Formulae for CSR Microbunching in a Bunch Compressor Chicane*

Zhirong Huang† and Kwang-Je Kim

Argonne National Laboratory, Argonne, IL 60439, USA

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Abstract

A microbunching instability driven by coherent synchrotron radiation (CSR) in a bunch compressor chicane is studied using an iterative solution of the integral equation that governs this process. By including both one-stage and two-stage amplifications, we obtain analytical expressions for CSR microbunching that are valid in both low-gain and high-gain regimes. These formulae can be used to explore the dependence of CSR microbunching on compressed beam current, energy spread, and emittance, and to design stable bunch compressors required for an x-ray free-electron laser.

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†Electronic address: zrh@aps.anl.gov

I. INTRODUCTION

A microbunching instability driven by coherent synchrotron radiation (CSR) in a bunch compressor chicane [1] is under intense study [2–5] as it may impact the design of an x-ray free-electron laser (FEL) calling for kiloampere, subpicosecond electron bunches [6, 7]. A klystron-like mechanism of amplification of parasitic density modulations in a bunch compressor is studied in Ref. [4] under the high-gain assumption and in the absence of electron energy chirp. A self-consistent treatment of CSR microbunching, including the electron energy chirp and the emittance effect, is developed in Ref. [5], and the microbunching process is described by an integral equation. Numerical solution of the integral equation for beam parameters and lattice functions corresponding to the second bunch compressor of the Linac Coherent Light Source (LCLS) [6] yields very low gain (<3) over a wide wavelength range.

In this paper we analyze the microbunching process in a typical bunch compressor and obtain the iterative solution of the integral equation that is valid in both high-gain and low-gain regimes. In Section II, we present a compact derivation of the integral equation for CSR microbunching, originally derived in Ref. [5] using the linearized Vlasov equation. In Section III, we discuss the iterative solution and express CSR microbunching initiated from either density or energy modulation in terms of beam energy, current, emittance, energy spread and chirp, initial lattice parameters, as well as basic chicane parameters. In Section IV, we apply these results to study the stability of the LCLS bunch compressors and to illustrate various amplification processes. Concluding remarks are given in Section V.

II. INTEGRAL EQUATION FOR CSR MICROBUNCHING

Consider a beam distribution function $f(x, x', z, \delta; s)$ in the transverse $(x, x' \equiv dx/ds)$ and longitudinal $(z, \delta \equiv \Delta E/E)$ phase spaces at location s along a bunch compressor chicane. (The vertical plane is irrelevant here.) If N is the total number of electrons, we have

$$\int d\mathbf{X} f(\mathbf{X}; s) = N, \tag{1}$$

where $\mathbf{X} = (x, x', z, \delta)$ denotes the set of phase-space variables at s.

In the absence of CSR, the evolution of f is given by

$$f(\mathbf{X}; s) = f(\mathbf{R}^{-1}(\tau \to s)\mathbf{X}; \tau), \tag{2}$$

where $\mathbf{X} = \mathbf{R}(\tau \to s)\mathbf{X}_\tau$, $\mathbf{X}_\tau$ is the set of phase-space variables at τ, and the symplectic transfer matrix $\mathbf{R}$ between τ and s is

$$\mathbf{R}(\tau \to s) = \begin{pmatrix} C(\tau \to s) & S(\tau \to s) & 0 & \eta(\tau \to s) \\ C'(\tau \to s) & S'(\tau \to s) & 0 & \eta'(\tau \to s) \\ R_{51}(\tau \to s) & R_{52}(\tau \to s) & 1 & R_{56}(\tau \to s) \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

Here $C(\tau \to s)$ and $S(\tau \to s)$ are the cosine- and sine-like solutions of the focusing equation

$$x'' + K_x(s)x = 0 \tag{4}$$

with the boundary conditions $C(\tau \to \tau) = 1$ and $S(\tau \to \tau) = 0$, $(') = d/ds$, $K_x(s)$ is the horizontal focusing function,

$$\eta(\tau \to s) = S(\tau \to s) \int_\tau^s d\zeta \frac{C(\tau \to \zeta)}{\rho(\zeta)} - C(\tau \to s) \int_\tau^s d\zeta \frac{S(\tau \to \zeta)}{\rho(\zeta)} \tag{5}$$

is the dispersion function, $\rho(s)$ is the bending radius, and the transfer function

$$(R_{51}, R_{52}, R_{56})(\tau \to s) = -\int_\tau^s \frac{d\zeta}{\rho(\zeta)} (C, S, \eta)(\tau \to s) \tag{6}$$

connects an offset in transverse phase space or energy at τ to a change in z at s. Thus, the distribution function $f(\mathbf{X}; s)$ is completely determined by the initial distribution $f_0(\mathbf{X}_0)$ at the chicane entrance $\tau = 0$ because

$$f(\mathbf{X}; s) = f(\mathbf{R}^{-1}(s)\mathbf{X}; 0) = f_0(\mathbf{X}_0), \tag{7}$$

where $\mathbf{R}(s) \equiv \mathbf{R}(0 \to s)$ for abbreviation.

Suppose coherent synchrotron radiation is emitted and the electron energy is changed by an amount $\Delta\delta$ at τ. The distribution function immediately after the emission (at $\tau{+}0$) is related to that immediately before (at $\tau{-}0$) by

$$f(\mathbf{X}_\tau; \tau{+}0) = f(\mathbf{X}_\tau - \Delta\mathbf{X}; \tau{-}0) \approx f(\mathbf{X}_\tau; \tau{-}0) - \Delta\delta \frac{\partial f(\mathbf{X}_\tau; \tau{-}0)}{\partial \delta_\tau}, \tag{8}$$

where $\Delta\mathbf{X} = (0, 0, 0, \Delta\delta)$, and only terms up to the first order in $\Delta\delta$ has been kept in Eq. (8). Summing up CSR contributions over the entire trajectory and using

$$f(\mathbf{X}; s) = f(\mathbf{X}_\tau; \tau{+}0), \quad f(\mathbf{X}_\tau; \tau{-}0) = f_0(\mathbf{X}_0), \tag{9}$$

the evolution of the distribution function under the influence of CSR is

$$f(\mathbf{X};s) = f_0(\mathbf{X}_0) - \int_0^s d\tau \frac{\partial f(\mathbf{X}_\tau;\tau-0)}{\partial \delta_\tau}\frac{d\delta}{d\tau}. \tag{10}$$

The rate of CSR energy change $d\delta/d\tau$ is determined from the beam density modulation as

$$\frac{d\delta}{d\tau} = -\frac{r_e}{\gamma}\int \frac{dk_1}{2\pi} Z(k_1;\tau) N b(k_1;\tau) e^{ik_1 z_\tau}. \tag{11}$$

Here r_e is the classical electron radius, γ is the electron energy in units of mc^2, $Z(k;s)$ is the one-dimensional, steady-state synchrotron radiation impedance [8, 9] given by [5]

$$Z(k;s) = -iA\frac{k^{1/3}}{\rho(s)^{2/3}}, \quad \text{with} \quad A = 1.63i - 0.94. \tag{12}$$

$b(k;s)$ is a complex bunching parameter for density modulation at wavelength $\lambda = 2\pi/k$:

$$b(k;s) = \frac{1}{N}\int d\mathbf{X} e^{-ikz} f(\mathbf{X};s). \tag{13}$$

Note that both k and the beam current vary in a chicane due to compression, but the bunching parameter defined above is independent of the current because of Eq. (1).

Equation (10) can now be cast into an integral equation for the bunching parameter. First, we write

$$\begin{aligned} b(k;s) =& b_0(k;s) - \frac{1}{N}\int d\tau \int d\mathbf{X}_\tau e^{-ikz(\mathbf{X}_\tau)} \frac{\partial f(\mathbf{X}_\tau;\tau-0)}{\partial \delta_\tau}\frac{d\delta}{d\tau} \\ =& b_0(k;s) - \frac{ik}{N}\int d\tau R_{56}(\tau\to s)\int d\mathbf{X}_\tau e^{-ikz(\mathbf{X}_\tau)} f(\mathbf{X}_\tau;\tau-0)\frac{d\delta}{d\tau}, \end{aligned} \tag{14}$$

where

$$b_0(k;s) = \frac{1}{N}\int d\mathbf{X}_0 e^{-ikz} f_0(\mathbf{X}_0) \tag{15}$$

is the bunching without CSR, and we have integrated the second term by parts over δ_τ using

$$z(\mathbf{X}_\tau) = z_\tau + R_{51}(\tau\to s)x_\tau + R_{52}(\tau\to s)x'_\tau + R_{56}(\tau\to s)\delta_\tau. \tag{16}$$

Changing variables from $\mathbf{X}_\tau$ to $\mathbf{X}_0$ with $f(\mathbf{X}_\tau, \tau-0) = f_0(\mathbf{X}_0)$ for the second term of Eq. (14) and inserting Eq. (11), we obtain

$$\begin{aligned} b_0(k;s) =& b_0(k;s) + \frac{ikr_e}{\gamma}\int d\tau R_{56}(\tau\to s)\int \frac{dk_1}{2\pi} Z(k_1;\tau) b(k_1;\tau) \\ & \times \int d\mathbf{X}_0 e^{-ikz(\mathbf{X}_0)+ik_1 z_\tau(\mathbf{X}_0)} f_0(\mathbf{X}_0), \end{aligned} \tag{17}$$

where $z(\mathbf{X}_0) = z_0 + R_{51}(s)x_0 + R_{52}(s)x_0' + R_{56}(s)\delta_0$.

We now write $f_0(\mathbf{X}_0)$ as

$$f_0(\mathbf{X}_0) = \bar{f}_0(\mathbf{X}_0) + \hat{f}_0(\mathbf{X}_0), \tag{18}$$

where $\bar{f}_0(\mathbf{X}_0)$ represents the average distribution and $\hat{f}_0(\mathbf{X}_0)$ represents an arbitrary but small perturbation. For modulation wavelengths much smaller than the electron bunch length, we may assume that the average beam distribution is uniform in z and Gaussian in transverse and energy variables:

$$\bar{f}_0(\mathbf{X}_0) = \frac{n_0}{2\pi\varepsilon\sqrt{2\pi}\sigma_\delta}\exp\left[-\frac{x_0^2 + (\beta_0 x_0' + \alpha_0 x_0)^2}{2\varepsilon_0\beta_0} - \frac{(\delta_0 - hz_0)^2}{2\sigma_\delta^2}\right]. \tag{19}$$

Here n_0 is the initial line density of electrons, α_0 and β_0 are the lattice functions at $s = 0$, ε_0 and σ_δ are the initial beam emittance and incoherent energy spread, respectively, and $h > 0$ is the initial energy chirp. Neglecting $\hat{f}_0$ in the second term of Eq. (17) and integrating over $d\mathbf{X}_0$, we obtain [5]

$$b(k(s); s) = b_0(k(s); s) + \int_0^s d\tau K(\tau, s) b(k(\tau); \tau), \tag{20}$$

with the kernel of the integral equation as

$$\begin{aligned} K(\tau, s) =& ik(s)R_{56}(\tau \to s)\frac{I(\tau)Z(k(\tau); \tau)}{\gamma I_A} e^{-k_0^2 U^2(s,\tau)\sigma_\delta^2/2} \\ & \times \exp\left[-\frac{k_0^2\varepsilon_0\beta_0}{2}\left(V(s,\tau) - \frac{\alpha_0}{\beta_0}W(s,\tau)\right)^2 - \frac{k_0^2\varepsilon_0}{2\beta_0}W^2(s,\tau)\right]. \end{aligned} \tag{21}$$

Here $k(\tau)/B(\tau) = k(s)/B(s) = k_0$, $B(s) = (1 + hR_{56}(s))^{-1}$, k_0 is the modulation wavenumber at $s = 0$, $I(\tau) = ecn_0 B(\tau)$ is the peak current at τ, $I_A = ec/r_e = 17045$ A is the Alfven current, and [10]

$$\begin{aligned} U(s, \tau) =& B(s)R_{56}(s) - B(\tau)R_{56}(\tau), \\ V(s, \tau) =& B(s)R_{51}(s) - B(\tau)R_{51}(\tau), \\ W(s, \tau) =& B(s)R_{52}(s) - B(\tau)R_{52}(\tau). \end{aligned} \tag{22}$$

The physical meaning of Eqs. (20) and (21) is very clear: Density modulation at τ induces energy modulation through CSR impedance and is subsequently turned into density modulation at s through the transfer function $R_{56}(\tau \to s)$.

III. STAGED AMPLIFICATION OF CSR MICROBUNCHING

Equation (20) can be solved numerically for given beam parameters and chicane optics [5]. Here we seek an approximate analytical solution that may provide insight into the amplification process and simplify microbunching calculations. First, we iterate Eq. (20) to obtain

$$\begin{aligned} b(k(s);s) =& b_0(k(s);s) + \int_0^s d\tau K(\tau,s) b_0(k(\tau);\tau) \\ &+ \int_0^s d\tau K(\tau,s) \int_0^\tau d\zeta K(\zeta,\tau) b_0(k(\zeta);\zeta) + ... \end{aligned} \tag{23}$$

For definiteness, we study a symmetric chicane that consists of three rectangular dipoles only. The length of both the first and the last dipoles is L_b, while the middle dipole is twice as long. In general, L_b is much smaller than the dipole separation distance ΔL. In the absence of horizontal focusing (i.e., $K_x(s) = 0$ in Eq. (4)), we have $C(s) = 1$ and $S(s) = s$. The dispersion and transfer functions are determined from Eqs. (5) and (6). In particular,

$$R_{56}(\tau \to s) = \begin{cases} O\left(\frac{L_b^3}{\rho_0^2}\right) & \text{within the same dipole,} \\ O\left(\frac{\Delta L L_b^2}{\rho_0^2}\right) & \text{from one dipole to another,} \end{cases} \tag{24}$$

where $\rho_0 = |\rho(s)|$ is the same for all dipoles. Thus, we may neglect the induced bunching from the energy modulation in the same dipole [4] (i.e., we may put $K(\tau,s) = O\left(\frac{L_b}{\Delta L}\right) \approx 0$ for $(s-\tau) < \Delta L$ in Eq. (23)) and consider staged amplification from one dipole to another as follows.

A. Microbunching Due to Initial Density Modulation

We first consider that CSR microbunching is initiated by a small deviation of the beam current such as from shot noise fluctuations and rf nonlinearity. For simplicity, we take a special form of $\hat{f}_0(\mathbf{X}_0) = \epsilon(z_0)\bar{f}_0(\mathbf{X}_0)$ ($|\epsilon(z_0)| \ll 1$ with $\int dz_0 \epsilon(z_0) = 0$). The initial density modulation is

$$b_0(k_0;0) = \frac{n_0}{N} \int dz_0 \epsilon(z_0) e^{-ik_0 z_0}. \tag{25}$$

Without CSR, the bunching degradation can be calculated from Eqs. (15) and (19) as

$$
\begin{aligned}
b_0(k(s);s) =& b_0(k(s);0)e^{-k^2(s)R_{56}^2(s)\sigma_\delta^2/2} \\
&\times \exp\left[-\frac{k^2(s)\varepsilon_0\beta_0}{2}\left(R_{51}(s)-\frac{\alpha_0}{\beta_0}R_{52}(s)\right)^2-\frac{k^2(s)\varepsilon_0}{2\beta_0}R_{52}^2(s)\right]
\end{aligned} \tag{26}
$$

for $k(s) = k_0 B(s)$ at s.

We now apply Eq. (23) to obtain CSR microbunching in each dipole:

$$
b(k(s_1);s_1) \approx b_0(k(s_1);s_1), \quad 0 \le s_1 \le L_b, \tag{27}
$$

$$
b(k(s_2);s_2) \approx b_0(k(s_2);s_2) + \int_0^{L_b} ds_1 K(s_1,s_2) b_0(k(s_1);s_1), \quad 0 \le s_2 \le 2L_b, \tag{28}
$$

$$
\begin{aligned}
b(k(s_3);s_3) \approx& b_0(k(s_3);s_3) + \int_0^{L_b} ds_1 K(s_1,s_3) b_0(k(s_1);s_1) + \int_0^{2L_b} ds_2 K(s_2,s_3) b_0(k(s_2);s_2), \\
&+ \int_0^{2L_b} ds_2 K(s_2,s_3) \int_0^{L_b} ds_1 K(s_1,s_2) b_0(k(s_1);s_1), \quad 0 \le s_3 \le L_b,
\end{aligned} \tag{29}
$$

where s_j (j=1,2,3) is measured from the beginning of the j^{th} dipole, and $b(k(s_j);s_j)$ represents the bunching parameter at s_j in the j^{th} dipole. The transfer functions are

$$
\begin{aligned}
&R_{51}(s_1) = \frac{s_1}{\rho_0}, \quad R_{52}(s_1) = \frac{s_1^2}{2\rho_0}, \quad R_{56}(s_1) = \frac{s_1^3}{6\rho_0^2}, \\
&R_{51}(s_2) = \frac{L_b - s_2}{\rho_0}, \quad R_{52}(s_2) \approx -\frac{\Delta L s_2}{\rho_0}, \quad R_{56}(s_2) \approx -\frac{\Delta L L_b}{\rho_0^2} s_2, \\
&R_{51}(s_3) = -\frac{L_b - s_3}{\rho_0}, \quad R_{52}(s_3) \approx \frac{2\Delta L (s_3 - L_b)}{\rho_0}, \quad R_{56}(s_3) \approx -\frac{2\Delta L L_b^2}{\rho_0^2} \equiv R_{56}, \\
&R_{56}(s_1 \to s_2) \approx -\frac{\Delta L}{\rho_0^2}(L_b - s_1)s_2, \quad R_{56}(s_2 \to s_3) \approx -\frac{\Delta L}{\rho_0^2}(2L_b - s_2)s_3, \\
&R_{56}(s_1 \to s_3) \approx -\frac{2\Delta L}{\rho_0^2}\left[(L_b - s_1)L_b + s_1 s_3\right].
\end{aligned} \tag{30}
$$

For a typical chicane, we have $\beta_0 \gg L_b$, $|\alpha_0| \sim 1$ and $R_{51}(s_1) \gg |\alpha| R_{52}(s_1)/\beta_0 \sim R_{52}(s_2)/\beta_0$. Since $R_{56}(s_1)$ is much smaller than the R_{56} generated between dipoles, we set $R_{56}(s_1) \approx 0$, $k(s_1) \approx k_0$ in Eq. (26) to obtain

$$
b_0(k(s_1);s_1) \approx b_0(k_0;0)e^{-k_0^2 R_{51}^2(s_1)\varepsilon_0\beta_0/2}. \tag{31}
$$

If the induced bunching $\int_0^{L_b} ds_1 K(s_1,s_2) b_0(k_0;s_1)$ in the middle dipole is much larger than $b_0(k_0;s_1)$ and $b_0(k(s_2);s_2)$ (i.e., if the gain is much larger than 1), the bunching in the last dipole is determined mainly from the induced bunching in the middle dipole (i.e., the last term on the right side of Eq. (29)). This situation corresponds to the two-stage amplification discussed in Ref. [4] under the high-gain assumption. However, the gain is usually not very high when both the emittance and the energy spread are taken into account; then one-stage amplifications from the first and the middle dipoles to the last dipole (i.e., the second and the third terms on the right side of Eq. (29)) are also important and may even dominate the two-stage process (see numerical examples in Section IV). Thus, the final bunching at the chicane exit can be calculated from Eq. (29) for $s_3 = L_b$ (denoted as "f"). Here the initial bunching degrades to

$$b_0(k_f; f) = \exp\left[-\frac{\bar{\sigma}_\delta^2}{2(1+hR_{56})^2}\right] b_0(k_0; 0), \tag{32}$$

where $\bar{\sigma}_\delta = k_0 R_{56}\sigma_\delta$, and $k_f = k_0/(1+hR_{56})$, and the emittance degradation effect is absent because of the achromatic condition $R_{51}(f) = R_{52}(f) = 0$. The one-stage amplification from the first dipole can be computed from Eqs. (29), (30), and (31) as

$$\int_0^{L_b} ds_1 K(s_1, s_3) b_0(k_0; s_1) = A\bar{I}_f \left[F_0(\bar{\sigma}_x) + \frac{1-e^{-\bar{\sigma}_x^2}}{2\bar{\sigma}_x^2}\right] \exp\left[-\frac{\bar{\sigma}_\delta^2}{2(1+hR_{56})^2}\right] b_0(k_0; 0), \tag{33}$$

where

$$\bar{I}_f = \frac{I_f k_0^{4/3} R_{56} L_b}{\gamma I_A \rho_0^{2/3}}, \tag{34}$$

I_f is the compressed beam current, $\bar{\sigma}_x = k_0 L_b \sqrt{\varepsilon_0 \beta_0}/\rho_0$, and

$$F_0(\bar{\sigma}_x) = \frac{e^{-\bar{\sigma}_x^2} + \bar{\sigma}_x \sqrt{\pi}\,\mathrm{erf}(\bar{\sigma}_x) - 1}{2\bar{\sigma}_x^2}, \tag{35}$$

with the error function $\mathrm{erf}(x) = 2\pi^{-1/2}\int_0^x dt \exp(-t^2)$. Similarly, the one-stage and the two-stage amplification from the middle dipole to the chicane exit can be computed as

$$\int_0^{2L_b} ds_2 K(s_2, s_3) b_0(k(s_2); s_2) = A\bar{I}_f F_1(hR_{56}, \bar{\sigma}_x, \alpha_0, \phi, \bar{\sigma}_\delta) b_0(k_0; 0), \tag{36}$$

$$\int_0^{2L_b} ds_2 K(s_2, s_3) \int_0^{L_b} ds_1 K(s_1, s_2) b_0(k_0; s_1) \approx A^2 \bar{I}_f^2 F_0(\bar{\sigma}_x) F_2(hR_{56}, \bar{\sigma}_x, \alpha_0, \phi, \bar{\sigma}_\delta) b_0(k_0; 0),$$

where $\phi = \frac{2\Delta L}{\beta_0} \approx \frac{-\rho_0^2 R_{56}}{\beta_0 L_b^2}$, and

$$F_1 = 2\int_0^1 dt \frac{(1-t)}{(1+hR_{56}t)^{4/3}} H(t), \quad F_2 = 2\int_0^1 dt \frac{(1-t)t(1+hR_{56})}{(1+hR_{56}t)^{7/3}} H(t),$$

$$H(t) = \exp\left[-\bar{\sigma}_x^2 \frac{(1-2t+\alpha_0\phi t)^2 + \phi^2 t^2}{(1+hR_{56}t)^2} - \frac{\bar{\sigma}_\delta^2}{2(1+hR_{56}t)^2}\left(t^2 + \frac{(1-t)^2}{(1+hR_{56})^2}\right)\right]. \tag{37}$$

Defining the final gain of density modulation in a chicane as $G_f = |b(k_f; f)/b_0(k_0; 0)|$, we obtain from Eqs. (32), (33), and (36)

$$G_f \approx \Bigg| \exp\left[-\frac{\bar{\sigma}_\delta^2}{2(1+hR_{56})^2}\right] + A\bar{I}_f \Bigg[\left(F_0(\bar{\sigma}_x) + \frac{1-e^{-\bar{\sigma}_x^2}}{2\bar{\sigma}_x^2}\right) \exp\left(-\frac{\bar{\sigma}_\delta^2}{2(1+hR_{56})^2}\right) + F_1(hR_{56}, \bar{\sigma}_x, \alpha_0, \phi, \bar{\sigma}_\delta) \Bigg] + A^2 \bar{I}_f^2 F_0(\bar{\sigma}_x) F_2(hR_{56}, \bar{\sigma}_x, \alpha_0, \phi, \bar{\sigma}_\delta) \Bigg|. \tag{38}$$

The first term on the right side of Eq. (38) represents the loss of microbunching in the limit of vanishing current, the second term (linear in current) is the one-stage microbunching amplification at low current (low gain), and the last term (quadratic in current) corresponds to the two-stage amplification at high current (high gain).

It is often useful to know the electron energy spectrum for beam diagnostics. The induced energy modulation at wavelength $\lambda = 2\pi/k$ can be obtained from Eq. (11) as

$$\Delta p_k(s) = -\int_0^s d\tau \frac{I(\tau)}{\gamma I_A} Z(k(\tau), \tau) b(k(\tau), \tau), \tag{39}$$

where $b(k(\tau), \tau)$ is determined by Eqs. (27), (28), and (29).

B. Microbunching Due to Initial Energy Modulation

CSR microbunching can also be seeded by an initial energy deviation $\Delta p_0(z_0)$ originated from upstream wakefield and CSR effects [11]. In this case, we write

$$\hat{f}_0(\mathbf{X}_0) = \bar{f}_0(\mathbf{X}_0 - \Delta\mathbf{X}_0) - \bar{f}_0(\mathbf{X}_0) \approx \frac{(\delta_0 - hz_0)\Delta p_0}{\sigma_\delta^2} \bar{f}_0(\mathbf{X}_0), \tag{40}$$

where $\Delta\mathbf{X}_0 = (0, 0, 0, \Delta p_0)$. In view of Eqs. (15) and (19), the density modulation at s in the absence of CSR is

$$b_0^p(k(s), s) = -ik(s) R_{56}(s) \Delta p_{k0} e^{-k^2(s) R_{56}^2(s) \sigma_\delta^2/2} \times \exp\left[-\frac{k^2(s)\varepsilon_0\beta_0}{2}\left(R_{51}(s) - \frac{\alpha_0}{\beta_0} R_{52}(s)\right)^2 - \frac{k^2(s)\varepsilon_0}{2\beta_0} R_{52}^2(s)\right], \tag{41}$$

where $\Delta p_{k0} = \frac{n_0}{N}\int dz_0 e^{-ik_0 z_0} \Delta p_0(z_0)$ is the Fourier amplitude of the energy modulation at $s = 0$.

We can now repeat the staged calculation as before. Since $R_{56}(s_1) \approx 0$ and induced bunching in the first dipole is negligible, we have $b^p(k(s_1), s_1) \approx 0$. Equation (29) reduces to

$$b^p(k(s_3); s_3) \approx b_0^p(k(s_3); s_3) + \int_0^{2L_b} ds_2 K(s_2, s_3) b_0^p(k(s_2); s_2). \qquad (42)$$

Thus, the final bunching at the chicane exit due to an initial energy modulation is

$$b^p(k_f; f) = -ik_f R_{56} \Delta p_{k0} \left[\exp\left(-\frac{\bar{\sigma}_\delta^2}{2(1+hR_{56})^2} \right) + A\bar{I}_f F_2(hR_{56}, \bar{\sigma}_x, \alpha_0, \phi, \bar{\sigma}_\delta) \right]. \qquad (43)$$

The induced energy modulation can also be calculated according to Eq. (39).

Finally, we note that the results of this section are equally applicable to a four-dipole chicane where two closely spaced dipoles (length L_b each) play the role of the middle dipole in a three-dipole configuration.

IV. NUMERICAL EXAMPLES

In this section, we apply the previous results to study the stability of the LCLS bunch compressors and to illustrate different amplification processes discussed in Section III. Two bunch compressors (BC1 and BC2) are incorporated in the LCLS design in order to increase the peak current by a factor of about 40. The basic beam and chicane parameters are listed in Table I for both BC1 and BC2. In Fig. 1 we compute the amplification factor G_{f1} in density modulation for wavelengths from 1 to 100 μm at the exit of BC1 and show that it is determined by one-stage amplifications as the gain is low. We also calculate the induced energy modulation Δp_{k1} (in units of initial bunching) at the end of BC1 by integrating Eq. (39) (see Fig. 2). In Figs. 3 and 4 we compute the amplification of density modulation G_{f2} in BC2 as a function of the initial modulation wavelength for four cases that are studied in Ref. [5]. Good agreement between the analytical results and the numerical solutions of the integral equation is found. Figure 4 also indicates that the two-stage amplification is the dominant process when the gain is very high.

In order to determine the total amplification factor G_T after a bunch (with some initial density modulation) passing through both BC1 and BC2, one should in principle transfer CSR energy kicks in both compressors to density modulations at the end of BC2. To simplify the calculation and to estimate G_T, we approximate CSR energy kicks in BC1 as an effective energy modulation at the entrance of BC2 given by $\Delta p_{k0} = \frac{E_1}{E_2} \Delta p_{k1}$ (E_1 is the energy in BC1 and E_2 is the energy in BC2). We also assume that the density modulation of BC1 is preserved to the entrance of BC2. Using Eqs. (38) and (43), we add up CSR microbunching originating from both density and energy modulation in BC2 and obtain G_T as shown in Fig. 5. The calculation assumes $\gamma\varepsilon_0 = 1$ μm in both compressors and $\sigma_\delta = 1.2 \times 10^{-5}$ at the beginning of BC1. Such an incoherent energy spread will change to 3×10^{-6} prior to the entrance of BC2 due to BC1 compression and acceleration between the two compressors. As seen in Fig. 5 (case 1), the total gain of the two-compressor system can be significant. To reduce the instability, σ_δ at the beginning of BC2 can be increased to 3×10^{-5} with the addition of a superconducting wiggler prior to BC2 [12]. Figure 5 (case 2) shows that the increased energy spread in BC2 improves the stability of the two-compressor system against the microbunching. It is interesting to note that the peak gain of the two-compressor system with the wiggler (case 2 of Fig. 5) is still larger than BC2 gain without the wiggler (case 3 of Fig. 3), in qualitative agreement with the numerical simulation results [12].

V. CONCLUSION

In this paper, we show that both one-stage and two-stage (klystron-like) amplifications are important processes for CSR microbunching in a bunch compressor chicane. Based on the assumption that the dipole separation is much larger than the length of the individual dipoles, we investigate the bunching process in a typical chicane and derive Eqs. (38) and (43) for CSR microbunching initiated by density and energy modulation. These results are applied to the study of the LCLS bunch compressors in order to determine the stability of the system. The method and formulae presented here should be useful to facilitate the design of bunch compressors in order to reach the challenging beam parameters required for an x-ray FEL.

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[1] M. Borland *et al.*, in Proceedings of the 2001 Particle Accelerator Conference, 2707 (IEEE, Piscataway, NJ, 2001); to be published in Proceedings of the 23rd International Free Electron Laser conference (Darmstadt, Germany, 2001).

[2] E.L. Saldin, E.A. Schneidmiller, and M.V. Yurkov, to be published in Proceedings of the 23rd International Free Electron Laser conference (Darmstadt, Germany, 2001).

[3] S. Heifets and G. Stupakov, "Micorbunching due to Coherent Synchrotron Radiation in a Bunch Compressor," SLAC-PUB-8988 (2001).

[4] E.L. Saldin, E.A. Schneidmiller, and M.V. Yurkov, "Klystron Instability of a Relativistic Electron Beam in a Bunch Compressor," DESY TESLA-FEL-2002-02 (2002).

[5] S. Heifets, S. Krinsky, and G. Stupakov, "CSR Instability in a Bunch Compressor," SLAC-PUB-9165 (2002).

[6] Linac Coherent Light Source Design Study Report, SLAC-R-521 (1998).

[7] TESLA Techinical Design Report, DESY TESLA Report 2001-23, DESY TESLA-FEL 2001-05 (2001).

[8] J.B. Murphy, S. Krinsky, and R.L. Gluckstern, in Proceedings of the 1995 Particle Accelerator Conference, 2980 (IEEE, Piscataway, NJ, 1995); Particle Accelerators, **57**, 9 (1997).

[9] Y.S. Derbenev *et al.*, "Microbunch Radiative Tail-Head Interaction," DESY TESLA-FEL 95-05 (1995).

[10] We use the standard notations for R_{51} and R_{52} (they are designated as r_{51} and r_{52} in Ref. [5]), and introduce U, V, W to avoid any confusion with the $\mathbf{R}$-matrix elements.

[11] G. Stupakov, private communications.

[12] P. Emma, private communications.

TABLE I: Basic beam and chicane parameters for the LCLS bunch compressors [12].

Parameter	BC1	BC2
E [GeV]	0.25	4.54
I_f [A]	480	4000
$\gamma\varepsilon_0$ [μm]	1	1
β_0 [m]	15	105
α_0	2	5
σ_δ	1.2e-5	3e-6(5)
h [m^{-1}]	21.4	40
R_{56} [mm]	-36	-22
ρ_0 [m]	2.5	12.2
L_b [m]	0.2	0.4
ΔL [m]	2.6	10

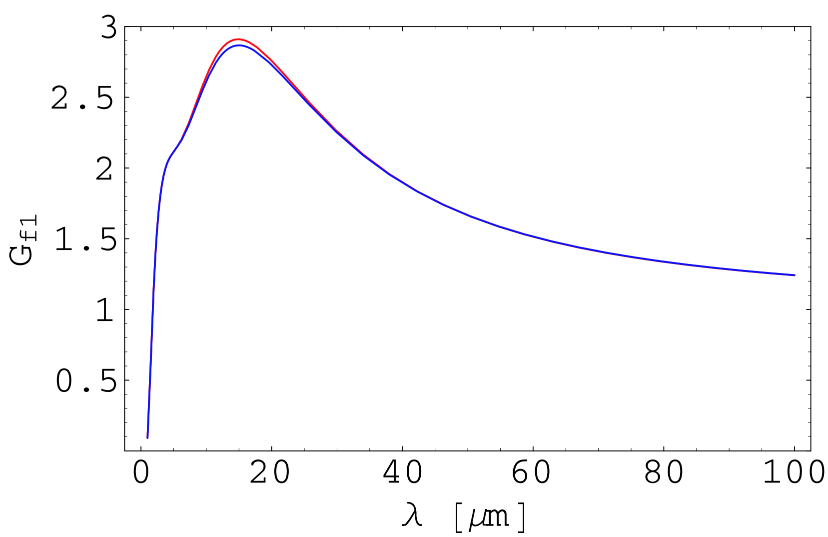

FIG. 1: (Color) BC1 gain G_{f1} of density modulation as a function of modulation wavelength at the exit of BC1 as calculated from Eq. (38) with (in red) and without (in blue) the last term (the two-stage amplification).

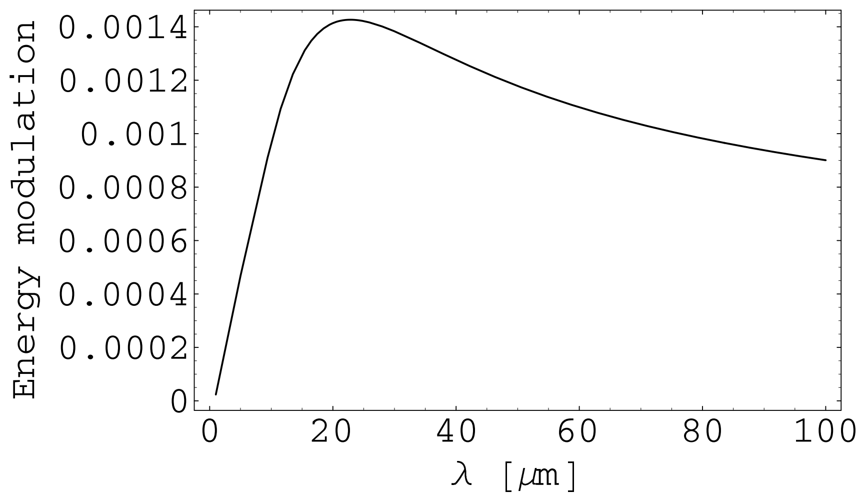

FIG. 2: Energy modulation amplitude $|\Delta p_{k1}|$ (in units of initial bunching) as a function of modulation wavelength at the exit of BC1.

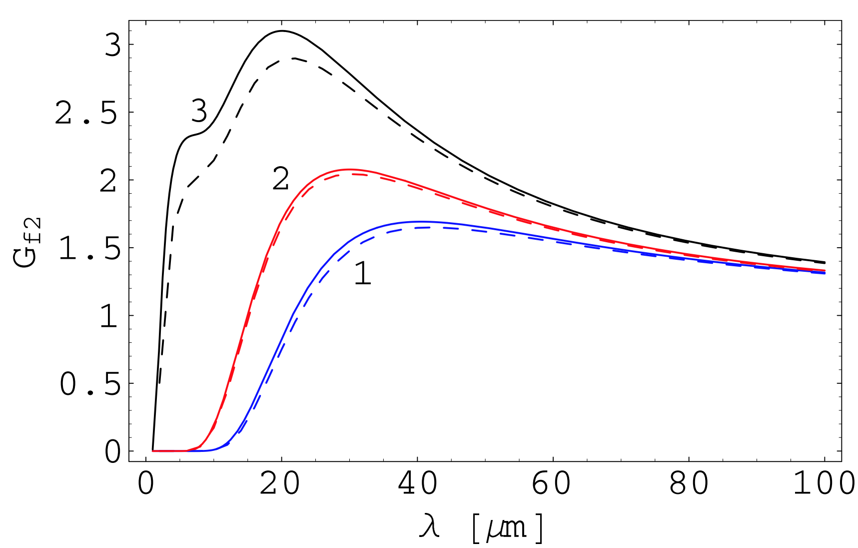

FIG. 3: (Color) BC2 gain G_{f2} of density modulation as a function of modulation wavelength at the entrance of BC2 for (1) $\sigma_\delta = 3 \times 10^{-5}$, $\gamma\varepsilon_0 = 1\ \mu$m (in blue); (2) $\sigma_\delta = 3 \times 10^{-5}$, $\gamma\varepsilon_0 = 0\ \mu$m (in red); (3) $\sigma_\delta = 3 \times 10^{-6}$, $\gamma\varepsilon_0 = 1\ \mu$m (in black). Solid curves are calculated from Eq. (38) and dashed curves are numerical solutions of the integral equation found in Ref. [5].

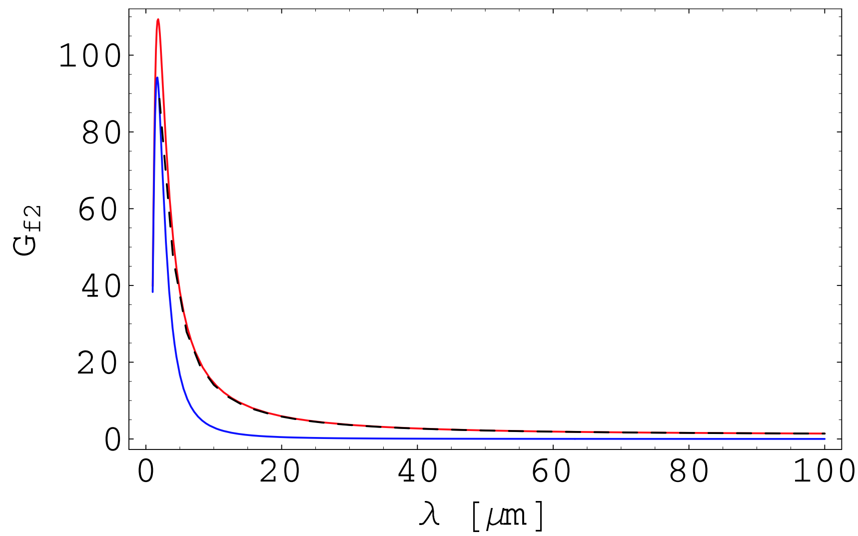

FIG. 4: (Color) BC2 gain G_{f2} of density modulation as a function of modulation wavelength at the entrance of BC2 for $\sigma_\delta = 3 \times 10^{-6}$, $\gamma\varepsilon_0 = 0$ μm, as calculated from Eq. (38) (in red) and the last term of Eq. (38) only (in blue). The dashed curve is the numerical solution of the integral equation found in Ref. [5].

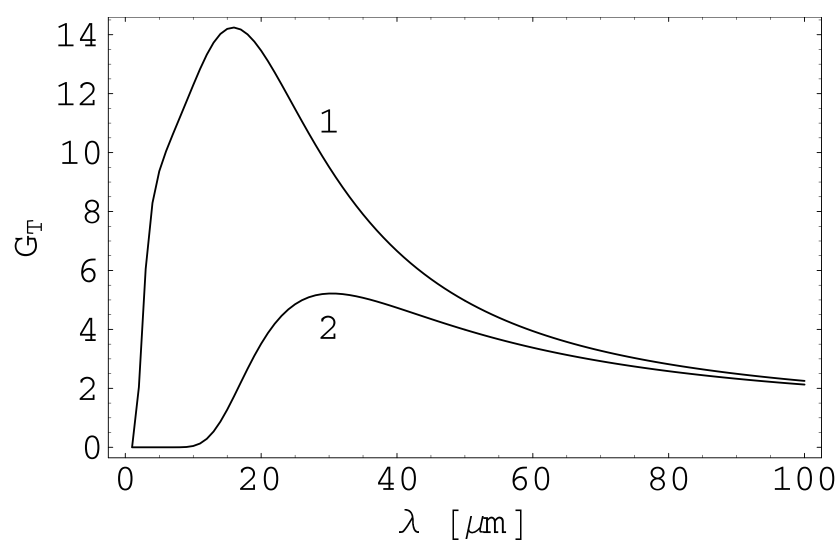

FIG. 5: Total amplification factor G_T of BC1 and BC2 as a function of modulation wavelength at the entrance of BC2 (1) without the wiggler; (2) with the wiggler.