PRIOR PROBABILITIES: AN INFORMATION-THEORETIC APPROACH

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Abstract

General theoretical principles that enable the derivation of prior probabilities are of interest both in data analysis and, more broadly, in the foundations of probability theory [1]. The aim of this paper is to present a derivation of a multinomial prior based on a novel theoretical principle, to show how priors of interest in both data analysis and physics can be obtained from multinomial priors, and to investigate the reasons for the dissimilarities between this multinomial prior and other multinomial priors that have been suggested.

The paper is structured as follows. First, a novel derivation of the multinomial prior $\Pr(P_1, P_2, \ldots, P_N) \propto 1/\sqrt{P_1 P_2 \ldots P_N}$ is presented that proceeds from an intuitively plausible information-theoretic invariance principle.

Second, it is shown that any multinomial prior induces a prior over any parameters of which a probability distribution is a function. In particular, it is shown that the above-mentioned multinomial prior leads to the well-known priors, $\Pr(\mu|I)$, and $\Pr(\sigma|I)$, for the mean and standard deviation of a Gaussian probability distribution.

Third, the above multinomial prior is compared with Jaynes' multinomial prior [1], $\Pr(P_1, P_2, \ldots, P_N) \propto 1/P_1 P_2 \ldots P_N$, and with the entropic prior [2], $\Pr(P_1, P_2, \ldots, P_N) \propto 1/\sqrt{P_1 P_2 \ldots P_N} \exp(-\alpha \sum P_i \ln P_i)$, with a view to discovering the reasons for their dissimilarities. In particular, the validity of Jaynes multinomial prior is brought into question by showing that it leads to priors over μ and σ for a Gaussian probability distribution which are in conflict with the broadly-accepted priors over these quantities. Since Jaynes' derivation of his multinomial prior proceeds from a transformation group principle that is theoretically well-founded, this conflict suggests that Jaynes' derivation contains particular assumptions that are incorrect or implausible. Consequently, an analysis of this derivation is made with a view to uncovering these problematic assumptions.

References:

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