# Measurement of high-frequency rotational transitions of $\mathrm{H}_{2} \mathrm{O}^{+}$in its ground state by far-infrared laser magnetic resonance (LMR) spectroscopy 

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#### Abstract

Thirteen new rotational transitions of $\mathrm{H}_{2} \mathrm{O}^{+}$in the $(0,0,0)$ level of the $\tilde{X}^{2} B_{1}$ state have been measured in the wavenumber region between 80 and $200 \mathrm{~cm}^{-1}$ ( 50 and $120 \mu \mathrm{~m}$ ) by far-infrared laser magnetic resonance (LMR) spectroscopy. LMR data measured previously between 25 and 90 $\mathrm{cm}^{-1}(110$ and $400 \mu \mathrm{~m})$, as well as optical and infrared combination differences, have been combined with the new LMR data in a weighted least-squares analysis using an A-reduced expression of the rotational-fine structure Hamiltonian. Thirty-two molecular constants were simultaneously determined, some sextic centrifugal distortion parameters and some quartic and sextic spin-rotation parameters for the first time. From this improved set of molecular parameters, very accurate calculations of rotational term values and zero-field predictions of the $1_{11}-0_{00}$ transition, including hyperfine structure, have been performed. Moreover, the electronic $g$-tensors and the hyperfine coupling constants are consistent with ab initio calculations which had been carried out for these constants. © 1998 American Institute of Physics. [S0021-9606(98)00746-6]


## I. INTRODUCTION

The water cation $\left(\mathrm{H}_{2} \mathrm{O}^{+}\right)$has been of interest to spectroscopists and astrophysicists since the early 1950's, when it was postulated to be present in comet tails. The first highresolution spectroscopy of $\mathrm{H}_{2} \mathrm{O}^{+}$was done by Lew and Heiber, ${ }^{1,2}$ when they investigated the $\widetilde{A}^{2} A_{1}-\widetilde{X}^{2} B_{1}$ electronic emission spectrum. It resulted in the identification of visible emission lines from $\mathrm{H}_{2} \mathrm{O}^{+}$in Comet Kohoutek ${ }^{3,4}$ and subsequently in other comets. Moreover, $\mathrm{H}_{2} \mathrm{O}^{+}$plays an important role in elementary processes in interstellar space ${ }^{5}$ and in the earth's upper atmosphere. ${ }^{6}$

Several other high-resolution absorption measurements have been performed on $\mathrm{H}_{2} \mathrm{O}^{+}$. The visible system of $\mathrm{H}_{2} \mathrm{O}^{+}$ has been observed using laser absorption spectroscopy. ${ }^{7}$ Some pure rotational transitions have been measured by farinfrared laser magnetic resonance spectroscopy (LMR), ${ }^{8}$ which permitted the observation of the proton hyperfine structure. A few lines of the rotational spectrum around 430 $\mathrm{cm}^{-1}$, measured by diode lasers, are reported in, Ref. 9, but those lines could not be assigned up to now. In the IR region, the $\nu_{3}$ band has been characterized using a difference frequency spectrometer, ${ }^{10}$ the $\nu_{2}$ and the $2 \nu_{2}-\nu_{2}$ bands using a diode laser spectrometer, ${ }^{11}$ and the $\nu_{1}$ and the $\nu_{2}+\nu_{3}-\nu_{2}$ bands by color-center laser spectroscopy. ${ }^{12}$ Recently, new vibronic bands in the near-infrared region were observed. ${ }^{13}$

A number of ab initio calculations have been carried out for $\mathrm{H}_{2} \mathrm{O}^{+}$, see, for example, Ref. 14 and references therein. Some of them investigated the hyperfine coupling constants of the ground state. ${ }^{14,15}$ Recently, a multireference configu-

[^0]ration interaction calculation of electronic $g$-tensors has been carried out. ${ }^{16}$

In this work, we measure 13 new rotational transitions of $\mathrm{H}_{2} \mathrm{O}^{+}$in its ground state, which prompts a reinvestigation of

TABLE I. Summary of the FIR laser lines used and the $\mathrm{H}_{2} \mathrm{O}^{+}$transitions observed.

| Laser line |  |  |  | $\mathrm{H}_{2} \mathrm{O}^{+}$transition |
| :---: | :---: | :---: | :---: | :---: |
| Calculated |  |  |  |  |
| [ $\mu \mathrm{m}$ ] | [MHz] | Gas | $\mathrm{CO}_{2}$ laser | Assignment |
| 115.823 | $2588361.7^{\text {a }}$ | ${ }^{13} \mathrm{CH}_{3} \mathrm{OH}$ | 10R(16) | $5_{24}{ }^{-1_{15}}$ |
| 100.806 | 2973 940.6 ${ }^{\text {b }}$ | $\mathrm{CH}_{3} \mathrm{OH}$ | 9R(14) | $\begin{aligned} & 2_{20}-1_{11} \\ & 6_{25}-6_{16} \\ & 5_{15}-3_{22} \end{aligned}$ |
| 88.948 | $3370404.93{ }^{\text {c }}$ | ${ }^{13} \mathrm{CH}_{3} \mathrm{OH}$ | 9P(44) | $\begin{aligned} & 7_{26}-7_{17} \\ & 3_{22} 2_{11} \\ & 4_{04}-2_{11} \end{aligned}$ |
| 81.917 | $3659704.49^{\text {d }}$ | $\mathrm{CH}_{3} \mathrm{OH}$ | 9 P (26) | $\begin{aligned} & 4_{40}-4_{31} \\ & 5_{42}-5_{33} \end{aligned}$ |
| 81.557 | $3675859.9^{\text {b }}$ | $\mathrm{CD}_{3} \mathrm{OH}$ | 10R(16) | $\begin{aligned} & 4_{40}-4_{31} \\ & 5_{42}-5_{33} \end{aligned}$ |
| 77.905 | $3848185.5^{\text {b }}$ | $\mathrm{CH}_{3} \mathrm{OH}$ | 10R(16) | $\begin{aligned} & 3_{21}-2_{12} \\ & 4_{23}-3_{12} \end{aligned}$ |
| 63.096 | $4751340.9^{\text {b }}$ | ${ }^{13} \mathrm{CH}_{3} \mathrm{OH}$ | 9 P (12) | $4_{22}-3_{13}$ |
| 62.966 | $4761182.4{ }^{\text {b }}$ | $\mathrm{CH}_{3} \mathrm{OH}$ | 10R(16) | $4_{22}-3_{13}$ |
| 56.728 | $5284711.68{ }^{\text {d }}$ | $\mathrm{CH}_{3} \mathrm{OH}$ | 9R(16) | $4_{31}-3_{22}$ |

${ }^{\mathrm{a}}$ Taken from Ref. 23 (there is a misprint in Ref. 24).
${ }^{\mathrm{b}}$ Taken from Ref. 24.
${ }^{\mathrm{c}}$ Remeasured during this work (the measurement of 3363537.3 MHz in Ref. 19 is incorrect, see text).
${ }^{\mathrm{d}}$ Measured during this work (it had not been measured previously).

TABLE II. Zeeman resonances observed and calculations of tuning rates and transition frequencies for zero field.

| $N^{\prime}$ | $K^{\prime}$ | $P^{\prime}$ | $2 J^{\prime}$ | $2 M_{J}^{\prime}$ | $2 M_{I}^{\prime}$ | $N^{\prime \prime}$ | $K^{\prime \prime}$ | $P^{\prime \prime}$ | $2 J^{\prime \prime}$ | $2 M_{J}^{\prime \prime}$ | $2 M_{I^{\prime \prime}}$ | $\begin{gathered} B_{o b s} \\ {\left[10^{-4} \mathrm{~T}\right]} \end{gathered}$ | $\begin{gathered} \Delta \nu^{\mathrm{a}} \\ {\left[10^{-5} \mathrm{~cm}^{-1}\right]} \end{gathered}$ | $\begin{gathered} \Delta \nu_{o b s}{ }^{\mathrm{b}} \\ {\left[10^{-5} \mathrm{~cm}^{-1}\right]} \end{gathered}$ | $\begin{gathered} T R^{\mathrm{c}} \\ {\left[10^{4} \mathrm{MHz} / \mathrm{T}\right]} \end{gathered}$ | $\begin{gathered} \nu_{c a l}(B=0)^{\mathrm{d}} \\ {\left[\mathrm{~cm}^{-1}\right]} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\nu_{L}=86.338453 \mathrm{~cm}^{-1}$ |  |  |  | (115.823 $\mu \mathrm{m},{ }^{13} \mathrm{CH}_{3} \mathrm{OH}, \mathrm{CO}_{2}$ pump: 10R(16)) |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 | 2 | 1 | 11 | 9 | -2 | 5 | 1 | -1 | 11 | 9 | -2 | 5738.8 | 0.03 | 4.2 | 1.80 | 86.15910 |
| 5 | 2 | 1 | 11 | 9 | 0 | 5 | 1 | -1 | 11 | 9 | 0 | 5768.7 | -0.43 | 4.2 | 1.80 | 86.15919 |
| 5 | 2 | 1 | 11 | 9 | 2 | 5 | 1 | -1 | 11 | 9 | 2 | 5798.5 | 0.60 | 4.2 | 1.80 | 86.15930 |
| 5 | 2 | 1 | 11 | 7 | -2 | 5 | 1 | -1 | 11 | 7 | -2 | 6270.3 | -1.14 | 3.2 | 1.31 | 86.15910 |
| 5 | 2 | 1 | 11 | 7 | 0 | 5 | 1 | -1 | 11 | 7 | 0 | 6298.4 | -0.83 | 3.2 | 1.31 | 86.15919 |
| 5 | 2 | 1 | 11 | 7 | 2 | 5 | 1 | -1 | 11 | 7 | 2 | 6326.6 | -0.39 | 3.2 | 1.31 | 86.15930 |
| 5 | 2 | 1 | 11 | 5 | -2 | 5 | 1 | -1 | 11 | 5 | -2 | 7105.6 | -1.10 | 2.9 | 0.95 | 86.15910 |
| 5 | 2 | 1 | 11 | 5 | 0 | 5 | 1 | -1 | 11 | 5 | 0 | 7131.8 | 0.07 | 2.9 | 0.95 | 86.15919 |
| 5 | 2 | 1 | 11 | 5 | 2 | 5 | 1 | -1 | 11 | 5 | 2 | 7158.6 | -0.33 | 2.9 | 0.95 | 86.15930 |
| 5 | 2 | 1 | 9 | 9 | -2 | 5 | 1 | -1 | 9 | 9 | -2 | 16765.2 | -1.16 | 2.8 | -0.33 | 86.89147 |
| 5 | 2 | 1 | 9 | 9 | 0 | 5 | 1 | -1 | 9 | 9 | 0 | 16804.8 | 0.66 | 2.8 | -0.33 | 86.89135 |
| 5 | 2 | 1 | 9 | 9 | 2 | 5 | 1 | -1 | 9 | 9 | 2 | 16845.1 | 3.16 | 2.8 | -0.33 | 86.89120 |

$\nu_{L}=99.199981 \mathrm{~cm}^{-1}\left(100.806 \mu \mathrm{~m}, \mathrm{CH}_{3} \mathrm{OH}, \mathrm{CO}_{2}\right.$ pump: 9R(14))

| 2 | 2 | 1 | 5 | -1 | -2 | 1 | 1 | -1 | 3 | -3 | -2 | 2776.5 | -2.76 | 2.6 | 0.94 | 99.10609 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 2 | 1 | 5 | -1 | 0 | 1 | 1 | -1 | 3 | -3 | 0 | 2801.0 | -0.56 | 2.6 | 0.94 | 99.10503 |
| 2 | 2 | 1 | 5 | -1 | 2 | 1 | 1 | -1 | 3 | -3 | 2 | 2826.1 | 0.48 | 2.6 | 0.94 | 99.10355 |
| 2 | 2 | 1 | 5 | 1 | -2 | 1 | 1 | -1 | 3 | -1 | -2 | 3402.6 | -0.99 | 2.6 | 0.90 | 99.10412 |
| 2 | 2 | 1 | 5 | 1 | 0 | 1 | 1 | -1 | 3 | -1 | 0 | 3426.8 | 0.26 | 2.6 | 0.90 | 99.10306 |
| 2 | 2 | 1 | 5 | 1 | 2 | 1 | 1 | -1 | 3 | -1 | 2 | 3451.9 | -0.65 | 2.6 | 0.90 | 99.10355 |
| 2 | 2 | 1 | 5 | 3 | -2 | 1 | 1 | -1 | 3 | 1 | -2 | 4574.3 | -0.71 | 2.6 | 0.88 | 99.10293 |
| 2 | 2 | 1 | 5 | 3 | 0 | 1 | 1 | -1 | 3 | 1 | 0 | 4596.3 | 2.87 | 2.6 | 0.87 | 99.10306 |
| 2 | 2 | 1 | 5 | 3 | 2 | 1 | 1 | -1 | 3 | 1 | 2 | 4619.8 | 2.42 | 2.6 | 0.87 | 99.10355 |
| 6 | 2 | -1 | 11 | 7 | 0 | 6 | 1 | 1 | 11 | 5 | 0 | 3591.5 | 0.07 | 2.7 | -1.12 | 99.28624 |
| 6 | 2 | -1 | 11 | 5 | 0 | 6 | 1 | 1 | 11 | 3 | 0 | 3806.2 | -0.24 | 2.5 | -0.97 | 99.28624 |
| 6 | 2 | -1 | 11 | 3 | 0 | 6 | 1 | 1 | 11 | 1 | 0 | 4112.5 | -0.38 | 2.4 | -0.82 | 99.28624 |
| 6 | 2 | -1 | 11 | 7 | 0 | 6 | 1 | 1 | 11 | 7 | 0 | 4288.3 | -0.35 | 2.8 | -1.10 | 99.28624 |
| 6 | 2 | -1 | 11 | 5 | 0 | 6 | 1 | 1 | 11 | 7 | 0 | 5466.4 | 1.03 | 2.7 | -0.89 | 99.28624 |
| 6 | 2 | -1 | 11 | 3 | 0 | 6 | 1 | 1 | 11 | 5 | 0 | 5976.4 | -0.89 | 2.5 | -0.70 | 99.28624 |
| 6 | 2 | -1 | 11 | -3 | 0 | 6 | 1 | 1 | 11 | -5 | 0 | 6108.9 | -0.99 | 4.5 | -0.40 | 99.28624 |
| 5 | 1 | -1 | 11 | 7 | -2 | 3 | 2 | 1 | 7 | 5 | -2 | 13260.9 | 9.10 | 6.0 | -1.03 | 99.75148 |
| 5 | 1 | -1 | 11 | 7 | 0 | 3 | 2 | 1 | 7 | 5 | 0 | 13285.9 | 3.84 | 6.0 | -1.03 | 99.75118 |
| 5 | 1 | -1 | 11 | 7 | 2 | 3 | 2 | 1 | 7 | 5 | 2 | 13311.1 | -0.93 | 6.0 | -1.03 | 99.75096 |

$\nu_{L}=112.424607 \mathrm{~cm}^{-1}\left(88.948 \mu \mathrm{~m},{ }^{13} \mathrm{CH}_{3} \mathrm{OH}, \mathrm{CO}_{2}\right.$ pump: $\left.9 \mathrm{P}(44)\right)$


TABLE II. (Continued.)

| $N^{\prime}$ | $K^{\prime}$ | $P^{\prime}$ | $2 J^{\prime}$ | $2 M_{J}^{\prime}$ | $2 M_{I}^{\prime}$ | $N^{\prime \prime}$ | $K^{\prime \prime}$ | $P^{\prime \prime}$ | $2 J^{\prime \prime}$ | $2 M_{J}^{\prime \prime}$ | $2 M_{I^{\prime \prime}}$ | $\begin{gathered} B_{o b s} \\ {\left[10^{-4} \mathrm{~T}\right]} \end{gathered}$ | $\begin{gathered} \Delta \nu^{\mathrm{a}} \\ {\left[10^{-5} \mathrm{~cm}^{-1}\right]} \end{gathered}$ | $\begin{gathered} \Delta \nu_{o b s}{ }^{\mathrm{b}} \\ {\left[10^{-5} \mathrm{~cm}^{-1}\right]} \end{gathered}$ | $\begin{gathered} T R^{\mathrm{c}} \\ {\left[10^{4} \mathrm{MHz} / \mathrm{T}\right]} \end{gathered}$ | $\begin{gathered} \nu_{c a l}(B=0)^{\mathrm{d}} \\ {\left[\mathrm{~cm}^{-1}\right]} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\nu_{L}=122.074602 \mathrm{~cm}^{-1}\left(81.917 \mu \mathrm{~m}, \mathrm{CH}_{3} \mathrm{OH}, \mathrm{CO}_{2}\right.$ pump: 9P(26)) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 4 | 1 | 9 | 9 | -2 | 4 | 3 | -1 | 9 | 7 | -2 | 19153.7 | -7.72 | 13.1 | 1.46 | 121.58380 |
| 4 | 4 | 1 | 9 | 9 | 0 | 4 | 3 | -1 | 9 | 7 | 0 | 19174.8 | -10.15 | 13.1 | 1.46 | 121.58405 |
| 4 | 4 | 1 | 9 | 9 | 2 | 4 | 3 | -1 | 9 | 7 | 2 | 19195.8 | -12.00 | 13.1 | 1.46 | 121.58435 |
| 5 | 4 | 1 | 11 | 9 | -2 | 5 | 3 | -1 | 11 | 11 | -2 | 15353.3 | 1.92 | 5.9 | -0.65 | 122.28327 |
| 5 | 4 | 1 | 11 | 9 | 0 | 5 | 3 | -1 | 11 | 11 | 0 | 15387.5 | 3.01 | 5.9 | -0.65 | 122.28343 |
| 5 | 4 | 1 | 11 | 9 | 2 | 5 | 3 | -1 | 11 | 11 | 2 | 15421.6 | 3.75 | 5.9 | -0.65 | 122.28363 |
| $\nu_{L}=122.613488 \mathrm{~cm}^{-1}\left(81.557 \mu \mathrm{~m}, \mathrm{CD}_{3} \mathrm{OH}, \mathrm{CO}_{2}\right.$ pump: 10R(16)) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 4 | 1 | 7 | 5 | 2 | 4 | 3 | -1 | 7 | 5 | 0 | 17761.0 | 8.95 | 62.0 | -0.93 | $122.97353 \%$ |
| 4 | 4 | 1 | 7 | 5 | 0 | 4 | 3 | -1 | 7 | 3 | 0 | 19485.0 | 45.94 | 47.1 | -0.72 | 122.97353 \% |
| 5 | 4 | 1 | 11 | 11 | -2 | 5 | 3 | -1 | 11 | 9 | -2 | 15670.3 | 4.28 | 10.6 | 1.24 | 122.28327 |
| 5 | 4 | 1 | 11 | 11 | 0 | 5 | 3 | -1 | 11 | 9 | 0 | 15692.5 | 4.89 | 10.6 | 1.24 | 122.28343 |
| 5 | 4 | 1 | 11 | 11 | 2 | 5 | 3 | -1 | 11 | 9 | 0 | 15715.4 | 2.72 | 10.6 | 1.24 | 122.28363 |
| 5 | 4 | 1 | 11 | 9 | -2 | 5 | 3 | -1 | 11 | 7 | -2 | 16550.4 | 3.65 | 8.5 | 0.96 | 122.28327 |
| 5 | 4 | 1 | 11 | 9 | 0 | 5 | 3 | -1 | 11 | 7 | 0 | 16573.1 | 0.74 | 8.5 | 0.96 | 122.28343 |
| 5 | 4 | 1 | 11 | 9 | 2 | 5 | 3 | -1 | 11 | 7 | 2 | 16595.5 | -1.13 | 8.5 | 0.96 | 122.28363 |
| 5 | 4 | 1 | 11 | 7 | -2 | 5 | 3 | -1 | 11 | 5 | -2 | 17938.9 | -2.08 | 7.1 | 0.76 | 122.28327 |
| 5 | 4 | 1 | 11 | 7 | 0 | 5 | 3 | -1 | 11 | 5 | 0 | 17960.8 | -3.98 | 7.1 | 0.76 | 122.28343 |
| 5 | 4 | 1 | 11 | 7 | 2 | 5 | 3 | -1 | 11 | 5 | 2 | 17983.0 | -6.57 | 7.1 | 0.76 | 122.28363 |
| $\nu_{L}=128.361651 \mathrm{~cm}^{-1}\left(77.905 \mu \mathrm{~m}, \mathrm{CH}_{3} \mathrm{OH}, \mathrm{CO}_{2}\right.$ pump: 10R(16)) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 | 2 | $-1$ | 5 | -3 | 0 | 2 | 1 | 1 | 5 | -3 | 0 | 8498.3 | -14.81 | 9.5 | 2.36 | 127.78729 |
| 3 | 2 | -1 | 5 | -1 | 0 | 2 | 1 | 1 | 5 | -1 | 0 | 10762.9 | -10.73 | 10.3 | 2.21 | 127.78729 |
| 3 | 2 | -1 | 5 | 1 | 0 | 2 | 1 | 1 | 5 | -1 | 0 | 12449.6 | 0.50 | 10.4 | 2.12 | 127.78729 |
| 3 | 2 | -1 | 5 | 1 | 0 | 2 | 1 | 1 | 5 | 1 | 0 | 13771.5 | -9.55 | 11.8 | 2.17 | 127.78729 |
| 3 | 2 | -1 | 5 | 3 | 0 | 2 | 1 | 1 | 5 | 1 | 0 | 16082.1 | -0.13 | 12.6 | 2.15 | 127.78729 |
| 3 | 2 | -1 | 5 | 3 | 0 | 2 | 1 | 1 | 5 | 3 | 0 | 17609.9 | -12.07 | 14.6 | 2.25 | 127.78729 |
| 4 | 2 | -1 | 7 | 3 | 0 | 3 | 1 | 1 | 7 | 1 | 0 | 14255.9 | -30.54 | 14.9 | 2.17 | 127.83222 |
| 4 | 2 | -1 | 7 | 5 | 0 | 3 | 1 | 1 | 7 | 3 | 0 | 17303.3 | -29.66 | 15.2 | 2.29 | 127.83222 |
| $\nu_{L}=158.487673 \mathrm{~cm}^{-1}\left(63.096 \mu \mathrm{~m},{ }^{13} \mathrm{CH}_{3} \mathrm{OH}, \mathrm{CO}_{2}\right. \text { pump: 9P(12)) }$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 2 | 1 | 9 | 5 | -2 | 3 | 1 | -1 | 7 | 3 | -2 | 3207.9 | -0.49 | 4.3 | 0.96 | 158.42612 |
| 4 | 2 | 1 | 9 | 5 | 0 | 3 | 1 | -1 | 7 | 3 | 0 | 3233.2 | -1.76 | 4.3 | 0.96 | 158.42619 |
| 4 | 2 | 1 | 9 | 5 | 2 | 3 | 1 | -1 | 7 | 3 | 2 | 3258.8 | -3.39 | 4.3 | 0.96 | 158.42637 |
| 4 | 2 | 1 | 9 | 7 | -2 | 3 | 1 | -1 | 7 | 5 | -2 | 3564.1 | -0.79 | 4.9 | 1.12 | 158.42612 |
| 4 | 2 | 1 | 9 | 7 | 0 | 3 | 1 | -1 | 7 | 5 | 0 | 3590.6 | -3.35 | 4.9 | 1.11 | 158.42619 |
| 4 | 2 | 1 | 9 | 7 | 2 | 3 | 1 | -1 | 7 | 5 | 2 | 3616.6 | -3.37 | 4.9 | 1.11 | 158.42637 |
| 4 | 2 | 1 | 9 | 5 | 0 | 3 | 1 | -1 | 7 | 5 | 0 | 5102.4 | 0.55 | 7.9 | 1.05 | 158.42619 |
| 4 | 2 | 1 | 9 | 5 | 2 | 3 | 1 | -1 | 7 | 5 | 2 | 5130.1 | 0.53 | 7.9 | 1.04 | 158.42637 |
| 4 | 2 | 1 | 9 | -3 | 0 | 3 | 1 | -1 | 7 | -3 | 0 | 5642.9 | -3.71 | 12.0 | 0.35 | $158.42619 \%$ |
| 4 | 2 | 1 | 9 | -5 | 0 | 3 | 1 | -1 | 7 | -5 | 0 | 6966.5 | -5.73 | 7.9 | 0.22 | $158.42619 \%$ |
| $\nu_{L}=158.815950 \mathrm{~cm}^{-1}\left(62.966 \mu \mathrm{~m}, \mathrm{CH}_{3} \mathrm{OH}, \mathrm{CO}_{2}\right.$ pump: 10R(16)) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 2 | 1 | 9 | 7 | -2 | 3 | 1 | -1 | 7 | 5 | -2 | 10294.5 | 2.89 | 5.1 | 1.23 | 158.42612 |
| 4 | 2 | 1 | 9 | 7 | 0 | 3 | 1 | -1 | 7 | 5 | 0 | 10324.6 | 3.12 | 5.1 | 1.23 | 158.42619 |
| 4 | 2 | 1 | 9 | 7 | 2 | 3 | 1 | -1 | 7 | 5 | 2 | 10354.4 | 4.93 | 5.1 | 1.23 | 158.42637 |
| 4 | 2 | 1 | 7 | 7 | -2 | 3 | 1 | -1 | 5 | 5 | -2 | 10709.8 | -2.52 | 5.1 | -1.16 | 159.21878 |
| 4 | 2 | 1 | 7 | 7 | 0 | 3 | 1 | -1 | 5 | 5 | 0 | 10737.5 | -4.46 | 5.1 | -1.16 | 159.21872 |
| 4 | 2 | 1 | 7 | 7 | 2 | 3 | 1 | -1 | 5 | 5 | 2 | 10765.8 | -4.38 | 5.1 | -1.16 | 159.21851 |
| 4 | 2 | 1 | 7 | 7 | -2 | 3 | 1 | -1 | 7 | 7 | -2 | 13120.4 | -1.42 | 5.1 | -0.87 | 159.61856 |
| 4 | 2 | 1 |  | 7 | 0 | 3 | 1 | -1 | 7 | 7 | 0 | 13156.9 | -1.42 | 5.1 | -0.87 | 159.62110 |
| 4 | 2 | 1 | 7 | 7 | 2 | 3 | 1 | -1 | 7 | 7 | 2 | 13193.7 | -0.81 | 5.1 | -0.87 | 159.62436 |
| $\nu_{L}=176.279007 \mathrm{~cm}^{-1}\left(56.728 \mu \mathrm{~m}, \mathrm{CH}_{3} \mathrm{OH}, \mathrm{CO}_{2}\right.$ pump: 9R(16)) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 3 | -1 | 7 | -3 | 0 | 3 | 2 | 1 | 5 | -5 | 0 | 9485.4 | -1.79 | 13.5 | -0.38 | $176.42473 \%$ |
| 4 | 3 | -1 | 7 | -1 | 0 | 3 | 2 | 1 | 5 | -3 | 0 | 9626.8 | 1.73 | 13.5 | -0.42 | $176.42597 \%$ |
| 4 | 3 | -1 | 7 | 1 | 0 | 3 | 2 | 1 | 5 | -1 | 0 | 9788.7 | -0.25 | 13.5 | -0.47 | $176.42597 \%$ |

${ }^{\text {a }}$ Differences between laser frequency and calculated transition frequency at the magnetic flux density $B_{o b s}$.
${ }^{\text {b }}$ Experimental uncertainties (used in the weighted fit).
${ }^{\text {c }}$ Calculated tuning rates.
${ }^{\mathrm{d}}$ Calculated transition frequencies for zero field.
${ }^{\star}$ Lines are unresolved triplets.
the structural properties of the molecule. The new data are of interest not only from a theoretical point of view, but also in relation to studies of interstellar space and of the upper atmosphere.

## II. EXPERIMENTAL DETAILS

The spectra were recorded by far-infrared LMR spectroscopy ${ }^{17}$ at the Boulder, Colorado laboratories of NIST. The experimental setup of the LMR apparatus has been described in detail previously. ${ }^{18}$ The far-infrared laser is transversely pumped by a $\mathrm{CO}_{2}$ laser with water-cooled end mirrors and a typical power output of 20 W on a single laser transition. The laser lines used in this work are given in Table I. The frequencies of two of these lines were measured for the first time during this study. One laser line, the 89.13 $\mu \mathrm{m}{ }^{13} \mathrm{CH}_{3} \mathrm{OH}$ laser line, was remeasured after the assignment of the $\mathrm{H}_{2} \mathrm{O}^{+}$signals seemed to be impossible. From the $\mathrm{H}_{2} \mathrm{O}^{+}$spectrum, the laser frequency was predicted to be about $0.2 \mathrm{~cm}^{-1}$ higher than that given in Ref. 19. The remeasurement showed that this prediction was right. The right frequency is given in Table II. A 15-in. electromagnet provides a maximum flux density of 2.0 T . The magnetic flux densities were measured with a Hall probe, which was calibrated with an NMR gaussmeter. The Zeeman modulation frequency was 39 kHz . A liquid helium-cooled germanium:gallium photoconductor was used as the detector.

The $\mathrm{H}_{2} \mathrm{O}^{+}$radicals were produced in the gas phase, using a cw microwave discharge source ( 50 W ). The discharge was run through ultrahigh purity helium (amounts of other gases $<1 \mathrm{ppm}$ ) at 190 Pa (1.4 torr), with a small amount of water vapor added under flow conditions. The pump system which was used for the cell consists of two parallel rotary pumps providing a maximum pumping speed of about 8 liter/s. The $\mathrm{H}_{2} \mathrm{O}^{+}$signals disappeared rapidly upon the addition of small amounts of $\mathrm{H}_{2}$, and were relatively unaffected by the addition of $\mathrm{N}_{2}$. This substitution was used to distinguish $\mathrm{H}_{2} \mathrm{O}^{+}$from other species.

The experimental uncertainties of the measurements are caused by the uncertainty of the laser line center $( \pm 2$ $\times 10^{-7}$ ), the uncertainty in the magnetic flux density ( 0.01 mT for $B<0.1 \mathrm{~T}$ and $10^{-4} B$ for $B>0.1 \mathrm{~T}$, and the readout error of the line positions. Thus, the total 1 -sigma uncertainty of our data is, at best, $2 \times 10^{-5} \mathrm{~cm}^{-1}(0.6 \mathrm{MHz})$, depending on the tuning rate of the transition.

## III. RESULTS AND ANALYSIS

Under the experimental conditions described above, 114 Zeeman components of 13 different rotational transitions, with $N$ up to 7 and $K_{a}$ up to 4, have been observed at wavelengths between 50 and $120 \mu \mathrm{~m}$. Figure 1 shows an energy level diagram of the ground state, in which all measured transitions are marked. The exact line positions of all the Zeeman components are listed in Table II. The maximum signal-to-noise was around 200:1, with a 100 ms time constant.

We observed both singlet and triplet hyperfine patterns according to levels with spin states $I=0\left(K_{a}+K_{c}\right.$ odd $)$ and $I=1\left(K_{a}+K_{c}\right.$ even $)$. Figure 2 shows a typical spectrum. In


FIG. 1. Energy level diagram of the $\mathrm{H}_{2} \mathrm{O}^{+}\left(\tilde{X}^{2} B_{1}, \nu=0\right)$ state. Transitions observed in this study are marked by solid arrows, and those of Ref. 8 by dotted arrows.
seven cases the triplets could not be resolved. These data are marked in Table II and are provided with low weights corresponding to the large readout errors.

The assignment of transitions, which involved either the $3_{22}$ or the $4_{04}$ states, turned out to be particularly difficult. This is due to the strong mixing between these two states. Thus, the pattern of the corresponding tuning diagrams is complex, and depends very sensitively on the term energies of these states. On the other hand, due to this mixing the $4_{04}-2_{11}$ and $5_{15}-3_{22}(\Delta N=2, \Delta J=2$, and 3) transitions gain intensity and can be observed.

Three of the rotational transitions $\left(4_{22}-3_{13}, 4_{40}-4_{31}\right.$, and $5_{42}-5_{33}$ ) were observed on two different laser lines, which allowed an accurate determination of the $g$-factors. Furthermore, a few transitions with $\Delta N \neq \Delta J$ could be measured, which enabled an accurate determination of the spinrotation constants.

The predictions and least-squares fitting of the data were both carried out using a computer program developed by Brown and Sears ${ }^{20}$ called ASYTOP. The effective Hamiltonian used by this program was developed by Bowater et al., ${ }^{21}$ who explained the derivation of the matrix elements and the assumptions inherent therein. The molecular Hamiltonian is expressed in a Hund's case (b) basis set of $\left|N K_{a} S J M_{J} I M_{I}\right\rangle$ (a nuclear spin-decoupled basis set). For the rotational Hamiltonian, the $A$ (asymmetric) reduction is used. ${ }^{22}$

The assignment of the LMR spectra was started by using the molecular parameters of Ref. 8 for predictions. Further


FIG. 2. Typical $\mathrm{H}_{2} \mathrm{O}^{+}\left(\tilde{X}^{2} B_{1}, \nu=0\right)$ LMR spectrum observed at $100.806 \mu \mathrm{~m} \mathrm{CH} \mathrm{C}_{3} \mathrm{OH}$ laser line in perpendicular $\left(\Delta M_{J}= \pm 1\right)$ polarization. The Zeeman components of the $2_{20}-1_{11}$ (triplets) and $6_{25}-6_{16}$ (singlets) transitions are shown.
transitions, especially the strongly perturbed and the transitions with $N=7$ and $K_{a}=4$, could be assigned only after analysis of the new data.

In the final analysis, previous measurements concerning $\mathrm{H}_{2} \mathrm{O}^{+}$in the ground state were included in addition to our LMR data. The data which have been used are listed below:
(1) 96 Zeeman components of 8 rotational transitions in the ground vibrational level of the $\widetilde{X}^{2} B_{1}$ state measured by laser magnetic resonance ${ }^{8}$ with an experimental uncertainty of 2 MHz
(2) 28 ground-state combination differences obtained from IR measurements of the $\nu_{3}$ band. ${ }^{10}$ The uncertainty of the combination differences was $0.008 \mathrm{~cm}^{-1}$.
(3) 251 ground-state combination differences obtained from the $\widetilde{A}^{2} A_{1}-\widetilde{X}^{2} B_{1}$ electronic emission spectrum. ${ }^{2}$ The accuracy of the measurements was $0.03 \mathrm{~cm}^{-1}$, giving the combination differences an uncertainty of $0.04 \mathrm{~cm}^{-1}$.

In the weighted nonlinear, least-squares fitting procedure, the weights were taken as the inverse squares of the assumed experimental precision (see Table II). As expected from the relative weights of the different data, the fit was
determined principally by the LMR data, although the inclusion of the combination differences obtained from the electronic emission spectrum reduced the correlations between the parameters and did not affect the parameter values. The inclusion of the combination differences from the IR data neither improved nor degraded the parameter values or standard deviations. During the analysis, it turned out that the indirect influence of the levels with $\Delta N= \pm 1$ and $\pm 2$ and with $\Delta K_{a}= \pm 2, \pm 4, \pm 6$ is considerable. Thus, these levels were all included in the basis set.

Thirty-two molecular parameters were simultaneously determined in the final fit of 375 lines and are listed in Table III. With this set of molecular parameters, all data are reproduced within their experimental precision. The overall standard deviation relative to the experimental uncertainties is 0.88 , whereas the deviation between observed and calculated line positions of our LMR data section is 2.2 MHz on average (deviation of LMR data of Ref. 8 is 2.4 MHz on average). Some of the parameters quoted in Table III ( $B, C, \varepsilon_{a a}$, $\varepsilon_{b b}, \varepsilon_{c c}, a_{F}, a_{I}, b_{I}, g_{S}^{a a}, g_{S}^{b b}, g_{S}^{c c}, g_{r}^{a a}, g_{r}^{b b}$, and $\left.g_{r}^{c c}\right)$ are linear combinations of parameters fitted by the ASYTOP program in order to give a comparison with parameters of pre-

TABLE III. Molecular parameters (in $\mathrm{cm}^{-1}$ ) for the ground-vibrational level of $\mathrm{H}_{2} \mathrm{O}^{+}$in the $\widetilde{X}^{2} B_{1}$ state (error limits are one standard deviation and refer to the last quoted digits). The A-reduced form of the Hamiltonian was used throughout, apart from Ref. 8, as marked.

| Parameter | [12] | [10] | [11] | [2] | [8] <br> (S-reduced) | [8] refitted | This work |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 29.0359 9(21) | 29.037 0(27) | 29.036 6(37) | 29.025 6(26) | 29.038 365(14) | 29.038 99(11) | 29.039 508(19) |
| B | 12.422 98(58) | 12.423 0(24) | 12.417 0(15) | 12.422 4(14) | $12.416055(10)$ | 12.421 23(12) | 12.420 949(14) |
| C | 8.469 21(58) | 8.469 1(16) | 8.468 4(13) | 8.4693 (14) | $8.4720857(73)$ | 8.468 37(12) | 8.468 088(14) |
| $\Delta_{N} \times 10^{3}$ | 1.015(13) | 1.063(14) | 0.937(50) | 1.00(2) | 0.866 89(24) | 0.9917 7(38) | 0.973 96(30) |
| $\Delta_{N K} \times 10^{3}$ | -5.007(71) | -5.07(6) | -5.63(19) | -4.73(10) | -5.043 8(43) | -5.067(55) | -5.078 8(32) |
| $\Delta_{K} \times 10^{3}$ | 44.20(63) | 43.75(60) | 45.72(73) | 41.87(18) | 44.983 5(73) | 45.276(61) | 45.7162 (81) |
| $\delta_{N} \times 10^{3}$ | $0.3817(24)$ | 0.3805 5(41) | 0.312(10) | 0.370(1) | 0.370 83(32) | 0.379 3(19) | 0.374 77(28) |
| $\delta_{K} \times 10^{3}$ | 1.795(50) | 1.91(10) | 1.60(12) | 1.97(19) | ... | 1.815(31) | $1.9308(52)$ |
| $H_{N} \times 10^{6}$ | 0.33(14) | 1.34(22) | $1.89(60)$ | ... | $\ldots$ | ... | ... |
| $H_{N K} \times 10^{6}$ | -5.1(11) | -14.3(17) | -9.7(29) | $\cdots$ | $\ldots$ | $\ldots$ |  |
| $H_{K N} \times 10^{6}$ | ... | 40(8) | 52(21) | $\ldots$ | -83.32(37) | -18.6(49) | -24.21(26) |
| $H_{K} \times 10^{6}$ | 151(44) | 70(31) | 203(27) | $\ldots$ | -259.55(70) | 205.7(59) | 259.35(95) |
| $H_{N}^{S} \times 10^{6}$ | ... | ... | ... | $\cdots$ | ... | ... | 0.126 4(52) |
| $H_{N K}^{S} \times 10^{6}$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | -1.440(64) |
| $H_{K}^{S} \times 10^{6}$ | $\ldots$ | $\ldots$ | $\cdots$ | $\ldots$ | $\cdots$ | $\ldots$ | 33.70(53) |
| $L_{K} \times 10^{6}$ | ... | ... | $\cdots$ | ... | ... | ... | 1.590(27) |
| $\varepsilon_{a a}$ | -1.088 2(43) | -1.082 8(51) | -1.104 9(72) | -1.092(18) | -1.089 817(83) | -1.089 199(84) | -1.088 215(63) |
| $\varepsilon_{b b}$ | -0.113 0(19) | -0.116 1(22) | $-0.1237(37)$ | -0.120(8) | -0.114 538(22) | -0.114 552(55) | -0.114 439(41) |
| $\varepsilon_{\text {cc }}$ | 0.003 2(19) | 0.002 82(5) | $-0.0061(25)$ | -0.004(8) | $0.001685(19)$ | 0.001 663(55) | 0.001 294(41) |
| $\Delta_{K}^{S} \times 10^{3}$ | $6.32(71)$ | 5.63(47) | 12.6(8) | 5(1) | 5.584(12) | 5.995(23) | 7.456(28) |
| $\left(\Delta_{N K}^{S}+\Delta_{K N}^{S}\right) \times 10^{3}$ | -0.72(23) | ... | -0.98(4) | ... | 0.170 8(33) | 0.033(13) | -0.543(12) |
| $\Delta_{N K}^{S} \times 10^{3}$ | ... | $\cdots$ |  | $\cdots$ | ... | ... | -0.266(22) |
| $\Delta_{N}^{S} \times 10^{3}$ | $\ldots$ | $\ldots$ |  | $\ldots$ | $\cdots$ | $\cdots$ | 0.023 4(15) |
| $\delta_{K}^{S}$ | $\ldots$ | $\ldots$ |  | $\ldots$ | $\cdots$ | $\ldots$ | ... |
| $\delta_{N}^{S} \times 10^{3}$ | $\ldots$ | $\ldots$ |  | $\ldots$ | $\ldots$ | $\ldots$ | 0.011 28(81) |
| $\Phi_{K}^{S} \times 10^{3}$ | $\ldots$ | $\ldots$ |  | $\ldots$ | $\ldots$ | $\cdots$ | -0.069 6(13) |
| $a_{F} \times 10^{3}$ | $\ldots$ | $\ldots$ |  | $\ldots$ | -2.525 4(70) | -2.518 0(72) | -2.506 2(91) |
| $a_{I} \times 10^{3}$ | $\ldots$ | $\ldots$ |  | $\ldots$ | 1.308(53) | 1.338(28) | 1.298(29) |
| $b_{1} \times 10^{3}$ | $\ldots$ | $\cdots$ |  | $\ldots$ | -0.577 1(63) | -0.586(24) | -0.600(26) |

${ }^{\#} d_{2}=-0.5162(12) \times 10^{-4} \mathrm{~cm}^{-1}$ (S-reduction).
vious work. The standard deviations of those parameters have been calculated from the deviation of the quoted parameters according to Gaussian quadrature formula (not taking into account the correlation between parameters).

## IV. DISCUSSION

The LMR data measured in this work give a significant extension to the measurements of Ref. 8 (see Fig. 1). The analysis of these new rotational transitions in combination with other data published previously leads to an improved set of ground-state molecular parameters (see Tables III and IV).

TABLE IV. $g$-factors for the ground-vibrational level of $\mathrm{H}_{2} \mathrm{O}^{+}$in the $\widetilde{X}^{2} B_{1}$ state (error limits are one standard deviation and refer to the last quoted digits).

|  |  | [8] refitted <br> (A-reduced) | This work | $[16]$ <br> (ab initio) |
| :--- | :---: | :---: | :---: | :---: |
| Parameters | $[8]$ | $2.01646(21)$ | $2.01833(14)$ | $2.01886(15)$ |
| $g_{S}^{a a}$ | $2.00650(13)$ | $2.00656(15)$ | $2.00657(13)$ | 2.006424 |
| $g_{S}^{b b}$ | $1.99918(21)$ | $2.00109(15)$ | $2.00211(13)$ | 2.002070 |
| $g_{S}^{c c}$ | $-6.892(72)$ | $-7.182(39)$ | $-6.742(43)$ |  |
| $g_{r}^{a a} \times 10^{3}$ | $-0.461(32)$ | $-0.586(30)$ |  |  |
| $g_{r}^{b b} \times 10^{3}$ | $-0.370(39)$ | $-0.492(32)$ | $0.421(30)$ |  |
| $g_{r}^{c c} \times 10^{3}$ | $0.501(32)$ | $0.396(32)$ |  |  |

Some parameters are determined for the first time and several others are refined. For a comparison, the molecular parameters of earlier studies are also given in Tables III and IV. The constants of Ref. 8 cannot be compared directly with ours because the S-reduced form of the rotational Hamiltonian was used in that work. Thus, we refitted the dataset of Ref. 8 with our program, which uses the A-reduced form. The data fitted slightly better than in Ref. 8, which could be seen from the residuals of each line. But the parameters have larger standard deviations than those for S-reduced Hamiltonian, which is probably due to the higher correlations between the parameters (for example, the spinrotational Hamiltonian has, in A-reduction, only matrix elements for $\Delta K_{a}$ $=0, \pm 2)$. On the other hand, the Hamiltonian in A-reduced form is easier to bring to diagonal form by computer methods. As seen from the refitted parameters in comparison with ours, we improved the accuracy of the rotational and centrifugal distortion constants by about one order of magnitude. The spin-rotation and the hyperfine splitting parameters, as well as the $g$-factors, are of the same accuracy as previous work.

As mentioned before, a number of $a b$ initio calculations have been carried out on $\mathrm{H}_{2} \mathrm{O}^{+}$; some of them investigated the hyperfine coupling constants of the ground state. A good agreement with the experimental value $a_{F}=-75.13$ (27)

TABLE V. Rotational term values (in $\mathrm{cm}^{-1}$ ) for the $\widetilde{X}^{2} B_{1}$ ground state $\left[T\left(0_{00}\right)=0\right]$.

| $N$ | $K_{a}$ | $K_{c}$ | [2] | $\begin{aligned} & \quad F_{1} \\ & \text { this work } \end{aligned}$ | difference | [2] | $\begin{aligned} & \quad F_{2} \\ & \text { this work } \end{aligned}$ | difference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | 20.857 | 20.85524 | 0.002 | 20.950 | 20.93747 | 0.013 |
| 1 | 1 | 1 | 37.188 | 37.19741 | -0.009 | 38.010 | 38.01100 | -0.001 |
| 1 | 1 | 0 | 41.100 | 41.11005 | -0.010 | 42.010 | 42.01115 | -0.001 |
| 2 | 0 | 2 | 61.955 | 61.95144 | 0.004 | 62.109 | 62.09015 | 0.019 |
| 2 | 1 | 2 | 75.094 | 75.10327 | -0.009 | 75.612 | 75.60769 | 0.004 |
| 2 | 1 | 1 | 86.845 | 86.85337 | -0.008 | 87.508 | 87.50238 | 0.006 |
| 2 | 2 | 1 | 135.672 | 135.67974 | -0.008 | 137.516 | 137.50442 | 0.012 |
| 2 | 2 | 0 | 136.293 | 136.30047 | -0.007 | 138.139 | 138.12768 | 0.011 |
| 3 | 0 | 3 | 122.117 | 122.11128 | 0.006 | 122.329 | 122.30162 | 0.027 |
| 3 | 1 | 3 | 131.478 | 131.48612 | -0.008 | 131.899 | 131.88851 | 0.010 |
| 3 | 1 | 2 | 154.892 | 154.89891 | -0.007 | 155.516 | 155.50345 | 0.013 |
| 3 | 2 | 2 | 198.465 | 198.47342 | -0.008 | 199.900 | 199.88923 | 0.011 |
| 3 | 2 | 1 | 201.483 | 201.49353 | -0.011 | 202.900 | 202.88966 | 0.010 |
| 3 | 3 | 1 | 288.757 | 288.71477 | 0.042 | 291.565 | 291.48641 | 0.079 |
| 3 | 3 | 0 | 288.820 | 288.77799 | 0.042 | 291.628 | 291.54944 | 0.079 |
| 4 | 0 | 4 | 200.025 | 200.01810 | 0.007 | 200.313 | 200.27990 | 0.033 |
| 4 | 1 | 4 | 205.979 | 205.98667 | -0.008 | 206.362 | 206.34565 | 0.016 |
| 4 | 1 | 3 | 244.589 | 244.59297 | -0.004 | 245.235 | 245.21233 | 0.023 |
| 4 | 2 | 3 | 281.534 | 281.54461 | -0.011 | 282.742 | 282.73042 | 0.012 |
| 4 | 2 | 2 | 289.900 | 289.91232 | -0.012 | 291.119 | 291.10722 | 0.012 |
| 4 | 3 | 2 | 373.658 | 373.62281 | 0.035 | 375.952 | 375.88588 | 0.066 |
| 4 | 3 | 1 | 374.086 | 374.05256 | 0.033 | 376.379 | 376.31520 | 0.064 |
| 4 | 4 | 1 | 495.619 | 495.63116 | -0.012 | 499.316 | 499.28331 | 0.033 |
| 4 | 4 | 0 | 495.625 | 495.63661 | -0.012 | 499.321 | 499.28873 | 0.032 |
| 5 | 0 | 5 | 294.755 | 294.75126 | 0.004 | 295.064 | 295.02871 | 0.035 |
| 5 | 1 | 5 | 298.215 | 298.22460 | -0.010 | 298.584 | 298.56418 | 0.020 |
| 5 | 1 | 4 | 354.940 | 354.94082 | -0.001 | 355.631 | 355.59952 | 0.031 |
| 5 | 2 | 4 | 384.372 | 384.38380 | -0.012 | 385.471 | 385.45552 | 0.015 |
| 5 | 2 | 3 | 401.892 | 401.90610 | -0.014 | 403.024 | 403.00826 | 0.016 |
| 5 | 3 | 3 | 479.658 | 479.62753 | 0.030 | 481.661 | 481.60218 | 0.059 |
| 5 | 3 | 2 | 481.310 | 481.28466 | 0.025 | 483.298 | 483.24382 | 0.054 |
| 5 | 4 | 2 | 601.896 | 601.91096 | -0.015 | 605.041 | 605.02309 | 0.018 |
| 5 | 4 | 1 | 601.943 | 601.95841 | -0.015 | 605.088 | 605.07030 | 0.018 |
| 5 | 5 | 1 | 753.374 | 754.10064 | -0.727 | 757.847 | 758.56849 | -0.721 |
| 5 | 5 | 0 | 753.375 | 754.10108 | -0.726 | 757.847 | 758.56893 | -0.722 |
| 6 | 0 | 6 | 405.959 | 405.96298 | -0.004 | 406.295 | 406.26257 | 0.032 |
| 6 | 1 | 6 | 407.850 | 407.86446 | -0.014 | 408.216 | 408.19591 | 0.020 |
| 6 | 1 | 5 | 484.628 | 484.62744 | 0.001 | 485.391 | 485.35102 | 0.040 |
| 6 | 2 | 5 | 506.463 | 506.47296 | -0.010 | 507.503 | 507.48216 | 0.021 |
| 6 | 2 | 4 | 537.054 | 537.06921 | -0.015 | 538.163 | 538.14097 | 0.022 |
| 6 | 3 | 4 | 606.602 | 606.57058 | 0.031 | 608.411 | 608.35111 | 0.060 |
| 6 | 3 | 3 | 611.193 | 611.17539 | 0.018 | 612.988 | 612.94083 | 0.047 |
| 6 | 4 | 3 | 729.401 | 729.40645 | -0.005 | 732.181 | 732.16135 | 0.020 |
| 6 | 4 | 2 | 729.625 | 729.63319 | -0.008 | 732.405 | 732.38738 | 0.018 |
| 6 | 5 | 2 | 881.026 | 881.71322 | -0.687 | 884.938 | 885.63219 | -0.694 |
| 6 | 5 | 1 | 881.031 | 881.71785 | -0.687 | 884.943 | 885.63679 | -0.694 |
| 6 | 6 | 1 | 1058.043 | 1061.18641 | -3.143 | 1063.144 | 1066.44230 | -3.298 |
| 6 | 6 | 0 | 1058.043 | 1061.18645 | -3.143 | 1063.144 | 1066.44234 | -3.298 |
| 7 | 0 | 7 | 533.622 | 533.63891 | -0.017 | 533.977 | 533.95252 | 0.024 |
| 7 | 1 | 7 | 534.618 | 534.64207 | -0.024 | 534.986 | 534.97097 | 0.015 |
| 7 | 1 | 6 | 632.208 | 632.20692 | 0.001 | 633.028 | 632.98227 | 0.046 |
| 7 | 2 | 6 | 647.235 | 647.24129 | -0.006 | 648.246 | 648.21722 | 0.029 |
| 7 | 2 | 5 | 694.445 | 694.46266 | -0.018 | 695.570 | 695.54222 | 0.028 |
| 7 | 3 | 5 | 754.111 | 754.06797 | 0.043 | 755.800 | 755.72758 | 0.072 |
| 7 | 3 | 4 | 764.440 | 764.43060 | 0.009 | 766.114 | 766.06994 | 0.044 |
| 7 | 4 | 4 | 878.125 | 878.10375 | 0.021 | 880.642 | 880.60015 | 0.042 |
| 7 | 4 | 3 | 878.891 | 878.87692 | 0.014 | 881.417 | 881.38257 | 0.034 |
| 7 | 5 | 3 | 1029.806 | 1030.41256 | -0.607 | 1033.319 | 1033.94360 | -0.625 |
| 7 | 5 | 2 | 1029.832 | 1030.43888 | -0.607 | 1033.345 | 1033.96977 | -0.625 |
| 7 | 6 | 2 | 1207.209 | 1210.18753 | -2.979 | 1211.762 | 1214.89537 | -3.133 |
| 7 | 6 | 1 | 1207.210 | 1210.18796 | -2.978 | 1211.762 | 1214.89579 | -3.134 |
| 7 | 7 | 1 | 1404.729 | 1413.24240 | -8.513 | 1410.279 | 1419.33848 | -9.059 |
| 7 | 7 | 0 | 1404.729 | 1413.24241 | -8.513 | 1410.279 | 1419.33848 | -9.059 |
| 8 | 0 | 8 | 677.812 | 677.84256 | -0.031 | 678.180 | 678.16476 | 0.015 |
| 8 | 1 | 8 | 678.325 | 678.36124 | -0.036 | 678.699 | 678.69030 | 0.009 |
| 8 | 1 | 7 | 796.481 | 796.47789 | 0.003 | 797.359 | 797.30741 | 0.052 |

TABLE V. (Continued.)

|  |  |  | $F_{1}$ <br> $N$ |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $K_{a}$ | $K_{c}$ | $[2]$ | this work ${ }^{\mathrm{a}}$ | difference | $F_{2}$ <br> this work | difference |  |  |
| 8 | 2 | 7 | 806.133 | 806.13224 | 0.001 | 807.133 | 807.09234 | 0.041 |
| 8 | 2 | 6 | 872.817 | 872.83927 | -0.022 | 873.972 | 873.93947 | 0.033 |
| 8 | 3 | 6 | 921.690 | 921.62226 | 0.068 | 923.306 | 923.20489 | 0.101 |
| 8 | 3 | 5 | 941.393 | 941.39527 | -0.002 | 942.998 | 942.95742 | 0.041 |
| 8 | 4 | 5 | 1047.905 | 1047.82940 | 0.076 | 1050.263 | 1050.16996 | 0.093 |
| 8 | 4 | 4 | 1050.072 | 1050.02039 | 0.052 | 1052.417 | 1052.34681 | 0.070 |
| 8 | 5 | 4 | 1199.684 | 1200.15716 | -0.473 | 1202.906 | 1203.40505 | -0.499 |
| 8 | 5 | 3 | 1199.788 | 1200.26410 | -0.476 | 1203.010 | 1203.51155 | -0.502 |
| 8 | 6 | 3 | 1377.378 | 1380.08500 | -2.707 | 1381.521 | 1384.38775 | -2.867 |
| 8 | 6 | 2 | 1377.380 | 1380.08780 | -2.708 | 1381.524 | 1384.39053 | -2.867 |
| 8 | 7 | 2 | 1575.591 | 1583.66732 | -8.076 | 1580.621 | 1589.21377 | -8.593 |
| 8 | 7 | 1 | 1575.591 | 1583.66736 | -8.076 | 1580.621 | 1589.21381 | -8.593 |
| 8 | 8 | 1 | 1787.638 | 1805.01081 | -17.373 | 1793.423 | 1812.12896 | -18.706 |
| 8 | 8 | 0 | 1787.638 | 1805.01081 | -17.373 | 1793.423 | 1812.12896 | -18.706 |

${ }^{\mathrm{a}}$ The estimated uncertainties of the term values are about $0.00004 \mathrm{~cm}^{-1}$ for levels up to $N=7$ and $K_{a}=4$. All other term energies are extrapolated.

MHz is achieved in Ref. 14. This group calculated the hyperfine coupling constant $a_{F}$ to be -81.0 MHz by the multireference configuration method, which is within $10 \%$ of the experimental value. Recently, a multireference configuration interaction calculation of electronic $g$-tensors was carried out. ${ }^{16}$ This group obtained values for the anisotropic $g$-tensor components $\Delta g_{l}^{a a}, \Delta g_{l}^{b b}$, and $\Delta g_{l}^{c c}$ (differences between the actual $g$-tensor components along molecule-fixed axes and the free electron $g$-factor $g_{S}=2.002319$ ) of 0.015733 , 0.004105 , and -0.000249 , respectively, which are in excellent agreement with the experimental values of $0.01654(15)$, $0.00425(13)$, and $-0.00021(13)$, respectively (see also Table IV).

From our set of molecular parameters, we recalculated the term values of the ground-state rotational level up to $N$ $=8$ and compared them to those of Ref. 2 (see Table V).

TABLE VI. Zero-field predictions of the $1_{11}-0_{00}$ transition frequencies (in $\mathrm{cm}^{-1}$ ) including hyperfine splitting.

| $J^{\prime}$ | $J^{\prime \prime}$ | $F^{\prime}$ | $F^{\prime \prime}$ | Transition frequency $^{\text {a }}$ | Relative intensity calculated $^{\mathrm{b}}$ |
| :--- | :--- | :--- | :--- | :---: | :---: |
| $3 / 2$ | $1 / 2$ | $5 / 2$ | $1 / 2$ | 37.19544 | (not allowed) |
| $3 / 2$ | $1 / 2$ | $3 / 2$ | $1 / 2$ | 37.19741 | 20 |
| $3 / 2$ | $1 / 2$ | $1 / 2$ | $1 / 2$ | 37.19860 | 16 |
| $3 / 2$ | $1 / 2$ | $5 / 2$ | $3 / 2$ | 37.19920 | 54 |
| $3 / 2$ | $1 / 2$ | $3 / 2$ | $3 / 2$ | 37.20117 | 16 |
| $3 / 2$ | $1 / 2$ | $1 / 2$ | $3 / 2$ | 37.20236 | 2 |
| $1 / 2$ | $1 / 2$ | $1 / 2$ | $1 / 2$ | 38.01100 | 2 |
| $1 / 2$ | $1 / 2$ | $3 / 2$ | $1 / 2$ | 38.01165 | 16 |
| $1 / 2$ | $1 / 2$ | $1 / 2$ | $3 / 2$ | 38.01475 | 16 |
| $1 / 2$ | $1 / 2$ | $3 / 2$ | $3 / 2$ | 38.01541 | 20 |

[^1]Large deviations exist for levels with $K_{a}=3$ and with $K_{a}$ $>4$ (a difference larger than the absolute accuracy of 0.05 $\mathrm{cm}^{-1}$ mentioned in Ref. 2). The large differences for $K_{a}$ $>4$ levels (for $K_{a}=5$ about $0.7 \mathrm{~cm}^{-1}$, for $K_{a}=6$ about 3 $\mathrm{cm}^{-1}$, for $K_{a}=7$ about $9 \mathrm{~cm}^{-1}$, and for $K_{a}=8$ about 17 $\mathrm{cm}^{-1}$ ) are because those term values are only extrapolated from measurements involving levels with $K_{a} \leqslant 4$, but the differences of the $K_{a}=3$ levels (up to $0.08 \mathrm{~cm}^{-1}$ ) cannot be explained.

Table VI gives zero-field predictions of the $1_{11}-0_{00}$ transition including hyperfine splitting (see Fig. 3). Calculations of relative intensities are also given. Measurements using a TUFIR spectrometer would give more accurate line frequencies, which in turn would help astrophysicists identify $\mathrm{H}_{2} \mathrm{O}^{+}$from far-infrared lines in comets or interstellar clouds.


FIG. 3. Energy level diagram (including hyperfine splitting) of the $1_{11}-0_{00}$ transition of $\mathrm{H}_{2} \mathrm{O}^{+}$in the $(0,0,0)$ level of the $\widetilde{X}^{2} B_{1}$ state.

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[^1]:    ${ }^{a}$ Absolute uncertainties of the transition wavenumbers are about 0.00006 $\mathrm{cm}^{-1}$. Relative uncertainties are smaller than this.
    ${ }^{\mathrm{b}}$ Calculated according to the relation:
    Intensity $\propto\left(2 F^{\prime}+1\right)(2 F+1)\left\{\begin{array}{lll}I & J^{\prime} & F^{\prime} \\ 1 & F & J\end{array}\right\}^{2}\left(2 J^{\prime}+1\right)(2 J+1)$

    $$
    \times\left\{\begin{array}{ccc}
    S & N^{\prime} & J^{\prime} \\
    1 & J & N
    \end{array}\right\}^{2}\left(2 N^{\prime}+1\right)(2 N+1)\left(\begin{array}{ccc}
    N^{\prime} & 1 & N \\
    -K^{\prime} & 1 & K
    \end{array}\right)^{2}
    $$

