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Combination of CDF and DØ Results on the W Boson Mass and Width

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Abstract

The results on the direct measurements of the W-boson mass and width, based on the data collected by the Tevatron experiments CDF and DØ at Fermilab are summarised and combined. The CDF Run-0 (1988-1989) and Run-I (1992-1995) results have been re-averaged using the BLUE method and combined with Run-I DØ results and the latest published results from CDF taken from the first period of Run-II (2001-2004). The results are corrected to have consistency between the parton distribution functions and electroweak parameters. The resulting Tevatron averages for the mass and total decay width of the W boson are: $M_W = 80432 \pm 39$ MeV and $\Gamma_W = 2056 \pm 62$ MeV. The inclusion of a preliminary Run-II measurement of Γ_W from DØ gives $\Gamma_W = 2050 \pm 58$ MeV.

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1 Introduction

The experiments CDF and DØ, taking data at the Tevatron proton-antiproton collider located at the Fermi National Accelerator Laboratory, have made several direct measurements of the mass M_W and total decay width Γ_W of the W boson. These measurements use $e\nu$ and $\mu\nu$ decay modes.

Mass measurements have been published by CDF in Run-0 [1], Run-I [2, 3] and recently Run-II [4] and by DØ in Run-I [5, 6, 7]. Total decay width measurements have been published by CDF in Run-I [8, 9] and recently Run-II [10] and by DØ [11] in Run-I. In 2004 DØ presented a preliminary width result based on 177 pb^{-1} of Run-II data [12].

This note reports on the combination of these measurements. The combination takes into account the statistical and systematic uncertainties as well as the correlations between systematic uncertainties, and replaces our previous combinations [13, 14]. The measurements are combined using a program implementing a numerical χ^2 minimization as well as the analytic BLUE method [15, 16]. The two methods used are mathematically equivalent, and are also equivalent to the method used in a previous combination [13, 14]. In addition, the BLUE method yields the decomposition of the error on the average in terms of the error categories specified for the input measurements [16].

This analysis has three significant changes over previous averages:

- The individual $e\nu$ and $\mu\nu$ results for CDF Run-0, Run-Ia and Run-Ib data are now combined for each run period using the BLUE method to achieve a consistent statistical treatment across all the results. The Run-I DØ and Run-II CDF results were already internally combined using the BLUE method.
- For the mass measurements, the central values are corrected to use the same parton distribution functions (PDFs) and the same SM value for the W width. The W mass error arising from an uncertainty in the W width is thus now consistently treated across all measurements.
- For the width measurements, the values are corrected back to the same assumed W mass value to achieve consistency across all results.

The values of M_W and Γ_W quoted here correspond to a definition based on a Breit-Wigner denominator with a mass-dependent width, $|M^2 - M_W^2 + iM^2\Gamma_W/M_W|$.

2 Mass of the W Boson

The five measurements of M_W to be combined are given in Table 1. The CDF Run-0, Run-Ia and Run-Ib values are each themselves averages of two individual measurements where internal correlated systematic errors e.g. momentum scale, are accounted for in the averaging. The Run-I $D\bar{O}$ measurement combines 10 individual measurements using the BLUE method. The Run-II CDF measurement combines six individual measurements using the BLUE method. The early CDF measurements from Run-0 and Run-I were not combined using the BLUE method, instead a simpler formulation was used. These measurements were combined using only the uncorrelated errors, and then the correlated errors were added in quadrature. In the case here, where the correlated errors are small with positive correlation coefficients, this method gives a very similar result to the BLUE method but we have taken this opportunity to recombine the CDF Run-0 and Run-I results using the BLUE method to achieve a consistent statistical treatment across all the results. Firstly, the combination of $e\nu$ and $\mu\nu$ results for the Run-0, Run-Ia and Run-Ib CDF data are combined internally using the BLUE method. This changes the Run-0, Run-Ia and Run-Ib CDF M_W values by -3.5 MeV, -3.5 MeV and 0.1 MeV respectively. These BLUE corrections are listed in Table 1. When these corrected values are further combined using the BLUE method, the Run-0/I CDF combination is changed from 80433 ± 79 MeV quoted in [3] and used in previous combinations [13] to 80436 ± 81 MeV. This combination uses the same PDF corrections (see below) and width errors as used in the original CDF Run-0/1 combination.

In fitting for M_W from the measured transverse mass, charged lepton and neutrino p_T distributions a fixed value for Γ_W is used. A variety of values have been used in the five measurements. Here we have corrected the mass values so that they all use the SM prediction of $\Gamma_W = 2093 \pm 2$ MeV [17] corresponding to $M_W = 80399 \pm 25$ MeV which is the world average resulting from this latest combination. The results are corrected using the relation $\Delta M_W = -(0.15 \pm 0.05) \times \Delta \Gamma_W$ which is the average, including the variance, of the empirical shifts in M_W determined by CDF and $D\bar{O}$ when Γ_W is varied. The M_W uncertainty from the 2 MeV uncertainty [17] in the SM value of Γ_W is 0.3 MeV (which we round to 0.5 MeV); this is added in quadrature with the uncertainty in the Γ_W correction i.e. $0.05 \times \Delta \Gamma_W$ and used across all measurements. Previous quoted Γ_W errors, where they existed, are subtracted in quadrature from the total error. 9.9 MeV is subtracted in quadrature from the $D\bar{O}$ Run-I combination, and 19.9 MeV is subtracted in quadrature from the CDF Run-Ia result. 0.5 MeV is added in quadrature to the CDF Run-0, Run-Ib and Run-II combinations since these three combinations did not include an uncertainty from Γ_W . The Γ_W correction is listed in Table 1.

The CDF Run-0 and Run-Ia results were obtained from very old PDF sets (MRS-B [18] and MRSD-' [19] respectively) that did not utilise the W charge asymmetry results and so provide somewhat offset predictions from the more modern PDF sets used in the later analyses. The predictions based on the more modern MRS [20] and CTEQ [21] sets used in Run-Ib and Run-II analyses have a variance smaller than the common PDF errors assumed in these analyses ie ≈ 10 MeV. We therefore only apply PDF corrections to the CDF Run-0 and Run-Ia results

since the shifts for these data are larger than 10 MeV. Corrections of +20 MeV and -25 MeV are applied to the Run-0 and Run-Ia published results respectively. We however retain the PDF uncertainties of 60 MeV and 50 MeV quoted in the original publications. We note that these corrections were also applied in the Run-I CDF combination presented in [3]. The realisation of a common PDF uncertainty, utilising modern PDFs, will be the subject of a future study.

The M_W values and revised errors after these three corrections (Γ_W , PDF and BLUE) are listed as “ M_W (corrected)” and “Total BLUE Error (Γ_W corrected)” in Table 1. After these corrections the following experiment averages are determined:

- CDF Run-I, $M_W = 80436 \pm 81$ MeV to be compared with $M_W = 80433 \pm 79$ MeV in [3].
- CDF Run-I/II, $M_W = 80421 \pm 43$ MeV to be compared with $M_W = 80418 \pm 42$ MeV quoted in [4]. This shift is almost entirely due to the 3 MeV change resulting from the consistent use of the BLUE method across the Run-0 and Run-I CDF results.
- DØ Run-I and hence DØ average, $M_W = 80478 \pm 83$ MeV compared to $M_W = 80483 \pm 84$ MeV quoted in [7]. This change is due to the Γ_W correction.

Three systematic errors are assumed to be fully correlated between all measurements, namely: (i) parton distribution functions and parton luminosity (PDF), (ii) Electroweak radiative corrections (EWK RC), and (iii) the width of the W boson (Γ_W). Further details on the sources of systematic uncertainties are given in [13] and the individual publications of the two experiments [1, 2, 3, 4, 5, 6, 7].

The combined Tevatron value for the W-boson mass is:

$$M_W = 80432 \pm 39 \text{ MeV}, \quad (1)$$

where the total error of 39 MeV contains the following components: an uncorrelated error of 35 MeV; and correlated systematic error contributions of: parton distribution functions 13 MeV, electroweak radiative corrections 11 MeV and W-boson width 0.7 MeV, for a total correlated systematic error of 17 MeV. The global correlation matrix for the 5 measurements is shown in Table 2.

The χ^2 of this average is 2.4 for 4 degrees of freedom, corresponding to a probability of 66%, showing that all measurements are in good agreement with each other which can also be seen in Figure 1. The χ^2 with respect to the Tevatron average of the three experiment averages (CDF-I/O, DØ-I, CDF-II) shown in Figure 1 is 0.5 for 2 degrees of freedom.

A combination with the latest LEP-2 value [22] yields a world average of $M_W = 80399 \pm 25$ MeV.

	Run-0	Run-I			Run-II
	CDF	CDF-Ia	CDF-Ib	DØ	CDF
M_W published	79910	80410	80470	80483	80413
Total Error published	390	180	89	84	47.9
Γ_W used in publication	2100	2064	2096	2062	2094
Γ_W correction applied	1.1	-4.4	0.5	-4.7	0.2
PDF correction applied	20	-25	0	0	0
BLUE correction applied	-3.5	-3.5	-0.1	0	0
M_W (corrected)	79927.6	80377.1	80470.4	80478.3	80413.2
Total BLUE error (Γ_W corrected)	390.9	181.0	89.3	83.4	47.9
Uncorrelated BLUE error (Γ_W corrected)	386.1	172.8	87.9	82.1	44.7
PDF Error (published/this analysis)	60	50	15	8.1	12.6
EWK RC Error (published/this analysis)	10	20	5	12	11.6
Γ_W Error (published)	0	20	0	10	2
Γ_W Error (this analysis)	0.5	1.5	0.5	1.5	0.5

Table 1: Summary of the five measurements of M_W performed by CDF and DØ. All numbers are in MeV. The published values and the corrected values used in the average are shown. The three sources of correlated systematic error (PDF, EWK RC, Γ_W) are explicitly given.

	Run-0	Run-I			Run-II
	CDF	CDF-Ia	CDF-Ib	DØ	CDF
CDF Run-0	1.0	0.05	0.03	0.02	0.05
CDF-Ia		1.0	0.05	0.04	0.10
CDF-Ib			1.0	0.02	0.06
DØ				1.0	0.06
CDF-II					1.0

Table 2: Matrix of global correlation coefficients between the 5 measurements of Table 1.

3 Width of the W Boson

As for the mass combination, we have made two corrections to achieve consistency across all the width results. The CDF Run-Ib results have been recombined using the BLUE method

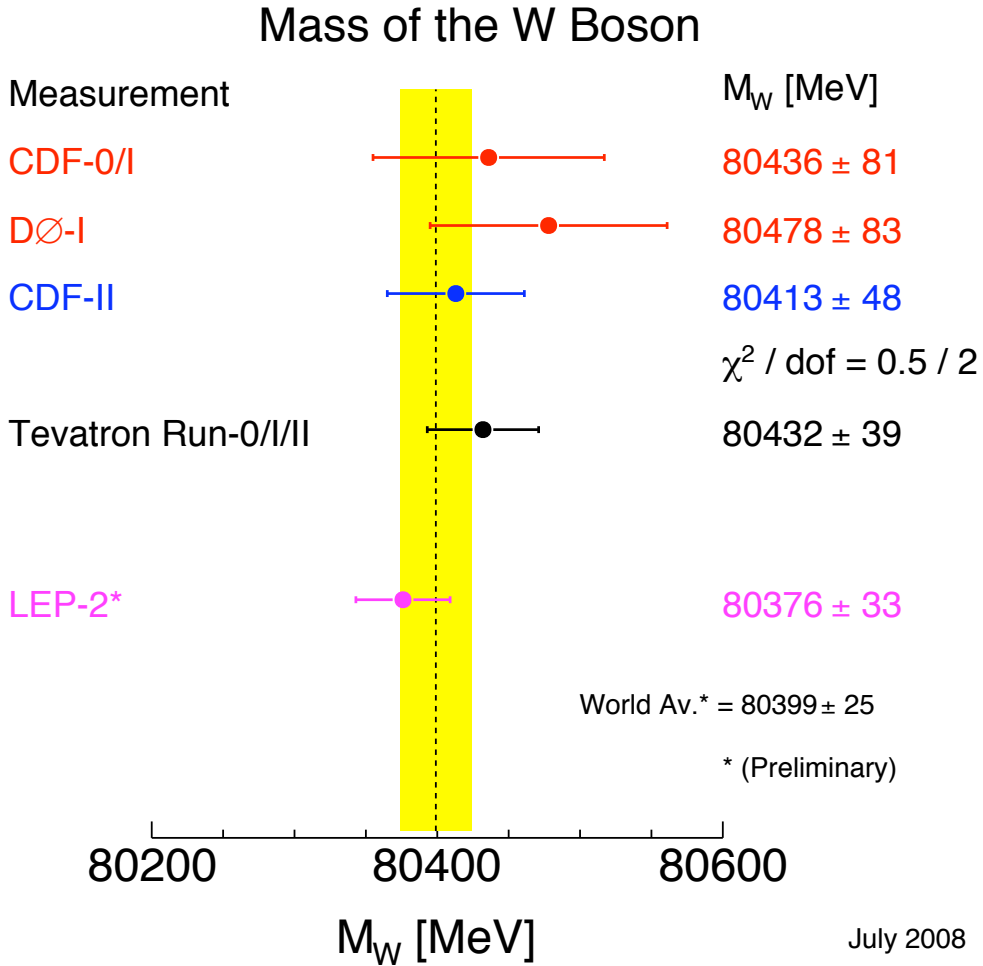


Figure 1: Comparison of the measurements of the W-boson mass and their average. The most recent preliminary result from LEP-2 [22] is also shown. The Tevatron values shown are the corrected values.

and all results have been corrected so that the values correspond to the same assumed $M_W = 80399 \pm 25$ MeV. The use of the BLUE method results in a negligible change to the CDF Run-Ib result. No PDF corrections have been applied. The results are corrected to a consistent mass value of $M_W = 80399$ MeV using the relation $\Delta\Gamma_W = -0.3\Delta M_W$. This is the CDF and DØ average of the empirically determined shift in Γ_W when M_W is varied. A common error of 9 MeV from the uncertainty in M_W is now used across all measurements and supercedes the published Γ_W error. 4.35 MeV is subtracted from the CDF-Ib results in quadrature and 12 MeV in quadrature from the DØ results. After these corrections, we obtain the following experiment averages:

- CDF Run-I, $\Gamma_W = 2041 \pm 128$ MeV

	Run-I			Run-II	
	CDF-Ia	CDF-Ib	DØ-Ib	CDF	DØ
Γ_W published	2110	2042.5	2231	2032	2011
Total Error published	329	138.3	172.8	72.4	142.1
M_W used in publication	80140	80400	80436	80403	80426
M_W correction applied	-78	0.3	11.1	1.2	8.1
Γ_W (corrected)	2032	2042.8	2242.1	2033.2	2019.1
Total BLUE error (corrected)	329.3	138.3	172.4	72.4	141.6
Uncorrelated BLUE error (Γ_W corrected)	327.6	136.8	167.4	68.7	138.7
PDF Error (published)	0	15	39	20	27
PDF Error (this analysis)	15	15	39	20	27
EWK RC (published/this analysis)	28	10	10	6	3
M_W Error (published)	0	10	15	9	15
M_W Error (this analysis)	9	9	9	9	9

Table 3: Summary of the five measurements of Γ_W performed by CDF and DØ. All numbers are in MeV. The published values and the corrected values used in the average are shown. The three sources of correlated systematic error (PDF, EWK RC, M_W) are explicitly given.

- CDF Run-I/II, $\Gamma_W = 2035 \pm 64$ MeV
- Preliminary DØ Run-I/II, $\Gamma_W = 2108 \pm 112$ MeV

The combined Tevatron value for Γ_W is:

$$\Gamma_W = 2056 \pm 62 \text{ MeV}, \quad (2)$$

from published results and $\Gamma_W = 2050 \pm 58$ MeV if the preliminary DØ result is included. The combination of the published Tevatron results has a χ^2 of 1.3 (1.4) for 3 (4) degrees of freedom, corresponding to a probability of 72% (84%) depending of whether the preliminary DØ result is included or not. All measurements are in good agreement with each other and this can also be seen in Figure 2 where the CDF-I, CDF-II, DØ-I and DØ-II averages are shown which has a χ^2 of 1.4 for 3 degrees of freedom with respect to the Tevatron average including the DØ preliminary result.

The combination of published results which has a total error of 62 MeV contains the following components: an uncorrelated error of 57 MeV; and correlated systematic error contributions of: parton distribution functions 21 MeV, electroweak radiative corrections 8 MeV, and W-boson

mass 9 MeV, for a total correlated systematic error of 24 MeV. The global correlation matrix for the 5 measurements is shown in Table 4. Figure 2

	Run-I			Run-II	
	CDF-Ia	CDF-Ib	DØ-Ib	CDF	DØ
CDF-Ia	1.0	0.01	0.02	0.02	0.01
CDF-Ib		1.0	0.03	0.04	0.03
DØ-I			1.0	0.07	0.05
CDF-II				1.0	0.06
DØ-II					1.0

Table 4: Matrix of global correlation coefficients between the 5 measurements of Table 3.

A combination with the latest LEP-2 value [22] gives a preliminary world average of $\Gamma_W = 2098 \pm 48$ MeV which is in excellent agreement with the SM prediction of $\Gamma_W = 2093 \pm 2$ MeV [17].

4 Summary

Combinations of the direct CDF and DØ measurements of the mass and total decay width of the W boson are presented. Corrections have been made to achieve a consistent treatment across all measurements and the values are corrected to consistent values for the SM parameters. The Tevatron averages of published results are: $M_W = 80432 \pm 39$ MeV and $\Gamma_W = 2056 \pm 62$ MeV or $\Gamma_W = 2050 \pm 58$ MeV if preliminary results are included.

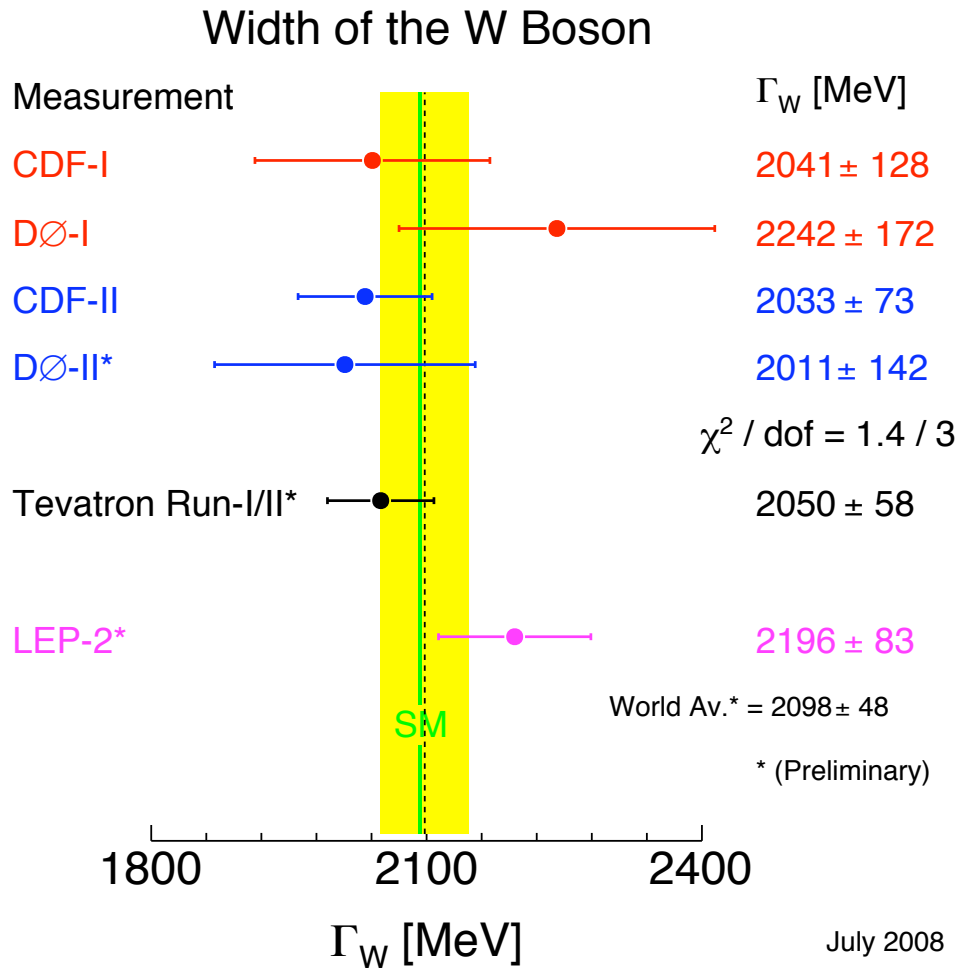


Figure 2: Comparison of the measurements of the W-boson width and their average. The most recent preliminary result from LEP-2 [22] and the Standard Model prediction are also shown. The Tevatron values shown are the corrected values.

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