The Third Generation Quarks

Sally Dawson, BNL Lecture 2 Maria Laach School September, 2004

Tevatron Run II

- Electroweak physics : W, Z's have large cross sections → high statistics, precision measurements
 - W, Z masses, widths, σ , W helicity
 - Gauge boson pair production New physics effects $\approx s/\Lambda^2$
- Many new results appearing
 - Higgs searches
 - More top, Single top soon
 - New physics searches
 - B physics
 - QCD



M. Kruse, 2004 FNAL User's Mrg

Physics landscape changes in 2007

Event Rates:

Process	10 fb ⁻¹ at LHC	Previous
		experiments
W→ev	108	10 ⁷ Tevatron
Z→e+e-	107	10 ⁷ LEP
tt	107	10 ⁴ Tevatron
bb	10 ¹² -10 ¹³	10 ⁹ B factories
Higgs, M _h =130 GeV	10 ⁵	?
g̃g, m=1 TeV	104	

Top and Bottom Quarks

- The Third Generation and the Standard Model
- Top Quarks
 - Top Quark Decay
 - Pair Production
 - Single Top production
- Bottom Quarks
 - Pair Production



Who Needs a Top Quark?

- Top needed for *anomaly cancellation*
- Triangle diverges; grows with energy

 $T^{\mu\nu\sigma} \approx \int \frac{d^n k}{\left(2\pi\right)^n} \frac{1}{k^3}$



• Anomaly independent of mass; depends only on gauge properties

 $T^{\mu\nu\sigma} \approx Tr[\eta_i t^a, \{t^b, t^c\}]$ $\eta_i = \pm 1 \ for \ \psi_{L,R}$

Sensible theories can't have anomalies

Anomalies, continued

• Standard Model Particles:

Particle	SU(3)	$SU(2)_L$	U(1) _Y
(u_R, d_R)	3	2	1/6
$\overline{u_R}$	3	1	-2/3
$\overline{d_R}$	3	1	1/3
(v_L,e_L)	1	2	-1/2
$\overline{e_{R}}$	1	1	1

- V= γ , A=W_a (SU(2) gauge bosons) $\sum T_3 Q_{em}^2 = N_c N_g \left(\frac{1}{2}\right) \left(\frac{2}{3}\right)^2 + N_c N_g \left(-\frac{1}{2}\right) \left(-\frac{1}{3}\right)^2 + N_g \left(-\frac{1}{2}\right) (1)^2 = 0$
- $N_c=3$ is number of colors; N_g is number of generations
- Anomaly cancellation requires complete generation

Top Quark Properties

- Top is weak isospin partner of b quark
 - b quark properties well measured at LEP/SLD
 - Example: $Z \rightarrow b\overline{b}$:



- b quark is $T_3 = -1/2$ particle, so it must have $T_3 = 1/2$ partner
 - **DEFINE** this to be the top
- SM: $Q_{em} = (T_3 + Y)/2 \implies Q_{em} = 2/3|e|$
- Color triplet
- Spin-1/2

Top Couplings

• Top-gluon coupling $g_s \lambda^a_{ij} \bar{t}_i \gamma^\mu t_j G^a_\mu$

 λ^{a} are 8 SU(3) color matrices

- Top -W coupling
 - Flavor changing with CKM matrix V, and left-handed $\frac{g}{2\sqrt{2}}\bar{t}\gamma^{\mu}(1-\gamma_5)V_{tq}qW_{\mu}^{+}$
- Top -Z coupling
 - Violates parity symmetry

$$\frac{g}{4\cos\theta_W}\bar{t}\left((1-\frac{8}{3}\sin^2\theta_W)\gamma^\mu-\gamma^\mu\gamma_5\right)tZ_\mu$$

• Top Higgs coupling

$$- \text{ Of O(1)} \frac{M_t}{v} \bar{t} th$$

Top Quark Decay Formula

- Dominant decay of top quark is $t \rightarrow W^+b$
- Theoretical error is $\sim 1\%$

$$\Gamma_{0} = \frac{G_{F}M_{t}^{2}}{8\pi\sqrt{2}} |V_{tb}|^{2} \left(1 - \frac{M_{W}^{2}}{M_{t}^{2}}\right) \left(1 + \frac{2M_{W}^{2}}{M_{t}^{2}}\right)$$
$$= |V_{tb}|^{2} 1.42 GeV$$

Corrections to Γ_0 :

$M_W \neq 0$	-11.5%
$\alpha_s, M_W = 0$	-9.5%
$\alpha_s, M_W \neq 0$	+1.8%
$\alpha_s^2, M_W = 0$	-2.0%
$\alpha_s^2, M_W \neq 0$	+0.1%
EW	+1.7%

Theory well understood

No Free Quarks

QCD color confinement

Non-perturbative binding

Top Quark is Different

- Top quark decays before it can form bound states
 - Top quark lifetime: $\tau_t \approx 5 \ge 10^{-25} \text{ sec}$
 - $\tau_{QCD} \approx 3 \times 10^{-24} \text{ sec}$
- Very different from b system: $\tau_b \approx 1.55 \pm 0.06$ ps
- Top quark interactions may be first indication of new physics beyond the SM
 - Top quark mass critical for precision measurements
 - Top quark couples strongly to Higgs, so it is window to electroweak symmetry breaking

Top Quark Discovery at Fermilab in 1995

- Run I, $p\overline{p}$ at $\sqrt{s}=1.8$ TeV, L~125 pb⁻¹
- Run II, $p\overline{p}$ at $\sqrt{s}=1.96$ TeV, eventually L~4-6 fb⁻¹
- Top quark produced by $gg \rightarrow t\bar{t}$





- Strongly produced; sensitive to $\alpha_s(\mu_R)$
- $M_t = 178 \pm 4.3 \text{ GeV} >> M_b$







Jassasser

New CDF/D0 Combined Top Quark Mass

Combination of Run I results:





Combination dominated by D0 lepton + jets result

CDF/D0 hep-ex/0404010

Why is the top quark mass so important?

 In QED, running of α at scale M not affected by heavy quarks with M_q>> M

$$\alpha(q^2) = \alpha(M^2) \left[1 + \frac{\alpha(M^2)}{12\pi} \ln\left(\frac{q^2}{M^2}\right) \right]$$
 Running of α from electron loop

- Decoupling theorem: diagrams with heavy virtual particles don't contribute at scales M << M_q if
 - Couplings don't grow with M_q
 - Gauge theory with heavy quark removed is still renormalizable

Why is the top quark mass so important?

- Spontaneously broken SU(2) x U(1) theory violates both conditions
 - Longitudinal modes of gauge bosons grow with mass ($\mathcal{E}_L \approx p_V / M_V$)

$$\frac{g}{2\sqrt{2}}\bar{t}\gamma^{\mu}(1-\gamma_5)V_{tq}qW_{\mu}^{+} \longrightarrow \frac{gM_t}{2\sqrt{2}M_W}\bar{t}(1-\gamma_5)V_{tq}qW_{L}^{+}$$

- Theory without top quark is not renormalizable

• Effects from top quark grow with M_t^2

Expect M_t to have large effect on precision observables

Why do we care so much about M_t ?

- M_t is fundamental parameter
- M_t measurement gives limit on M_h
- δM_t must match $\delta M_{W:}$
 - Tevatron Run II with 2 fb⁻¹
 - $\delta M_W \sim 27 \text{ MeV}$
 - $-\delta M_t \sim 3 \text{ GeV}$



$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2 \theta_W} \frac{1}{1 - \Delta r(M_t, M_h)}$$

Simple Example

• ρ parameter

$$\rho \equiv \frac{M_{W}^{2}}{M_{Z}^{2} \cos^{2} \theta_{W}}$$

= 1 + ...
= $\frac{A_{WW}(0)}{M_{W}^{2}} - \frac{A_{ZZ}(0)}{M_{Z}^{2}}$

 $i\Pi_{VV}^{\mu\nu}(p) = A_{VV}(p^2)g^{\mu\nu} + B_{VV}p^{\mu}p^{\nu}$

• Calculate in N=4-2ɛ dimensions



$$\delta \rho = \frac{G_F}{\sqrt{2}} \frac{N_c}{8\pi^2} M_t^2$$

Quadratic dependence on M_t

Same calculation gives logarithmic dependence on M_h

How heavy is a heavy quark?

> A simple criterion:

Production and decay of heavy quarks can be calculated perturbatively

> QCD running coupling constant:



Reliable perturbative calculation in QCD requires $\alpha_s(\mu) \ll 1$

Renormalization scale μ ~ order of the hard scale

 $\alpha_{s}(M_{Z})=.118$

Top Quark Cross Section

QCD factorization:

 $\sigma_{H}(S) = \sum_{i,j} \int dx_{1} dx_{2} F_{i}(x_{1}, \mu_{F}) F_{j}(x_{2}, \mu_{F}) \hat{\sigma}_{ij}(x_{1}, x_{2})$ •Hadronic energy, S •Partonic sub-energy, s=x_{1}x_{2}S •Parton distribution functions, F_{i}(x, \mu_{F})

•To any given order in α_s , σ_H independent of μ

• μ dependence of σ_H often used to estimate theoretical uncertainty

•Typically see reduced scale dependence at higher orders

Heavy flavor pair production



$d\hat{\sigma}_{_{q\overline{q}}}$ _	1		2
$-d\hat{t}$	$\overline{16\pi\hat{s}^2}$	$q\bar{q} \rightarrow Q\bar{Q}$	

 $\sum \left| \overline{M}_{q\overline{q} \to Q\overline{Q}} \right|^2 = \frac{(4\pi\alpha_s)^2 C_F}{N_c} \left(\frac{u_1^2 + t_1^2}{s^2} + \frac{2M_Q^2}{s} \right)$

Dominates at Tevatron for top production (90% of cross section)

≻Use to generate differential cross sections

$$s = 2k_1 \bullet k_2, \quad t_1 = -2k_1 \bullet p_1, \quad u_1 = -2k_1 \bullet p_2 = -s - t_1$$

$$\hat{\sigma}_{q\bar{q}\to Q\bar{Q}} = \left(\frac{2}{9}\right) \frac{4\pi\alpha_s^2}{3\hat{s}} \left[1 + \frac{2m_Q^2}{\hat{s}}\right] \sqrt{1 - \frac{4m_Q^2}{\hat{s}}}$$

Same calculation valid for b production Threshold suppression, $\sim\beta$

Hadronic Cross Sections at NLO in QCD

- NLO corrections from $\hat{\sigma}_{ij}^{NLO} = \hat{\sigma}_{ij}^{LO} + \delta \hat{\sigma}_{ij}^{NLO}$
- NLO corrections contain one loop virtual diagrams and real gluon emission
- Sample virtual diagrams:



- In N=4-2ε dimensions, virtual diagrams have 1/ε² and 1/ε poles
- Add wavefunction, mass, coupling renormalization

Hadronic Cross Sections at NLO in QCD, 2

• Sample real gluon emission diagram:



Gluon propagator: $\frac{1}{(p-k)^2} = \frac{-1}{\sqrt{sE_g(1-\cos\theta)}}$ Soft singularity: E_g→0
Collinear singularity: cosθ →0

≻At NLO, also small contribution from qg initial state

- Need to
 - Renormalize UV divergences in σ_{virt}
 - Cancel IR divergences in $\sigma_{virt+}\sigma_{real}$
 - NLO result has form:

$$\hat{\sigma}_{ij} = \alpha_s^{2}(\mu_R) \left(h_{ij}^{0} + \frac{\alpha_s(\mu_R)}{4\pi} h_{ij}^{1} + \frac{\alpha_s(\mu_R)}{4\pi} h_{ij}^{2} \log\left(\frac{\mu_R^{2}}{M_t^{2}}\right) \right)$$

- $\alpha_s(M_t)$ ~0.1 so perturbation theory converges well

Hadronic Cross Sections at NLO in QCD, 3

- Need to
 - Check μ dependence of σ^{NLO} :
 - To any given order in α_s , $d\sigma_H/d\mu=0$
 - Using $\mu^2 \frac{\partial}{\partial \mu^2} F_i(x,\mu_F) = \frac{\alpha_s(\mu_F)}{2\pi} \sum_i \int_{-\infty}^{1} \frac{dz}{z} P_{ij}(z) F_j\left(\frac{x}{z},\mu_F\right)$

Scale dependence formally $O(\alpha_s^4)$

$$\frac{d\alpha_s(\mu_R)}{d\ln(\mu^2)} = -b_0\alpha_s^2 + O(\alpha_s^3), \quad b_0 = \frac{1}{4\pi} \left(\frac{11}{3}N_c - \frac{2}{3}n_{lf}\right)$$

• µ dependence predicted

$$h_{ij}^{2}(x_{1}, x_{2}) = 2 \left\{ 4\pi b_{0} h_{ij}^{0}(x_{1}, x_{2}) - \sum_{k} \left[\int_{4M_{t}^{2}/s}^{1} P_{ik}(z_{1}) h_{kj}^{0}(x_{1}z_{1}, x_{2}) + (1 \leftrightarrow 2, i \leftrightarrow j) \right] \right\}$$

- Near threshold, corrections take form: $\sigma \approx \alpha_s \left(c_1^2 \ln^2 \beta^2 + c_1^1 \ln \beta^2 \right)$
- Large logarithms can be summed to improve convergence of perturbation theory

Theoretical predictions for top pair production

- Ingredients
 - Parton level cross section with NLO matrix elements plus resummation to all orders of soft logarithms
 - Resummation improves stability with respect to scale variations
 - Very sensitive to M_t
 - PDFs





Scale variation

Top Pair Production, continued

- PDF uncertainties (fix $\mu=M_t$, $\sqrt{S}=1.96$ TeV)
 - CTEQ6 family of PDFs
 - $M_t=175 \text{ GeV: } \sigma=6.70\pm0.45 \text{ pb}$
 - $M_t=180 \text{ GeV: } \sigma=5.75\pm0.38 \text{ pb}$
- Full range of uncertainties
 - $M_t/2 < \mu < 2M_t$, CTEQ6, MRST PDFs, $\sqrt{S}=1.96$ TeV
 - M_t =175 GeV: 5.82< σ <7.41 pb
 - M_t =180 GeV: 5.0 < σ <6.34 pb

 $\sigma(1.96)/\sigma(1.8)=1.295\pm0.15$ stable to scale choices, PDFs, M_t

≻Updated σ 's from summer conferences (see Quadt talk)

➢Good agreement between theory/experiment



Cacciari, Frixione, Mangano, Nason, Ridolfi, hep-ph/0303085

Need Monte Carlos which include NLO

- Match NLO with parton shower
- Subtract terms which are included in parton shower from NLO result
 - At high p_T , NLO
 - At low p_T , MC

MC@NLO



pp→tt at LHC

Frixione, Nason & Webber, hep-ph/0305252

Top Quark Mixing

• Weak interaction eigenstates are not mass eigenstates

$$L = -\frac{g}{2\sqrt{2}} \sum_{q=d,s,b} \bar{t} \gamma^{\mu} (1-\gamma_5) V_{tq} q W_{\mu}^{+} + h.c.$$

• Interactions mix down-type quarks with Cabbibo-Kobayashi-Maskawa Matrix V:

$$\begin{pmatrix} d'\\ s'\\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d\\ s\\ b \end{pmatrix}$$



• Top quark emits W boson and become a b quark (V_{tb}) , an s quark (V_{ts}) or a d quark (V_{td})

$$V = \begin{pmatrix} 0.9741 - 0.9756 & 0.219 - 0.226 & 0.002 - 0.005 \\ 0.219 - 0.226 & 0.9732 - 0.9748 & 0.038 - 0.044 \\ 0.004 - 0.014 & 0.037 - 0.044 & V_{tb} \end{pmatrix}$$

Top Quark Decays

- Unitarity of CKM matrix (VV+=1) and assuming 3 generations: $\begin{bmatrix} 1 = |V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 \\ V_{tb} \approx .9990 - .9993 \end{bmatrix}$
- Dominant decay of top quark is $t \rightarrow Wb$
- More than 3 generations, almost no limit on V_{tb}

	0.9730-0.9746	0.2174 - 0.2241	0.003 - 0.0044]
$V_{CKM} =$	0.213-0.226	0.968 - 0.975	0.039 - 0.044	
	< 0.08	< 0.11	0.07-0.9993	
	L :	:	:	·

Top and CKM

- **Predict from unitarity:** $|V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2 = 1$
- V_{ts} and V_{td} measured indirectly from b physics

>If 3 generations:
$$R = \frac{|V_{tb}|^2}{|V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2} = |V_{tb}|^2$$

Measure $R = \frac{BR(t \rightarrow Wb)}{BR(t \rightarrow Wq)}$

D0 Run II (158-169 pb⁻¹) in e-jets and μ -jets channels (2 different lifetime triggers):

SVT: R= $0.70^{+0.27}_{-0.24}$ (stat)^{+0.11}-_{0.10}(syst)

CSIP: $R=0.65^{+0.34}-0.30$ (stat)+0.17-0.12(syst)

Reasonable agreement with 3 generation Standard Model



D0 note 4586-CONF

Top Quark Decay

• Decay amplitude for $t \rightarrow Wb$

$$A = -\frac{g}{2\sqrt{2}} V_{tb} \overline{u}_b \mathcal{E}_W (1 - \gamma_5) u_t$$

• Substitute expressions for momenta & polarization vectors

Longitudinal W (helicity 0):

$$\begin{array}{c} \overset{b}{\longleftrightarrow} & \overset{t}{\longleftarrow} & \overset{\bullet}{\swarrow} & \overset{\bullet}{W} \\ & & & \\ p_{W} = (E_{W}, 0, 0, \mid p_{W}^{-} \mid) \\ & & \\ \epsilon_{L}^{W} = \frac{1}{M_{W}} (\mid p_{W}^{-} \mid, 0, 0, E_{W}) \\ & & = \frac{p_{W}^{-}}{M_{W}} + \mathcal{O}(\frac{M_{W}}{M_{t}}) \\ & & \\ \mathcal{A}(t \rightarrow bW_{L}) = -\frac{g}{2\sqrt{2}} \overline{b} \frac{p_{W}}{M_{W}} (1 - \gamma_{5}) t \\ & & = -\frac{m_{t}}{\sqrt{2}v} \overline{b} (1 + \gamma_{5}) t \end{array}$$

Left-handed fermion: spin is opposite from direction of motion

$$\Gamma(t o b W_L) \sim rac{m_t^3}{v^2}$$



left- or right-handed



Decay amplitude for t \rightarrow Wb $M = -\frac{ig}{2\sqrt{2}}V_{tb}\overline{u}_b\varepsilon_W(1-\gamma_5)u_t$

- Transverse W: $\epsilon^{\pm} = (1/\sqrt{2})(0, 1, \pm i, 0)$
 - $\Gamma(t \rightarrow W_T b) \sim g^2 M_t$
 - No enhancement

Top Quark Decay, 3

$$F_{0} = \frac{\Gamma(t \to W_{L}b)}{\Gamma(t \to Wb)_{tot}}$$
$$= \frac{m_{t}^{2}/2M_{w}^{2}}{1 + m_{t}^{2}/2M_{w}^{2}}$$
$$= 69.5\% (NLO)$$

CDF Run I: $F_0=91\pm37\pm13\%$ D0 Run I: $F_0=56\pm31\%$

- W helicity correlated with momentum of decay leptons
 - h^{W} = + gives harder charged leptons than $h^{W=}$ -
 - 2 fb⁻¹ give 5% measurement

Tests V-A structure of tbW vertex

LHC is Top Factory



$$\hat{s} \approx x_1 x_2 S \approx (350 \ GeV)^2$$

 $x_1x_2 \sim 10^{-3}$ for $\sqrt{S}=14$ TeV

Production dominated by gluon fusion (~90% from gg)

 \succ Large statistics \Rightarrow Precision measurements of:

• Production cross section

 $\succ \sigma \sim 800 \text{ pb} \implies 10^7 \text{ events/year}$

• Mass

➢Probe rare decays of top and BSM top decays

Top is background for new physics

Top at LHC

Precision measurement of top mass from leptons plus jet:

 $t\bar{t} \to W^+W^-b\bar{b} \to (l\nu)(jj)(b\bar{b})$

>Including e, μ , BR~30%

•Signal from MC@NLO

•Most important bkgd: W + jets (tree)

•Largest systematic uncertainties: jet energy scale, b fragmentation, initial and final state radiation



Single Top in the SM

- Single top produced at hadron colliders through interactions involving a W boson and b quark.
- Rates directly proportional to $|V_{tb}|^2$
- At tree level there are three modes:
 - s-channel W exchange
 - Large rates at Tevatron, small at LHC.
 - t-channel W exchange
 - Dominant mode at Tevatron and LHC.
 - t-W associated production
 - Small at Tevatron, large rate at LHC.







Sensitive to b quark PDFs

Single Top



σ	Run I Limits	Tevatron Run 2	LHC
σ_t (NLO)	< 13.5 pb	1.98±0.28 pb	247±12 pb
σ_{s} (NLO)	< 12.9 pb	0.88±0.12 pb	10.7±1.1 pb
$\sigma_{tW}(LL)$		0.09±0.02 pb	56±8 pb
Total		2.95±0.33 pb	314±15 pb

Uncertainties from PDFs, δM_t

Single top rate $\sim 1/2$ tt rate at Tevatron

Z. Sullivan, hep-ph/0408049

Single top at the LHC

- Large background from $\sigma(tt)=830 \text{ pb}, \sigma(Wbb)>300 \text{ pb}$
- Large QCD multi-jet background
- Largest uncertainty is PDF uncertainty in theory
 - Assume 6% theory uncertainty

 $\delta |V_{tb}|$ to:

.4% from Wg

1.4% from Wt

2.7% from W*

•V_{tb} can be constrained using unitarity

•This assumes the SM, with 3 generations

•Physics beyond the SM can modify these results

b-quarks at Tevatron



We cannot really measure b-quark momentumLong-standing puzzle

Usually model fragmentation by writing hadronic B meson σ as:

$$\frac{d\sigma}{dp_T} = \int d\hat{p}_T dz \frac{d\sigma}{d\hat{p}_T} D(z)\delta(1-z\hat{p}_T)$$

Often use Peterson form:

$$D(z) \approx \frac{z(1-z)^2}{\left[(1-z)^2 + \varepsilon z\right]^2}$$

b-mesons at Tevatron



Cacciari, Nason, PRL 89 (2002)

Conclusion

- Exploration of t- and b- quark properties sheds light on structure of SM
- So far, everything looks like the SM

Still hoping for something new