Extended modelling for weekly intertechnique combination

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EGU - 1st General Assembly Session G7, Nice, April 26, 2004

Introduction and objectives

IERS Combination Pilot Project (CfP):

Output: Generation of weekly combined inter-technique solutions
Input: Generation of weekly intra-technique solutions

Objectives:

- **Presentation of the extended combination model (ECM)**
- **Feasibility analysis for the automated processing of the weekly ECM with up-to-date input solutions**
- **Status of S/W development**
- **First experiences**
- **Recommendations**

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Basic combination model
General

$$E(\tilde{\boldsymbol{p}}) = \boldsymbol{f}(\boldsymbol{p}) \quad \text{with} \quad \tilde{\boldsymbol{p}} = \begin{bmatrix} \tilde{p}_1 \\ \cdots \\ \tilde{p}_n \end{bmatrix}$$

$$\boldsymbol{D}(\tilde{\boldsymbol{p}}) = \boldsymbol{C} \quad \text{with} \quad \boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_{11} & \cdots & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{C}_{ii} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \cdots & \boldsymbol{C}_{nn} \end{bmatrix}$$

$$\boldsymbol{C} = s_i^2 \boldsymbol{Q}_{ii}$$

Linearized Gauss-Markov-Model

$$E[\delta \tilde{\boldsymbol{p}}] = \boldsymbol{A} \ \delta \boldsymbol{p} \quad \text{with} \quad \delta \tilde{\boldsymbol{p}} = \tilde{\boldsymbol{p}} - \boldsymbol{f}(\boldsymbol{p}_0) \quad \text{and} \quad \boldsymbol{A} = \partial \boldsymbol{f} / \partial \boldsymbol{p} \,|\, \boldsymbol{p}_0$$

$\boldsymbol{p}_0$: a priori parameter

Extended combination model
General

* Iteration process with

  - Variance Component Estimation (VCE)

  - Robust Estimation (RE)

  - Selection and quality control

Extended combination model
Iteration step (1)

(1) Basic combination model: $\delta \boldsymbol{p} = (\boldsymbol{A}^T \boldsymbol{C}^- \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{C}^- \delta \tilde{\boldsymbol{p}}$

Note: $\boldsymbol{C}^-$ **generalised inverse**

(2) Variance component estimation

(2a) $\boldsymbol{W} = \boldsymbol{N} - \boldsymbol{N}\boldsymbol{A}(\boldsymbol{A}^T \boldsymbol{N}\boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{N}$ with $\boldsymbol{C}_{ii} = s_i^2 \boldsymbol{Q}_{ii}$ and $\boldsymbol{N} = \boldsymbol{C}^-$

(2b) $\boldsymbol{q} = (q_i) = \boldsymbol{v}^T \boldsymbol{N} \bar{\boldsymbol{C}}_{ii} \boldsymbol{N} \boldsymbol{v}$ with $\boldsymbol{v} = \boldsymbol{A}\,\delta \boldsymbol{p} - \delta \tilde{\boldsymbol{p}}$ and $\bar{\boldsymbol{C}}_{ii} = \begin{bmatrix} \mathbf{0} & \dots & \mathbf{0} & \dots & \mathbf{0} \\ \dots & \dots & \dots & \dots & \dots \\ \mathbf{0} & \dots & \boldsymbol{C}_{ii} & \dots & \mathbf{0} \\ \dots & \dots & \dots & \dots & \dots \\ \mathbf{0} & \dots & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix}$

(2c) $\boldsymbol{S} = (S_{ij}) = tr(\boldsymbol{W} \bar{\boldsymbol{C}}_{ii} \boldsymbol{W} \bar{\boldsymbol{C}}_{jj})$

(2d) $s_i^2 (\boldsymbol{S}^{-1} \boldsymbol{q})_i \rightarrow s_i^2$ or stop iteration if $s_i^2 (\boldsymbol{S}^{-1} \boldsymbol{q})_i \approx s_i^2$

Extended combination model
Iteration step (2)

(3) Robust estimation: Huber, Schweppes, Biber ...

(3a) $\boldsymbol{C}_{\boldsymbol{vv}} = \boldsymbol{C} - \boldsymbol{A} (\boldsymbol{A}^{\boldsymbol{T}} \boldsymbol{N} \boldsymbol{A})^{-1} \boldsymbol{A}^{\boldsymbol{T}}$

(3b) $(s_v)_i = \sqrt{(\boldsymbol{C}_{\boldsymbol{vv}})_i}$

(3c) $w_i = \dfrac{v_i}{(s_v)_i}$

(3d) if $-c < w_i < c \ \forall i$ stop robust estimation iteration

Extended combination model
Selection and quality control

Select automatically:

* **Helmert transformation parameter set for the correct position datum definition**

* **Bias parameters (offset, drift ...) for correct EOP datum**

Control the estimation quality

* **before, during and after estimation process**

Input data analysis
General

Up-to-date weekly data set with intra-technique combined products:

- **week Oct 26 – Nov 1 , 2003: newest data sets for all 4 techniques**
- **GPS: IGS combination product**
- **SLR: inofficial ILRS combination product**
- **DORIS: Analysis Center product**
- **VLBI: Analysis Center product composed to one data set**

Input data analysis GPS

Solution name: igs03P1242.snx

Station number: 234

Parameter types: STAX, STAY, STAZ, XPO, YPO, XPOR, YPOR, LOD, XGC, YGC, ZGC

Parameter number: 743

Position refence epoch: slightly varying about midst of week

EOP reference epochs: daily midst of day

Input data analysis
SLR

Solution name:	dgfi.pos+eop.031110.snx
Station number:	25
Parameter types:	STAX, STAY, STAZ, XPO, YPO, LOD
Parameter number:	162
Position reference epoch:	fixed epoch near midst of week
EOP reference epochs:	daily at the beginning of day

Input data analysis
DORIS

Solution name:	ign03299wd05.snx
Station number:	42
Parameter types:	STAX, STAY, STAZ, XPO, YPO, UT, XPOR, YPOR, LOD
Parameter number:	168
Position reference epoch:	midst of week
EOP reference epochs:	daily midst of day

Input data analysis
VLBI

Solution name:	03OCT27XA_gsfd0001.snx, 03OCT28XN_gsfd0001.snx, 03OCT30XE_gsfd0001.snx (composed!)
Station number:	14
Parameter types:	STAX, STAY, STAZ, XPO, YPO, UT, XPOR, YPOR, LOD, NUT_LN, NUT_OB
Parameter number:	78
Position reference epochs:	varying from station to station
EOP reference epochs:	midst of 24h session (3 seessions)

Input data analysis
Local ties

Collocation site statistics

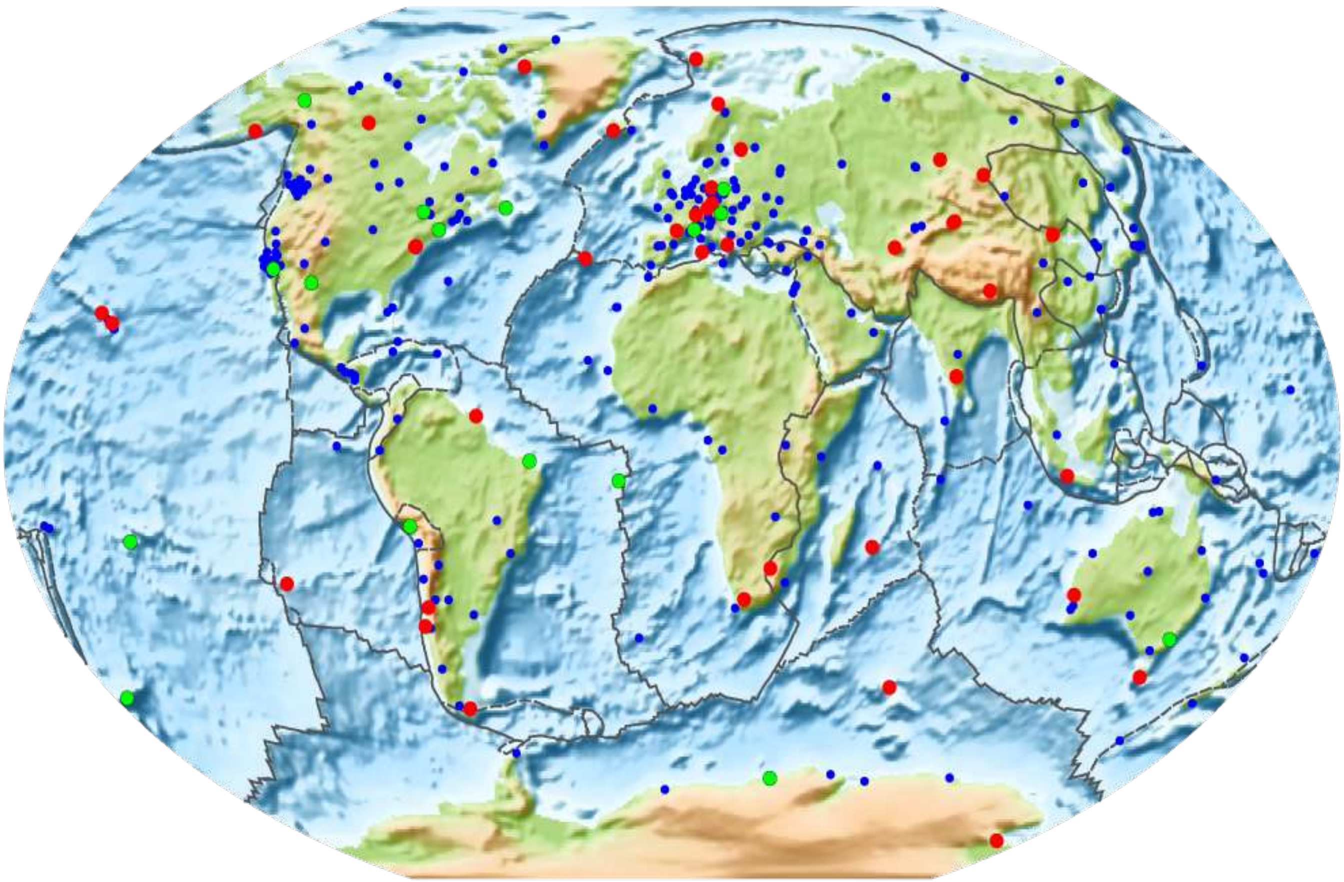
	number
collocation sites:	57
local ties required:	79
local ties known:	27
local ties missing:	52 (66%)

sites without known local ties:

GPS – DORIS:	12
GPS – GPS:	9
GPS – SLR:	4
GPS – DORIS – SLR:	2
GPS – SLR – VLBI:	1

Local tie differences(c-o)	>10 cm:	4
	< 1cm:	6
	rest:	17

Input data analysis
Local ties



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Conclusions for modelling

Output parameter

$$\boldsymbol{p} = [\,\boldsymbol{x}_1^T \ldots \boldsymbol{x}_n^T \;\; \boldsymbol{e}_1^T \ldots \boldsymbol{e}_m^T \;\; \boldsymbol{dx}_1^T \ldots \boldsymbol{dx}_{nl}^T, \boldsymbol{h}^T, \boldsymbol{b}^T\,]^T$$

Local ties

$$\boldsymbol{E}[\boldsymbol{dx}_i] = {}_m\boldsymbol{x}_i - {}_l\boldsymbol{x}_i$$

$$\boldsymbol{D}[\boldsymbol{dx}_i] = {}_{dx}\boldsymbol{C}_{ii}$$

Optimal case: Full covariance matrix !

Selection parameters for position => datum definition

$$E\left[{}_k\boldsymbol{x}_i(t_l)\right] = \left[\boldsymbol{x}_i + \delta T_k + \boldsymbol{R}(\boldsymbol{x}_i)\,\delta\,\boldsymbol{a}_k + \boldsymbol{x}_i\,\delta\,s_k + (t_l - t)\,{}_{\boldsymbol{0}}\dot{\boldsymbol{x}}_i\right]$$

Selection parameters for EOP => datum definition

$$E[\,{}_o e(t_i + \delta\,t_i)] = e_0(t_i) + \delta\,e(t_i) + \delta\,g_0(\delta\,t_i) + \delta\,\boldsymbol{b}_k(\delta\,\boldsymbol{a}_k)$$

First experiences
Variance component estimation

variance factors (vf)

	dgfi.040327 5 iterations		dgfi.040403 5 iterations		dgfi.040410 10 iterations	
	vf	var(vf)	vf	var(vf)	vf	var(VF)
asi	16.4	0.5	7.7	0.3	10.6	0.7
dgfi	4.2	0.8	2.0	0.1	3.6	0.2
gfz	7.6	0.4	8.0	0.3	12.6	0.7
jcet	2.7	0.1	2.5	0.1	381.8	8.8
nerc	5.3	0.2	23.6	0.8	3.7	0.2

=> VCE downweighting versus robust estimation (fail criterium)

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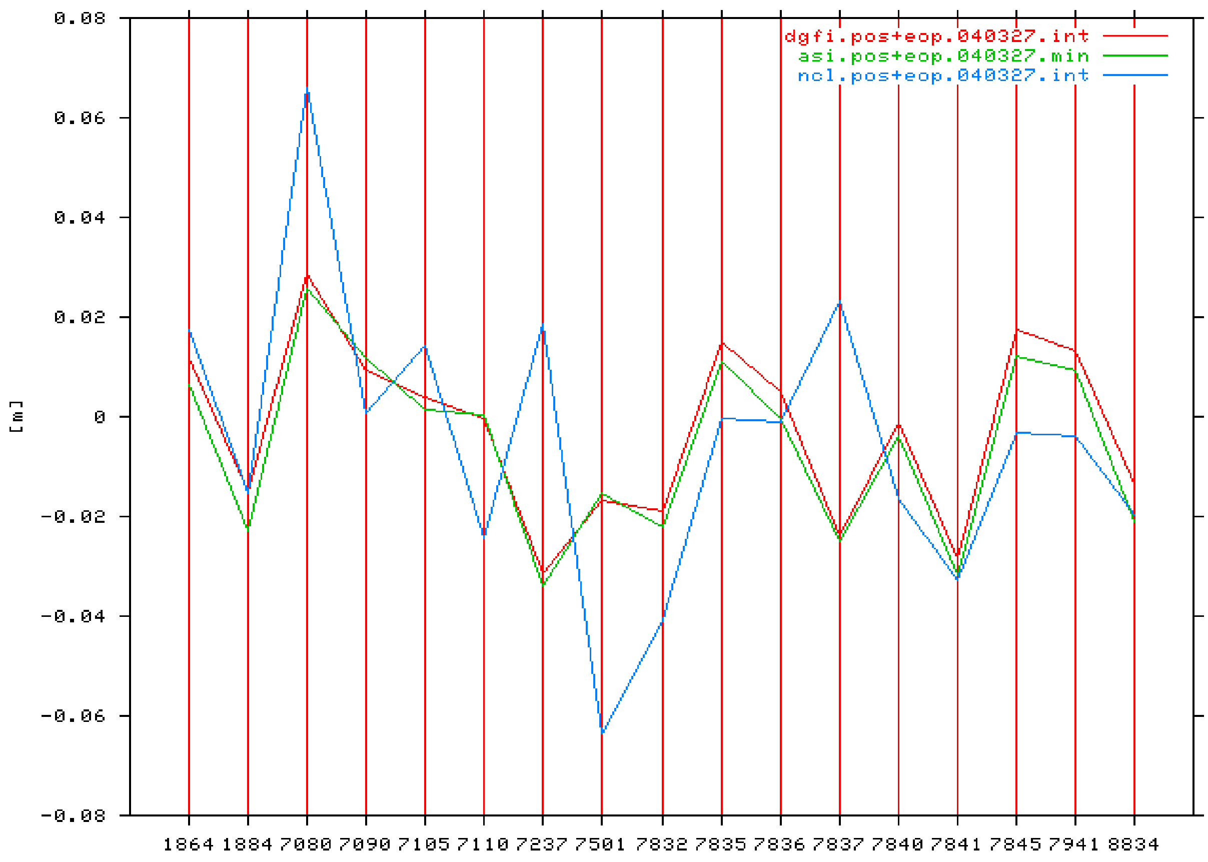
First experiences
Datum definition (SINEX combination file)

SLR combined solutions: X coordinates



Red: min.constraints on unconstrained NEQ

Green: min. constraints on loosely constrained NEQ

Blue: claimed as loosely constrained, but tight constraints given in SINEX file

=> clear conventions required

Status and outlook of extended modelling

S/W

* S/W modules finished and tested
* script for automated processing in development

Test strategy

* Tests with simulated random noise on actual input solutions
  - sufficient local ties?
  - sensitivity anylysis of VCE and robust estimation
  - sensitivity analysis for parameter selection control
  - sensitivity analysis quality control before, during and after iteration process

Application within IERS Combination Pilot Project

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Recommendations

Local ties

- **optimal case: unconstrained local tie network with full covariance matrix or (even better) NEQ sytem (SINEX file) for all collocation sites**
  => Site Survey and Collocation Pilot Project: local to global frame alignment

- **urgent case: observing, adjusting and storing local ties to open database in near online mode**
  => Task for Site Survey and Collocation Pilot Project

Input solutions

- ***optimal case: unique product per technique in unconstrained mode (free NEQ or full covariance matrix with well-defined loose constraints)***