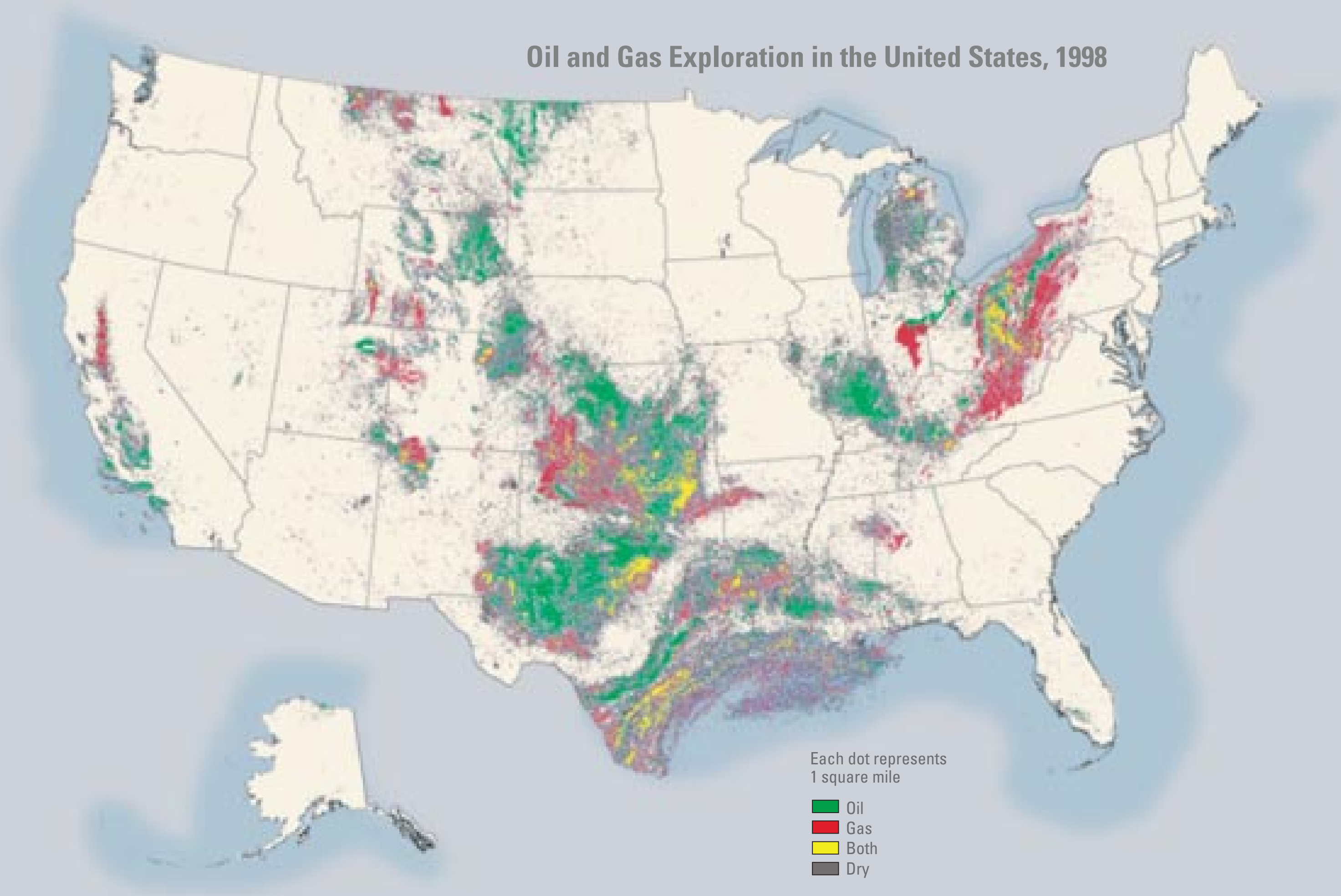
Probabilistic Method for Estimating Future Growth of Oil and Gas Reserves

Chapter C *of*
Geologic, Engineering, and Assessment Studies of Reserve Growth

U.S. Geological Survey Bulletin 2172–C



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Cover. This map represents historical oil and gas exploration and production data for the conterminous United States. It was derived from data used in U.S. Geological Survey Geologic Investigations Series I-2582.* The map was compiled using Petroleum Information Corporation's (currently IHS Corporation) database of more than 2.2 million wells drilled in the U.S. as of June 1993. The area of the U.S. was subdivided into 1 mi^2 grid cells for which oil and gas well completion data were available. Each colored symbol represents a 1 mi^2 cell (to scale) for which exploration has occurred. Each cell is identified by color as follows: red, a gas-producing cell; green, an oil-producing cell; yellow, an oil- and gas-producing cell; gray, a cell that has been explored through drilling, but no production has been reported. Mast and others (1998) gives details on map construction.

*Mast, R.F., Root, D.H., Williams, L.P., Beeman, W.R., and Barnett, D.L., 1998, Areas of historical oil and gas exploration and production in the conterminous United States: U.S. Geological Survey Geologic Investigations Series I-2582, one sheet.

Probabilistic Method for Estimating Future Growth of Oil and Gas Reserves

By Robert A. Crovelli *and* James W. Schmoker

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Edited by T.S. Dyman, J.W. Schmoker, *and* Mahendra Verma

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Introduction

In the United States, the estimated size (cumulative production plus remaining reserves) of oil and gas fields typically increases through time as fields are discovered, developed, and produced. (See, for example, Arrington, 1960; Attanasi and Root, 1994.) This phenomenon is usually referred to as reserve growth or field growth; the term "reserve growth" is used here.

Reserve-growth patterns of individual fields are highly variable. Indeed, the sizes of some fields are observed to decrease through time. However, for United States fields as a whole, collective reserve growth is strongly positive and is a major component of remaining United States oil and natural-gas resources (Gautier and others, 1995; U.S. Geological Survey National Oil and Gas Resource Assessment Team, 1995; Schmoker and Attanasi, 1997).

International oil and gas fields also show clear evidence of reserve growth (for example, Root and Attanasi, 1993; Oil & Gas Journal, 1996, p. 37; U.S. Geological Survey World Energy Assessment Team, 2000), even though criteria for estimating and reporting field sizes can be quite different from those of the United States. Worldwide, reserve growth has the potential to become important in the future, especially for gas, as demand increases and opportunities for new large-field discoveries decrease. Projections of the future reserve growth of known fields have thus become important, and in our view, necessary components of petroleum resource assessments.

Many algorithms for estimating the future growth of known fields utilize the age of fields (years since discovery) as a predictive variable, on the assumption that age is a surrogate for the degree of the development activity that generates reserve growth. Two additional assumptions implicit in most reserve-growth models are as follows: (1) reserve growth of a field is proportional to its field size, and (2) patterns of past reserve growth provide some basis for forecasting future reserve growth.

Schmoker and Crovelli (1998) presented an algorithm (a deterministic method) for estimating future reserve growth of oil and gas fields that incorporates the fundamental reserve-growth assumptions listed in the preceding paragraph, but which is programmed for a personal computer in the form of formulas for a Microsoft® Excel spreadsheet. Major advantages of this spreadsheet program include its simplicity and ease of use. However, like all other published reserve-growth models of which we are aware, this program (Schmoker and Crovelli, 1998) generates single-value (point) estimates of future reserve growth, in contrast to estimates in the form of probability distributions.

The purpose of this report is to explain an analytic probabilistic method and spreadsheet software system called probabilistic reserve growth spreadsheet (PREGS). The probabilistic method herein is a probabilistic extension of the deterministic method of Schmoker and Crovelli (1998). The PREGS method is based upon mathematical equations derived from probability theory. The PREGS spreadsheet can be used to calculate probabilistic estimates of reserve growth of oil and gas reserves.

Acknowledgments

The authors wish to acknowledge the helpful reviews of T.S. Dyman and Mahendra Verma of the U.S. Geological Survey.

Deterministic Method

The spreadsheet that implements the deterministic method (Schmoker and Crovelli, 1998) for estimating future growth of oil and gas reserves is shown in figure 1. The data used to illustrate the spreadsheet system are based on successive annual estimates made between 1977 and 1991 of the sizes of Lower 48 United States gas fields. These data were compiled by the U.S. Department of Energy and summarized in table 2 of Attanasi and Root (1994).

Input Data

Column A of figure 1*A* and 1*B* identifies the age classes to which the input data of columns B and C apply, and is not used in the spreadsheet calculations. Age (column A) is defined as years since discovery.

In the deterministic method, a growth function is composed of a set of growth *factors*. A growth factor is a dimensionless parameter, that is, a multiplicative constant representing proportional growth. The growth function of column B is composed of seventeen 10-year growth factors (rows 2 through 18). Each 10-year growth factor applies to fields of a particular age or range of ages, as indicated by column A. For example, the 10-year growth factor of 1.529 in cell B7 applies to fields that are 5 years old. The size of fields 5 years of age would be multiplied by 1.529 to forecast the size of these same fields 10 years later, when they have become 15 years old.

Rows 2 through 18 of column C (fig. 1*A* and 1*B*) list the volumes of petroleum that will be "grown" by the 10-year growth factors of column B. In this illustration, the values in column C are the totals of the estimated sizes of Lower 48 United States gas fields as of 1977 for each of the 17 field-age classes of column A. For example, the size as estimated in 1977 of Lower 48 gas fields 5 years of age (those discovered in 1972) is 5,400 billion cubic feet of gas (bcfg) (cell C7).

Calculations

The formulas shown in columns D through L (fig. 1*A*) are used to calculate future growth of the petroleum volumes tabulated in column C. The headings of columns D through L (row 1) indicate that the spreadsheet program calculates reserve growth from 10 to 90 years beyond the date associated with the field-size estimates of column C, at 10-year increments.

The spreadsheet algorithm can be explained by considering how reserve growth is calculated for the fields of a single age class. Consider the fields that were 5 years old (row 7) in 1977, when the field sizes of column C were estimated. To calculate the first growth increment of 10 years, the petroleum volume in cell C7 is multiplied by the 10-year growth factor that applies to 5-year-old fields (cell B7). This multiplication is done in cell D7. In the example used here, the total estimated size of 5-year-old fields of 5,400 bcfg grows to 8,257 bcfg (fig. 1*B*, cell D7), after the first 10-year growth period.

After 10 years of growth, fields that were 5 years old have become 15 years old. Therefore, to calculate the second growth increment of 10 years, the petroleum volume in cell D7 is multiplied by the 10-year growth factor that applies to 15-year-old fields, which is in cell B10. This multiplication is done in cell E7. The total estimated size of 5-year-old fields of 5,400 bcfg grows to 10,106 bcfg (fig. 1*B*, cell E7), after two 10-year growth periods.

Fields that were 5 years old have become 25 years old after two 10-year growth periods. The 10-year growth factor that applies to 25-year-old fields is in cell B11 and the multiplication process continues. The estimated size of 5-year-old fields grows from 5,400 bcfg to 20,129 bcfg (fig. 1*B*, cell L7) after nine 10-year growth periods. Reserve-growth calculations for the other age classes proceed similarly.

Rows 2 through 18 of each column (except columns A and B) are summed in row 20 (fig. 1). Thus, cell C20 contains the total initial volume of all the fields that are to be grown (463,656 bcfg in this example); cell D20 contains the total volume of these same fields increased by 10 years of reserve growth (557,163 bcfg); cell E20 contains the total volume of these same fields increased by 20 years of reserve growth (629,585 bcfg), and so on. The total volume of petroleum attributable to reserve growth following each 10-year growth increment is calculated in row 22 by subtracting cell C20 (the total initial volume) from cell D20 (the total volume after 10 years of reserve growth), from cell E20 (the total volume after 20 years of reserve growth), and so on. In this example, total reserve growth in the 90 years between 1977 and 2067 is projected to be 480,816 bcfg (fig. 1*B*, cell L22).

Probabilistic Method

In the probabilistic method, a growth function is composed of a set of growth *variables*. A growth variable is a dimensionless random variable having a probability distribution. In contrast, a growth factor (previous section) is a multiplicative constant. It is important to the understanding of this paper to keep these two definitions in mind. Each 10-year growth variable, like the corresponding 10-year growth factor, applies to fields of a particular age or range of ages. The growth variable in the probabilistic method is related to the growth factor in the deterministic method as follows: the mean value of a growth variable is set equal to its corresponding growth factor.

The mean-based left-triangular probability distribution (fig. 2) is used here as a probability model for the random variable *X*: growth variable (dimensionless). The right-skewed shape of the left-triangular probability distribution is appropriate as a probability model for a growth variable, whereby high probability of low growth values tapers off to low probability of high growth values. The left-triangular probability distribution is described in Law and Kelton (1991, p. 516). The defining parameters of the mean-based left-triangular probability distribution are the minimum (a) and mean (μ). From the defining parameters, the standard characterizing parameters of the left-triangular probability distribution are obtained: minimum (a) and maximum (b). Namely, given the formula for the mean in terms of a and b

$$\mu_X = a + (b - a)/3$$

We solve for the maximum, b, and get

$$b = 3\mu_X - 2a$$

The probability density function for the left-triangular distribution is given by

$$f(x) = \frac{2(b-x)}{(b-a)^2} \qquad a \le x \le b$$

The graph of the probability density function for the left-triangular distribution of the growth variable is displayed in figure 2.

The standard deviation is equal to

$$\sigma_X = (b-a)/(3\sqrt{2})$$

The fractiles can be computed from

$$F100p = b - \sqrt{p}(b-a) \qquad 0 \le p \le 1$$

$$\text{where } P(X \ge F100p) = p$$

1	age, yr	grwth funct	ESF known	ESF + 10yr	ESF + 20yr	ESF + 30yr	ESF + 40yr	ESF + 50yr	ESF + 60yr	ESF + 70yr	ESF + 80yr	ESF + 90yr
2	0	4.460	2024	=C2*B2	=D2*B10	=E2*B11	=F2*B12	=G2*B13	=H2*B14	=I2*B15	=J2*B16	=K2*B17
3	1	2.484	2794	=C3*B3	=D3*B10	=E3*B11	=F3*B12	=G3*B13	=H3*B14	=I3*B15	=J3*B16	=K3*B17
4	2	2.077	3890	=C4*B4	=D4*B10	=E4*B11	=F4*B12	=G4*B13	=H4*B14	=I4*B15	=J4*B16	=K4*B17
5	3	1.809	7009	=C5*B5	=D5*B10	=E5*B11	=F5*B12	=G5*B13	=H5*B14	=I5*B15	=J5*B16	=K5*B17
6	4	1.696	5603	=C6*B6	=D6*B10	=E6*B11	=F6*B12	=G6*B13	=H6*B14	= I6*B15	=J6*B16	=K6*B17
7	5	1.529	5400	=C7*B7	=D7*B10	=E7*B11	=F7*B12	=G7*B13	=H7*B14	= I7*B15	=J7*B16	=K7*B17
8	6	1.449	5334	=C8*B8	=D8*B10	=E8*B11	=F8*B12	=G8*B13	=H8*B14	=I8*B15	=J8*B16	=K8*B17
9	7-9	1.304	11402	=C9*B9	=D9*B10	=E9*B11	=F9*B12	=G9*B13	=H9*B14	=I9*B15	=J9*B16	=K9*B17
10	10-19	1.224	77058	=C10*B10	=D10*B11	=E10*B12	=F10*B13	=G10*B14	=H10*B15	=I10*B16	=J10*B17	=K10*B18
11	20-29	1.167	105915	=C11*B11	=D11*B12	=E11*B13	=F11*B14	=G11*B15	=H11*B16	=I11*B17	=J11*B18	=K11*B18
12	30-39	1.130	81263	=C12*B12	=D12*B13	=E12*B14	=F12*B15	=G12*B16	=H12*B17	=I12*B18	=J12*B18	=K12*B18
13	40-49	1.100	47880	=C13*B13	=D13*B14	=E13*B15	=F13*B16	=G13*B17	=H13*B18	=I13*B18	=J13*B18	=K13*B18
14	50-59	1.090	93233	=C14*B14	=D14*B15	=E14*B16	=F14*B17	=G14*B18	=H14*B18	=I14*B18	=J14*B18	=K14*B18
15	60-69	1.080	14748	=C15*B15	=D15*B16	=E15*B17	=F15*B18	=G15*B18	=H15*B18	=I15*B18	=J15*B18	=K15*B18
16	70-79	1.080	102	=C16*B16	=D16*B17	=E16*B18	=F16*B18	=G16*B18	=H16*B18	=I16*B18	=J16*B18	=K16*B18
17	80-89	1.080	0	=C17*B17	=D17*B18	=E17*B18	=F17*B18	=G17*B18	=H17*B18	=I17*B18	=J17*B18	=K17*B18
18	90+	1.000	0	=C18*B18	= D18*B18	=E18*B18	=F18*B18	=G18*B18	=H18*B18	=I18*B18	=J18*B18	=K18*B18
19												
20	sum of columns		=SUM(C2:C18)	=SUM(D2:D18)	=SUM(E2:E18)	=SUM(F2:F18)	=SUM(G2:G18)	=SUM(H2:H18)	=SUM(I2:I18)	=SUM(J2:J18)	=SUM(K2:K18)	=SUM(L2:L18)
21												
22	reserve growth		=C20-C20	=D20-C20	=E20-C20	=F20-C20	=G20-C20	=H20-C20	=I20-C20	=J20-C20	=K20-C20	=L20-C20
23	*A*											
24												
25												
26	age, yr	grwth funct	ESF known	ESF + 10yr	ESF + 20yr	ESF + 30yr	ESF + 40yr	ESF + 50yr	ESF + 60yr	ESF + 70yr	ESF + 80yr	ESF + 90yr
27	0	4.460	2024	9027	11049	12894	14571	16028	17470	18868	20377	22007
28	1	2.484	2794	6940	8495	9914	11202	12323	13432	14506	15667	16920
29	2	2.077	3890	8080	9889	11541	13041	14345	15636	16887	18238	19697
30	3	1.809	7009	12679	15519	18111	20466	22512	24538	26501	28621	30911
31	4	1.696	5603	9503	11631	13574	15338	16872	18391	19862	21451	23167
32	5	1.529	5400	8257	10106	11794	13327	14660	15979	17257	18638	20129
33	6	1.449	5334	7729	9460	11040	12475	13723	14958	16155	17447	18843
34	7-9	1.304	11402	14868	18199	21238	23999	26399	28775	31077	33563	36248
35	10-19	1.224	77058	94319	110070	124379	136817	149131	161061	173946	187862	187862
36	20-29	1.167	105915	123603	139671	153638	167466	180863	195332	210959	210959	210959
37	30-39	1.130	81263	91827	101010	110101	118909	128422	138695	138695	138695	138695
38	40-49	1.100	47880	52668	57408	62001	66961	72318	72318	72318	72318	72318
39	50-59	1.090	93233	101624	109754	118534	128017	128017	128017	128017	128017	128017
40	60-69	1.080	14748	15928	17202	18578	18578	18578	18578	18578	18578	18578
41	70-79	1.080	102	110	119	119	119	119	119	119	119	119
42	80-89	1.080	0	0	0	0	0	0	0	0	0	0
43	90+	1.000	0	0	0	0	0	0	0	0	0	0
44												
45	sum of columns		463655	557162	629584	697456	761286	814308	863299	903745	930550	944470
46												
47	reserve growth		0	93507	165929	233801	297631	350653	399644	440090	466895	480815

B

Figure 1. Spreadsheet program for estimating future oil and gas reserve growth. *A*, Calculation formulas and example of input data (in this case representing Lower 48 United States gas fields). *B*, Results of calculations. See text for detailed explanation; grwth funct, growth function (dimensionless); ESF, estimated size of fields (cumulative production plus remaining reserves); units for ESF and reserve growth in this example are billion cubic feet of gas. (Modified from Schmoker and Crovelli, 1998.)

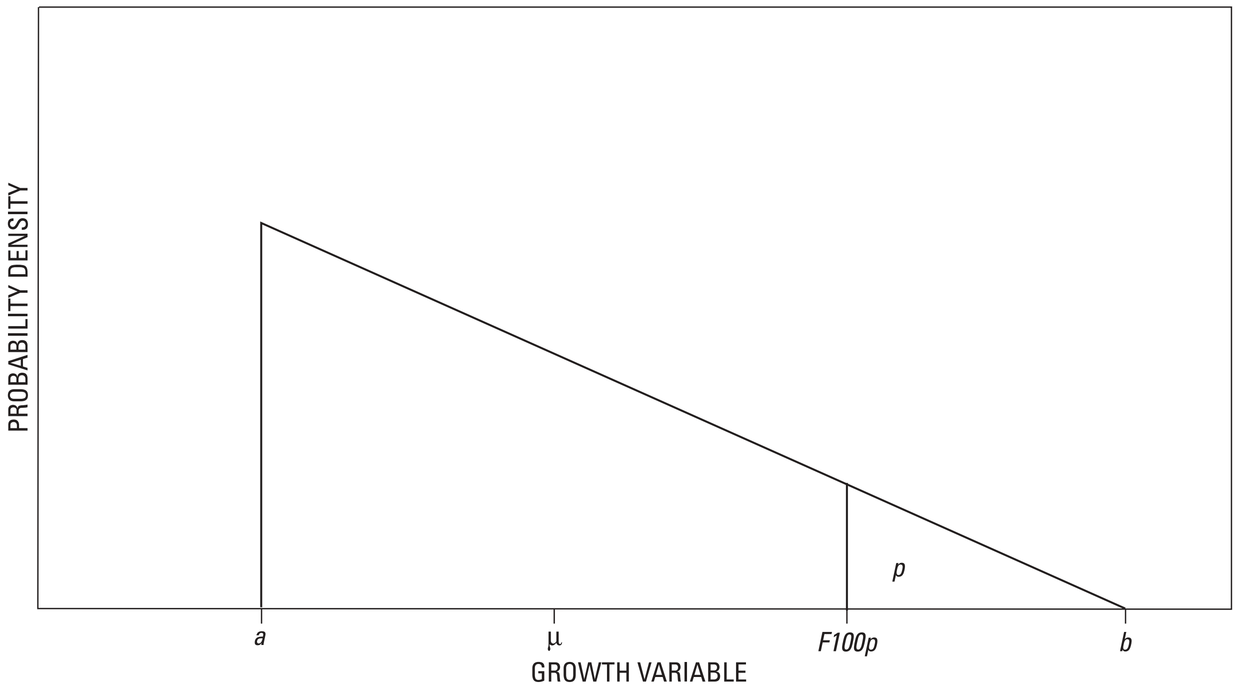

Figure 2. Left-triangular probability distribution of growth variable (dimensionless) where *a*: minimum, *b*: maximum, μ: mean, and fractile $F100p$ for $0 \le p \le 1$.

For example, the median (where $p = 0.50$) can be shown to be

$$F50 = a + 0.293(b - a)$$

Suppose we denote the parameter *c*: initial petroleum volume, that is, the volume of petroleum that will be "grown" (for example, column C of fig. 1). Then the random variable cX represents the grown petroleum volume (total estimated size) after the first growth period. It can be proved that the random variable cX is also distributed as a left-triangular probability distribution with characterizing parameters: minimum (ca) and maximum (cb). The mean, standard deviation, and fractiles of cX are the following:

$$\mu_{cX} = c\mu_X$$

$$\sigma_{cX} = c\sigma_X$$

$$F100p = c[b - \sqrt{p}(b - a)] \qquad 0 \le p \le 1$$

In general, the calculated future growth is based upon a series of multiplications by the appropriate growth variables, as in the case of the growth factors in the previous section and shown in figure 1*A*. Let us denote the random variable Y_i : grown petroleum volume (total estimated size) after the *i*th growth period. The growth process can be described as a stochastic process consisting of the set of random variables $\{Y_i\}$ where $i = 0, 1, 2, 3, \ldots, n$. Note that Y_0 represents the initial petroleum volume *c*. Also, X_i represents the growth variable that is applied in the *i*th growth period, where $i = 1, 2, 3, \ldots, n$. Therefore, we have the following sequence of grown petroleum volumes after *i* growth periods for $i = 0, 1, 2, 3, \ldots, n$:

$$Y_0 = c$$

$$Y_1 = Y_0 X_1 = cX_1$$

$$Y_2 = Y_1 X_2 = cX_1 X_2$$

$$Y_3 = Y_2 X_3 = cX_1 X_2 X_3$$

$$\ldots$$

$$Y_i = Y_{i-1} X_i = cX_1 X_2 X_3 \cdots X_i$$

$$\ldots$$

$$Y_n = Y_{n-1} X_n = cX_1 X_2 X_3 \cdots X_n = c\prod_{i=1}^{n} X_i$$

This stochastic process can be viewed as an example of the *law of proportionate effect*. The law of proportionate effect as stated by Aitchison and Brown (1957) is:

A variate subject to a process of change is said to obey the law of proportionate effect if the change in the variate at any step of the process is a random proportion of the previous value of the variate.

The importance of the law is embodied in the following theorem:

A variate subject to the law of proportionate effect tends, for large *n*, to be distributed as a two-parameter lognormal distribution, that is, asymptotically lognormally distributed in a two-parameter form.

The lognormal probability distribution is a good approximate distribution for a product of independent random variables—the approximation becoming better with more variables (multiplicative central limit theorem for independent random variables). Hence, the fractiles of Y_i where i = 2, 3, …, n can be approximated by using the lognormal distribution. As derived in Crovelli (1992), the characterizing parameters of the lognormal distribution, namely mu (μ) and sigma (σ), can be calculated from the mean μ_Y and standard deviation σ_Y of a lognormal random variable Y as follows:

$$\mu = \ln\left(\frac{\mu_Y^2}{\sqrt{\mu_Y^2 + \sigma_Y^2}}\right)$$

$$\sigma = \sqrt{\ln\left(\sigma_Y^2/\mu_Y^2 + 1\right)}$$

Knowing the lognormal characterizing parameters, the lognormal fractiles can be calculated from the formula

$$F100p = e^{\mu + z_p\sigma} \qquad 0 \le p \le 1$$

where Z is a standard normal random variable and $P(Z > z_p) = p$.

The two fractiles of most interest in this report are

$$F95 = e^{\mu - 1.645\sigma} \quad \text{and} \quad F5 = e^{\mu + 1.645\sigma}$$

At the core of the above stochastic process is the product of independent random variables in the form of YX, where Y represents a petroleum volume and X represents a growth variable. The mean, standard deviation, minimum, and maximum of YX are as follows:

$$\mu_{YX} = \mu_Y \mu_X$$

$$\sigma_{YX} = \sqrt{\sigma_Y^2\,\sigma_X^2 + \sigma_Y^2\,\mu_X^2 + \sigma_X^2\,\mu_Y^2}$$

$$Min(YX) = Min(Y)Min(X)$$

$$Max(YX) = Max(Y)Max(X)$$

Suppose there are m initial petroleum volumes, that is, we are now interested in m volumes of petroleum that will be "grown." For example in the deterministic case, figure 1 (column C) lists m = 17 initial petroleum volumes, two of which are zero. Let us denote the random variable Y_{ij} : jth grown petroleum volume (total estimated size) after the ith growth period, where j = 1, 2, 3, …, m. Note that Y_{0j} represents the jth initial petroleum volume c_j. Then the total grown petroleum volume after the ith growth period (S_i) would be the sum

$$S_i = \sum_{j=1}^{m} Y_{ij} \quad \text{for } i = 0, 1, 2, 3, ..., n$$

Notice that the total initial petroleum volume is

$$S_0 = \sum_{j=1}^{m} Y_{0j} = \sum_{j=1}^{m} c_j$$

For example in the deterministic case, S_0 corresponds to cell C20 of figure 1.

Finally, the reserve growth after the ith growth period (R_i) = (total grown petroleum volume after the ith growth period) – (total initial petroleum volume). That is,

$$\text{reserve growth}(i) = R_i = S_i - S_0 = \sum_{j=1}^{m} Y_{ij} - \sum_{j=1}^{m} c_j$$

$$\text{for } i = 0, 1, 2, 3, ..., n$$

Also, we have total reserve growth after the nth growth period = *reserve growth*(n).

A simplifying assumption for purposes of mathematical tractability will be made concerning the nature of the Y_{ij} series for j = 1, 2, 3, …, m. The assumption of perfect positive correlation is made for these random variables. Under the assumption of perfect positive correlation, the standard deviations and fractiles are additive. (The means are always additive.) That is, the mean, standard deviation, and fractiles of S_i are

$$\mu_i = \sum_{j=1}^{m} \mu_{ij}$$

$$\sigma_i = \sum_{j=1}^{m} \sigma_{ij}$$

$$(F100p)_i = \sum_{j=1}^{m} (F100p)_{ij} \qquad 0 \le p \le 1$$

Because of the simplifying assumption, we have

$$P\{(F95)_i\} \le S_i \le (F5)_i\} \ge 0.90$$

The probability of S_i being within the range from $(F95)_i$ to $(F5)_i$ is at least 0.90.

Also under the assumption of perfect positive correlation, the mean, standard deviation, and fractiles of R_i are

$$\mu_i = \sum_{j=1}^{m} \mu_{ij} - \sum_{j=1}^{m} c_j$$

$$\sigma_i = \sum_{j=1}^{m} \sigma_{ij}$$

$$(F100p)_i = \sum_{j=1}^{m} (F100p)_{ij} - \sum_{j=1}^{m} c_j \qquad 0 \le p \le 1$$

Because of the simplifying assumption, we have

$$P\{(F95)_i\} \leq R_i \leq ((F5)_i\} \geq 0.90$$

The probability of R_i being within the range from $(F95)_i$ to $(F5)_i$ is at least 0.90.

Spreadsheet System

The analytic probabilistic method described in the previous section is incorporated into a spreadsheet software system called probabilistic reserve growth spreadsheet (PREGS). PREGS consists of a series of 10 panels in the spreadsheet. A panel is a set of approximately 11 columns of related calculations.

Note that column A of Panel 1 in the probabilistic spreadsheet is the same as column A in the deterministic spreadsheet (fig. 1). Panel 1 includes the input of the defining parameters of the mean-based left-triangular probability distribution for each growth variable *X* in the growth function: mean (column B) and minimum (column D). Note that column B in the probabilistic spreadsheet is the same as column B in the deterministic spreadsheet. From the defining parameters, the maximum (column J) is computed, and then the other descriptive parameters: standard deviation (column C) and fractiles F95, F75, F50, F25, and F5 (columns E through I). For example, the 10-year growth variable with parameters in cell B9 (mean of 1.529) through cell J9 applies to fields that are 5 years old. The known estimated size of fields to be grown is also entered (column K).

Note that column K in the probabilistic spreadsheet is the same as column C in the deterministic spreadsheet. For example, the size as estimated in 1977 of Lower 48 gas fields 5 years of age (those discovered in 1972) is 5,400 billion cubic feet of gas (bcfg) (cell K9). The total initial petroleum volume S_0 is 463,656 bcfg (cell K22).

Panel 2 comprises the computed descriptive parameters of the left-triangular probability distribution for the grown petroleum volumes after the first growth period Y_{1j}, and the corresponding estimates of reserve growth R_1 (row 24): mean, standard deviation, and fractiles. Note that column L in the probabilistic spreadsheet is the same as column D in the deterministic spreadsheet (fig. 1 (values, 1*B*)). For example, the total estimated initial size of 5-year-old fields of 5,400 bcfg grows to 8,257 bcfg (mean estimate in cell L9, rounded) with a range from 5,617 bcfg (F95 estimate in cell O9) to 12,054 bcfg (F5 estimate in cell S9), after the first 10-year growth period. The reserve growth is projected to be 93,507 bcfg (mean estimate in cell L24) with a range from 7,103 bcfg (F95 estimate in cell O24) to 217,794 bcfg (F5 estimate in cell S24), after the first 10-year growth period.

Panel 3 includes the computed descriptive parameters of the *lognormal* probability distribution for the grown petroleum volumes after the second growth period Y_{2j}, and the corresponding estimates of reserve growth R_2: mean, standard deviation, and fractiles. The mean (column U) and standard deviation (column V) are used to compute the characterizing parameters of the lognormal distribution: mu (column W) and sigma (column X), which, in turn, are used to compute the fractiles. Note that column U in the probabilistic spreadsheet is the same as

	A	B	C	D	E	F	G	H	I	J	K
1	PREGS: Probabilistic Reserve Growth Spreadsheet							R.A. Crovelli			(Panel 1)
2		Growth Function									ESF,known
3	Age,yr	mean,E(X)	SD(X)	F100	F95	F75	F50	F25	F5	F0	(bcfg)
4	0	4.46	2.446589463	1	1.26282747	2.390656309	4.040231611	6.19	9.058961439	11.38	2024
5	1	2.484	1.049346463	1	1.112727158	1.596454902	2.30396061	3.226	4.456502536	5.452	2794
6	2	2.077	0.761554003	1	1.081810747	1.43287192	1.94633799	2.6155	3.508526436	4.231	3890
7	3	1.809	0.572049386	1	1.061453013	1.325156345	1.710851842	2.2135	2.884306302	3.427	7009
8	4	1.696	0.49214632	1	1.052869341	1.279738957	1.611561041	2.044	2.621109006	3.088	5603
9	5	1.529	0.374059487	1	1.040183737	1.212617684	1.464821538	1.7935	2.232136012	2.587	5400
10	6	1.449	0.317490945	1	1.034106802	1.180463781	1.394527166	1.6735	2.045801643	2.347	5334
11	7–9	1.304	0.214960461	1	1.023092356	1.122184832	1.267118616	1.456	1.7080706	1.912	11402
12	10–19	1.224	0.158391919	1	1.01701542	1.090030929	1.196824243	1.336	1.521736232	1.672	77058
13	20–29	1.167	0.118086832	1	1.012685603	1.067121273	1.146739503	1.2505	1.388972994	1.501	105916
14	30–39	1.13	0.091923882	1	1.009875021	1.052250093	1.114228355	1.195	1.302793349	1.39	81263
15	40–49	1.1	0.070710678	1	1.00759617	1.040192379	1.087867966	1.15	1.232917961	1.3	47880
16	50–59	1.09	0.06363961	1	1.006836553	1.036173141	1.079081169	1.135	1.209626165	1.27	93233
17	60–69	1.08	0.056568542	1	1.006076936	1.032153903	1.070294373	1.12	1.186334369	1.24	14748
18	70–79	1.08	0.056568542	1	1.006076936	1.032153903	1.070294373	1.12	1.186334369	1.24	102
19	80–89	1.08	0.056568542	1	1.006076936	1.032153903	1.070294373	1.12	1.186334369	1.24	0
20	90+	1	0	1	1	1	1	1	1	1	0
21											
22	Sum of columns										463656
23											
24	Reserve growth										0

Panel 1 of PREGS. Probabilistic spreadsheet system for estimating future oil and gas reserve growth: example of input data (in this case the same as that used in figure 1 and representing Lower 48 United States gas fields). ESF, estimated size of fields (cumulative production plus remaining reserves); units for ESF and reserve growth in this example are billion cubic feet of gas. In all panels, SD, standard deviation.

	L	M	N	0	P	Q	R	S	T
1									(Panel 2)
2	Estimated Size of Fields, +10yr								
3	mean,E(Y)	SD(Y)	F100	F95	F75	F50	F25	F5	F0
4	9027.04	4951.897073	2024	2555.962799	4838.688369	8177.428781	12528.56	18335.33795	23033.12
5	6940.296	2931.874018	2794	3108.959679	4460.494997	6437.265945	9013.444	12451.46809	15232.888
6	8079.53	2962.445073	3890	4208.243807	5573.87177	7571.254781	10174.295	13648.16784	16458.59
7	12679.281	4009.494146	7009	7439.724165	9288.020822	11991.36056	15514.4215	20216.10287	24019.843
8	9502.688	2757.495829	5603	5899.226917	7170.377376	9029.576512	11452.532	14686.07376	17302.064
9	8256.6	2019.921231	5400	5616.992182	6548.135495	7910.036307	9684.9	12053.53446	13969.8
10	7728.966	1693.496699	5334	5515.925681	6296.593808	7438.407902	8926.449	10912.30597	12518.898
11	14868.208	2450.979182	11402	11665.29904	12795.15145	14447.68645	16601.312	19475.42099	21800.624
12	94318.992	12205.36449	77058	78369.17424	83995.6033	92224.88252	102949.488	117261.9506	128840.976
13	123603.972	12507.28495	105916	107259.6084	113025.2167	121458.0612	132447.958	147114.4637	158979.916
14	91827.19	7470.010387	81263	82065.4738	85508.99927	90545.53884	97109.285	105868.8959	112955.57
15	52668	3385.627268	47880	48243.7046	49804.4111	52087.1182	55062	59032.11196	62244
16	101623.97	5933.311788	93233	93870.39232	96605.53045	100605.9746	105819.455	112777.0762	118405.91
17	15927.84	834.2728647	14748	14837.62265	15222.20576	15784.70141	16517.76	17496.05927	18287.52
18	110.16	5.769991334	102	102.6198474	105.2796981	109.170026	114.24	121.0061056	126.48
19	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0
21									
22	557162.733	66119.24499	463656	470758.9301	501238.5804	545818.464	603916.0995	681449.9756	744176.199
23									
24	93506.733	66119.24499	0	7102.930078	37582.58039	82162.46403	140260.0995	217793.9756	280520.199

Panel 2 of PREGS. Probabilistic spreadsheet system for estimating future oil and gas reserve growth: after one growth period (in this application, 10 years).

	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
1											(Panel 3)
2	Estimated Size of Fields, +20yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	11049.09696	6276.682824	9.17026423	0.528847333	2024	4025.121464	6726.573315	9607.162853	13721.33682	22930.38332	38511.37664
5	8494.922304	3781.829456	8.956817213	0.42522153	2794	3855.776902	5826.768876	7760.617721	10336.2925	15619.98761	25469.38874
6	9889.34472	3873.758204	9.127840092	0.37781761	3890	4945.978612	7138.022265	9208.111804	11878.5456	17143.08323	27518.76248
7	15519.43994	5340.534566	9.593891156	0.334537145	7009	8464.003376	11712.51399	14674.86076	18386.44876	25443.22453	40161.1775
8	11631.29011	3721.295844	9.312727251	0.312175971	5603	6628.924004	8976.0989	11078.11978	13672.39144	18513.52311	28929.05101
9	10106.0784	2815.195707	9.183524988	0.2733765	5400	6209.406797	8097.164581	9735.409565	11705.10967	15263.64796	23357.5056
10	9460.254384	2422.250171	9.123104777	0.25199117	5334	6054.63457	7733.081692	9164.611568	10861.14288	13872.03542	20931.59746
11	18198.68659	3833.633531	9.787395261	0.208371994	11402	12640.02104	15474.55804	17807.86088	20492.98651	25088.55865	36450.64333
12	110070.2637	18138.64562	11.59547717	0.163688898	77058	82967.88928	97260.75593	108605.4831	121273.4863	142165.2527	193390.305
13	139672.4884	18170.54505	11.83866419	0.129548493	105916	111921.9369	126924.6017	138505.3436	151142.7254	171402.7719	220982.0832
14	101009.909	10486.15497	11.51761415	0.10353505	81263	84736.2686	93697.94376	100469.9691	107731.443	119125.0789	146842.241
15	57408.12	4989.92533	10.95417767	0.086756696	47880	49585.95504	53944.10302	57192.47848	60636.46276	65965.84842	79049.88
16	109753.8876	8615.253872	11.60292439	0.078375602	93233	96181.90495	103787.3351	109417.3102	115352.685	124474.0139	146823.3284
17	17202.0672	1275.100856	9.750045114	0.074023344	14748	15188.23984	16320.11018	17155.00272	18032.60609	19376.44661	22676.5248
18	118.9728	8.818842373	4.776155168	0.074023344	102	105.0447833	112.8730159	118.647293	124.7169665	134.0112255	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	629584.822	93749.62485			463656	493511.1061	563732.5044	620500.9894	685348.3796	796517.8676	1051250.7
23											
24	165928.822	93749.62485			0	29855.10615	100076.5044	156844.9894	221692.3796	332861.8676	587594.7004

Panel 3 of PREGS. Probabilistic spreadsheet system for estimating future oil and gas reserve growth: after two growth periods (in this application, 20 years).

	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP
1											(Panel 4)
2	Estimated Size of Fields, +30yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	12894.29615	7477.014425	9.319607072	0.538392537	2024	4600.642965	7759.944813	11154.59803	16034.2709	27045.14527	57805.57634
5	9913.574329	4547.942337	9.106160055	0.437035893	2794	4390.665155	6711.634977	9010.62806	12097.11469	18491.82645	38229.55249
6	11540.86529	4691.430613	9.277182934	0.391066707	3890	5618.826997	8214.071293	10691.27144	13915.54577	20342.90877	41305.66248
7	18111.18641	6526.802271	9.743233998	0.349431143	7009	9589.459134	13463.22815	17038.55503	21563.35422	30274.11177	60281.92742
8	13573.71556	4575.927169	9.462070093	0.328086665	5603	7497.819537	10310.72279	12862.48344	16045.76939	22065.54577	43422.50556
9	11793.79349	3511.142452	9.33286783	0.291413337	5400	6998.792341	9287.783755	11303.50157	13756.68848	18255.88495	35059.61591
10	11040.11687	3052.93383	9.272447619	0.271452708	5334	6808.369309	8861.652107	10640.76458	12777.06115	16630.39499	31418.32778
11	21237.86725	4983.832371	9.936738103	0.231529502	11402	14127.40875	17688.80476	20676.19057	24168.10307	30260.67017	54712.41564
12	124379.3979	22918.75353	11.71439691	0.182728883	77058	90563.62294	108146.0216	122320.131	138351.9637	165212.1897	268812.5239
13	153639.7372	22331.52479	11.93191251	0.144590903	105916	119857.6985	137924.0208	152042.0654	167605.2475	192868.6262	287276.7082
14	110100.8008	13130.51068	11.60209034	0.118838183	81263	89913.33388	100910.9177	109326.0892	118443.0194	132930.1592	186489.6461
15	62000.7696	6298.293463	11.02976885	0.101323502	47880	52213.25609	57611.46145	61683.3211	66042.97142	72870.99842	98021.8512
16	118534.1986	11196.32452	11.67851556	0.094246818	93233	101060.9973	110745.8491	118008.9281	125748.3438	137799.0271	182060.9272
17	18578.23258	1687.764019	9.825636291	0.090659711	14748	15938.63104	17405.32801	18502.04029	19667.85658	21477.72254	28118.89075
18	118.9728	8.818842373	4.776155168	0.074023344	102	105.0447833	112.8730159	118.647293	124.7169665	134.0112255	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	697457.5249	116939.0153			463656	529284.5687	615154.3143	685379.2151	766342.027	906659.2225	1413172.966
23											
24	233801.5249	116939.0153			0	65628.56869	151498.3143	221723.2151	302686.027	443003.2225	949516.9662

Panel 4 of PREGS. Probabilistic spreadsheet system for estimating future oil and gas reserve growth: after three growth periods (in this application, 30 years).

	AQ	AR	AS	AT	AU	AV	AW	AX	AY	AZ	BA
1											(Panel 5)
2	Estimated Size of Fields, +40yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	14570.55465	8559.402447	9.43852681	0.544483529	2024	5129.95109	8704.060422	12563.19529	18133.3617	30767.13074	80349.75111
5	11202.33899	5236.062753	9.225079793	0.444517898	2794	4884.625271	7521.153003	10148.48582	13693.61379	21084.88549	53139.07796
6	13041.17778	5423.597225	9.396102672	0.399410764	3890	6242.100139	9199.457716	12041.36004	15761.18463	23228.4565	57414.87085
7	20465.64065	7584.625377	9.862153736	0.358744914	7009	10636.19784	15068.46855	19190.17554	24439.30091	34623.5415	83791.87912
8	15338.29858	5335.818677	9.580989831	0.337989126	5603	8308.195523	11535.50743	14486.75164	18193.04216	25260.11484	60357.28273
9	13326.98665	4125.686775	9.451787568	0.302518632	5400	7739.902884	10382.63534	12730.90229	15610.28274	20940.29804	48732.86611
10	12475.33206	3606.924431	9.391367357	0.283341423	5334	7519.618745	9901.041557	11984.47519	14506.31682	19100.38933	43671.47562
11	23998.79	5978.095229	10.05565784	0.245360347	11402	15553.48598	19737.68374	23287.16994	27474.97076	34866.28556	76050.25773
12	136817.3377	26749.82856	11.80764523	0.193684181	77058	97639.24139	117842.2441	134275.0024	152999.2611	184657.0705	349456.2811
13	167467.3135	26270.18389	12.0163887	0.155915158	105916	128015.7095	148940.4564	165444.1079	183776.4801	213815.5773	364841.4194
14	118908.8649	15506.1954	11.67768152	0.129854694	81263	95231.92828	108029.4823	117910.5443	128695.3909	145989.866	231247.1611
15	66960.83117	7661.421776	11.10536003	0.11404464	47880	55146.94318	61604.73053	66526.79169	71842.11301	80254.92905	121547.0955
16	128016.9345	13841.22414	11.75410674	0.107806263	93233	106592.1807	118355.1724	127275.1732	136867.4422	151971.4636	225755.5497
17	18578.23258	1687.764019	9.825636291	0.090659711	14748	15938.63104	17405.32801	18502.04029	19667.85658	21477.72254	28118.89075
18	118.9728	8.818842373	4.776155168	0.074023344	102	105.0447833	112.8730159	118.647293	124.7169665	134.0112255	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	761287.6065	137575.6495			463656	564683.7564	664340.2946	746484.8228	841785.3344	1008171.742	1824630.694
23											
24	297631.6065	137575.6495			0	101027.7564	200684.2946	282828.8228	378129.3344	544515.7423	1360974.694

Panel 5 of PREGS. Probabilistic spreadsheet system for estimating future oil and gas reserve growth: after four growth periods (in this application, 40 years).

	BB	BC	BD	BE	BF	BG	BH	BI	BJ	BK	BL
1											(Panel 6)
2	Estimated Size of Fields, +50yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	16027.61012	9490.864037	9.531775131	0.548257267	2024	5596.473391	9530.473999	13791.05029	19956.30732	33984.44963	104454.6764
5	12322.57289	5825.661428	9.318328114	0.449132362	2794	5321.472761	8230.589361	11140.3409	15078.77382	23321.96385	69080.80135
6	14345.29555	6048.973542	9.489350994	0.404540079	3890	6794.593921	10063.70816	13218.21384	17361.51071	25714.73425	74639.33211
7	22512.20471	8484.630864	9.955402058	0.36444702	7009	11566.7133	16477.7253	21065.71376	26931.16241	38365.63461	108929.4429
8	16872.12844	5980.68011	9.674238152	0.344035414	5603	9029.93001	12611.42317	15902.60405	20052.67862	28006.06598	78464.46755
9	14659.68531	4644.234261	9.54503589	0.309259179	5400	8402.66789	11345.71255	13975.14802	17213.97058	23243.18475	63352.72594
10	13722.86526	4072.493429	9.484615678	0.290527243	5334	8157.54352	10816.2003	13155.76939	16001.39269	21216.46889	56772.9183
11	26398.669	6804.478012	10.14890616	0.253624559	11402	16843.05632	21546.38237	25563.12501	30328.68112	38797.78987	98865.33505
12	149130.8981	30477.18466	11.89212143	0.202278445	77058	104754.3521	127489.0495	146110.9328	167452.8499	203794.9188	443809.477
13	180864.6986	29948.49432	12.09197988	0.164466605	105916	136139.0831	159712.3056	178435.0373	199352.5948	233871.5805	452403.3601
14	128421.5741	18068.39487	11.75327269	0.140007033	81263	101008.6186	115717.5924	127169.0641	139753.7793	160104.8613	286746.4798
15	72317.69766	9110.45933	11.18095121	0.125482698	47880	58368.53075	65931.78109	71750.57915	78082.91423	88200.70579	150718.3984
16	128016.9345	13841.22414	11.75410674	0.107806263	93233	106592.1807	118355.1724	127275.1732	136867.4422	151971.4636	225755.5497
17	18578.23258	1687.764019	9.825636291	0.090659711	14748	15938.63104	17405.32801	18502.04029	19667.85658	21477.72254	28118.89075
18	118.9728	8.818842373	4.776155168	0.074023344	102	105.0447833	112.8730159	118.647293	124.7169665	134.0112255	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	814310.0396	154494.3559			463656	594618.8923	705346.3173	797173.4394	904226.6312	1092205.556	2242268.691
23											
24	350654.0396	154494.3559			0	130962.8923	241690.3173	333517.4394	440570.6312	628549.5555	1778612.691

Panel 6 of PREGS. Probabilistic spreadsheet system for estimating future oil and gas reserve growth: after five growth periods (in this application, 50 years).

	BM	BN	BO	BP	BQ	BR	BS	BT	BU	BV	BW
1											(Panel 7)
2	Estimated Size of Fields, +60yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	17470.09503	10412.73648	9.616251324	0.551352009	2024	6058.861975	10348.94716	15006.68914	21760.73717	37168.81487	132657.4391
5	13431.60445	6408.943508	9.402804307	0.452904941	2794	5754.720069	8933.346723	12122.32784	16449.69537	25535.70469	87732.61772
6	15636.37215	6667.406119	9.573827186	0.408724458	3890	7342.799501	10919.95252	14383.35891	18945.23014	28174.67828	94791.95177
7	24538.30314	9374.12274	10.03987825	0.36908622	7009	12490.59723	17874.20941	22922.59193	29396.837	42067.26156	138340.3924
8	18390.62	6617.731243	9.758714344	0.348946089	5603	9746.836281	13677.736	17304.36991	21892.60108	30721.88855	99649.87379
9	15979.05699	5155.943086	9.629512082	0.31471296	5400	9061.67503	12300.50439	15207.01452	18800.3096	25519.9276	80457.96195
10	14957.92314	4531.526609	9.569091871	0.29632598	5334	8792.33532	11723.70525	14315.41018	17480.05126	23307.91095	72101.60624
11	28774.5492	7617.089101	10.23338236	0.260246852	11402	18129.14754	23341.21537	27816.4362	33149.69297	42680.11616	125558.9755
12	161061.37	34022.95275	11.9677126	0.208940893	77058	111748.3596	136883.6318	157583.7911	181414.3218	222219.3803	550323.7515
13	195333.8745	33966.26511	12.16757105	0.172594878	105916	144878.7523	171312.0638	192446.0348	216187.2052	255630.8341	560980.1665
14	138695.3	20847.31159	11.82886387	0.149471392	81263	107257.0353	124010.3307	137154.5774	151692.0244	175385.9601	355565.6349
15	72317.69766	9110.45933	11.18095121	0.125482698	47880	58368.53075	65931.78109	71750.57915	78082.91423	88200.70579	150718.3984
16	128016.9345	13841.22414	11.75410674	0.107806263	93233	106592.1807	118355.1724	127275.1732	136867.4422	151971.4636	225755.5497
17	18578.23258	1687.764019	9.825636291	0.090659711	14748	15938.63104	17405.32801	18502.04029	19667.85658	21477.72254	28118.89075
18	118.9728	8.818842373	4.776155168	0.074023344	102	105.0447833	112.8730159	118.647293	124.7169665	134.0112255	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	863300.9061	170270.2947			463656	622265.5075	743130.7976	843909.0419	961911.636	1170196.38	2702910.046
23											
24	399644.9061	170270.2947			0	158609.5075	279474.7976	380253.0419	498255.636	706540.3803	2239254.046

Panel 7 of PREGS. Probabilistic spreadsheet system for estimating future oil and gas reserve growth: after six growth periods (in this application, 60 years).

	BX	BY	BZ	CA	CB	CC	CD	CE	CF	CG	CH
1											(Panel 8)
2	Estimated Size of Fields, +70yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	18867.70263	11304.4517	9.691842501	0.55383099	2024	6508.019873	11142.92909	16185.03778	23508.67047	40251.1752	164495.2245
5	14506.13281	6972.668595	9.478395484	0.455919525	2794	6175.887434	9615.250631	13074.19193	17777.43516	27677.72186	108788.446
6	16887.28193	7264.71899	9.649418364	0.412062387	3890	7876.002344	11750.93763	15512.76269	20478.85997	30554.30862	117542.0202
7	26501.36739	10232.51915	10.11546943	0.372779245	7009	13389.78751	19229.79574	24722.5096	31784.13798	45646.91414	171542.0866
8	19861.8696	7232.162824	9.834305522	0.35284997	5603	10444.88143	14712.9692	18663.13603	23673.85139	33347.68794	123565.8435
9	17257.38155	5648.841455	9.705103259	0.319036008	5400	9703.957061	13227.76078	16401.09303	20335.70586	27720.22287	99767.87281
10	16154.55699	4973.267586	9.644683048	0.3009133	5334	9411.434549	12605.23502	15439.47852	18910.99187	25328.49756	89405.99174
11	31076.51314	8397.009302	10.30897353	0.265458381	11402	19385.76856	25085.73367	30000.62616	35878.46311	46427.74762	155693.1296
12	173946.2796	37906.3868	12.04330378	0.215397365	77058	119249.7265	146990.8996	169957.5162	196512.5555	242227.4512	682401.4518
13	210960.5845	38359.78275	12.24316223	0.1803572	105916	154272.321	183799.6306	207557.1976	234385.6195	279246.4002	695615.4065
14	138695.3	20847.31159	11.82886387	0.149471392	81263	107257.0353	124010.3307	137154.5774	151692.0244	175385.9601	355565.6349
15	72317.69766	9110.45933	11.18095121	0.125482698	47880	58368.53075	65931.78109	71750.57915	78082.91423	88200.70579	150718.3984
16	128016.9345	13841.22414	11.75410674	0.107806263	93233	106592.1807	118355.1724	127275.1732	136867.4422	151971.4636	225755.5497
17	18578.23258	1687.764019	9.825636291	0.090659711	14748	15938.63104	17405.32801	18502.04029	19667.85658	21477.72254	28118.89075
18	118.9728	8.818842373	4.776155168	0.074023344	102	105.0447833	112.8730159	118.647293	124.7169665	134.0112255	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	903746.8076	183787.3871			463656	644679.2089	773976.6272	882314.5669	1009681.245	1235597.99	3169132.782
23											
24	440090.8076	183787.3871			0	181023.2089	310320.6272	418658.5669	546025.2452	771941.9904	2705476.782

Panel 8 of PREGS. Probabilistic spreadsheet system for estimating future oil and gas reserve growth: after seven growth periods (in this application, 70 years).

	CI	CJ	CK	CL	CM	CN	CO	CP	CQ	CR	CS
1											(Panel 9)
2	Estimated Size of Fields, +80yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	20377.11884	12272.04496	9.767433678	0.556298925	2024	6990.601948	11997.91547	17455.9122	25396.81759	43588.35947	203974.0783
5	15666.62343	7585.321851	9.553986662	0.458914307	2794	6628.094435	10349.34404	14100.79788	19212.08726	29998.44113	134897.673
6	18238.26448	7914.515312	9.725009541	0.415373493	3890	8448.296864	12645.38755	16730.84902	22136.23804	33133.4603	145752.105
7	28621.47678	11167.35217	10.1910606	0.376436041	7009	14354.5654	20688.69537	26663.75962	34364.47123	49528.22028	212712.1874
8	21450.81917	7901.730713	9.909896699	0.356711128	5603	11193.70553	15827.0123	20128.59458	25599.29265	36195.37056	153221.6459
9	18637.97207	6186.619496	9.780694437	0.323301256	5400	10392.75149	14225.4713	17688.93243	21995.63896	30107.36194	123712.1623
10	17446.92155	5455.572984	9.720274226	0.30543173	5334	10075.26843	13553.67815	16651.81046	20458.12129	27521.13191	110863.4298
11	33562.63419	9249.790287	10.38456471	0.270569547	11402	20732.91485	26962.46076	32356.32213	38829.22968	50496.11158	193059.4808
12	187861.9819	42159.39748	12.11889496	0.221665858	77058	127293.9963	157864.4716	183302.8455	212840.3739	263955.3643	846177.8002
13	210960.5845	38359.78275	12.24316223	0.1803572	105916	154272.321	183799.6306	207557.1976	234385.6195	279246.4002	695615.4065
14	138695.3	20847.31159	11.82886387	0.149471392	81263	107257.0353	124010.3307	137154.5774	151692.0244	175385.9601	355565.6349
15	72317.69766	9110.45933	11.18095121	0.125482698	47880	58368.53075	65931.78109	71750.57915	78082.91423	88200.70579	150718.3984
16	128016.9345	13841.22414	11.75410674	0.107806263	93233	106592.1807	118355.1724	127275.1732	136867.4422	151971.4636	225755.5497
17	18578.23258	1687.764019	9.825636291	0.090659711	14748	15938.63104	17405.32801	18502.04029	19667.85658	21477.72254	28118.89075
18	118.9728	8.818842373	4.776155168	0.074023344	102	105.0447833	112.8730159	118.647293	124.7169665	134.0112255	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	930551.5345	193747.7059			463656	658643.9388	793729.5524	907438.0388	1041652.844	1280940.085	3580301.278
23											
24	466895.5345	193747.7059			0	194987.9388	330073.5524	443782.0388	577996.8445	817284.0848	3116645.278

Panel 9 of PREGS. Probabilistic spreadsheet system for estimating future oil and gas reserve growth: after eight growth periods (in this application, 80 years).

	CT	CU	CV	CW	CX	CY	CZ	DA	DB	DC	DD
1											(Panel 10)
2	Estimated Size of Fields, +90yr										
3	mean,E(Y)	SD(Y)	Mu	Sigma	F100	F95	F75	F50	F25	F5	F0
4	22007.28835	13321.94045	9.843024855	0.558755959	2024	7509.10305	12918.59906	18826.57765	27436.41354	47201.37994	252927.8571
5	16919.95331	8251.110163	9.629577839	0.461889672	2794	7113.63987	11139.62889	15208.01452	20762.24512	32512.70937	167273.1145
6	19697.32564	8621.34853	9.800600718	0.418658412	3890	9062.566437	13608.16077	18044.58138	23927.32733	35928.77574	180732.6103
7	30911.19492	12185.31631	10.26665178	0.380057655	7009	15389.74935	22258.80462	28757.43962	37153.40277	53736.43941	263763.1124
8	23166.8847	8631.289763	9.985487876	0.360530937	5603	11997.03104	17025.88351	21709.12322	27680.56241	39283.5552	189994.841
9	20129.00984	6773.268947	9.856285614	0.327510962	5400	11131.45409	15299.00734	19077.895	23790.17602	32697.08294	153403.0812
10	18842.67527	5982.072594	9.795865403	0.309884284	5334	10787.09462	14574.13113	17959.33659	22130.84045	29900.33759	137470.6529
11	36247.64493	10182.0432	10.46015589	0.275585934	11402	22177.13373	28981.44183	34896.99101	42019.99296	54912.41549	239393.7561
12	187861.9819	42159.39748	12.11889496	0.221665858	77058	127293.9963	157864.4716	183302.8455	212840.3739	263955.3643	846177.8002
13	210960.5845	38359.78275	12.24316223	0.1803572	105916	154272.321	183799.6306	207557.1976	234385.6195	279246.4002	695615.4065
14	138695.3	20847.31159	11.82886387	0.149471392	81263	107257.0353	124010.3307	137154.5774	151692.0244	175385.9601	355565.6349
15	72317.69766	9110.45933	11.18095121	0.125482698	47880	58368.53075	65931.78109	71750.57915	78082.91423	88200.70579	150718.3984
16	128016.9345	13841.22414	11.75410674	0.107806263	93233	106592.1807	118355.1724	127275.1732	136867.4422	151971.4636	225755.5497
17	18578.23258	1687.764019	9.825636291	0.090659711	14748	15938.63104	17405.32801	18502.04029	19667.85658	21477.72254	28118.89075
18	118.9728	8.818842373	4.776155168	0.074023344	102	105.0447833	112.8730159	118.647293	124.7169665	134.0112255	156.8352
19	0	0			0						0
20	0	0			0						0
21											
22	944471.6809	199963.1481			463656	664995.5121	803285.2446	920141.0195	1058561.908	1306544.323	3887067.541
23											
24	480815.6809	199963.1481			0	201339.5121	339629.2446	456485.0195	594905.9084	842888.3233	3423411.541

Panel 10 of PREGS. Probabilistic spreadsheet system for estimating future oil and gas reserve growth: after nine growth periods (in this application, 90 years).

column E in the deterministic spreadsheet (fig. 1 (values, 1*B*)). For example, the total estimated initial size of 5-year-old fields of 5,400 bcfg grows to 10,106 bcfg (mean estimate in cell U9) with a range from 6,209 bcfg (F95 estimate in cell Z9) to 15,264 bcfg (F5 estimate in cell AD9), after the second 10-year growth period. The reserve growth is projected to be 165,929 bcfg (mean estimate in cell U24) with a range from 29,855 bcfg (F95 estimate in cell Z24) to 332,862 bcfg (F5 estimate in cell AD24), after the second 10-year growth period.

Panels 4 through 10 are computationally similar in their composition to Panel 3. They compute the respective probabilistic estimates of reserve growth after the *i*th growth period for $i = 3, 4, \ldots, 9$.

In the example used to illustrate PREGS, the system is applied to data based on successive annual estimates made between 1977 and 1991 of the sizes of Lower 48 United States gas fields. The results of this application (Panels 1–10) are summarized in the graph displayed in figure 3. The probabilistic estimates of future reserve growth of Lower 48 United States gas fields are in the form of a mean estimate with a range from a low F95 estimate to a high F5 estimate for nine growth periods (10-year increments).

Summary

The objective of this report is the development and description of a probabilistic method and spreadsheet system, called the PREGS system, for estimating future growth of oil and gas reserves. The primary advantages of the PREGS system are several-fold. The probabilistic method utilizes the same data as required by its deterministic predecessor in Schmoker and Crovelli (1998); most importantly, no additional data are needed. All the advantages of the deterministic system carry over to the probabilistic system. The mean estimates computed by the probabilistic system equal the point estimates computed by the deterministic system. Additionally, the probabilistic system also computes a range of low and high estimates. Many operational benefits accrue from incorporating an analytic probabilistic method into a computer software spreadsheet.

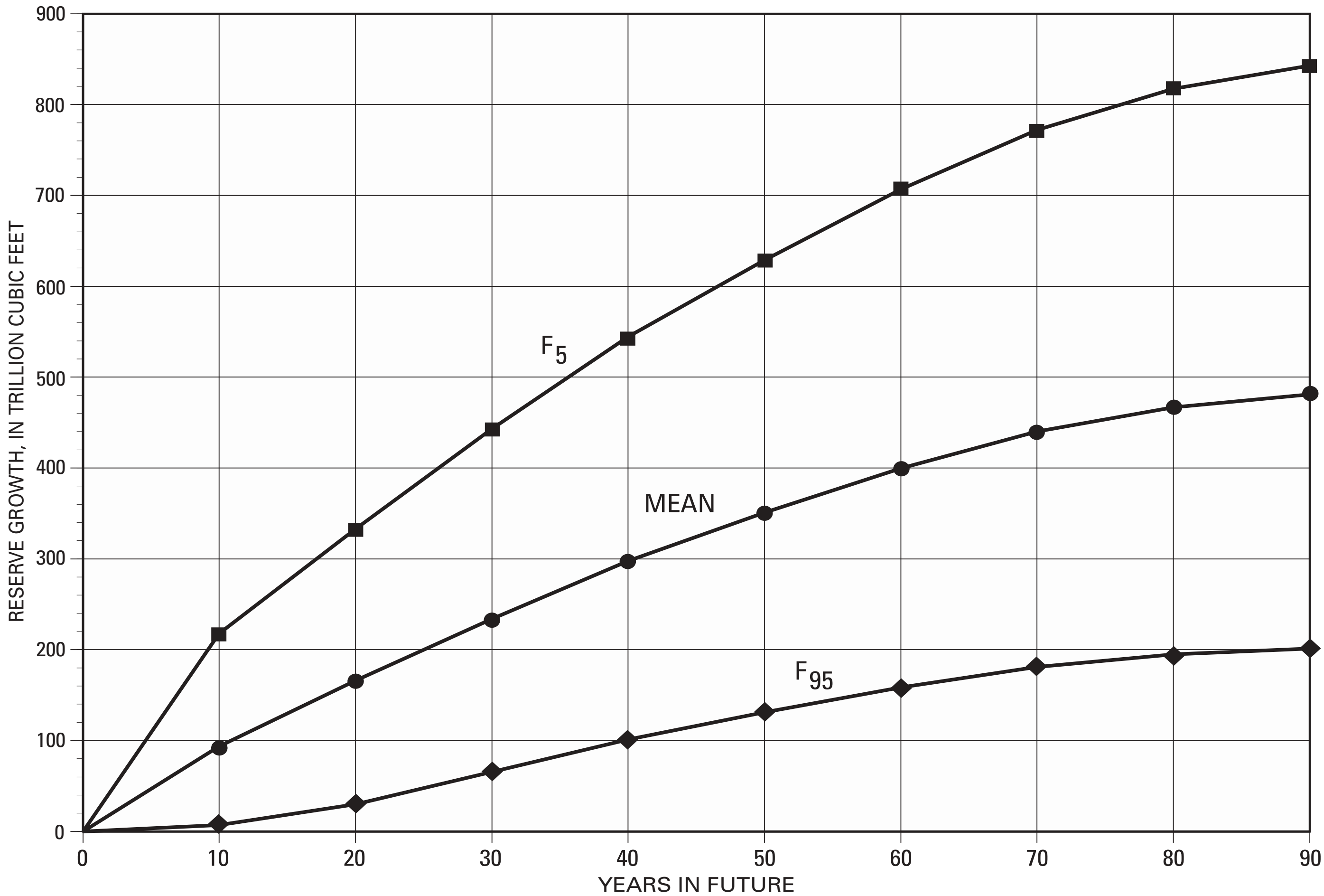

Figure 3. Graph of probabilistic estimates of future reserve growth of Lower 48 United States gas fields in the form of a mean estimate together with a range from a low F95 estimate to a high F5 estimate for nine growth periods (10-year increments). Units for reserve growth are trillion cubic feet of gas. This example is derived from the results shown in Panels 1–10.

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