Scission configurations and the spin of fission fragments

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Abstract.
After a brief presentation of the orientation pumping mechanism as a mean to generate finite average angular momenta in oriented systems, some consequences are drawn for the spin of fission fragments. Through a crude model approximation for the scission configurations, the results of microscopic calculations of fragment deformabilities are then used to deduce from the above mechanism, a distribution of fission fragment spins as a function of the total fragment excitation energy. A fair qualitative agreement with available data is demonstrated.

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1. INTRODUCTION

It has been well known for a long time from various experimental sources that fission fragments (even when resulting from the spontaneous fission of an even-even nucleus) possess quite sizeable angular momenta, typically up to about 10 $\hbar$ [1]. Recent detailed data from GAMMASPHERE experiments for the spontaneous fission of ^{252}Cf have exhibited an increase of this angular momentum with the total fragment excitation energy [2].

Fission fragment spins and related theoretical explanations

The current theoretical explanation takes stock on the thermal excitation of collective angular momentum carrying modes of which the most effective one for this purpose seems to be the bending mode [3]. Refined calculations along these lines have been performed by M. Zielinska-Pfabe and K. Dietrich years ago [4] and are still utilized [5]. This approach meets with, at least, three difficulties. First to account for the average spin values one has to resort to a vastly too high temperature (about 3 MeV) at least for reasonable collective phonon energies as given in [4]. Then it is unable to provide any explanation on the above quoted dependence [2] of the spin with the excitation energy. Finally it is inconsistent with the observed similarities between angular gamma ray distributions of both binary and ternary fissions [6] while the latter should clearly perturb the bending mode excitation with respect to what is expected in the former case.

It is the point of our approach to insist that in most cases the bulk of the fission fragment spins is due to quantal fluctuations rather than to thermal fluctuations. Actually there is more to angular momentum than a mere rotation of a matter spatial distribution, as experienced in so-called magnetic rotations [7] or intrinsic vortical modes [8] for instance. Here, we make use of the Heisenberg uncertainty principle as applied to systems whose orientation is somewhat fixed. Since, in that case, some angular information is known, one gets, as a result, a quantal distribution of the canonically conjugated variable, hence a finite average angular momentum. It is in that sense that one may say that the orientation "pumps" angular momentum [9]. This may be illustrated in the example of a quantal pendulum in the small oscillation angle θ limit, whose Hamiltonian writes [10]:

$$H = -\frac{\hbar^2}{2\mu}\left[\frac{1}{\theta}\frac{\partial}{\partial\theta}\left(\theta\,\frac{\partial}{\partial\theta}\right) + \frac{1}{\theta^2}\frac{\partial^2}{\partial\phi^2}\right] + \frac{1}{2}C\theta^2\,. \tag{1}$$

The corresponding ground state wave-function is given by $\Psi(\theta) = N_0 \exp(-\gamma\theta^2)$, where γ is proportional to the rigidity parameter C of the oscillator. The orbital momentum (l) expansion of the latter

$$\Psi(\theta) = \sum_{l=0}^{\infty} a_l Y_l^0(\theta) \tag{2}$$

exhibits a weighted gaussian distribution in l for large l values with respect to $\sqrt{\gamma}$

$$a_l \sim \left(l + \frac{1}{2}\right) \exp\left(-\frac{(l+1/2)^2}{4\gamma}\right)\,. \tag{3}$$

This is an example of this angular moment pumping for such a system constrained to move in a restricted θ angular domain.

Orientation pumping mechanism

Let us come back now to our fission context and assume that the scission configuration is reasonably well described as a product of two separated wave-functions, as BCS wave-functions. Upon projecting on good angular momentum each of these wave-functions, and coupling them to a total zero angular momentum (we assume that we describe here the spontaneous fission of an even-even nucleus and that the relative angular momentum is vanishing so as to minimize any other sources of angular moment generation in the fragments but the orientation pumping) we have obtained in [9] for a pure rotor distribution of projected energies that

$$\langle J^2 \rangle = \left(\frac{1}{\langle J_1^2\rangle_{\text{intr}}} + \frac{1}{\langle J_2^2\rangle_{\text{intr}}}\right)^{-1} \tag{4}$$

where $\langle J_i^2\rangle_{\text{intr}}$ stands for the intrinsic expectation value of the $\hat{\mathbf{J}}^2$ operator computed for the wave-function of fragment i. It is to be noted that this value is i-independent as it should for angular momentum conservation reasons. Furthermore if one of the fragment is spherical then this formula yields vanishing fragment spins. However we clearly reach one limit of our model assumptions, namely the rotational character of the projected spectra which should not be valid in this case.

2. SEMI-MICROSOPIC DESCRIPTION OF SCISSION CONFIGURATIONS

Let us consider a nucleus undergoing a fission process all along which axially symmetrical shapes are assumed. We decompose the total energy of the fissioning nuclear system in the following way:

$$E_{\mathrm{tot}}(Q_{20}^{(1)},Q_{20}^{(2)},D) = \sum_{i=1}^{2} E_i(Q_{20}^{(i)}) + E_{\mathrm{mut}}^{(Coul)}(Q_{20}^{(1)},Q_{20}^{(2)},D) + E_{\mathrm{mut}}^{(nucl)}(Q_{20}^{(1)},Q_{20}^{(2)},D) \quad (5)$$

where $Q_{20}^{(i)}$ denotes the axial quadrupole moment of the fragment i – chosen to represent the elongation of the fragment – and D is the distance between the fragments centers of mass. With obvious notations, $E_{\mathrm{mut}}^{(Coul)}$ and $E_{\mathrm{mut}}^{(nucl)}$ stand for the mutual Coulombian and nuclear interaction energies, respectively, whereas E_i denotes the (deformation) energy of the fragment i. In the expression (5) of E_{tot}, it is assumed that E_i depends only on $Q_{20}^{(i)}$, which means that the polarization of both fragments would potentially be only considered through this quadrupole mode only. In the present preliminary crude study, however, we go as far as to neglect any polarization altogether by computing E_i, for given $Q_{20}^{(i)}$ values corresponding to fully separated nuclei, in the Skyrme–Hartree–Fock plus BCS approach used in [11] for fission barriers calculations and in another contribution to this workshop [12]. In the calculation of E_i, the pairing force parameters for the corresponding nucleus and the factor $1/A_i$ in the one-body term of the center of mass correction are used. Moreover, we have assumed axial and left-right symmetries. The latter hypothesis sounds reasonable since the ground state shapes of the considered nuclei, namely ^{106}Mo and ^{146}Ba, do not exhibit any energetically significant octupole distortions [13] and that we neglect any such polarization effet which would arise essentially from the Coulomb repulsion of the fission partner. The deformation energy curves $E_i(Q_{20}^{(i)})$ obtained for the ^{106}Mo and ^{146}Ba isotopes are displayed in Fig. 1.

To arrive at a quantitative definition of scission, we start from the idea that, at scission, the nuclear interaction acting between two nucleons whose wave functions are localized in different fragments becomes negligible. Moreover we consider that a scission configuration should be accessible from the initial state, chosen to be the ground state of the fissioning nucleus since we are interested in spontaneous fission. This means that the total energy of such a configuration should be lower than the ground state one. Any

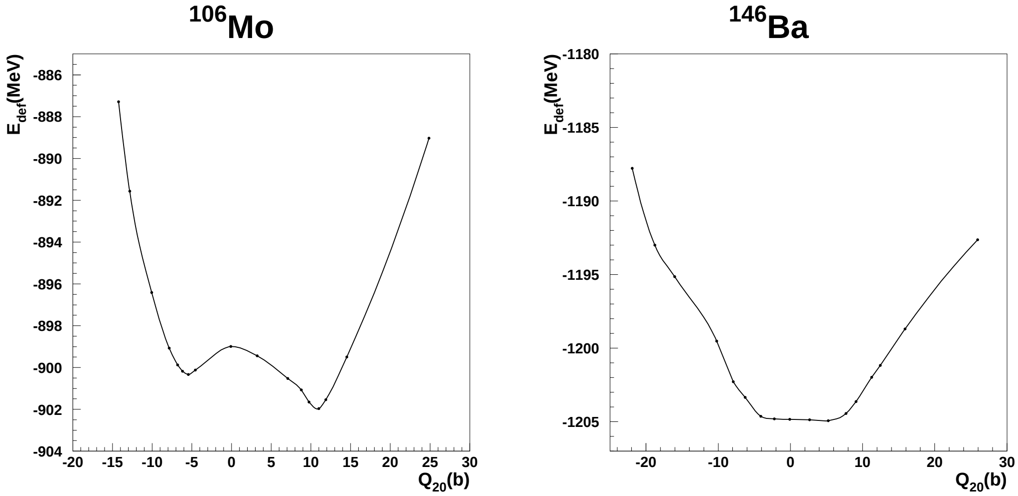

FIGURE 1. Deformation energy curves of the ^{106}Mo (left) and ^{146}Ba (right) nuclei.

scission configuration should then satisfy the following two relations:

$$\left| \frac{E_{\rm mut}^{(nucl)}}{E_{\rm mut}^{(Coul)}} \right| = \varepsilon \ll 1 \tag{6}$$

$$E_{\rm tot}^{(sc)} \leqslant E_{\rm tot}^{(GS)} . \tag{7}$$

In the above Eq. (6), ε is chosen to be 1%. This quantitative condition for scission is of course somewhat arbitrary. Eqs. (6) and (7) lead to a set of solutions $(Q_{20}^{(1)}, Q_{20}^{(2)}, D)$ where D is a function of $(Q_{20}^{(1)}, Q_{20}^{(2)})$ which both vary in limited ranges because of (7).

As a first approach to the above problem, we approximate $E_{\rm mut}^{(Coul)}$ by the expression given in Ref. [14] obtained by likening the fragments to homogeneously charged, spheroidal droplets with collinear symmetry axes – namely the fission direction:

$$E_{\rm mut}^{(Coul)}(Q_{20}^{(1)}, Q_{20}^{(2)}, D) = \frac{Z_1 Z_2 e^2}{D} S(x_1, x_2) \tag{8}$$

where the eccentricitiy-like variable x_i is defined through its square:

$$x_i^2 = \frac{c_i^2 - a_i^2}{D^2} \tag{9}$$

and the (dimensionless) function S, expressing the departure from two spherical fragments, takes the form:

$$\begin{aligned} S(x,y) = \frac{3}{40} \Bigg[& \frac{1 + 11(x^2 + y^2)}{x^2 y^2} + P_x P_y \left(\frac{(1+x+y)^3}{x^3 y^3} \times \right. \\ & \left. \ln(1+x+y) \left(1 - 3(x+y) + 12xy - 4(x^2+y^2)\right) \right) \Bigg] \end{aligned} \tag{10}$$

in which $P_x[f(x)]$ means the even part of the function $f(x)$. In Eq. (9) c_i and a_i denote the semi-axes along the axial symmetry – corresponding to the fission direction – and in the perpendicular direction of the fragment i, respectively. As can be easily seen, prolate shapes ($c_i > a_i$) lead to real x_i-values, whereas x_i is purely imaginary for oblate shapes ($c_i < a_i$). As expected, we have $S(0,0) = 1$ for two spherical shapes. As for the attractive nuclear interaction between both fragments we chose the proximity potential of Blocki and Swiatecki [15, 16] expressed as, with their notations:

$$E_{\mathrm{mut}}^{(nucl)} = 4\pi\bar{R}\gamma b\,\Phi(\zeta)\,. \tag{11}$$

In this expression, $\bar{R}$ is a kind of mean curvature radius and has been calculated exactly for two spheroids from Eq. (4) of Ref. [15] as:

$$\bar{R} = \left(\frac{a_1}{c_1^2} + \frac{a_2}{c_2^2}\right)^{-1} . \tag{12}$$

The values of the γ and b parameters of Eq. (11) have been taken from Ref. [15], whereas the parametrisation of the universal proximity fucntion Φ is the one from Ref. [16].

It is worth mentioning that the above modelisation of scission configurations is formarly similar to the one sketched in Ref. [17]. An important difference lies in the way D is treated (or equivalently, the tip distance d). In the present model, the condition (6) leads to a distribution of D-values, thence a distribution of tip distances, whereas in the work of Ref. [17] a particular d-value has been considered.

3. FISSION FRAGMENT ANGULAR MOMENTA

In the above described "orientation pumping" mechanism and in the absence of any other source of angular momentum, both fragments have the same spin J_{frag} defined as:

$$J_{\mathrm{frag}}(J_{\mathrm{frag}}+1) = \langle J^2 \rangle \tag{13}$$

where $\langle J^2 \rangle$ is calculated within the crude approximation of Eq. (4). In the same spirit as what has been done for the energy E_i of the fragment i in Sect. 2, we have neglected the polarization effects in the calculation of $\langle J^2 \rangle_{\mathrm{intr}}^{(i)}$ entering Eq. (4) and used the BCS wave-function of the isolated nucleus i to compute the expectation value of the $\hat{J}^2$ operator. This has been done for a number deformations $Q_{20}^{(i)}$, for both fragments. The resulting J_{frag}-value has been obtained as a function of $(Q_{20}^{(1)}, Q_{20}^{(2)})$ and plotted as a contour map in Fig. 2.

In view of calculating the average J_{frag}-value as a function of the fragment deformation energy $E_{\mathrm{def.\,frag}}$ defined by:

$$E_{\mathrm{def.\,frag}} = \sum_{i=1}^{2}\left(E_i - E_i^{(GS)}\right), \tag{14}$$

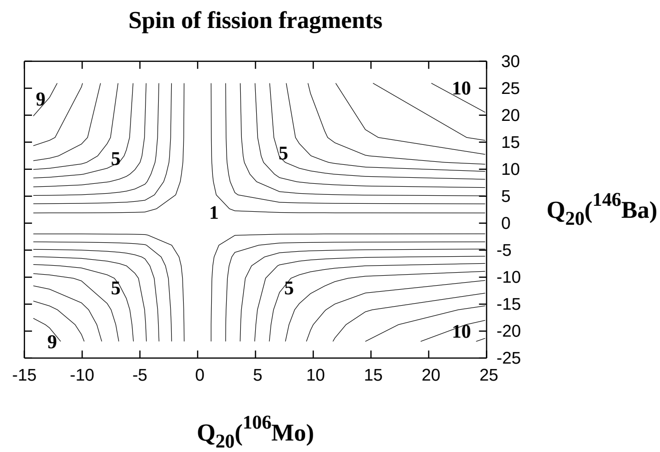

FIGURE 2. Contour map of $J_{\rm frag}$ as a function of $\left(Q_{20}^{(1)}, Q_{20}^{(2)}\right)$ for the fragmentation ^{106}Mo+^{146}Ba.

we need to assign a weight to each configuration satisfying the two scission conditions (6) and (7), and corresponding to a fixed $E_{\rm def.\,frag}$-value. Before further discussing this, it is worth noting that $E_{\rm def.\,frag}$ depends only on the fragment deformations $Q_{20}^{(i)}$. Lines of equal total deformation energies in the $\left(Q_{20}^{(1)}, Q_{20}^{(2)}\right)$ plane are reported in Figure 3 for configurations satisfying the scission conditions (6) and (7). The distribution over

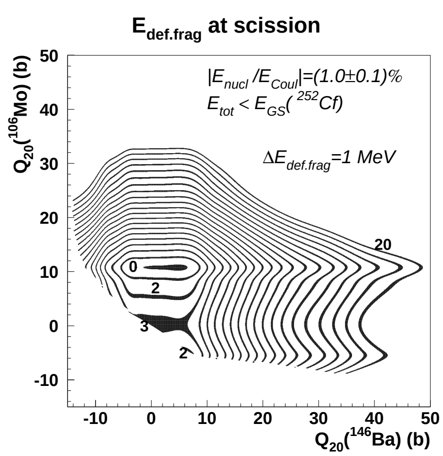

FIGURE 3. Contour lines of equal $E_{\rm def.\,frag}$-values from 0 to 20 MeV for ^{106}Mo+^{146}Ba scission configurations.

various scission configurations having the same total deformation energy should result ultimately from an appropriate quantal calculation for the collective motion. To mock it up, in the present preliminary stage, we resort rather to a “Boltzman” weight $e^{-E_{\rm tot}/T}$ where T should be a priori of the order of 1 MeV (value chosen here). This enables to compute the mean $J_{\rm frag}$-value for a given $E_{\rm def.\,frag}$-value as:

$$\langle J_{\rm frag}\rangle = \frac{1}{N}\sum_{Q_{20}^{(1)},Q_{20}^{(2)}} e^{-E_{\rm tot}/T} J_{\rm frag}(Q_{20}^{(1)},Q_{20}^{(2)}) \tag{15}$$

where we have discretized the $(Q_{20}^{(1)},Q_{20}^{(2)},D)$ space and used the scission condition (6) to deduce D for each $(Q_{20}^{(1)},Q_{20}^{(2)})$ pair. The constant N is a normalization coefficient, equivalent to a partition function:

$$N = \sum_{Q_{20}^{(1)},Q_{20}^{(2)}} e^{-E_{\rm tot}/T}\,. \tag{16}$$

Note that even though the above simulation of the spreading among scission configurations seem to refer to the approach of Wilkins and collaborators [18], it is quite different in spirit. We are not resorting here to a thermal distribution but to a quantal fluctuation. The latter yields a Gaussian distribution provided that, at a given $E_{\rm def.\,frag}$, one makes a harmonic approximation around the minimum of $E_{\rm tot}$. Upon varying $E_{\rm def.\,frag}$, we have obtained an increasing trend for $J_{\rm frag}$ as shown in Fig. 4 where the reported error bars

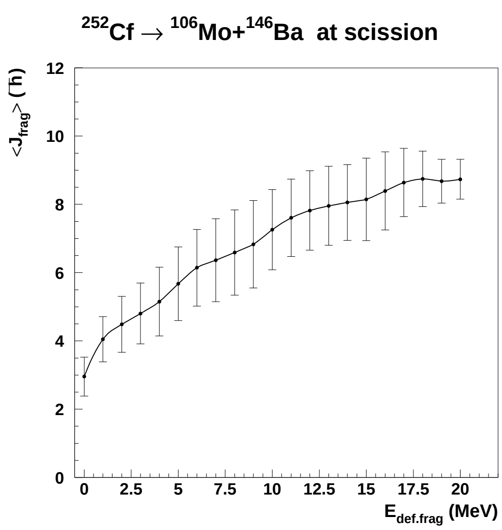

FIGURE 4. Variation of the average angular momentum $\langle J_{\rm frag}\rangle$ at scission with the total fragment deformation energy $E_{\rm def.\,frag}$ for the ^{106}Mo+^{146}Ba fragmentation.

are calculated as the variance of the corresponding spin distribution:

$$\Delta J_{\rm frag} = \sqrt{\langle (J_{\rm frag} - \langle J_{\rm frag}\rangle)^2\rangle}\,. \tag{17}$$

Assuming that the total excitation energy of fission fragments is stored in deformation, which sounds reasonable at low TXE-values, we can deduce from Fig. 4 that the mean fission fragment spin increases with TXE, which is compatible with the similar experimental trend reported in Ref. [2]. Interestingly, our average $J_{\rm frag}$-value obtained for cold fission events (no neutrons emitted) is finite and very close to the experimental one. It seems however that our spin values might rise somewhat too fast as a function of TXE. Nevertheless one should keep in mind the very crude approximations made here with respect to the fragment deformations, the angular momentum projection properties of their intrinsic wave-function descriptions and the over-simplified definition of TXE.

4. CONCLUSION

We have proposed a quantitative criterion for scission configurations (in terms of the nuclear and Coulomb mutual energies) and implemented the orientation pumping mechanism in a semi-microscopic scission point model. Upon identifying the total excitation energy of the fragments with their deformation energy, we have finally shown that the orientation pumping mechanism is able not only to account for the order of magnitude of the fission fragment spins, but also to reproduce the experimental increasing trend of the average fragment spin as a function of TXE.

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