The Elementary Dynamics Analysis of Rill Flow

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Abstract: The determination of the velocity in rill flow is the basis of studying the soil erosion. The velocity of rill flow is influenced by flow discharge, sediment concentration, underlying soil and so on. Under the steady discharge, by assuming the resistance of viscosity is directly proportional to the velocity of flow, the dynamics equation of rill flow is deduced. By nonlinear simulation of the experiment data, we obtained functions between velocity of flow and time, the friction coefficient and erosion intensity, and the viscosity force and sediment concentration in flow. The simulating procedure and results showed that the model is feasible when the velocity of flow is lower. Velocity of flow is mainly affected by slope and sediment concentration when slope and texture of soil are different. The steeper the slope is, the more time the velocity needs to achieve steady. When the friction between water flow and underlying soil is bigger than soil anti-erodibility, sediment concentration in rill flow will increase accordingly.
Keywords: rill flow, dynamic analysis, stream velocity, simulation

In the view of dynamics, the soil eroded by runoff is determined by the ability of counter-erosion of soil and the intensity of runoff erosion. To a kind of soil, its counter-erosion ability is definite [Lei Tingwu, 2000]. The erosion intensity of runoff is determined by the runoff discharge, velocity and the sediment concentration, and these three factors affect each other in the course of eroding soil. To different runoffs, if the velocity and sediment concentration are in the same way, the bigger the discharge, the smaller the cut force of runoff on soil, and if the discharge is definite, the bigger of the velocity of runoff, the bigger the ability of carrying sediment, the bigger of the cut-force of runoff on soil. So that the velocity of runoff is the main factor that affect the soil erosion.

The mean velocity of flow is mainly affected by the factors of its discharge, the rude of underlying soil, slope and so on. If the slope is steeper, the velocity of stream cost more time to reach a steady value, but the final velocity may be bigger accordingly. In the period of rill erosion or rill-interval erosion, the bigger the discharge, the wider the rill, then the depth of rill becomes shallow, so we often suppose the discharge of stream is definite in the studying the dynamics of rill erosion, if the slope is less than 15°, maybe we can neglect the influence of fossa made by stream. If the initial velocity is small we can consider the stream as the standard Norton stream.

1 The dynamics model of rill velocity

According to the principles of hydrodynamics and prevenient study, the bigger the velocity of runoff, the bigger the resistance, the velocity is direct proportion to the resistance. The resistance of runoff is the function of its velocity, discharge and sediment. If the underlying soil is uniform, we suppose the resistance of stream is related to the velocity linearly, the relation between the resistance force and the unit weight of stream is as follow:

$$f / m = kv + c \tag{1}$$

where f is the resistance force (N), m is the mass of mass (kg), v is the velocity of runoff (ms^{-1}), k and c are the coefficient in which will be calculated.

Use equation (1) we can know the acceleration of unit quality of runoff as follow:

$$a = g\sin\alpha - c - kv \tag{2}$$

In which g is the acceleration of gravity (GM^{-2}), α is the slope (Radian). The following extremely equation (3) is derived from equation (2):

$$\frac{\mathrm{d}^2S}{\mathrm{d}t^2} + k\frac{\mathrm{d}S}{\mathrm{d}t} = g\sin\alpha - c \tag{3}$$

Suppose the initial and boundary condition in equation (3) as follow:

$$S = 0 \qquad t=0 \tag{3.1}$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = v_0 \qquad t=0 \tag{3.2}$$

$$\frac{\mathrm{d}^2S}{\mathrm{d}t^2} = 0 \qquad t=\infty \tag{3.3}$$

The following equation can derived from the equation (3), (3.1), (3.2) and (3.3) using the Laplace transformation.

$$s^2L(s) + ksL(s) = \frac{g\sin\alpha - c}{s} + v_0 \tag{4}$$

In which $L(s) = \int_0^\infty S(t)\mathrm{e}^{-st}\mathrm{d}t \quad (k \neq 0)$, equation (4) can be simplified as follow:

$$\begin{aligned} L(s) &= \frac{g\sin\alpha - c}{s(s^2+sk)} + \frac{v_0}{s^2+sk} \\ &= -\frac{g\sin\alpha - c}{k^2}\left(\frac{1}{s} - \frac{k}{s^2} - \frac{1}{s+k}\right) + \frac{v_0}{k}\left(\frac{1}{s} - \frac{1}{s+k}\right) \end{aligned} \tag{5}$$

Use inverse transformation of Lapace:

$$S = -\frac{g\sin\alpha - c}{k^2}\left(1 - kt - \mathrm{e}^{-kt}\right) + \frac{v_0}{k}\left(1 - \mathrm{e}^{-kt}\right) \tag{6}$$

Using the experimental results and equation (6), we can calculate the value of k and c with the method of non-liner regression.

From the equation (6) , the velocity v can be derived:

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} = \frac{g\sin\alpha - c}{k}(1 - \mathrm{e}^{-kt}) + v_0\mathrm{e}^{-kt} \tag{7}$$

And the acceleration of runoff as follow:

$$a = (g\sin\alpha - c - kv_0)\mathrm{e}^{-kt} \tag{8}$$

If $k \to 0$

$$a = (g \sin \alpha - c)t \tag{9}$$

In the view of the principle of classical mechanics, this mathematic model for the runoff is reasonable. For the initial velocity is unacquainted, so there are three parameters(k, c, v_0) to be calculated in equation (6) or equation (7). The relation between the distance s and the time t can be measured with the method of dye tracing (Abrahams A. D, ate, 1986, Luk S. H, 1992), and calculate these parameters with simple mathematics methods.

2 Material and methods

In the experiment we use the heavy loam and light soil as the underlying soil that are sifted through 2mm diameter earth sieve. The two kinds soil are put in a gutter made of Lucite, which width is 15cm and the volume weights of these two soil are the 1,350kg • M^{-3} and 1,300kg • M^{-3}, the mechanical compositions of these two soil are as follow Table 1. These soil are eroded with the 0.09m^3 • s^{-1} discharge of runoff at the slope of 5°, 10°, 15° after these soil are saturated for some hours. When the rill take place, measure the time of dye passing the points of 0.5m, 1.0m, 1.5m, 2.0m, 2.5m, 3.0m, 3.5m, 4.0m, 4.5m, 5.0m away from the entrance. Each slope experiment repeats for three times. The dye is made of gasoline and varnish, and its proportion is 1090kg • m^{-3}. For the proportion is bigger than water's one, so the measured velocity is not the surface velocity of runoff but the mean value. At these measuring points, get some samples and measure the sediment concentration with oven drying method.

Table 1 The mechanical composition of soil samples (%)

Soil	2—0.25mm	0.25—0.05mm	0.05—0.01mm	0.01—0.005mm	0.005—0.001mm	<0.001mm	<0.01mm
Heavy loam	0.4	4.5	44.7	10.7	20.6	19.1	50.4
Light loam	0.16	22.57	60.6	4.2	3.1	9.4	16.7

3 Results and discussion

Using the measured results and non-liner regression, the parameters k, c and v_0 can be found as follow Table 2.

Table 2 The non-liner regression results

Soil simple	Slope	k	c	v_0	Regression Coefficient(R^2)
Light loam	5°	0.5241	0.0898	0.1254	0.91
	10°	0.8254	0.3211	0.1542	0.94
	15°	1.2351	0.08621	0.0956	0.89
Heavy loam	5°	0.4854	0.1154	0.1124	0.88
	10°	0.7241	0.4205	0.0985	0.86
	15°	0.9854	0.4421	0.1058	0.91

In the Table 2, we can found that though the soil is same, the value of k and c at different slope is different. the value of parameter k increase when the slope become steep. At the same slope, the value of parameter k of light loam is bigger than that of light loam. These show the parameter k is not only related the kind of soil, but also the slope. The rising of parameter k with the slope may be that when the slope raises, the fine fraction of sediment augment, that in the soil reduce. The hypothesis is certified in some extent by the parameter of k of light loam larger than of the heavy loam. The influence of soil particle composition on parameter *k* may be studied in the future.

Using the results of Table 2 and equation (7), we can calculate the velocity of runoff on two kinds of soil. Combined the measured the time at different distance and the velocity value and sediment concentration, the relationship of the velocity and distance, and the relationship of the sediment concentration and distance can be found as Fig. 1—3.

Contrasting the Fig. 1 with Fig. 2, we can see that the variation of velocity is different to different kind of soil at the same slope. To the heavy soil, the velocity rises rapidly at the initial distance, but when the distance approach about 2.0m, the velocity of run on two kinds of soil become approximately and the velocity augment slowly. These may be that with the augmentation of velocity, the resistance force enlarges, and then the acceleration of runoff steps down and the sediment concentration of runoff become steadily as Fig. 3.

Contrasting Fig. 3 with Fig. 1—2, we can see that the sediment concentration rises rapidly when the velocity of runoff enlarges quickly. These may be that the resistance force becomes largely when the velocity enlarges. This makes the shear stress of runoff enlarge, and strengthen the erosion force, so the sediment concentration eke out. When the runoff reach 2m, the velocity and sediment concentration become steadily. So the carrying capacity of runoff correlates the velocity closely. The measurement of velocity directly is very important, and it is valuable to study it.

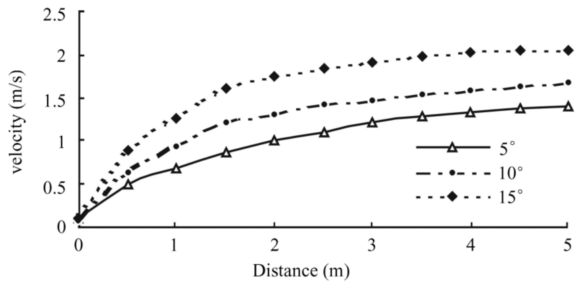

Fig. 1 The relationship of velocity and distance of light soil

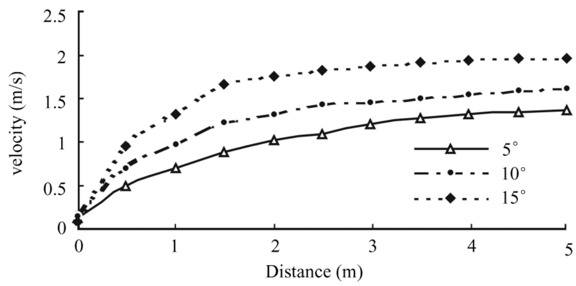

Fig. 2 The relationship of velocity and distance of heavy soil

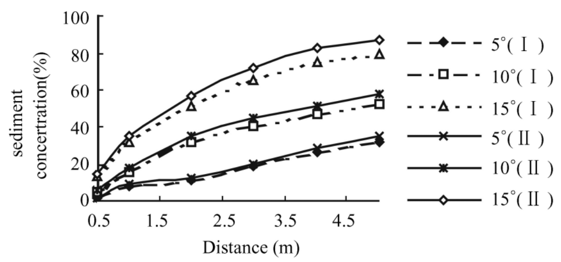

Fig. 3 The distribution of sediment concentration at different slope

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