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The Second Coming of Cold Dark Matter?

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Abstract

While the standard cold dark matter (CDM) model has received numerous marks against it on the basis of a variety of observational data, the prediction of high σ_v, the pairwise-velocity dispersion between galaxies, on small (megaparsec) scales was reported as perhaps one of its greatest failings. Here, we reexamine the case of CDM and σ_v with high-resolution numerical simulations. The σ_v statistic was measured in simulations, in artificial galaxy catalogs, and in the CfA North Forty redshift survey. In our reanalysis of the CfA data, we found that $\sigma_v \sim 500$ km/s, significantly higher than the original estimates of Davis & Peebles (1983). This new value, along with our estimate of the velocity bias, leads to a virial measure of the cosmological density parameter, $\Omega \sim 0.9$, which is more than four times the traditionally cited CfA result. Analysis of the simulations indicates that while the method of Davis and Peebles works well, the σ_v statistic itself is not generally robust: there is large scatter from catalog to catalog and evidence that the recovered σ_v value depends strongly on a small fraction of galaxies in the central regions of rare, massive clusters. Thus, at present the CDM model cannot be deemed incompatible with observed small-scale peculiar velocities.

1 Introduction

The biased CDM cosmogony reigned as the premier model of structure formation in the Universe from its inception in the early 1980's through the end of that decade. Its fall from favor began with evidence from angular galaxy-galaxy correlations in the APM survey (Maddox *et al.* 1990) that the CDM model could not fit the amplitude of clustering over the full range of observed scales. At that time, there was still freedom to choose the normalization of the primordial CDM power spectrum to match galaxy clustering data on either large or small scales (a length of 8,000 km/s or $8h^{-1}$ Mpc delineates these two regimes), but not both.

Then came the Cosmic Background Explorer (COBE) data. The detection of the cosmic microwave background anisotropy constrained the amplitude of of the initial CDM power spectrum to within 20%. This result gave the CDM model little trouble on large scales when compared to redshift data, although other problems persist on these scales, notably with angular correlations of galaxies and cluster-cluster correlation functions. The real difficulty for CDM seemed to lie at small scales, the issue we address here.

There are two key indicators of small-scale structure. One is σ_8, the rms fluctuation in mass averaged within spherical tophats of radius $8h^{-1}$ Mpc. Observations indicate that for galaxies this quantity is near unity. If the galaxies are biased tracers of matter then σ_8 for the total mass may have a different value. Conventionally, the total mass fluctuations are expressed by the so-called bias parameter b, where $1/b$ is σ_8 for total mass in units of σ_8 for galaxies.

The second indicator of small-scale structure is σ_v, the radial pairwise peculiar velocity dispersion of galaxies. In the seminal work of Davis & Peebles (1983), the observational estimate of σ_v on megaparsec scales was first determined to be 340 ± 40 km/s. By contrast, early simulations of CDM indicated $\sigma_v \geq 1200$ km/s. Prior to the COBE results, reparations could be made for this discrepancy by reducing the mass normalization of CDM at the expense of increasing the bias required to keep the galaxy distribution consistent with $\sigma_8 = 1$. The bias required to lower σ_v below 500 km/s appeared to be $b \approx 2.5$

The COBE data curtailed this freedom to "tune" the CDM model with the bias factor b. These data suggest that $\sigma_8 \approx 1.1$ for total mass in a CDM model, with an initial uncertainty level around 20%. In this case galaxies are likely to be slightly antibiased relative to the CDM distribution.

Taken at face value the COBE result and the small-scale constraints condemn the CDM cosmogony when compared with numerical simulations. However, we must be wary that historically the problems of CDM on small scales were mitigated somewhat as the resolution of CDM simulations was increased. A case in point is the pairwise velocity dispersion which fell from over 1200 km/s in the 1985 work of Davis *et al.* to a value around 500 km/s, as reported by Carlberg & Couchman (1989) and Gelb (1992). Furthermore, as we shall discuss, the opposite trend has occurred for the observed value of σ_v.

There is another issue that must be considered before the CDM model is dismissed: In an inflationary scenario it is possible that a fraction of the anisotropy signal detected by COBE resulted not from density perturbations but from gravity waves—relics of the transition from the inflationary epoch to the early radiation-dominated phase. In this event, the COBE constraint will admit lower-amplitude mass fluctuations so that a biased model may still be viable.

In view of these developments we have decided to simulate formation of structure in a CDM universe with a high-resolution N-body code. Our numerical methods and some of the related astrophysical results are discussed elsewhere (Zurek *et al.* 1994; Warren & Salmon 1993; Warren & Salmon 1992). Here we shall concentrate on pairwise radial peculiar velocities, their measurement in the simulations and in observations.

2 Simulations

We consider results from three simulations with the initial conditions appropriate for CDM cosmogony with mass fluctuations of $\sigma_8 \simeq 0.7, 1.05$ and 1.6; hereafter, we refer to these runs as undernormalized, COBE-normalized, and overnormalized, respectively. These simulations were realizations of the CDM density and velocity fields within a 250 Mpc diameter sphere.

To enable a comparison with redshift catalogs, we identified halos of CDM with galaxies. As our simulations had either $N \simeq 8.8 \times 10^6$ or $N \simeq 1.76 \times 10^7$ particles each, halos with hundreds of virialized particles formed abundantly in all of them; by $z = 0$ there were typically 10^4 such halos per simulation (Fig. 1 illustrates the halo distribution in one of the runs). High spatial resolution (20 kpc Plummer smoothing) of our models appears to have prevented massive overmerging (reported by some of the earlier simulations [White *et al.* 1987; Gelb *et al.* 1993]).

We still may be subject to some overmerging: Halo-halo correlation functions (not shown here, but reproduced in Zurek *et al.* 1993) follow the observed power law shape on scales of a few megaparsecs, but are a bit more shallow on smaller sub-megaparsec scales. We regard this as evidence for a possible deficiency of objects with small ($\sim$ 100 kpc) separations, which is most logically attributed to merging.

Correlation functions tend to be equal to unity on scales $\sim 4h^{-1}$ Mpc for σ_8 of 0.7 and 1.06 (i.e., somewhat less than the observed $5.4h^{-1}$ Mpc). Moreover, on submegaparsec scales halos are significantly less correlated than mass. This slight antibias of the halo distribution is probably also due to mergers, as the correlation between the average masses of the halos and the density of their environment indicates.

In all of the simulations the least massive halos resolved by the N-body code are most abundant. The peak in the halo mass function, which corresponds to a rotation velocity of $v_{rot} \sim 150$ km/s, is an artificial cutoff resulting from our finite resolution. On the other hand, there are very few halos with "excessive" rotation velocities, especially in the undernormalized ($\sigma_8 \simeq 0.7$) run which, as we

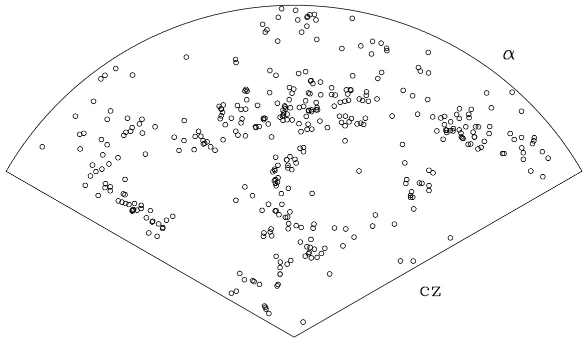

Figure 1: A 20° redshift slice of halos from the COBE-normalized CDM simulation. The slice spans 120° in right ascension and extends to 8000 km/s.

will remark in section 6, may be the best estimate of σ_8 on small scales given (i) COBE normalization on the horizon scale and (ii) "generic" inflationary scenarios for the origin of perturbations.

Before proceeding to the subject of peculiar velocities, we note that the sizes of galactic halos, their spatial distribution, and their rotation curves are all reasonably consistent with observations.

3 Small-Scale Peciliar Velocities in the Simulations

Observations of the spatial distribution of matter have been reasonably well-matched by the CDM cosmogony, as realized in our simulations. The excess of massive galaxy halos reported in previous numerical work seems to have been caused by poor resolution, as it has been greatly alleviated by the high resolution of our N-body code. The discrepancies which remain between the CDM model and the Universe could be caused by the serious uncertainties in processes such as star formation which determine how galaxies become luminous and how they are related to the underlying dark matter. The situation with the velocity distribution is quite different: it is commonly believed that small-scale peculiar velocity statistics are extremely problematic for the CDM model. We now consider on small-scale peculiar velocities in our simulations and in observational data sets.

The pairwise peculiar radial velocity dispersion σ_v is measured in simulations on the basis of the full 6D phase space; its values for halos on megaparsec scales are approximately 600, 800, and 1600 km/s for the simulations with $\sigma_8 \simeq 0.7, 1.06$, and 1.6, respectively. In the absence of error limits these values do not compare favorably with the commonly accepted observational estimate of 340 km/s.

To some degree, the disrepancy between the observed estimate of σ_v and previous numerical results has been decreased in our simulations by a velocity bias in the halo velocity dispersion, as detected by Carlberg & Couchman (1989). It is nearly a factor of two on kiloparsec scales, becoming somewhat smaller ($b_v \sim 0.7$) at megaparsec scales. There is preliminary evidence (*e.g.* Carlberg 1994; also see Bromley, Warren & Zurek, this volume) that a significant contribution to this bias is differences in the statistical weighting of halos compared to that of the mass particle. Nonetheless, this effect is still too small to bring our results into agreement with the Davis & Peebles estimate.

The velocity dispersion can be extracted from the data by computing the distribution of number of pairs as a function of their radial peculiar velocity differences. The reader should be cautioned that a naive estimate of σ_v from a simple average over pairwise peculiar velocities is not optimal because it spuriously mixes in the first moment of the distribution. In the models investigated here, the errors introduced by following the naive prescription for σ_v are of the order of 10%.

At Mpc separations the velocity distribution is well-approximated by an exponential,

$$f(\Delta v) \sim \exp(-\sqrt{2}|\Delta v|/\sigma_v) \; ; \tag{2}$$

when the mean infall velocity is not included in the pairwise peculiar velocity Δv (see Zurek *et al.* 1994 for a more complete discussion). Thus, the slopes of an isosceles triangle, generated by plotting the logarithm of f, can be used to obtain σ_v. We shall shortly see that this is in fact the basis of the method for extracting σ_v from the observations.

The situation becomes considerably more complicated at larger distances (5–10 Mpc). There the triangular profiles of $\log f$ no longer isosceles, hence the symmetric exponential form of the distribution equation (2) is no longer valid. This is an important conclusion for the observational estimates of σ_v, since the form of the velocity distribution function must be assumed in order to extract the value of σ_v.

4 Observations—The Davis & Peebles Method

In contrast to numerical experiments, which allow one to compute σ_v directly from complete position and velocity data, redshift surveys contain only the angular separations and proper recession velocities of galaxies. In the absence of peculiar motions, redshifts translate directly to the line-of-sight distances and the galaxy-galaxy correlations as measured in "redshift space" would be equivalent to their real-space counterparts. The effect of peculiar velocities is to smear out redshift-space correlations along the line of sight. Hence, the extent to which the line-of-sight correlations are distorted is a measure of peculiar motions.

The quantitative measure of second-order correlations in redshift space is $\xi(r_p, \pi)$, which gives the excess two-point clustering above a pure random distribution as a function of separations parallel (π) and perpendicular (r_p) to the line of sight:

$$\pi = v_1 - v_2, \quad r_p = \frac{1}{H_0}(v_1 + v_2)\tan(\theta_{12}/2) \; , \tag{3}$$

where v_1 and v_2 are recession velocities of two points in redshift space and θ_{12} is the sky angle between them. One estimator of $\xi(r_p, \pi)$ is

$$\frac{n_R DD(r_p, \pi)}{n_D DR(r_p, \pi)} - 1, \tag{4}$$

where DD is the number of redshift data pairs which are separated by (r_p, π); DR is the number of pairs which consist of one data object and one object from a catalog of Poisson distributed objects, subjected to the same selection criteria as the data catalog. The mean densities n_D and n_R correspond to the data and random catalogs respectively. Figure 2 shows $\xi(r_p, \pi)$ estimated in this way from the three simulations and the CfA North Forty survey.

The method of Davis & Peebles (DP) is to model the distribution of pairwise peculiar velocities, with σ_v as a parameter, and to fit the observed anisotropy in the redshift-space correlation function. However, there are assumptions in this procedure which which cannot be validated from redshift data alone; they must be tested independently through other means such as numerical experiments.

The first fundamental assumption is that the pairwise velocity distribution function of the galaxies is exponential and symmetric about the mean, as in equation (2). In our simulations this

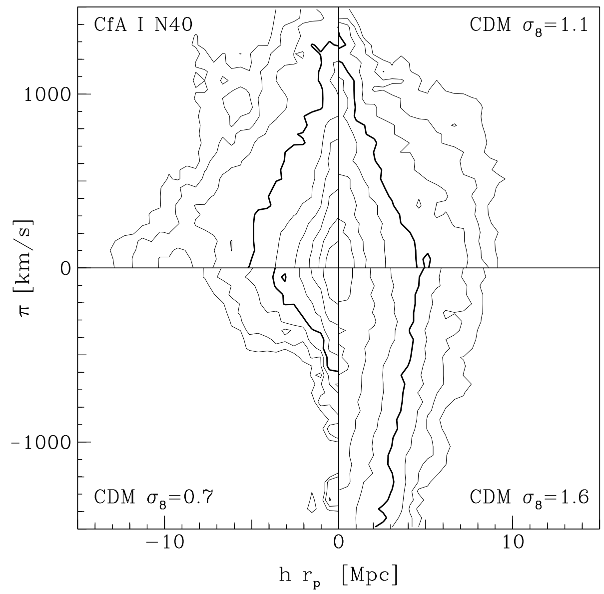

Figure 2: Contours of $\xi(r_p, \pi)$ for the CfA North Forty data and CDM halo surveys of equivalent geometry and magnitude selection. The contour spacing is logarithmic (0.2 dex); the heavy line indicates the $\xi = 1$ contour.

assumption is found to be quite accurate at small separations. The use of the method at 5–10 Mpc separations, however, would require modification of the assumed velocity distribution.

The second fundamental assumption is a model for the mean streaming velocity relative to the Hubble flow, since the goal is to extract the velocity dispersion about the mean pairwise peculiar infall. Davis & Peebles use the similarity solution derived in their 1977 work,

$$h(r) = F[1 + (r/r_0)^2]^{-1} . \tag{5}$$

The agreement between this model and the simulations is reasonable, but there are a discrepancies of 50–100 km/s at scales between 1 and 3 Mpc, where the model predicts less infall than is observed in the N-body data.

These two assumptions are sufficient to model the redshift-space correlations as the result of convolution of the real-space correlations with the pairwise velocity distribution along the line-of-sight direction (DP eq. [22]). In the case of power-law correlations in real-space, an exponential velocity distribution results in an approximately exponential profile in $\xi(r_p, \pi)$ along the π axis for values of r_p near or below megaparsec scales.

To test our implementation of this method of analysis, we have applied it to a redshift catalog of all halos from the undernormalized model. For this test case, unlike the analysis of the CfA data and artificial catalogs below, we use a majority of the halos and include no flux limits. The DP method derives a value for σ_v of 628 km/s as compared with 610 km/s obtained from the full phase-space analysis. Thus, the DP method works well in this optimal case.

The key observational result of $\sigma_v = 340$ km/s from the DP analysis comes from the North Forty slice of the CfA I survey. The survey boundaries are: sky mask, $\delta > 0°$, $b > 40°$; depth, $100 < cz < 10,000$ km/s; magnitude limit, $\mathrm{m_B} \leq 14.5$; and an absolute magnitude cut, $\mathrm{M_B} < -18.5$ for a Hubble parameter of 100 km/s/Mpc, to semi-volume limit the sample. To assess the statistical integrity of these data, we ran the DP analysis on equivalent surveys from our halo catalogs. Due to the limited size of the computational volume, the origin for these slices is placed near the edge of the computational sphere. The center of the volume tends to be oversampled, and data sets with different view-point origins are not completely independent. Thus, the formal uncertainty limits are underestimates as they neglect the "cosmic" variations expected from truly independent samples.

Pairwise velocity dispersions obtained from the full phase-space analysis of these CfA-like slices vary between 600 and 800 km/s for 1 Mpc separations in the $\sigma_8 \simeq 0.7$ model samples. An even greater variation of 450 to 850 km/s occurs when the DP algorithm is used to infer σ_v from sky and redshift coordinates. Moreover, both of these estimates are bound to depend critically on the treatment of the regions near rich clusters, which have velocity dispersions near 1500 km/s in our simulations.

Differences of σ_v among the artificial surveys are much larger than the statistical estimates of the error inferred from any one of them. For example, the standard deviation computed for σ_v estimates from ten undernormalized slices and eight COBE-normalized slices are 115 km/s and 197 km/s respectively. We emphasize that because the slices overlap, these limits are bound to underestimate the variations in independent slices of similar size. The uncertainties estimated from a single slice on the basis of a least squares fit, presumably the source of the limits quoted by Davis & Peebles, are typically half of these values.

5 Pairwise Velocities from the CfA Survey

We have analyzed the publicly available CfA North Forty redshift survey using the method described in the previous section. The raw redshift data were corrected for solar motion then subjected to the sky mask, redshift limits and apparent magnitude cuts as described above; they were also corrected for an infall of 440 km/s toward the Virgo cluster to determine their distance prior to application of absolute magnitude cuts. With these selection criteria we obtain a number of galaxies, 1259, similar that of Davis & Peebles. If the Virgo infall correction for the absolute magnitudes is not performed, the number of galaxies in the sample is only 1032.

We estimated $\xi(r_p, \pi)$ from these data (Fig. 2) to obtain σ_v. The value we recovered was 517 ± 60 km/s. A lower value, 457 km/s, obtains if we exclude all galaxies beyond 8000 km/s, reducing the total galaxy count to ~ 1130.

These values are still in excess of the original DP estimate. We can alter the recovered σ_v by either neglecting the Virgo infall correction or cutting out galaxies from within the core of Virgo. For example, we made another CfA sample by excluding ~ 50 galaxies within 10° of the core, from which σ_v is estimated to be approximately 365 km/s. This is surprising from the point of view that the measurement appears to be quite sensitive to this set of galaxies which constitutes less than 5% of the sample. However, it is less surprising when one notes that the method weights heavily pairs which are separated by about 1 Mpc: The galaxies which were eliminated from this latter sample had megaparsec separations and represented a good portion of the population with high peculiar velocity. This assessment is consistent with the recent reanalysis of the CfA data by Mo *et al.* (1993).

6 Discussion

The principal lesson from our analysis of small-scale velocity dispersions is that the σ_v statistic is highly sensitive to the presence of rare, massive clusters. As we have found, there is a large variance in estimates of σ_v from sample to sample of the same cosmological model that stems from how clusters appear given a set of selection criteria. Indeed, with uncertainty levels at a minimum of 200 km/s for the COBE-normalized simulation and of the order 100 km/s for the

CfA survey, the CDM model cannot be excluded with reasonable confidence. The undernormalized model fares even better, however, the same cannot be said of the overnormalized case for which σ_v is estimated at ~ 1600 km/s. This result, in conjunction with other recent studies (*e.g.* Brainerd & Villumsen 1994), suggests that the predictions of CDM at small scales are generally compatible with observations.

Our conclusion about the viability of the CDM model on the basis of small scale data depends not just on our reanalysis of the CfA data but on a velocity bias that appears in the simulations: The averaged pairwise velocities of halos on megaparsec scales are significantly smaller than the relative velocities of particles. This velocity bias decreases the discrepancy between the observational estimates of velocity dispersion and our numerical estimates for the CDM model. The bias we obtain is close to the one reported by Carlberg (1991) in a dissipationless simulation with a significantly lower resolution. A larger velocity bias $b_v = 0.37$ was given by Couchman & Carlberg (1992) for the "old" population of collapsed objects which had well-defined seeds at $z = 1$–3. On the other hand, our velocity bias is slightly more pronounced than the one reported by Cen & Ostriker (1992) who estimate $b_v \approx 0.8$ on megaparsec scales. This small discrepancy, which is of low statistical significance, could be certainly attributed to the difference in the resolution between the two codes. Overall, there seems to be a consensus between these estimates of b_v obtained in several rather different simulations, using different numerical schemes and somewhat different physical input.

The increase in σ_v from the reanalyzed CfA data and the detection of a velocity bias both have an impact on estimates of the cosmological density parameter Ω from the cosmic virial theorem. For clustering parameters appropriate to the CfA galaxies,

$$\Omega \approx 1.73 \times 10^{-6} b_1 \sigma_v^2 / b_v^2 \ ,$$

where b_1 is the bias in galaxy correlations at megaparsec scales. Whereas Davis & Peebles obtained $\Omega = 0.2$ with $\sigma_v = 340$ km/s and no biasing, we now derive $\Omega = 0.9$ with $\sigma_v = 500$ km/s, $b_1 = 1$, and $b_v = 0.7$. However, as we discuss next, the statistical significance of σ_v must be carefully considered when interpreting this result.

The degree of sensitivity that σ_v exhibits to the presence of galaxies within rich clusters is striking. We mentioned that the selective removal of a small fraction (5%) of galaxies in the CfA data induced a significant ($\gtrsim 40\%$) change in the estimated value of σ_v. Clearly σ_v is heavily dependent on the number density and velocities of galaxies in the centers of clusters. Biasing in the spatial distribution of these galaxies, as well as the velocity bias, can greatly affect the pairwise velocity dispersion. For example, an underabundance of galaxies in a cluster core would make these high-velocity objects relatively sparse, causing an underestimate in σ_v. Unfortunately, the biases in the phase-space distribution of galaxies in rich clusters depend on uncertain physics of galaxy formation and are presently not well determined, much less well understood. We thus find σ_v to be an unreliable measure of cosmic structure on small scales.

The dependence of σ_v on galaxies in the most massive clusters in a survey may be quite common for second-order statistical indicators of cosmological structure at megaparsec scales and below. For example, number counts of galaxies in spherical cells are known to have distribution functions with very broad tails, falling off in certain cosmological systems as a shallow power law. In this event, the variance in the cell counts can depend on the number of galaxies within the richest cluster in the survey (Bromley 1994). Although the velocity distribution itself falls off exponentially, the steepness may well depend on the dominant contributions of galaxies within the single most massive cluster.

These remarks concerning the reliability of σ_v should be kept in mind when analyzing other data sets. For example, Fisher *et al.* (1994) found $\sigma_v \approx 320$ km/s from the IRAS survey; (we find a value of about 330 km/s in the same data set; the two numbers are both well within each other's uncertainty bounds). It is expected that the dispersion derived from the IRAS survey is lower than in the CfA case, because the infrared selection criteria tend to exclude galaxies in the center of clusters. Tests of our simulations against the IRAS data at small scales will critically hinge on how we identify an equivalent halo population.

7 Conclusion

In an effort to examine small-scale cosmological indicators in CDM simulations, we have found that the σ_v statistic is inadequate, especially when derived from samples which are not representative of the Universe at large. Consequently, claims that COBE-normalized CDM are inconsistent with observed velocity dispersion on small scales carry little weight at this point in time.

With the caveat that the velocity statistic is unreliable, we propose that the undernormalized model, with its low σ_v prediction, may be realistic. It can, for example, match the observed abundance of Abell clusters (see White *et al.* 1993). This biased model ($b = 1.4$) is also consistent with large-scale flow measurements, for which observations indicate $\Omega/b^{1.7} \sim 0.4$–0.6 (Fisher *et al.* 1993; Nusser & Davis 1994).

The apparent discrepancy between the undernormalized model and the COBE data can be remedied by speculation about an inflationary scenario (Davis *et al.* 1992): Inflationary cosmology yields an exact Harrison-Zel'dovich power spectrum of fluctuations, $P(k) \sim k^n$, $n = 1$, when inflation is eternal. The roll-over from de Sitter expansion to the standard radiation-dominated phase at some early epoch must leave the spectral index $n < 1$. This "tilt" in the spectrum would lower the amplitude of CDM power from megaparsec scales to the Hubble radius. The tilt necessarily implies that a fraction of the fluctuations detected by COBE is the result of gravitational tensor modes which do not serve as seeds for structure formation. In two popular models for the inflaton field with quadratic and quartic potentials, the predicted small-scale fluctuation amplitude σ_8 is 0.85 and 0.75, respectively, values which are between the undernormalized and COBE normalized models discussed here.

The CDM cosmogony certainly still has hurdles to overcome in the large-scale regime. We have already mentioned a problem with angular correlations in the APM survey on scales corresponding to $\sim$ 100 Mpc. The CDM model predicts too little power compared with the optical dataset. Interestingly, there is no such problem of missing power on those same scales when compared to redshift data (*e.g.* Efstathiou 1993) although these data are sparse when compared to the two million APM galaxies. Even with a sample of a large number of galaxies, one must worry about selection effects. For example, the Great Wall, the 100 Mpc sheetlike structure found in the optically selected CfA II survey, is not similarly discernible in the IRAS data.

Other concerns include cluster-cluster correlations. The standard CDM model reportedly predicts low amplitude correlations between clusters of galaxies on scales greater than 10 Mpc (*e.g.* Dalton *et al.* 1992). Here again, selection effects are worrisome until we achieve a greater understanding of how galaxies light up within clusters.

We cannot sweep all of the outstanding problems of CDM under the carpet of uncertainty in the physics of galaxy formation. Indeed, the model's assumption that the density parameter $\Omega = 1$ would crumble if reliable measurements of the Hubble parameter lead to conflicts with direct age estimates from globular clusters. However, we do emphasize the importance of high-resolution numerical work in drawing conclusions about the viability of CDM and other theories of structure formation.

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