Precise Measurements of Small Linear and Angular Displacements with Capacitance Methods

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- Motivation: EDM G-2 Upgrade (by Gerald Bennet)
- Types of Displacement and Angle Sensors
- Advantages of Capacitive Displacement Sensors
- Readout Methods
- Limits to Sensitivity: Electrical and Mechanical Noise
- MEMS/Microelectronics Applications: Accelerometers

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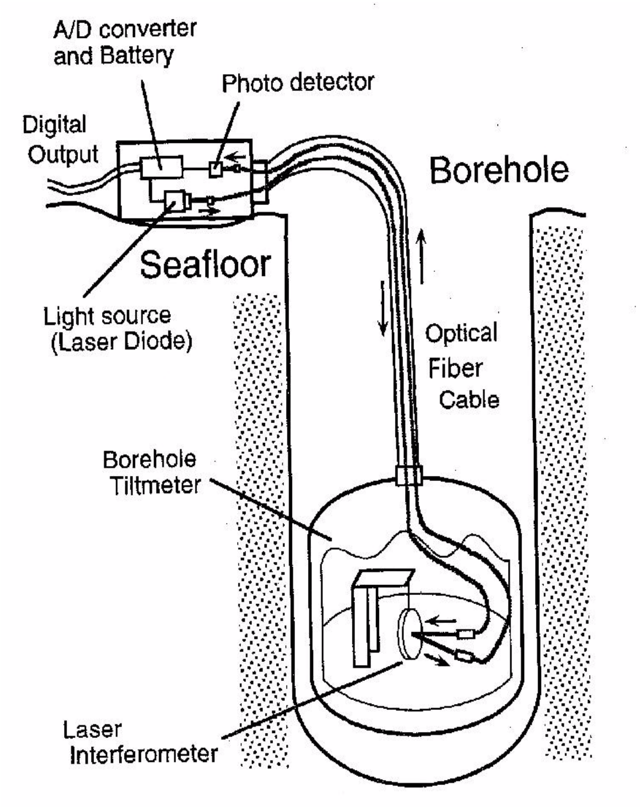
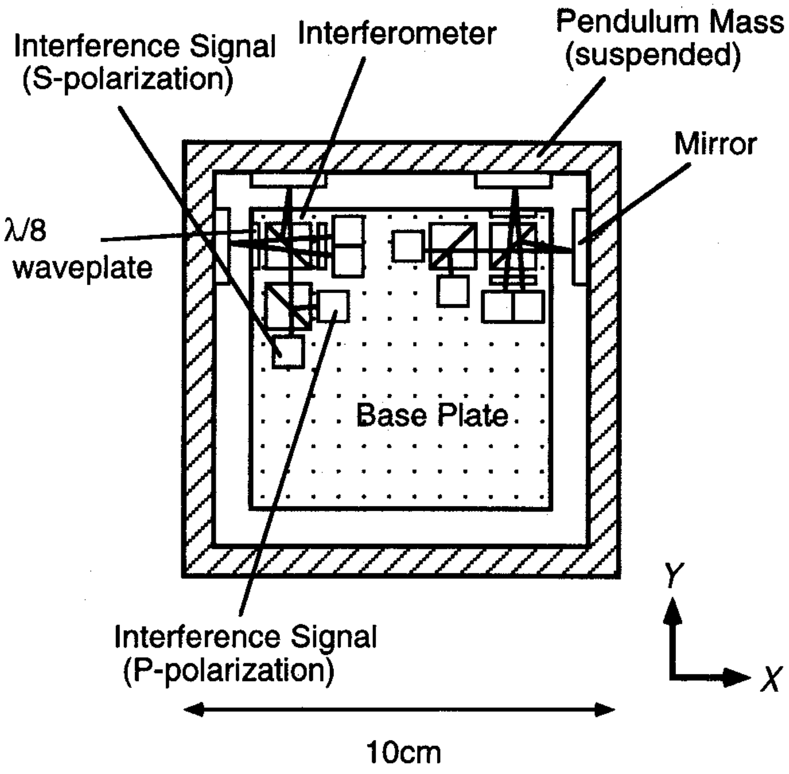
P. Rehak

Types of Displacement and Angle Sensors

1: Interferometric Sensor

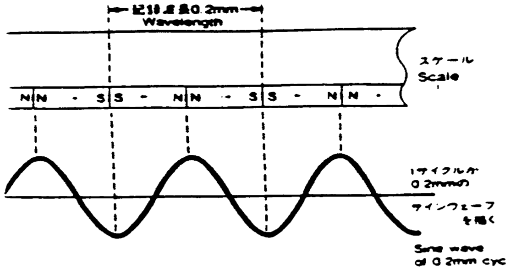


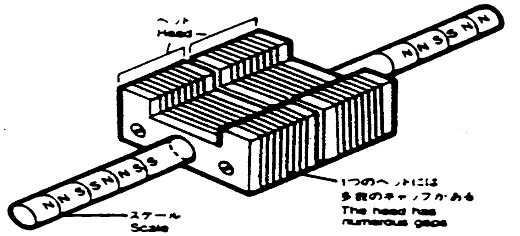


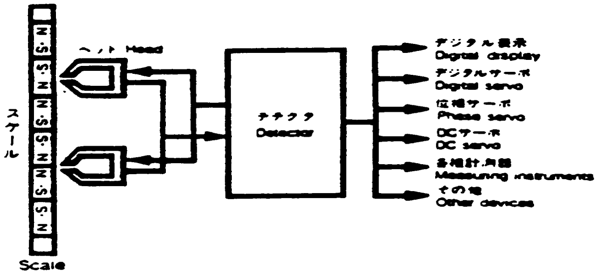
- **interferometric technique: optical wavelength-scale resolution**
- **Measures relative movements: laser light must be on all the time unless the reference point is lost**

Types of Displacement and Angle Sensors

2: Interpolating Sensors (e.g SONY MAGNESCALE)





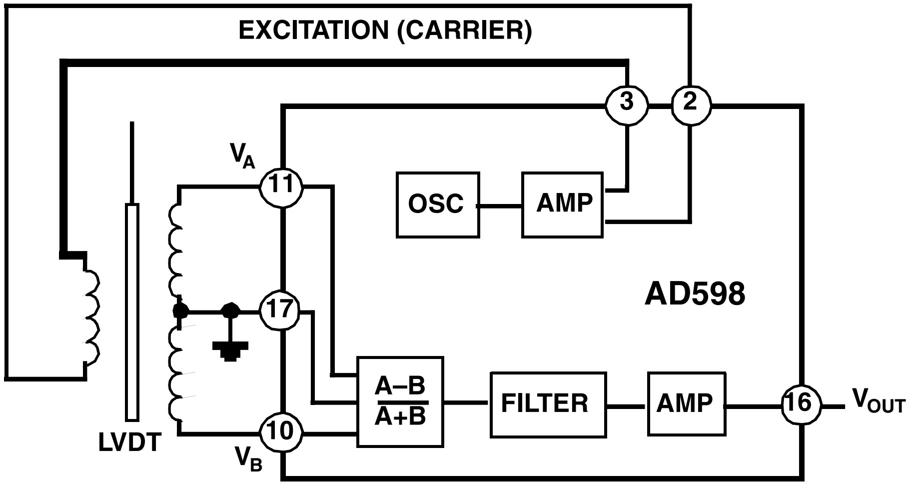


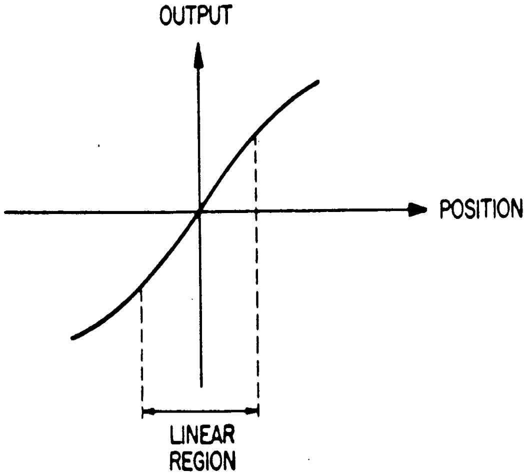
- **Uses spatial averaging: reduced differential non-linearity**
- **Large Dynamic range**
- **Integral non-linearity depends on accumulated error in N-S magnetization locations**

Types of Displacement and Angle Sensors (cont)

LVDT: Linear Variable Differential Transformer

A ferromagnetic core moves with respect to two opposite windings, changing the coupled flux.



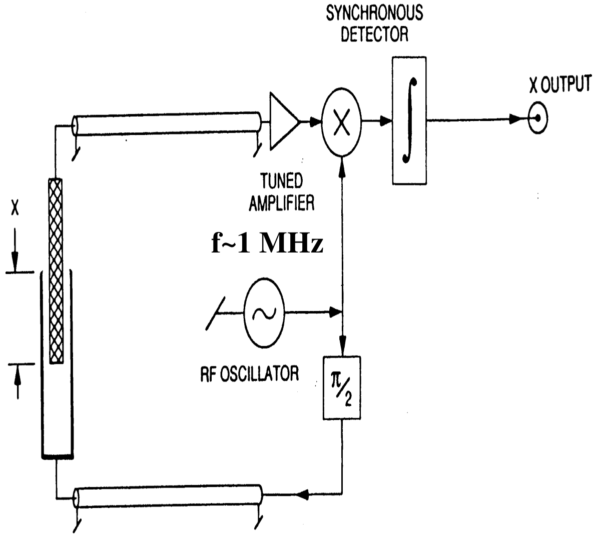


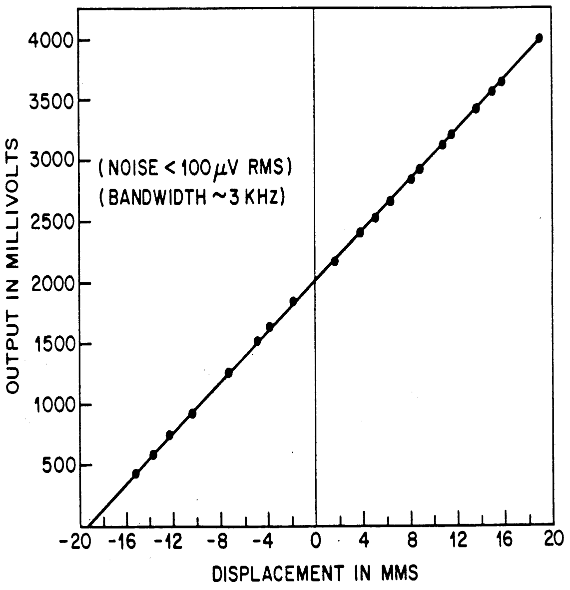
- S-shaped characteristics
- Bulky
- Slow response (uses low frequency)
- Sensitive to magnetic fields
- It is the most widely used displacement sensor

Types of Displacement and Angle Sensors (cont)

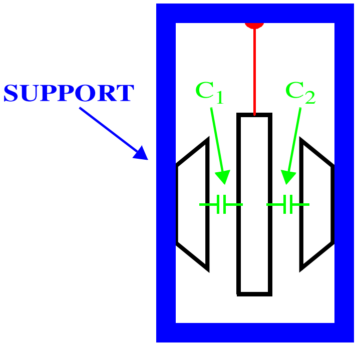
CAPACITIVE SENSORS

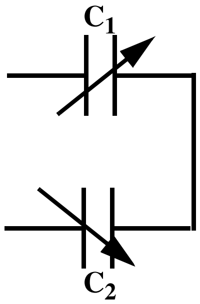
Piston type variable Capacitor





Differential Capacitors





Advantages of Capacitive Sensors

$$C = \varepsilon \cdot \frac{A}{d}$$

Vary A => better linearity

Vary d => better sensitivity over small displacement

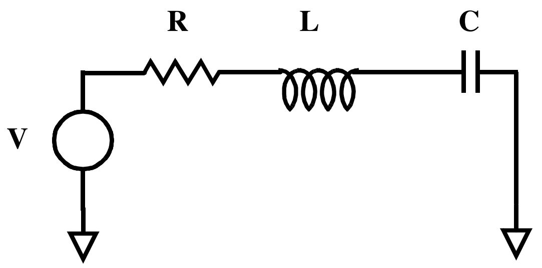
- **Excellent linearity over entire dynamic range when Area is changed (since stray electric fields are small)**
- **The system responds to average displacement of a large area of a moving electrode**
- **Freedom of electrode materials and geometry for demanding environments and applications**
- **Fractional change in capacitance can be made large**
- **Capacitive sensors can be made to respond to displacements in one direction only**
- **The forces exerted by the measuring apparatus are electrostatic, and usually small enough so that they can be disregarded**
- **Capacitors are noiseless: excellent S/N ratio can be obtained (or their dissipation factor D is large enough that the dominant noise sources are elsewhere)**

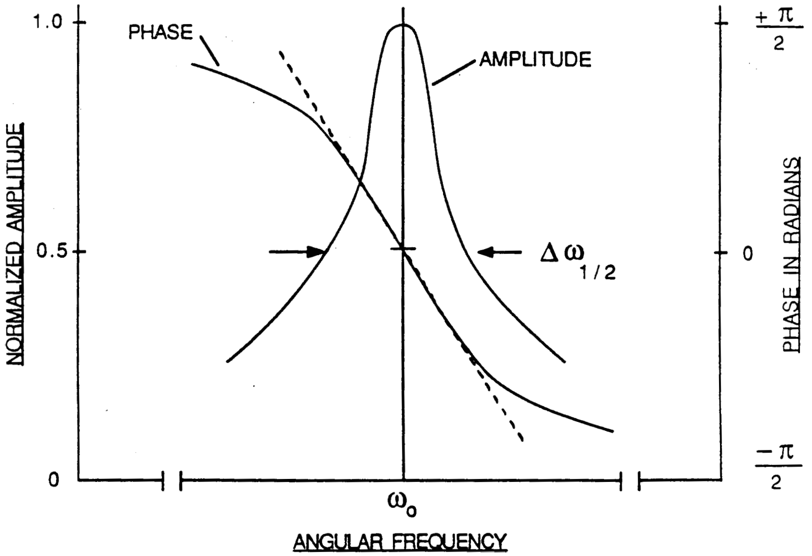
Types of Readout

Two major categories:

1. Readout based on Resonance
2. Readout based on Bridge method

Readout based on Resonance





1. Measure frequency change of an oscillator built around the variable capacitor
1. Excite at resonance, measure amplitude change
2. Excite at resonance, measure phase change
3. Use feedback loop and VCO oscillator to track resonance change

Readout based on Resonance (cont)

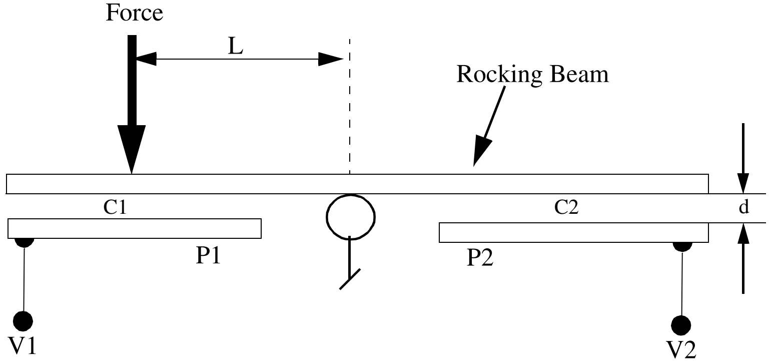
Advantages

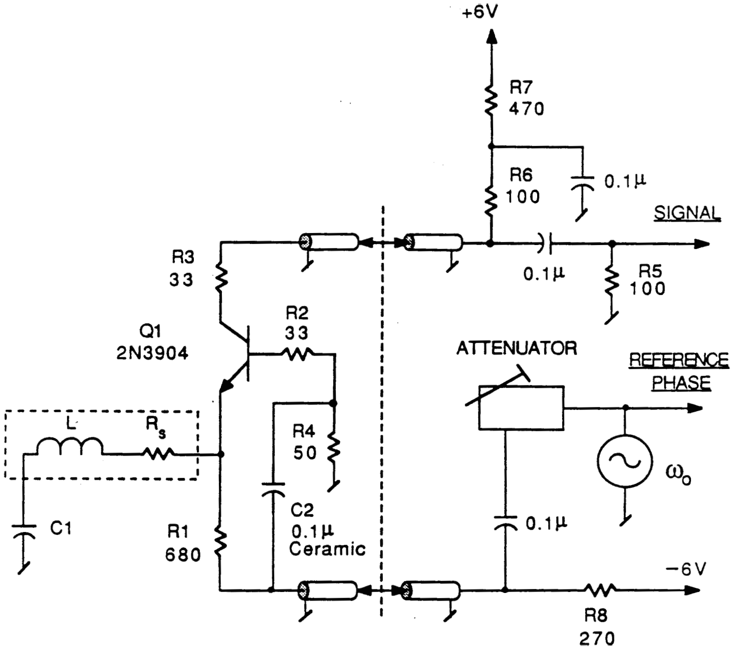
- **Makes use of the high Q of a resonant circuit: does not require a low noise preamplifier**
- **Sensitive**
- **Simple and straightforward**

Disadvantages

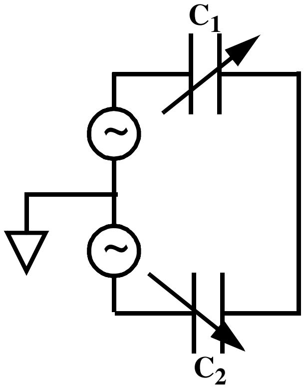
- **All resonating elements are created equal: it cannot distinguish a change in L from a change in C. The overall stability depends on the stability of BOTH L and C, with different temperature coefficients and stray effects.**
- **Cannot take advantage of differential capacitance change**

Readout Based on Resonance: Example
Rocking Beam Balancing in Atomic Force Microscope (G. L. Miller)





Types of Readout: AC Bridge



C_1 - C_2 = ε A/(δx - x_0) - ε A/(δx + x_0) = 2 C_0 δx/x_0

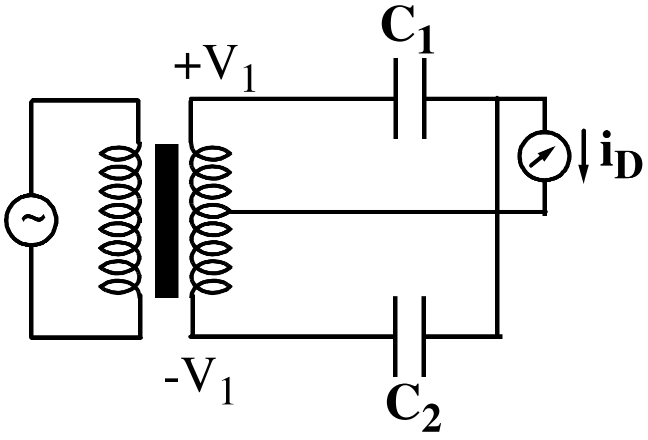
A=400 mm^2 x_0 = 75 μm C_1 = C2 = 50 pF

l = 30 mm   δθ= 1 nrad   δx = δθ l = 30 10^{-12} m

δC = C_1 - C_2 = 40 aF

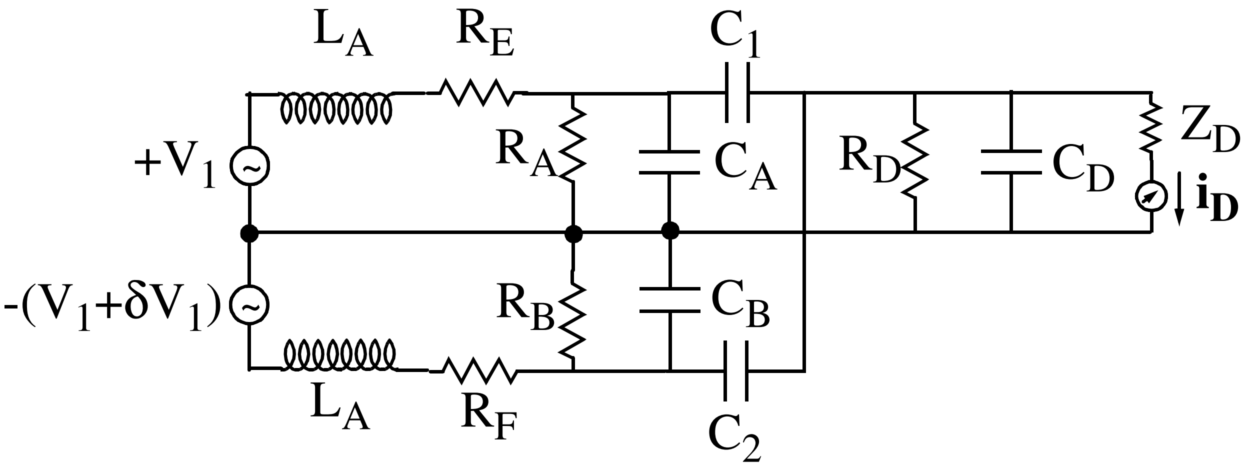
$$Force = \frac{1}{2} \cdot \frac{C_1}{x_0} \cdot V_1^2$$

F = 0 for a symmetric system



i_D = V1 ω (C_1 -C_2)

Types of Readout: AC Bridge
Readout Equivalent Circuit

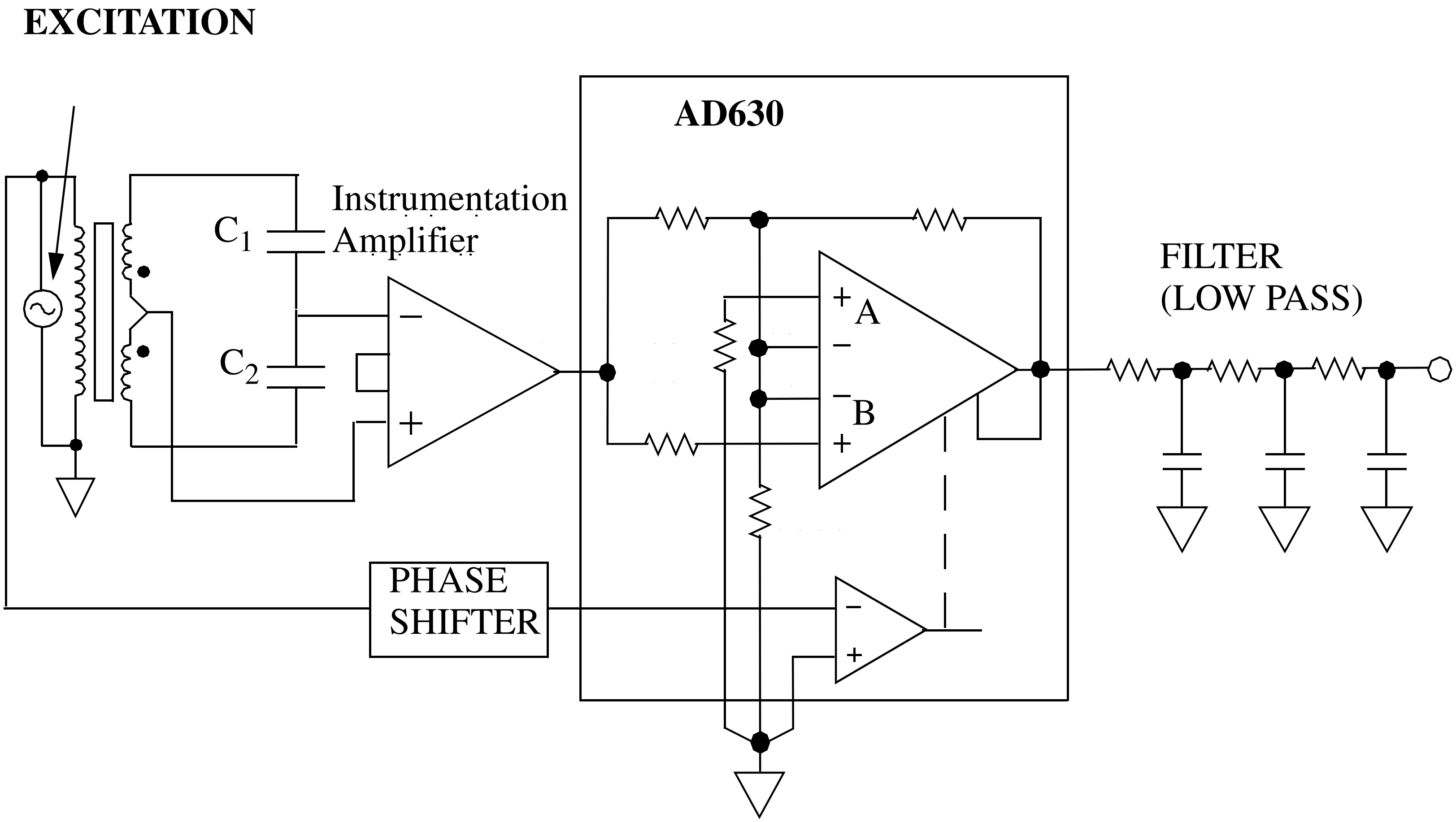


For zero output from the capacitance balance detector must be:

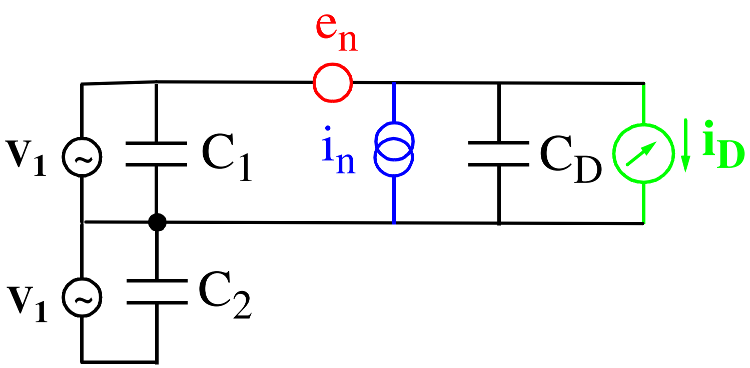
$$\frac{C_1}{C_2} = 1 + L_B(C_2 + C_B)\omega^2 - L_A(C_1 + C_A)\omega^2 + \frac{R_E}{R_A} - \frac{R_F}{R_B} + \frac{\delta V_1}{V_1}$$

L_A, L_B	0.4 mH	ratio transformer Leakage inductance
R_E, R_F	<1 Ω	transformer secondary DC resistance
R_A, R_B	>10 MΩ	Dielectric losses
C_A, C_B	~ 1000 pF	Stray capacitance to “ground” (cables, interwinding C)
R_D, C_D		Impedance shunting the preamplifier. Does not affect balance
Z_D	low	Current preamplifier input impedance

Readout Circuit



AC Bridge: Noise



$$\overline{i_{D,n}^2} = \left(\overline{i_n^2} + \frac{\overline{e_n^2}}{1/\omega_0^2 C_T^2}\right) BW$$

$$C_T = C_1 + C_2 + C_D$$

The “equivalent noise capacitance” can be calculated as:

$$\left(\overline{i_n^2} + \frac{e_n^2}{1/\omega_0^2 C_T^2}\right) BW = \frac{V_1^2}{1/(\omega_0{}^2 C_n^2)}$$

so that we have:

$$\overline{C_n^2} = \frac{1}{V_1^2}\left(\overline{e_n^2} C_T^2 + \frac{\overline{i_n^2}}{\omega_0{}^2}\right) BW$$

AC Bridge: Noise (cont)

To reduce noise:

- **Increase V_1**
- **Decrease Bandwidth (i.e. increase averaging time)**
- **Decrease C_T (depends mostly on connection length, strays etc.)**

Example: Tiltmeter

$$\left.\begin{array}{l} C_T = 1\ \mathrm{nF} \\ e_n = 1\ \mathrm{nV/Hz}^{1/2} \\ V_1 = 3\ \mathrm{V} \end{array}\right\} C_n = 4\ 10^{-19}\ \mathrm{F} = 400\ \mathrm{zF}$$

Minimum Signal (for 1 nrad angular displacement) = 40 aF= 20 10^{-18}F => S/N=100

Example: Monolithic MEMS circuit

$$\left.\begin{array}{l} C_T = 1\ \mathrm{pF} \\ e_n = 10\ \mathrm{nV/Hz}^{1/2} \\ V_1 = 5\ \mathrm{V} \end{array}\right\} C_n = 2\ 10^{-21}\ \mathrm{F} = 2\ \mathrm{zF}$$

Mechanical Noise

The mechanical rms fluctuation can be computed by means of the fluctuation dissipation theorem.
The rms fluctuation of the displacement of a suspended mass m is:

$$\overline{\delta x^2} \cong \frac{k_B T}{m \omega_0^2 \tau S}$$

Where:

k_B = 1.38 10^{23} J/K

m ~ 20 g

ω_0 = mechanical resonant frequency ~10^5 rad/sec

τ = damping time constant ~ 5-10 sec

S = Averaging period ~ 1 sec

So that $(\delta x^2)^{(1/2)}$ ~ 10^{-17} m giving a “noise” too small to be detected.

Absolute Limits:

Since the damping could be increased (e.g. using more dense fluid to increase friction) there is no fundamental limit to the mechanically generated noise

Other Sources of Errors

- **Temperature variations**

  Thermal expansion => cancelled in a symmetrical design

  Effects on the readout electronics: Gain Variation

  temperature dependence of dielectric constant (2 10^{-6}/°C for dry air at STP;

  700 10^{-6} for moist air

- **Humidity variations**

  at 20°C a change in humidity from 40 to 90% changes the dielectric constant by 200 ppm

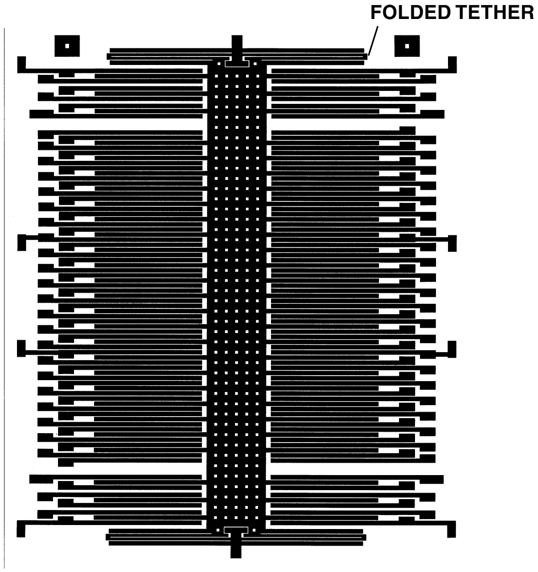
- **Pressure changes**

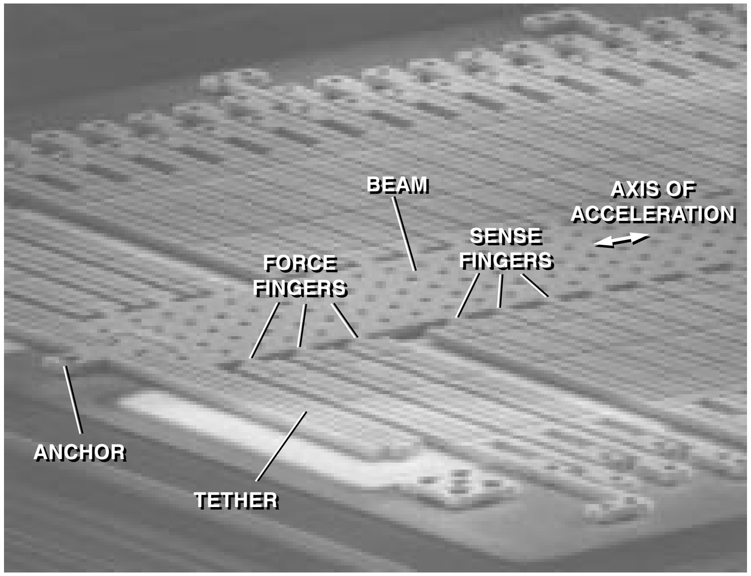
  a pressure change of 1 atm at 20°C changes the dielectric constant by 200 ppm

  causes dimensional changes (a brass cube of 1 cm contracts by 3 µm for a 1 atm change)

- **Oxidation of surfaces (rodium plating recommended)**
- **Stability of materials (70-30 brass gives good results)**
- **Creep of materials**
- **Relaxation of screw tension**
- **Microseismicity (about 2 µm peak to peak displacement, period 3-8 s)**

MEMS Accelerometers





It consist of multiple fingers on each side of a movable center member.

They constitute the center plates of a paralleled set of differential capacitors. Pairs of fixed fingers attached to the substrate interleave with the beam fingers to form the outer capacitor plates. The beam i9s supported by tethers which serve as mechanical spring.

“Force” fingers are used for calibration

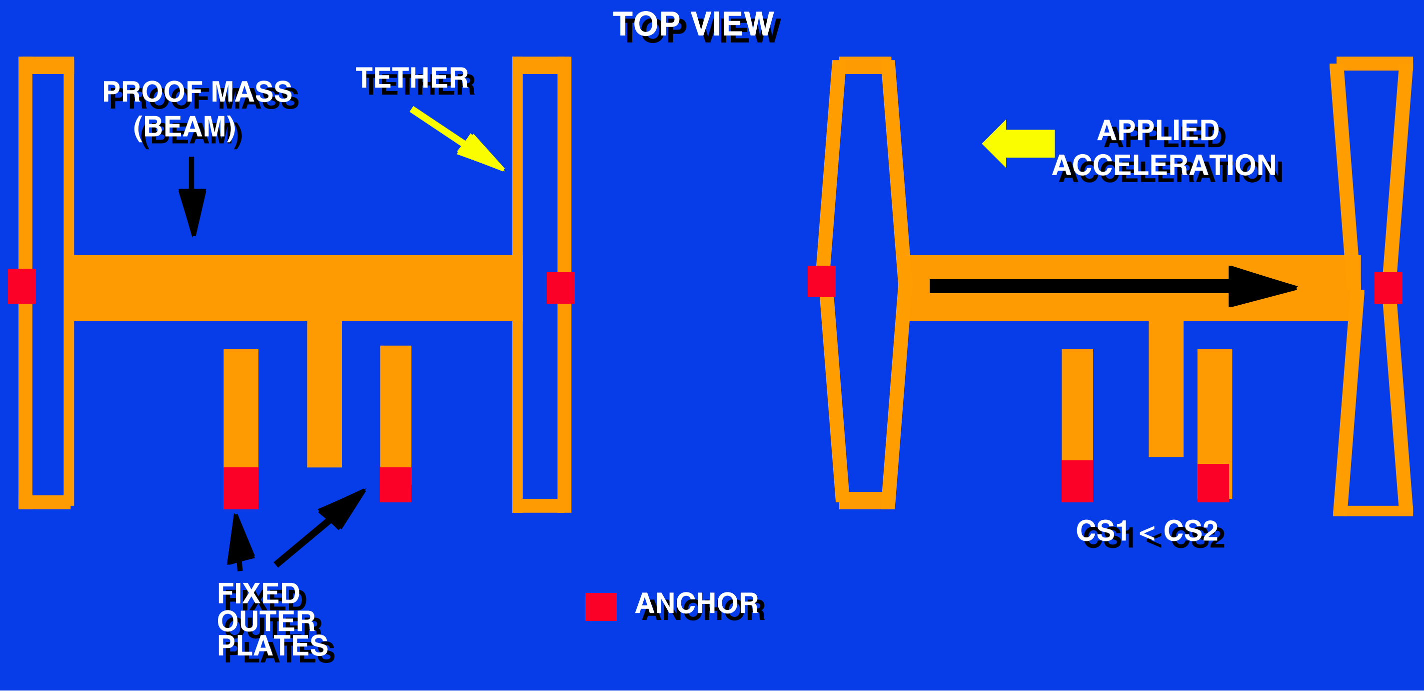
mass = 0.5 µg

SIZE: 0.5 mm x 0.4 mm, 2 µm thick,

- **Requirement: Avoid “stiction” => rigid cantilevered beam**

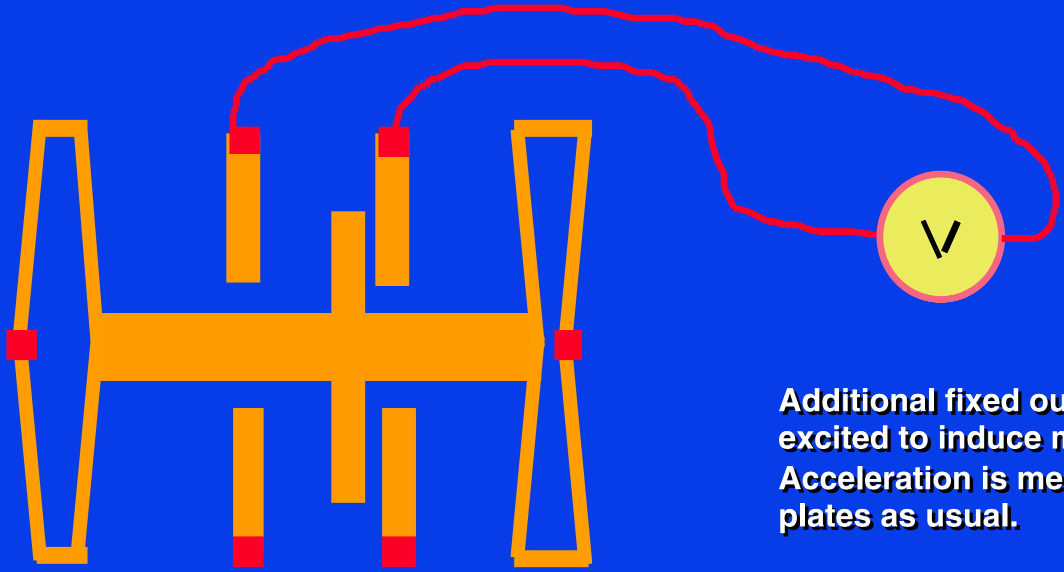
Sensor operation; ADI’s implementation

- **Folded tethers have more consistent spring constants, leading to better part to part consistency**



Self test operation

- Extra fixed outer plates may be added which when exited, force the proof mass to move. So you can electronically test the accelerometer



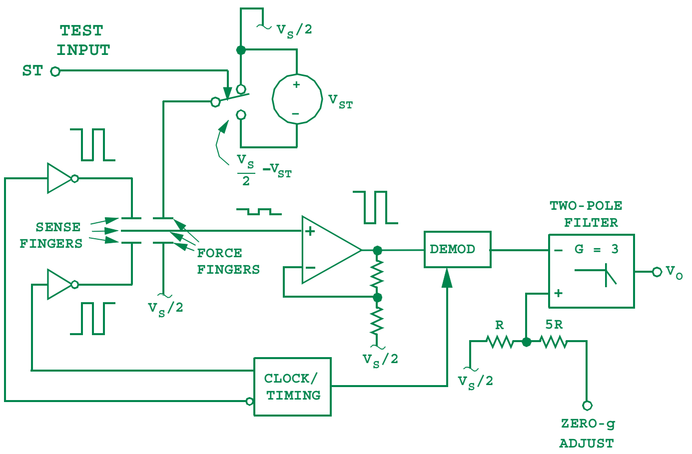
Additional fixed outer plates are electrically excited to induce movement of the proof mass. Acceleration is measured by the standard fixed plates as usual.

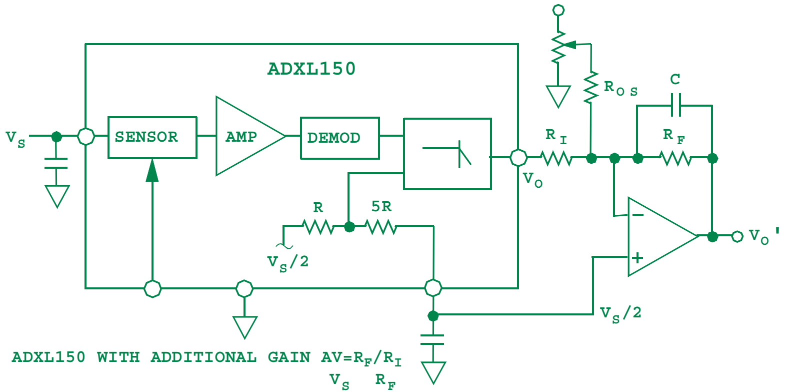
Interesting facts

- 0.1µgrams Proof Mass
- 0.1pF per Side for the Differential Capacitor
- 20aF (10^{-18}f) Smallest Detectable Capacitance Change
- Total Capacitance Change for Full-scale is 10fF
- 1.3µm Gaps Between Capacitor Plates
- 0.2Å Minimum Detectable Beam Deflection (one tenth of an Atomic diameter)
- 1.6 µm Between the Suspended Beam and Substrate
- 10 to 22kHz Resonant Frequency of Beam

MEMS Accelerometers:

Readout





ADXL150 WITH ADDITIONAL GAIN $AV=R_F/R_I$

OFFSET ADJUSTMENT RANGE OF $\frac{V_S}{2} \cdot \frac{R_F}{R_{OS}}$,

AND ADDITIONAL SINGLE POLE LPF,-3dB $= \frac{1}{2 R_F C}$.

Noise

Specifications	**ADXL105**	**ADXL202**	**ADXL05**	**ADXL50**
Range	**±5 *g***	±2 *g*	±5 *g*	±50 *g*
Noise ($\mu g/\sqrt{\text{Hz}}$)	**225**	500	500	6500
Bandwidth (kHz)	**10**	5	5	6
Supply Current (mA)	**2**	0.6	10	10
Number of Axes	**1**	2	1	1
Output Type	**Analog**	Analog/ Digital	Analog	Analog

ADXL105:
C = 150 fF
Sensitivity = 100 aF for 1g acceleration
noise = 225 g/Hz$^{1/2}$
“capacitance” noise = 22.5 10^{-21} F/Hz$^{1/2}$ = 22.5 zF/Hz$^{1/2}$

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