University of Wisconsin - Madison

MADPH-95-869

February 1995

A one parameter representation for the Isgur-Wise function

M.G. Olsson and Siniša Veseli

Department of Physics, University of Wisconsin, Madison, WI 53706

Abstract

We use a $1S$ lattice QCD heavy-light wavefunction to generate a single parameter, model independent description of the Isgur-Wise function. Using recent data we find the zero-recoil slope to be $\xi'(1) = -1.16 \pm 0.17$, while the second derivative turns out to be $\xi''(1) = 2.64 \pm 0.74$.

1 Introduction

The recent development of the Heavy Quark Effective Theory [1] yields an expression for the $\bar{B} \to D^{(*)} l \bar{\nu}_l$ decay rate in terms of a single unknown form factor, the Isgur-Wise function (IW). This function is absolutely normalized at zero recoil point up to corrections of order $\frac{1}{m_Q^2}$ [2]. It is currently believed that these corrections can be calculated with less than 5% uncertainty [3], which would allow a precise determination of the CKM matrix element $|V_{cb}|$ from the study of $\bar{B} \to D^{(*)} l \bar{\nu}_l$ decay as a function of the $D^{(*)}$ meson recoil.

In this note we take a somewhat different approach. We use an established LQCD result [4, 5] for the heavy-light wavefunction to express the IW function in terms of a single parameter, and then constrain that parameter using recent improved experimental results by the CLEO II group [6] in exclusive semileptonic $\bar{B} \to D^{(*)} l \bar{\nu}_l$ decay [7]. The procedure which we describe here is self contained and can be applied to newer LQCD results.

2 Numerical procedure and results

If one knows the heavy-light meson wavefunction in its rest frame, and the energy of light degrees of freedom E_q, then the IW function for a given $\omega = v \cdot v'$ (where v and v' are 4-velocities of two hadrons), can be computed [8] from

$$\xi(\omega) = \frac{2}{\omega + 1} \langle\, j_0(2E_q \sqrt{\frac{\omega - 1}{\omega + 1}} r) \rangle \ , \tag{1}$$

where

$$\langle A \rangle = \int_0^\infty dr\ r^2 R(r) A(r) R(r)\ . \tag{2}$$

From the above the first and second derivatives at the zero recoil point are [8, 9],

$$\xi'(1) \ = \ -(\frac{1}{2} + \frac{1}{3} E_q^2 \langle r^2 \rangle)\ , \tag{3}$$

$$\xi''(1) \;=\; \frac{1}{2} + \frac{2}{3} E_q^2 \langle r^2 \rangle + \frac{1}{15} E_q^4 \langle r^4 \rangle \; . \tag{4}$$

Higher derivatives could similarly be computed if desired.

The heavy-light wavefunction has been recently computed by a lattice simulation [4, 5]. These authors used a set of 50 configurations (separated by 2000 sweeps) generated on a $16^3 \times 32$ lattice at $\beta = 5.9$. The configurations were fixed to Coulomb gauge and light propagators with a hopping parameter $\kappa = 0.158$. In order to use the above formulas (1-4) with the LQCD "data", we need an analytic expression for the wavefunction. Instead of trying to find some specific analytic form that would describe the behavior of the lattice data close to the origin and at large r, we expand the lattice wavefunction in terms of a complete set of basis functions. We then truncate the expansion to the first N basis states, hoping that we would be able to find a good description with a small number of basis states. In other words,

$$R_{1S}(r) \simeq \sum_{i=0}^{N-1} c_i R_i \; . \tag{5}$$

The quasi-Coulombic basis set [10] which we have chosen is particularly well suited for relativistic hadron systems, and it is given (for s-waves) by

$$R_i(r) = \sqrt{\frac{8\beta_s^3}{(i+2)(i+1)}} e^{-\beta_s r} L_i^2(2\beta_s r) \; , \tag{6}$$

where L_i^2 are associated Laguerre polynomials and β_s is a scale parameter. Substituting (5) and (6) in the expression (1) yields

$$\xi(\omega) = \frac{2}{\omega+1} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} c_i c_j \frac{(i+2)!(j+2)!}{\sqrt{(i+2)(i+1)(j+2)(j+1)}} I_{ij} \; , \tag{7}$$

where

$$I_{ij} = \sum_{m=0}^{i} \sum_{n=0}^{j} \frac{(-1)^{m+n}(m+n+1)! \sin\left[(m+n+2)\arctan(a)\right]}{m!n!(i-m)!(m+2)!(j-n)!(n+2)!a(1+a)^{\frac{m+n+2}{2}}} \; , \tag{8}$$

and

$$a = \frac{E_q}{\beta_s}\sqrt{\frac{\omega-1}{\omega+1}} \ . \tag{9}$$

In deriving this expression we have used the Laguerre polynomial representation

$$L_i^\alpha(x) = \sum_{m=0}^{i} \frac{(-1)^m}{m!} \frac{(i+\alpha)!}{(i-m)!(m+\alpha)!} x^m \ . \tag{10}$$

Our radial basis states depend on the meson size parameter β_s, which we estimate from the exponential falloff of the lattice data ($\sim e^{-\beta_s r}$) to be somewhere between 0.35 and 0.45 GeV. Once β_s has been fixed, we vary the coefficients c_i from (5) in order to best fit the lattice data. Fortunately, for any β_s within the expected range we are able to find an excellent approximation to the lattice wavefunction with only 3 basis states, with χ^2 of about 1.1 per degree of freedom (we have assumed an uncertainty of 10% in the wavefunction values [11]). Since all fits were essentially equivalent, and the wavefunctions were nearly identical up to $r = 20\ GeV^{-1}$, we have chosen the intermediate value $\beta_s = 0.40\ GeV$. However, we stress that none of the details of the basis function representation are important to our final result. In Figure 1 we show our radial wavefunction for the $1S$ state, together with LQCD data points, for $\beta_s = 0.40\ GeV$. As one can see from the figure the agreement is excellent, even though there is some ambiguity in the data, especially at large r, where the effects of the small lattice size become large. We should emphasize though that ambiguities of this sort are not expected to significantly alter our results. The three basis state coefficients ($\beta_s = 0.40\ GeV$) are found to be

$$\begin{aligned} c_0 &= 0.9985 \ , \\ c_1 &= -0.0221 \ , \\ c_2 &= 0.0500 \ . \end{aligned} \tag{11}$$

Unfortunately, the energy eigenvalue associated with the lattice wavefunction is not easily interpreted. The value of E_q depends sensitively upon the lattice spacing [5, 11]. Therefore, in the calculation of the IW function we treat E_q as a parameter.

From (7) and (11) we can now compute $\xi(\omega)$ in terms of E_q. In Fig. 2 we show the IW function for the range of

$$E_q = 0.306 \pm 0.040 \; GeV \; , \tag{12}$$

which corresponds to a one standard deviation corridor for the seven CLEO II [6] data points (or χ^2 of about 0.65 per degree of freedom). We also show (full line) the IW function corresponding to the best fit. Finally, we use (3) and (4) with the allowed range of E_q (12) to evaluate the first and second derivatives at zero recoil point. The results are

$$\xi'(1) \;=\; -1.16 \pm 0.17 \; , \tag{13}$$

$$\xi''(1) \;=\; 2.64 \pm 0.74 \; . \tag{14}$$

The best fit corresponding to $E_q = 0.307 \; GeV$, $\xi'(1) = -1.15$, and $\xi''(1) = 2.56$, has χ^2 of 0.38 per degree of freedom.

3 Conclusion

Since the IW function is non-perturbative any parametrization necessarily must contain some physical input. We have used a reliable lattice result for the heavy-light meson wavefunction to compute the IW function in terms of a single parameter E_q. By comparing with experimental data we determine the allowed range of this parameter. Among the direct uses of this parametrization is a more believable extraction of the IW slope at zero recoil point and for the first time a reliable estimate of the second derivative. Previous slope estimates have assumed $\xi(\omega)$ can accurately approximated by $\xi(\omega) \simeq 1 + \xi'(1)(\omega - 1)$, but this inevitably leads to an overestimated (not sufficiently negative) value for the slope [6].

ACKNOWLEDGMENTS

We would like to thank T. Duncan for providing us with the lattice data and H. Thacker for several useful conversations. This work was supported in part by the U.S. Department of Energy under Contract No. DE-FG02-95ER40896 and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

References

[1] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); Phys. Lett. B **237**, 527 (1990).

[2] M. E. Luke, Phys. Lett. B **252**, 447 (1990).

[3] A. F. Falk, M. Neubert and M. E. Luke, Nucl. Phys. B **388**, 3363 (1992); A. F. Falk and M. Neubert, Phys. Rev. D **47**, 2965 (1993); M. Neubert, Phys. ReV. D **47**, 4063 (1993).

[4] A. Duncan, E. Eichten, and H. Thacker, Phys. Lett. B **303**, 109 (1993).

[5] A. Duncan et. al., *Properties of B-Mesons in Lattice QCD*, Fermilab preprint FERMILAB-PUB-94/164-T (hep-lat/9407025).

[6] B. Barish et al., CLEO Collaboration, *Measurement of the* $\bar{B} \to D^* l \bar{\nu}_l$ *Branching Fractions and* V_{cb}, Cornell Nuclear Studies Wilson Lab preprint, CLNS-94-1285.

[7] H. Albrecht et al., ARGUS Collaboration, Z. Phys. C **57**, 533 (1993).

[8] M. Sadzikowski and K. Zalewski, Z. Phys. C **59**, 677 (1993); H. Høgaasen and M. Sadzikowski, Z. Phys. C **64**, 427 (1994).

[9] M. G. Olsson and S. Veseli, Phys. Rev. D **51**, 2224 (1995).

[10] E. J. Weniger, J. Math. Phys. **26**, (1985); M. G. Olsson, S. Veseli and K. Williams, *Observations On the Potential Confinement of a Light Fermion*, UW-Madison preprint MADPH-94-852 (hep-ph/9410405), to be published in Phys. Rev. D.

[11] H. Thacker (private communication).

FIGURES

Figure 1: S-wave radial wavefunction obtained from the fits to the LQCD data [4, 5]. We used $\beta_s = 0.40\ GeV$ and $N = 3$ basis states. The wave function is normalized as in (2).

Figure 2: Two limiting cases of the IW function calculated from the $1S$ radial wavefunction shown on Fig. 1, corresponding to $E_q = 0.346\ GeV$ and $E_q = 0.266\ GeV$(dashed lines). The best fit to the data is for $E_q = 0.307\ GeV$ (full line). For the sake of clarity, the error bars are shown only for the CLEO II data.

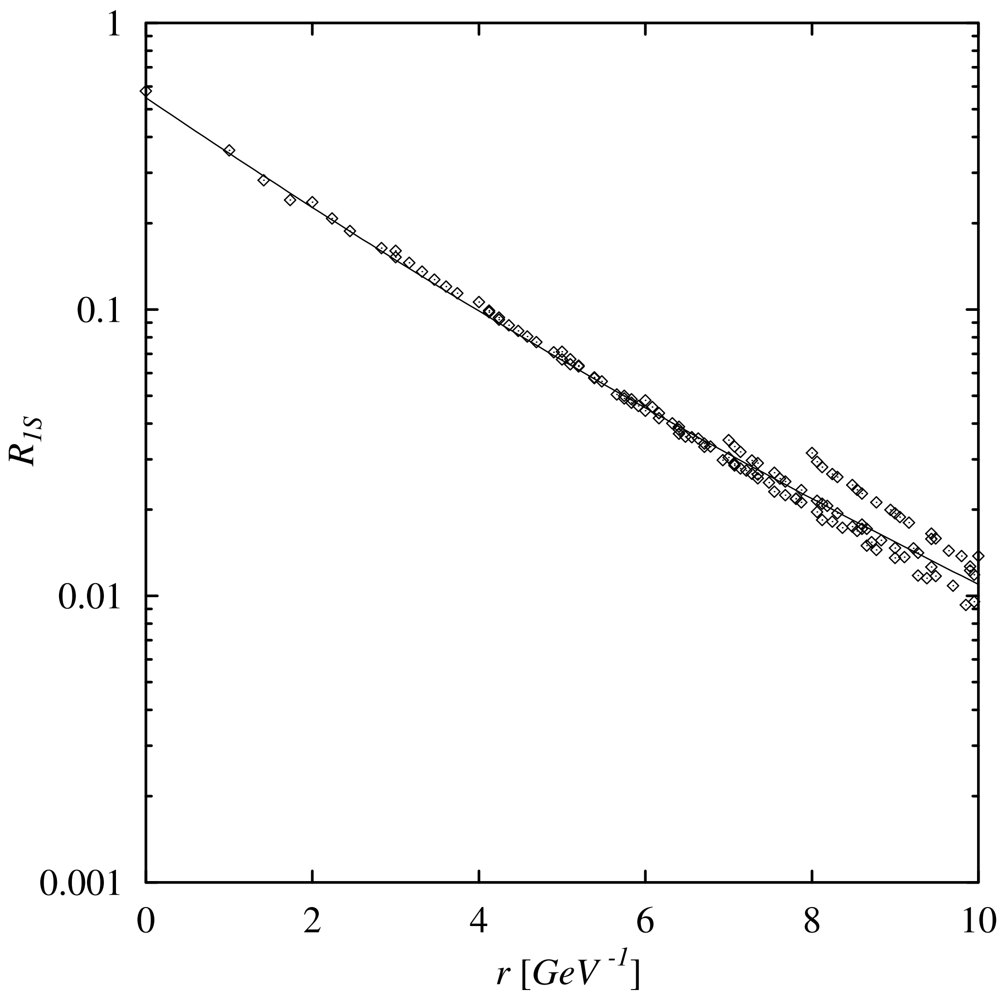

Figure 1

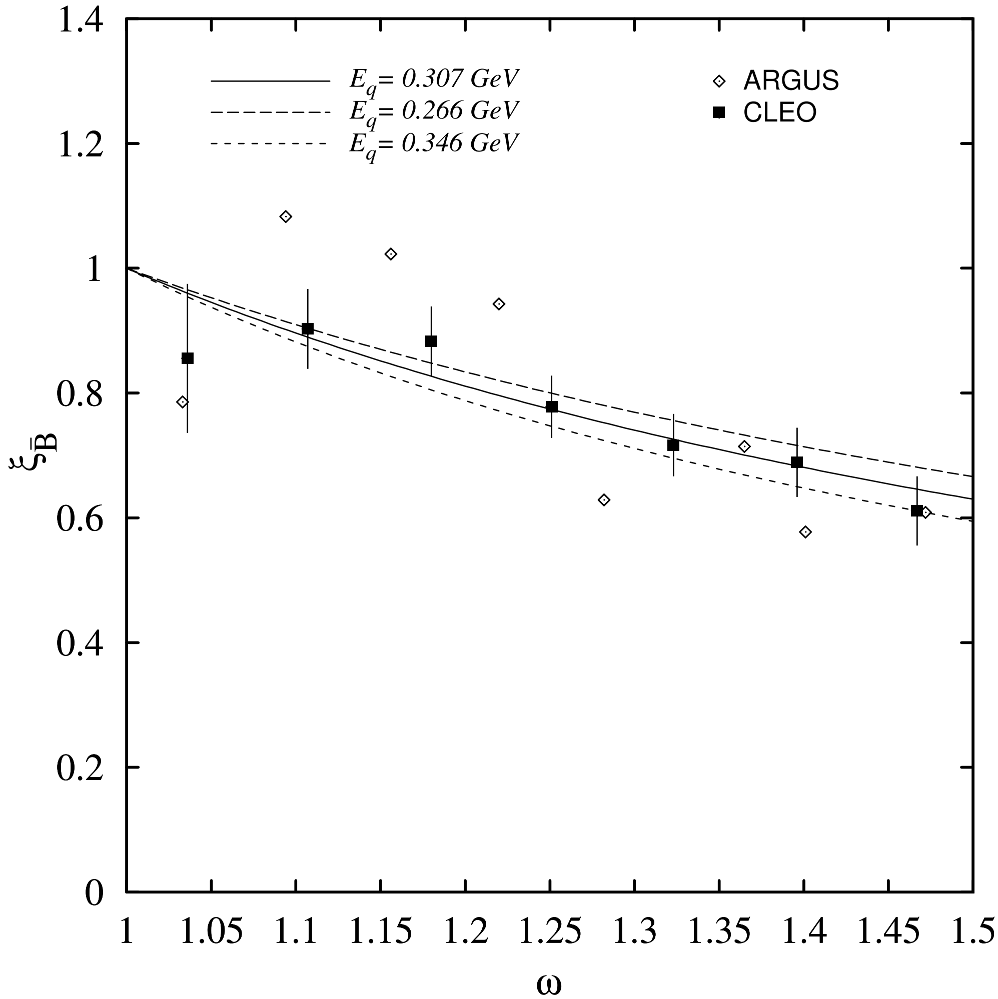
E_q= 0.307 GeV
E_q= 0.266 GeV
E_q= 0.346 GeV
ARGUS
CLEO
$\xi_{\bar{B}}$
ω
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5

Figure 2