# Quantum computation and quantum simulation with Coulomb crystals

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## Abstract

(I) **Quantum simulations:** ***Trapped ions*** **meet** ***Condensed Matter***

- ***Quantum spin models*** - A standing wave couples the ions' internal states to the vibrational modes in the pushing-gate scheme: it allows us to study quantum criticallity in **Ising** and **Heisenberg** models of quantum magnetism.

- ***Interacting bosons*** - Radial phonons in ion chains satisfy a ***Bose Hubbard Hamiltonian*** with tunable parameters.

- ***Spin-phonon models*** - By coupling a single ion to the vibrational modes of the crystal, a variety of models that describe **quantum dissipation in Solid State physics** can be implemented, while controlling initial states/parameters of the model.

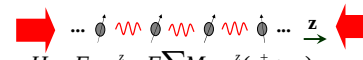
(II) **Quantum gates and simulations in Penning traps** - In Penning traps, ions arrange themselves in 2D Coulomb crystals (in a triangular lattice), where a large number of particles (N = $10^4$ -$10^6$) can be stored:

- By coupling the ions to the vibrations transverse to the crystal, one can induce *quantum gates between* nearest neighbors.

- This idea also allows us to perform **quantum simulations of frustrated magnetic systems**.

## Effective quantum spin models [1,2]

An always-on standing wave induces a state dependent force, which couples the internal states to the vibrational modes of the chain:



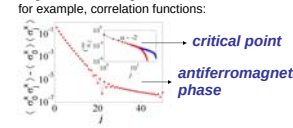
$$H_I = F z_i \sigma_i^z = F \sum_{j,q} M_{j,q} \sigma_j^z (a_q^+ + a_q)$$

This system is formally equivalent to a system of interacting spins, with an interaction that is transmitted by the collective vibrational modes. With only one standing-wave, we can implement an Ising-like Hamiltonian:

$$H_{eff} = \frac{1}{2}\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z + \frac{\Omega}{2}\sum_n \sigma_n^x$$

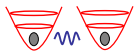
$$J_{ij} = J/|i-j|^3$$

single ion addressing allows us to measure, for example, correlation functions:




## The Phonon Hubbard Model in ion traps [1]

If the trapping energy is very large compared to the Coulomb interaction, phonons are localized at each ion, such that they are akin to atoms in a lattice: the number of phonons in an ion is equivalent to the number of atoms in a site.



$$H_0 = \sum_j \hbar\omega_0 a_j^\dagger a_j + H_{\text{Coul}}$$

The harmonic terms in the Coulomb interaction couple the position of different ions and induces tunneling of phonons between different sites:

$$H_{\text{Coul}} = \sum_{i,j} t_{i,j} x_i x_j \approx \sum_{i,j} t_{i,j}(a_i^\dagger a_j + a_i^\dagger a_j) + \sum_{i,j} t_{i,j}(a_i a_j + a_i a_j),$$

$$t_{i,j} = \frac{e^2}{d_{i,j}^3}$$

$$\text{if } \omega_0 \gg t_{i,j}$$ ***phonon non-conserving terms can be neglected (Rotating Wave Appr.)***

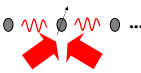
Furthermore, non-linearities in the position of the ions will induce effective phonon-phonon interactions. Nonlinear couplings appear, for example, in the interaction of the ions with a non-resonant standing wave (position dependent ac-Stark shift):

$$H_{\text{non-linear}} = \sum_n \sigma_n^z \frac{\Omega^2}{\Delta}\cos(k(x_n - x_{eq})) \approx \sum_n U\, a_n^{\dagger 2} a_n^2 + \text{shift of } \omega_n, \text{ if } \omega_0 \gg U$$

The phonon system turns out to be described by a ***Bose-Hubbard model***, and the sign and strength of the interaction can be chosen at will in the experiment.

## Spin-boson models in ion traps

When the internal state of only one ion is coupled by lasers to the vibrational modes, our system is an implementation of the celebrated spin-boson model:

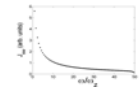


$$H = \sum_q \hbar\omega_q a_q^+ a_q + \sum_q \lambda_q \sigma^z (a_q^+ + a_q) + \frac{\Delta}{2}\sigma^x + \frac{\varepsilon}{2}\sigma^z$$

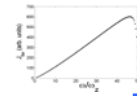
The characteristics of this model depend on the effective density of states, given by:

$$J(\omega) = |\lambda(\omega)|^2 \rho(\omega) \propto \omega^\alpha$$

In the case in which the internal state is coupled to the axial vibrational modes of the chain, we find two interesting regimes, depending on whether the coupling is induced by a standing wave (counter-propagating beams), or a travelling wave (co-propagating beams):



Standing-wave:
J(ω) = 1/ω
(Sub-ohmic effective density of states)



Travelling-wave:
J(ω) = ω
(Ohmic effective density of states)

-Describes dissipation in qubits in Josephson junctions (1/f) noise
-Not fully understood theoretically

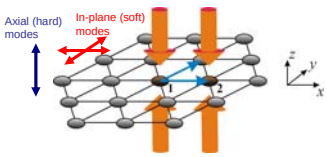
-Mapping to the Kondo effect
-Localization/Delocalization quantum phase transition as a function of the spin-phonon coupling strength
-Bath of vibrational modes is a "mesoscopic environment"

## Quantum computation, and quantum simulation with ions in Penning Traps [3]

### The idea

Penning Traps can store a huge number of ions in a 2D regular array, thus they seem to be ideally suited for scalable quantum computation and quantum simulation. However, the complicated vibrational level structure of this system raises two questions which have to be answered, to show the viability of this idea:

1) How to induce quantum gates without addressing a given vibrational mode? (use a pushing gate).
2) How important is the decoherence induced by the other vibrational modes? (Sit down and calculate it).




A state dependent force acts on two ions, and induce the following unitary evolution:

$$H_{sw}(t) = \sum_{j=1,2} F(t) Z_j \sigma_j^z$$

$$U_g = e^{-i\int dt\, J(t)\sigma_1^z\sigma_2^z}$$ (equivalent to a sign gate)

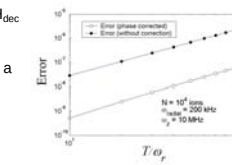
Ions are trapped by approximately harmonic potentials. The condition $\omega_z >> \omega_{x,y}$ has to be fulfilled, for the 2D structure to be stable. Temperatures of the order of T = 10 $\omega_z$ are reached in current experiments by means of Dopler cooling, but this still implies huge occupation numbers (100, 1000) in the in-plane vibrational modes.

The main mechanism of decoherence is the residual coupling between qubits, and vibrations in the x-y plane, induced by non-harmonic terms in the ion-ion Coulomb interaction:

$$H_{dec}(t) = (\sum_q F_q(t) X_q + \sum_q G_{qk}(t) X_q X_k)(\sigma_1^z - \sigma_2^z)^2$$

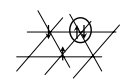
Formally, the problem consists in calculating the effect of $H_{dec}$ in the time evolution of the qubits, after tracing out the x-y vibrational modes.
Since the anharmonic terms are small, we have performed a calculation, up to fourth order in the coupling constants:

- The error is below the threshold for fault-tolerant QC.
- It can be reduced further by adjusting the gate-time.




### Simulation of quantum frustrated models

Under the effect of an axial state dependent force (always-on pushing gate acting simultaneously on all the ions), the 2D Coulomb crystal in a Penning trap implements a quantum frustrated Ising Model:



$$H_{\text{Ising}} = \frac{1}{2}\sum_{i,j} \frac{|J|}{|i-j|^3}\sigma_i^z\sigma_j^z + \frac{\Omega}{2}\sum_n \sigma_n^x$$

The implementation of this model allows us to study the fenomenology of frustrated spin models in a clean experimental system: macroscopic ground state degeneracy, "order" by desorder, etc.

[1] D. Porras and J.I. Cirac, Phys. Rev. Lett. **92**, 207901 (2004); Phys. Rev. Lett **93**, 263602 (2004).
[2] X. Deng, D. Porras and J.I. Cirac, Phys. Rev. A **72**, 063407 (2005).
[3] D. Porras and J.I. Cirac, *submitted to Phys. Rev. Lett.* (quant-ph/0601148).