Dispersive intensification of tsunami waves

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Abstract. A new mechanism of tsunami wave intensification is discussed. It is shown that the dispersive amplification of tsunami waves that leads to increased wave heights in some coastal areas may occur. Examples are found in water waves caused by two successive underwater volcano eruptions.

1. Introduction

During recent years multiple tsunami generation mechanisms (running displacement in the source area or sequential shock displacements of the bottom) have become of great interest due to the possibility of achieving significant wave heights and a narrow directivity diagram. The mathematical approach used to describe these mechanisms leans upon linear (or nonlinear) shallow water equations, the dispersion not being taken into account. However, when the dispersion effects are taken into account wave intensification occurs due to the interaction of the wave trains of different wavelengths propagating with different velocities. This effect can be explained as follows.

It is well known that in a dispersive medium (the free surface water waves can serve as an example of such a medium) any impulse signal spreads with time, approaching the form of a quasi-harmonic wave train with decaying amplitude. In this case both the wavelength and the amplitude of the wave diminish from the front of the wave to its tail. This problem is reversible, i.e., a quasi-harmonic signal in which the short waves run in front of the longer waves (with necessary delays between them) will transform at a definite distance into an impulse of higher amplitude (at larger distances it will transform again into a quasi-harmonic train).

So this effect can lead to a significant amplitude growth in several districts. In what follows we shall demonstrate this effect for the tsunami waves generated by two successive explosive volcano eruptions, where the first one is the weakest.

2. Evaluation of Tsunami Parameters

2.1 Source model

The theory developed for the description of explosion generated water waves has been successfully used for the description of water waves generated by ex-

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plosive volcano eruptions (Le Mehaute, 1971; Pelinovsky, 1982; Le Mehaute and Wang, 1996; Pelinovsky, 1996). At distances much larger than the wavelength, the real source can be replaced by an equivalent one. The best agreement between the experimental data and the results of the linear wave theory is achieved when using the following equivalent source from Le Mehaute (1971):

$$\eta(r,t) = \begin{cases} H_m \cdot \left[2(r/R)^2 - 1\right] & , \quad r \leq R \\ 0 & , \quad r > R \end{cases} , \tag{1}$$

where $\eta(r,t)$ is the water level displacement in the source and the parameters of the equivalent source, the maximum displacement in the source H_m and its dimension (characteristic radius) R, are defined by two parameters of the eruption, i.e., the eruption energy or the equivalent charge (yield) W and the depth at which the explosive volcano eruption occurred. Moreover, it is known that the last dependence, i.e., the dependence on explosion depth, can be ignored when the depth at which the eruption took place is of the order of the radius, R, of the equivalent source; then $H_m \approx 0.9\ W^{0.24}$, $R \approx 4\ W^{0.3}$, where W is defined in kg of TNT; H_m and R are measured in meters (Pelinovsky, 1982).

2.2 Water level evaluation

As known (Le Mehaute, 1971; Pelinovsky, 1982) the surface elevation generated by the source (1) in the approximation of stationary phase (i.e., far from the source at large time) is given by:

$$\eta(r,t) \approx \sqrt{2}\frac{H_m R}{r} J_3(k_* R) \cdot \cos\left(\frac{gt^2}{4r}\right) , \tag{2}$$

where $J_3(z)$ is the third order Bessel function and $k_* = gt^2/4r^2$.

The volcano energy is usually released in the form of successive shock pulses. Therefore, the examination of mutual effects caused by the interaction of the waves generated by several sources is of significant interest.

Let's examine the case of two successive volcano eruptions localized at the same point with time delay τ. When the first shock with an equivalent energy W_1 occurred and afterwards a much greater energy W_2 has been released in the second shock from the same point source after a time delay τ, then the surface level disturbance at time t at a large distance r from the source can be estimated as a sum of two waves of the form given by (2):

$$\eta_\Sigma(r,t) \approx \sum_{i=1}^{2} \sqrt{2}\frac{H_{m_i} R_i}{r} J_3(k_{*i} R_i) \cdot \cos\left(\frac{gt_i^2}{4r}\right) , \tag{3}$$

where $t_1 = t$, $t_2 = t - \tau$, and H_m, R_i are the maximum displacement and the radius of the ith source respectively ($R_1 < R_2$ in the case under consideration.)

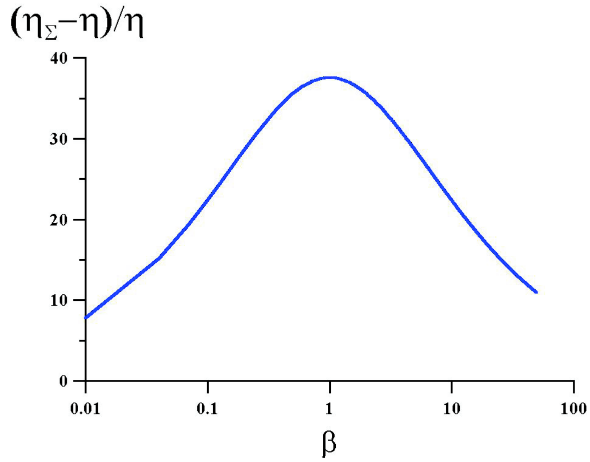

Figure 1: The relative gain as a function of $\beta = W_1/W_2$.

2.3 Maximum height time arrival

At some distance r_M the maximum distortion caused by the second (the strongest) shock will overtake and be imposed on the maximum elevation caused by the first (the weakest) shock. The distance r_M (measured from the source) where this occurs can be determined from (3) taking into account that this overlapping occurs at some moment t_M when the two wave envelopes achieve their maximal values. So we have:

$$r_M \approx 0.2\tau \frac{\sqrt{gR_1R_2}}{\sqrt{R_2} - \sqrt{R_1}} \tag{4}$$

and

$$t_M \approx \frac{4.1 r_M}{\sqrt{gR_1}} \; . \tag{5}$$

At this point and at this time the maximal disturbance in the resulting wave will exceed the maximal disturbance η in a wave generated by an explosion whose equivalent yield equals the algebraic sum of the equivalent yields of the separate shocks $W = W_1 + W_2$.

2.4 Intensification estimation

The dependence of relative amplitude gain on relative equivalent eruption yield is given by:

$$\frac{\eta_\Sigma - \eta}{\eta} = \frac{1 + \beta^{0.54}}{(1+\beta)^{0.54}} - 1 \; , \tag{6}$$

where $\beta = W_1/W_2$. This dependence is plotted in Fig. 1. As seen from Fig. 1, a significant “benefit” in amplitude can be achieved in the case of two sequential shocks in the source going one after another with time delay τ

when compared to the case of one shock having an energy equal to the sum of energies of separate shocks. For example, for $\beta = 0.02$ (i.e., $W_1 = 0.02\ W_2$) the benefit in amplitude is 10% . At the same time, the maximal elevation obtained from the source whose equivalent yield equals the sum of the equivalent yields of constituting eruptions (i.e., when the wave is generated by a shock of equivalent energy equal to $W = 1.02\ W_2$) is only 1% greater than the maximal elevation generated only by one shock of equivalent energy equal to W_2.

A more detailed illustration of the wave propagation caused by two successive eruptions can be obtained from Fig. 2, where mareograms of waves are presented: upper graph for the case of two volcano explosions with $\beta = W_1/W_2 = 0.5$ and time delay $\tau = 50$ s at $r = r_m$, where r_m is the distance where the effect begins; the bottom graph given for the case of one source of the energy equal to the sum of the energies in two sources of the upper case, i.e., for $W = W_1 + W_2$.

3. Evaluation of Dispersive Intensification Region

Estimation of the region where the effect of dispersive tsunami intensification can be observed is also of great interest. Thus, obviously, there exists a point in polar coordinates, i.e., a circular area of radius $r_{in} = \rho_1 r_m$, where the first minimum of the weak signal lays on the maximum of the second (strongest) signal. This occurs at the moment $t_{1\ \min}$ which can be calculated from the strong signal phase arrival by:

$$\begin{aligned} t_{1\ \min} &= t_{2\ \max}(r_{in}) + \tau = 4.1\left(\rho_1 - 1 + \sqrt{R_2/R_1}\right) \cdot r_M/\sqrt{gR_2} \\ &= t_M\left[1 + (\rho_1 - 1)\sqrt{R_1/R_2}\right] \ , \end{aligned} \tag{7}$$

or from the weak signal specific phase arrival by:

$$t_{1\ \min} = 2 \cdot \rho_1 \cdot r_M\sqrt{6.3/gR_1} = 1.22\rho_1 \cdot t_M \ . \tag{8}$$

Hence from (7) and (8), taking also into account the relation between the equivalent source dimension R and the equivalent eruption energy yield W given previously, we obtain:

$$\frac{r_{in}}{r_M} = \rho_1 = \frac{1 - \beta^{0.15}}{1.2 - \beta^{0.15}} \ . \tag{9}$$

Similarly, the dependence of the distance $r_{fin} = \rho_2 \cdot r_M$ can be obtained, where the effect of dispersive intensification disappears (i.e., $\rho_2 > 1$). The arrival of the first minimum of the strongest wave at the point where the first maximum of the weakest signal occurs has been taken as a criterion of the case when the strongest signal leaves behind it the weakest one.

The normalized r_{fin} dependence on β is defined by the following:

$$\frac{r_{fin}}{r_M} = \rho_2 = \frac{1 - \beta^{0.15}}{1 - 1.2 \cdot \beta^{0.15}} \ . \tag{10}$$

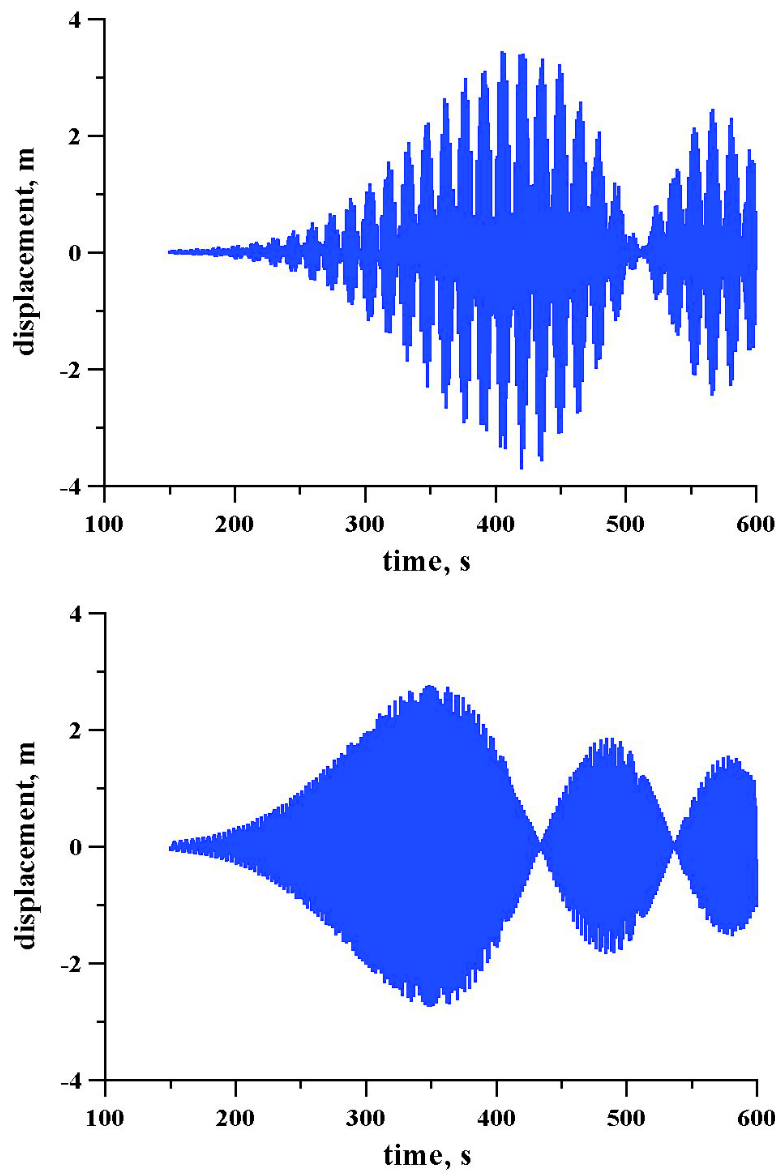

Figure 2: Mareograms of waves: upper graph for the case of two volcano explosions with $\beta = W_1/W_2 = 0.5$ and time delay $\tau = 50$ sec at $r = r_m$, where r_m is the distance where the effect begins; the bottom graph given for the case of one source of the energy equal to the sum of the energies in two sources of the upper case, i.e., for $W = W_1 + W_2$.

As seen from (10) the intensification obtained does not change significantly for $\beta > 0.3$ when the distance from the source increases.

4. Comparison of Results with Data on Volcano Tsunamis

We considered above the effect of dispersive intensification for tsunami waves in deep water. As is known, the short wave approximation (i.e., when the wave period can be estimated by $T = \sqrt{2\pi\lambda/g}$) is justified for $\lambda < 2h$ or $T \leq \sqrt{4\pi h/g}$ (where h is the water depth, λ is the wavelength). This results in $T \leq 2.5$ min for typical oceanic conditions. Such periods of tsunami waves (about 1.5 min) have been registered during the 1952–1953 Mijojin underwater volcano eruptions. At Hatijo Island, 130 km northward of the volcano, wave heights in the range of 0.2–0.9 m were measured (Soloviev and Go, 1984). For much stronger eruptions (i.e., with greater equivalent yields) the wavelength increases and the final water depth must be taken into account. In this case, the effect of dispersion intensification of the signal is also available (truly, the mutual effects of nonlinearity and dispersion being taken into account). A characteristic example of this situation is the formation of the freak wave in shallow water (Pelinovsky *et al.*, 2000). However, for shallow water the form of the equivalent source is not yet known. That is why it is possible to carry out the computations meanwhile only for the deep water case.

Thus for the water waves generated by successive volcano eruptions we have shown that the effect of dispersive intensification can take place for tsunami waves and bring an increase in wave height in several districts along the coast.

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