicknesses greater than 0.3925 ninety-five percent of the time. Not nearly as much uncertainty is associated with the 0.4014 LAI estimate due to the larger sample size. The Plate 3 and Plate 2 bounds are even closer to the point estimates since then 274 and 407 measurements respectively are available.

Note that even though the 2-parameter Weibull Plate 2 point estimate (0.4251) is greater than the Plate 3 estimate (0.4212), the Plate 2 bound is less than the Plate 3 bound. This is due to two things. The first is the smaller sample size (274 compared to 407) and therefore greater uncertainty in the estimated parameters. The second is as mentioned before that in Figure 9 the minimum wall thickness tail of the estimated Plate 2 distribution appears a bit heavier than that of the Plate 3 distribution, so uncertainties would result in stronger movements out into the tail of the Plate 2 distribution.

For the three cases other than the five points case, numerical methods generate uncertainties for the 3-parameter case as well. Slightly more uncertainty is involved when the uncertainty associated with the extra parameter also has to be considered. Note in comparing the closeness of the 2 parameter bounds to the 3 parameter bounds, results are directly related to the sample sizes: the larger the sample size, the less difference. The final column of 3-parameter Weibull bounds gives the proposed minimum measured wall thickness predictions.

The conclusion is that the respective 95% lower confidence bounds from the 3-parameter Weibull case (Type 3 extreme value) should adequately bound the expected measured minimum wall thicknesses in Plate 2, Plate 3, and the LAI. These bounds are the final column in the following and were given in the Summary and Conclusions sections at the beginning of this report:

Proposed Minimum Measured Wall Thickness

Case	Predicted Minimum	95% Confidence Bound
LAI	0.4014	0.3913
Plate 3	0.4213	0.4184
Plate 2	0.4251	0.4173

Note that only the two outlying values, that were obviously going to be difficult to model, one each in Plate 2 and Plate 3, are less than these predictions. The approach used to obtain these bounds is more rigorous than that applied in the SRP approach followed in Anantamula (2002) in that:

1. Alternative extreme value distribution fits were evaluated and used, and
2. Lower confidence bounds for the minimum measured wall thickness were generated

References:

Johnson, Norman L., Kotz, Samuel, and Balkrishnan, N. (1995). Continuous Univariate Distributions, Volumes 1 and 2, 2nd Edition, John Wiley & Sons, New York.

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JMP 2002, *JMP: The Statistical Discovery Software,* SAS Institute INC. Cary, NC.

Posakony, Gerald J., and Pardini, Allan F., *Ultrasonic Examination of Double-Shell Tank 241-AY-101*, PNNL letter report, August 21, 2001

Posakony, Gerald J., and Pardini, Allan F., *Ultrasonic Examination of Double-Shell Tank 241-AY-101 – Riser 91*, PNNL letter report, June 4. 2002

Posakony, Gerald J., and Pardini, Allan F., *Ultrasonic Examination of Double-Shell Tank 241-AY-101*, PNNL letter report, June 25, 2002

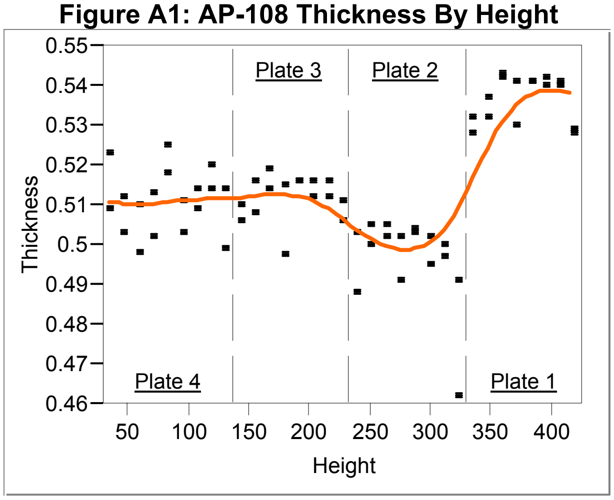
APPENDIX: Tank AP-108 Wall Thicknesses

The following data were obtained from *Ultrasonic Examination of Double Shell Tank 241-AP-108,* by Allan F. Pardini and Gerald J. Posakony, Pacific Northwest National Laboratory, September 2000

All AP-108 wall thickness data for Plates 1 through 4 are shown grouped together in Figure A1. Wall thicknesses are again on the vertical axis while the height in inches (above the top edge of the knuckle) on the tank wall is on the horizontal axis. The top of the tank is therefore to the right on the figures. Note thicknesses for Plates 1 and 2 are as shown since their nominal thickness is indeed 0.5 in. For Plate 3 nominal thickness is actually 0.5625 in., but 0.0625 was subtracted to "adjust" the nominal to 0.5 to preserve the scale on the figures across the plates of differing thickness. Actual wall thicknesses are therefore 0.0625 thicker than shown for Plate 3. Similar adjustment was made to Plate 4, which has nominal thickness 0.75 in. Actual wall thicknesses for Plate 4 are therefore 0.25 greater than the figures indicate.

Note the "significant" difference in the thicknesses of Plates 1 and 2. Similar differences were noted between Plates 2 and 3 for Tank AY-101 with the lower plate also being the thinner of the two. Here Plate 2 averages 0.497 (including the indicated outlying value 0.462 near the top edge of the plate) while the upper Plate 1 is thicker with a mean of 0.537. It was suggested that such dramatic differences between plates is likely due to differences in the original nominal thickness of the plates even though both had 0.5 inch target nominal thickness.

Figure A1: AP-108 Thickness By Height



In Figure A2, the two individual and adjacent vertical paths used for the UT measurements are indicated separately. Some concern had been expressed regarding potential significant calibration differences between days. The vertical paths would have been done on separate days, but the Figure A2 results indicate little difference between, so at least in this case, calibrations differences are not observed.

The red line (bottom line at the ends and top line in the middle) indicates Path 1 through the bolder dots while the green line (top line at the ends and bottom line in the middle) indicates Path 2 through the smaller dots. The paths would not be considered different from each other over Plates 1, 2, and 3, but a bit of systematic drift between the two paths might be indicated in Plate 4 with Path 2 having consistently greater values than Path 1.

Figure A2: AP-108 Thickness By Height by Vertical Path