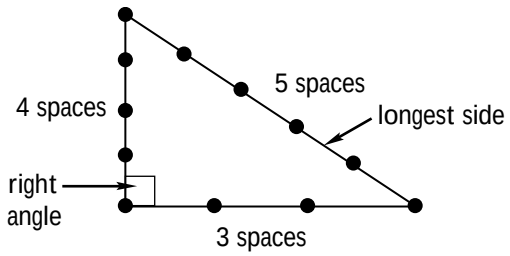
Flying With Pythagoras

Preparation: Students should be familiar with the concepts of squares and square roots. They should be able to use their calculators to square numbers and find the square roots of numbers.

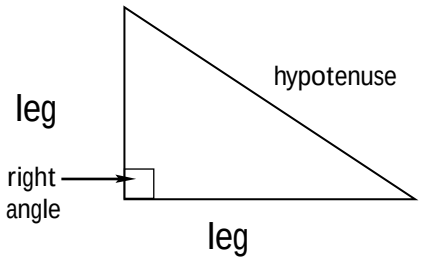
Background: In the 6th century BC, a Greek philosopher named Pythagoras lived in the village of Samos. He started a school where philosophy and religion were studied, in addition to astronomy, mathematics and music. The students from his school were called Pythagoreans. Central to Pythagoras' teaching was the idea that all physical relationships could be expressed by mathematical relations. One of the most famous discoveries of the Pythagoreans was a proof for a distance relationship that had been developed many years before by the Egyptians.

The Nile River flows through Egypt. This huge river is a source of life in an otherwise barren, desert land. During the rainy season, the Nile floods regularly. After each flood the surveyors would have to reset the boundaries of the farmers' fields. Land was sectioned into squares, so it was critical that the surveyors knew how to mark a right angle (because squares have four right angles).

The clever Egyptians took a rope and tied twelve evenly-spaced knots in it. They then made a triangle with the rope. One side had three spaces between the knots, another had four spaces between the knots, and the longest had five. This triangle was very special. The angle opposite from the longest side was always a right angle. Using this rope, the surveyors were able to show that the boundaries they marked were indeed in the shape of a square.



Many years later, the Pythagoreans named a triangle that contained a right angle, a "right triangle". They also named some of the parts of a right triangle. They called the longest side, opposite the right angle, the hypotenuse. The sides next to (or adjacent to) the right angle were called the legs.



The Pythagoreans discovered that the legs and hypotenuse of a right triangle did not always have to have lengths of 3, 4 and 5. But the numbers did have to work in a special formula. The special formula is called the Pythagorean Theorem. The Pythagorean Theorem goes like this:

If you take the length of a leg of a triangle (say "a" in the graphic above) and multiply it by itself (or "square" it);

$a \times a$ or a^2

then, do the same with the length of the other leg ("b" in the graphic above);

$b \times b$ or b^2

and add the results together;

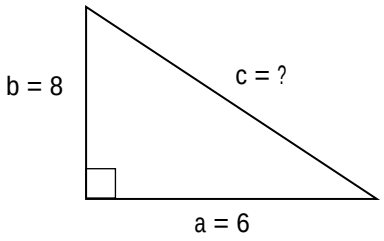
$a^2 + b^2 =$

your final result will be equal to the length of the hypotenuse ("c" in the graphic above) multiplied by itself (or "squared").

$a^2 + b^2 = c^2$

The Pythagoreans also discovered that if they knew the lengths of the two legs of a right triangle, they could use the Pythagorean Theorem to find the length of the hypotenuse.

Say that one leg of a right triangle has a length of 6 units and another has a length of 8 units. What is the length of the hypotenuse?



We know from the Pythagorean Theorem that

$a^2 + b^2 = c^2$

In our example, a = 6 and b = 8. So,

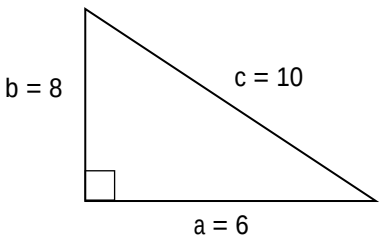
$a^2 + b^2 = c^2$

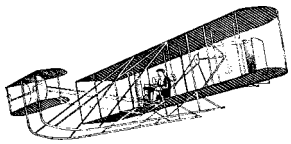
$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$100 = c^2$

Since the square root of 100 is 10 (that is, 10^2 equals 100) the length of the hypotenuse must equal 10.





Exercises

Directions: Use the information given and the Pythagorean Theorem to solve the following problems.

Problem 1: The length of leg "a" of a right triangle is 9, the length of leg "b" is 12. What is the length of the hypotenuse?

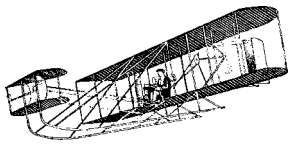
Draw the triangle. Make sure you mark the lengths of the two legs and the hypotenuse, and tell which angle is the right angle.

Problem 2: NASA Test Pilot Loren Haworth is instructed to fly the following mission in a brand new aircraft, the X-99. He will be testing the aircraft's ability to follow a flight path very precisely. Test Pilot Haworth is instructed to fly North from San Antonio, Texas to Sioux Falls, South Dakota, a distance of 1,000 miles. He is instructed to then fly east to Scranton, Pennsylvania, a distance of 1,200 miles. After reaching Scranton, he is supposed to fly directly back to San Antonio. What is the distance he must fly from Scranton to San Antonio? Hint: Draw his route and label the distances on the map below to help you find the return distance.

Sioux Falls ● Scranton ●

●

San Antonio



Problem 3: One day the Space Shuttle blasts off from the launch pad at the Kennedy Space Center in Florida. Unfortunately, a computer malfunctions and, after reaching an altitude of 4 miles, the Shuttle must return to earth and land on a runway. The runway is 5 miles away from the launch pad. How far must the Shuttle fly from its highest altitude to the runway? Hint: Draw a picture and label the mileages of the Space Shuttle's route to help you find the return distance.

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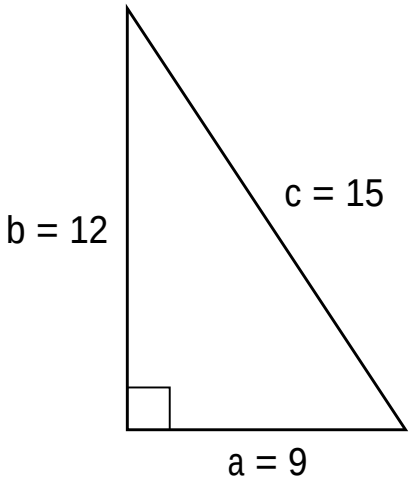
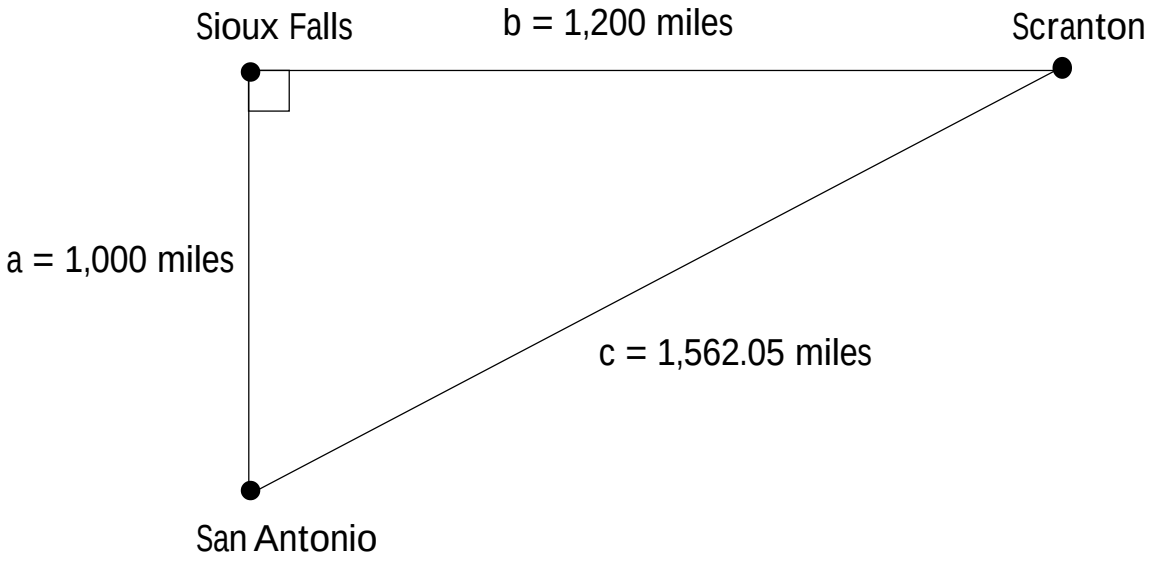
Exercises Key

1: *The length of the hypotenuse is 15.*



2: *The distance from Scranton to San Antonio is 1,562.05 miles.*



3: *The Shuttle must fly 6.4 miles from its highest altitude to the runway.*