CHEF: Convex Hull of Elliptic Features for 3D Blob Detection *

Qing Yang and Bahram Parvin
Computing Sciences
Lawrence Berkeley National Laboratory
Berkeley, CA 94720

Abstract

We present an efficient protocol for robust detection of 3D blobs from volumetric datasets. The approach has three steps. The first step of the process detects elliptic features by classifying the Hessian of the scale space representation of the volume data. These features are then grouped into 3D connected components, which are subsequently partitioned by computing a convex hull of each connected component. The proposed framework has been applied to a database of multicellular systems for detailed quantitative analysis.

Index terms: Convex hull, blob detection, cell segmentation.

1. Introduction

Automatic detection of 3D blobs from volumetric datasets is an important step in large-scale analysis of scientific data. These blobs may represent organization of nuclei in a cultured colony, homogeneous regions in geophysical data, tumor locations in MRI or CT data, etc. This paper presents an efficient approach for detection of individual nuclei in a cell cultured colony. In general the nucleus of a cell is ellipsoidal, but neighboring nuclei may overlap, thus making delineation a necessary component. Successful detection of each nucleus and its relationship to the organization of the colony provides the basis for better understanding of cell-cell communication under a variety of treatments and conditioning [11, 2, 3, 12]. Our experience indicates that detection of blobs in 2D is more complex due to inherent lack of 3D information, however, a more efficient technique is needed to detect blobs in 3D. In general, analysis of these images is complex due to the fact that nuclei of interest (1) do not respond uniformly to the fluorescent compounds, (2) may have many internal substructures, and (3) overlap each other as a result of cell division. Figure 1 shows an example of a cell culture colony that is imaged with a deconvolution microscope. The first step of our algorithm is extraction of elliptic features. A multiscale representation of the image is generated and the Hessian is computed to detect and classify each point in the image. If the Hessian is negative (positive) definite then the point is classified as a bright (dark) elliptic feature. This classification is then used to group similar features using a 3D connected component algorithm. Although false labeling is unavoidable, sufficient information can be gathered so that a higher-level technique can group partial information into a whole. This higher level constraint is expressed in terms of convexity and implemented as a convex hull for improved efficiency. An efficient implementation is also proposed.

Organization of this paper is as follows. Section 2 provides a summary of the method for detecting elliptic features. Section 3 presents the grouping technique using convex hulls. Section 4 gives the detailed algorithm and the experimental results. Section 5 concludes the paper.

2. Elliptic Features

The underlying assumption for detection of the nuclei in the colony is that each individual nucleus is convex. This is true in most cases. The first step is to detect elliptic features. Let the linear scale-space representation of the original image $I_0(x, y, z)$ at scale σ be given by:

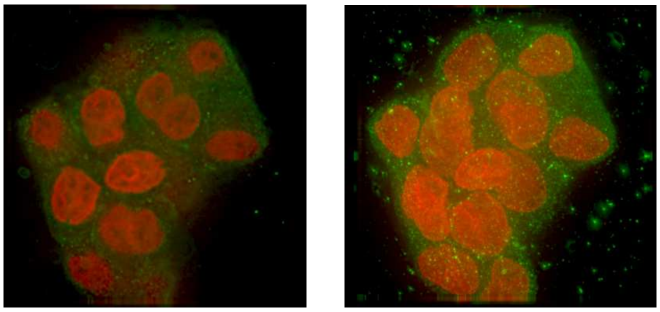
$$I(x, y, z; \sigma) = I_0(x, y, z) * G(x, y, z; \sigma) \tag{1}$$

where $G(x, y, z; \sigma)$ is the Gaussian kernel. Let $I(x, y, z; \sigma)$ represent $I(x, y, z)$ for simplicity. At each point (x, y, z), its iso-intensity surface is defined by

$$I(x + \Delta x, y + \Delta y, z + \Delta z) = I(x, y, z) \tag{2}$$

where $\delta := (\Delta x, \Delta y, \Delta z)$ is the displacement vector. Expanding and truncating the above equation using Taylor's

*Research was funded by Director of Lawrence Berkeley National Laboratory, Medical Sciences Division, and Mathematical and Information and Computing Sciences Division of the U.S. Department of Energy under Contract No. DE-AC0376F00098 with the University of California. The publication number is LBNL-50178. Email: qyang@media.lbl.gov. Web site is: http://vision.lbl.gov.



(a) (b)

Figure 1. An example of volume data indicates that the nuclei are noisy, have many internal substructures, and overlap in the colony: (a) a slice of 3D colony; (b) one view of the volume-rendered data.

series, we have the following estimation:

$$\frac{1}{2}\delta H(x,y,z)\delta^T + (I_x, I_y, I_z)\delta^T = 0 \tag{3}$$

where

$$H(x,y,z) = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{pmatrix}$$

is the Hessian matrix of $I(x,y,z)$. This quadratic equation represents a general quadric surface, embracing ellipsoids, cylinders, cones, paraboloids and hyperboloids. Equation 2 divides the entire image domain into two parts:

$$\frac{1}{2}\delta H(x,y,z)\delta^T + (I_x, I_y, I_z)\delta^T > 0 \tag{4}$$

and

$$\frac{1}{2}\delta H(x,y,z)\delta^T + (I_x, I_y, I_z)\delta^T < 0 \tag{5}$$

If $H(x,y,z)$ is positive definite, then region defined by Equation 4 is locally convex. Similarly, if $H(x,y,z)$ is negative definite, then region defined by Equation 5 is locally convex. In summary, we have:

1. $H(x,y,z) > 0$. Then we have $I_{xx} > 0$, $I_{yy} > 0$, $I_{zz} > 0$, and hence $I_{xx} + I_{yy} + I_{zz} > 0$. Positive Laplacian means that (x,y,z) is a "dark point".

2. $H(x,y,z) < 0$. Then we have $I_{xx} < 0$, $I_{yy} < 0$, $I_{zz} < 0$, and hence $I_{xx} + I_{yy} + I_{zz} < 0$. Negative Laplacian means that (x,y,z) is a "bright point".

Classifying each pixel as an elliptic point (either bright or dark) is a simple and efficient way to separate background from objects of interest in the presence of noise, shading artifacts, and variable thresholds in the image. From the above analysis we introduce the following definition:

Definition 1 *A point is a* bright(dark) elliptic feature *at scale σ if the Hessian matrix of $I(x,y,z;\sigma)$ is negative(positive) definite at this point.*

Restricting our discussion to 2D will help us understand this feature better. The Hessian matrix of 2D image $I(x,y)$ is

$$H(x,y) = \begin{pmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{pmatrix}$$

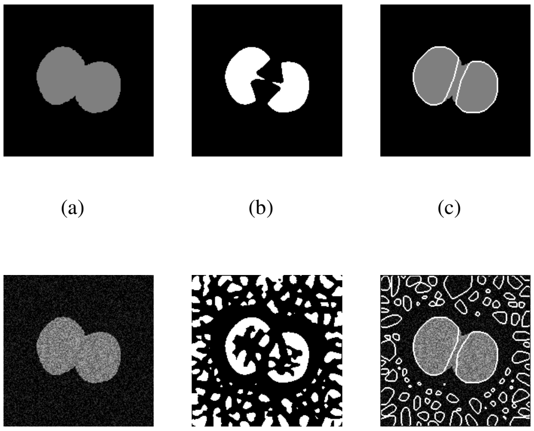
The set of bright elliptic features is $\{(x,y)|H(x,y) < 0\} = \{(x,y)|I_{xx} + I_{yy} < 0 \wedge I_{xx}I_{yy} > I_{xy}^2\}$. Similarly, the set of dark elliptic features is $\{(x,y)|H(x,y) > 0\} = \{(x,y)|I_{xx} + I_{yy} > 0 \wedge I_{xx}I_{yy} > I_{xy}^2\}$. In classic scale-space theory [7], $I_{xx}I_{yy} - I_{xy}^2$ is referred to as the elliptic feature, which has been extended to 3D in our paper.

3. Grouping and Completion

To demonstrate that elliptic features can be grouped into meaningful regions, we show an example in 2D. Figure 2 shows two touching blobs without and with added random noise. In both cases, regions are labeled, small regions are initially removed, and dominant regions are represented using a convex hull. Once a convex region is hypothesized, filtered small regions that are contained in each convex set are incorporated again to increment confidence in convexity. In general, grouping and visual organization can occur at many scales. Some of these techniques are at low level [6, 9, 10, 5, 4], while others are at mid to high level. Our formulation focuses on regions (as opposed to contours) by constructing a convex hull of each region. The critical parameter is the size of the known objects of interest that are determined empirically. This system is known as **CHEF**, that is, **C**onvex **H**ull of **E**lliptic **F**eatures.

The convex hull of a set of points is the smallest convex set that contains the points. It is a fundamental construction for mathematics and computational geometry[8, 1]. The computational complexity for the construction of the convex hull of N points is known to be $O(N \log N)$ for 2D and 3D datasets. And it is also known that for simple 2D polygons, the complexity can be as low is $O(N)$. An efficient algorithm, known as QHull [1] has also been recently proposed. From the endpoint application perspective, the scale of objects of interest can be used to bound the object between a set parallel line (2D) or parallel plane (3D) at equal interval. Let's denote the convex hull of point set V

$$\{v|\alpha v_1 + (1-\alpha)v_2, \forall v_1, v_2 \in V, \alpha \in [0,1]\} \tag{6}$$



(a) (b) (c)

(d) (e) (f)

Figure 2. Convex hull of elliptic regions: (column 1) Original images; (column 2) elliptic features; (3) convex hull of dominant regions.

as $hull(V)$. Consider all the planes with normal vector (n_x, n_y, n_z). Let

$$F_{\min}(n_x, n_y, n_z; V) = \min\{n_x x + n_y y + n_z z | (x, y, z) \in V\}$$

and

$$F_{\max}(n_x, n_y, n_z; V) = \max\{n_x x + n_y y + n_z z | (x, y, z) \in V\}$$

It is easy to prove

Lemma 1 *For any* $(x, y, z) \in hull(V)$, *we have*

$$F_{\min}(n_x, n_y, n_z; V) \le n_x x + n_y y + n_z z \le F_{\max}(n_x, n_y, n_z; V)$$

Recall that any 3D unit vector can be represented in polar coordinates as

$$\begin{cases} n_x &= \cos\theta \sin\varphi \\ n_y &= \sin\theta \sin\varphi \\ n_z &= \cos\varphi \end{cases}$$

where $\theta \in [0, 2\pi)$ and $\varphi \in [0, \pi)$. Next a set of unit vectors that are uniformly distributed in the half space $\{(x, y, z)|z \ge 0, (x, y, z) \in \Re^3\}$ are selected:

$$\begin{cases} n_x(i,j) &= \cos\frac{i\pi}{2N}\sin\frac{j\pi}{2N} \\ n_y(i,j) &= \sin\frac{i\pi}{2N}\sin\frac{j\pi}{2N} \\ n_z(i,j) &= \cos\frac{j\pi}{2N} \end{cases}$$

where $i = 0, 1, \cdots, 4N - 1$ and $j = 0, 1, \cdots, N$. Now consider the polyhedra defined by

$$\begin{aligned} &P(V, N) := \{(x, y, z)|F_{\min}(i, j) \le n_x(i, j)x + \\ &n_y(i, j)y + n_z(i, j)z \le F_{\max}(i, j), \forall i = 0, \\ &1, \cdots, 4N - 1; j = 0, 1, \cdots, N\} \end{aligned} \quad (7)$$

where $F_{\min}(i, j) := F_{\min}(n_x(i, j), n_y(i, j), n_z(i, j); V)$ and $F_{\min}(i, j) := F_{\max}(n_x(i, j), n_y(i, j), n_z(i, j); V)$. $P(V, N)$ has the following properties:

Lemma 2 *1.* $hull(V) \subseteq P(V, N)$.

2. If V *is a finite set, when* $N \to +\infty$, $P(V, N) \to hull(V)$.

which show that $P(V, N)$ is an approximation of $hull(V)$. This technique has good performance for degenerate cases. $P(V, N)$ is in fact the region bounded by a group of parallel planes. In our experimental dataset, a good approximation is obtained when $N = 8$.

4. Summary and Experimental Results

In this section, above techniques are extended to 3D, implementation details are expressed, and results on real examples are shown.

CHEF algorithm

Step 0) Select a scale σ, *and a positive integer* N*(e.g.,* 4 *or* 8*).*

Step 1) Convolve the original image $I_0(x, y, z)$ *with the Gaussian kernel at scale* σ*:* $I(x, y, z) := I_0(x, y, z) * G(x, y, z; \sigma)$.

Step 2) Compute the bright elliptic features $S = \{(x, y, z)|H(x, y, z) < 0\}$ *where* $H(x, y, z)$ *is the Hessian matrix of* $I(x, y, z)$.

Step 3) Decompose S *to disjoint connected components* $S = \bigcup_{i=1}^{n} S_i$ *where* $S_i \cap S_j = \phi, \forall i \ne j$.

Step 4) Compute $P(S_i, N), i = 1, 2, \cdots n$.

Step 5) For each pair $(i, j) \in 1..n \times 1..n$, *merge* $P(S_i, N)$ *and* $P(S_j, N)$ *if* $P(S_i, N) \bigcap P(S_j, N) \ne \phi$. *If no merging appears, then stop; otherwise goto Step 3.*

The proposed approach is structural and uses known knowledge of the size of the blobs of interest. The main advantages are:

1. The approach is a region-based method and is simpler to deal with than edge-based techniques.
2. Touching cells can be separated simultaneously.
3. Good performance is obtained for handling 3D data.

Figure 3 shows the segmentation of the image shown in Figure 1. We have applied our approach to 12 other datasets of cell cultured colony with results shown in Figure 4.

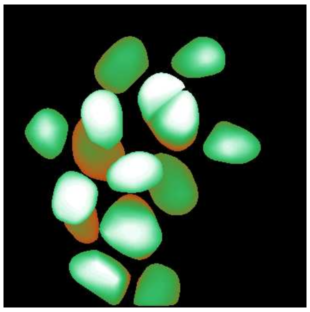


Figure 3. Visualization of 3D depth map of final segmentation of Figure 1.

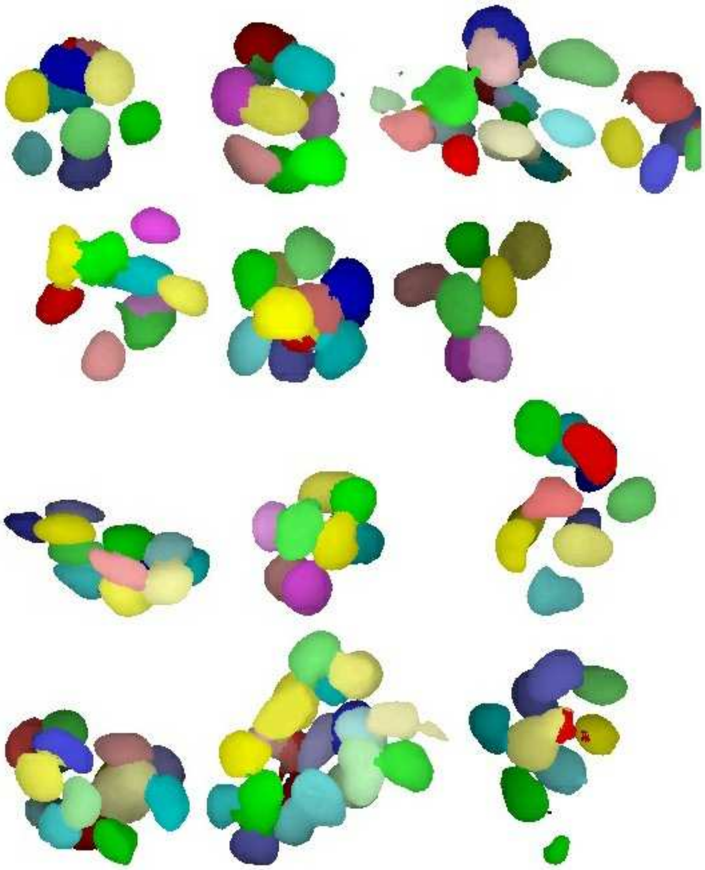


Figure 4. Segmentation results on a number of cultured colonies.

5. Conclusion

Detection of elliptic features is a critical step in extraction of blobs in biological datasets. They reveal pertinent structures such as nucleus and mitochondria. We have developed an efficient and robust technique for detection of blobs from 3D biological datasets. This is achieved by computing the Hessian matrices to generate reliable seed features which are grouped further by connectedness. 3D connected components are further represented by computing the convex hull of every connected seed region. The key parameter is the size of known objects of interest. If this can be determined, then elliptic features are reliable seeds for blob detection through convex hull representation.

References

[1] C.B. Barber, D.P. Dobkin, and H. Huhdanpaa. The quickhull algorithm for convex hulls. *TMS*, 22(4):469–483, December 1996.

[2] G. Cong and B. Parvin. Model-based segmentation of nuclei. In *CVPR99*, pages I:256–261, 1999.

[3] A. Garrido and N.P. de la Blanca. Applying deformable templates for cell image segmentation. *PR*, 33(5):821–832, May 2000.

[4] D. Geiger, H.K. Pao, and N. Rubin. Salient and multiple illusory surfaces. In *CVPR98*, pages 118–124, 1998.

[5] G. Guy and G.G. Medioni. Inferring global perceptual contours from local features. *IJCV*, 20(1/2):113–133, 1996.

[6] D.A. Huffman. A duality concept for the analysis of polyhedral scenes. *MI*, 8:475–492, 1977.

[7] T. Lindeberg. Scale-space theory: A basic tool for analyzing structures at different scales. *Journal of Applied Statistics*, pages 225–270, 1994.

[8] C. Ronse. A bibliography on digital and computational convexity (1961-1988). *PAMI*, 11(2):181–190, February 1989.

[9] S. Ullman. Filling the gaps: The shape of subjective contours and a model for their generation. In *MIT AI*, 1976.

[10] L.R. Williams and A.R. Hanson. Perceptual completion of occluded surfaces. In *CVPR94*, pages 104–112, 1994.

[11] H.S. Wu, J. Barba, and J. Gil. A parametric fitting algorithm for segmentation of cell images. *IEEE Trans. on Biomedical Engineering*, 45(3):400–7, 1998.

[12] H.S. Wu, J. Barba, and J. Gil. Iterative thresholding for segmentation of cells from noisy images. *Journal of Microscopy*, 197(3):296–304, 2000.