Liquidation Value and Capital Structure in Industry Equilibrium

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Abstract

This paper develops a model of firm value and capital structure with endogenous default and endogenous liquidation value. I use this to investigate the interrelationship of capital structure choices among firms. Debt overhang limits the amount a firm can raise to acquire a bankrupt competitor. Ex-ante, this increases the competitor's cost of debt. However, if it is anticipated that a firm will make acquisitions in a downturn, this serves to buffer its debt against default and lower the firm's cost of debt. The result is an industry equilibrium with heterogeneity in the leverage of firms. It also leads to a non-monotonic relationship between an entrant's choice of leverage and that of incumbents.

The question of optimal capital structure has been one of the most popular topics in corporate finance, and has been the subject of a tremendous number of theoretical and empirical studies. Traditionally, research had focused on factors inherent in a firm's industry, such as technology, operating leverage, volatility of demand, and asymmetric information between management and outsider investors. Empirical tests of these factors left much unexplained. MacKay and Phillips (2005) find that industry fixed effects explain only 13 percent of variation in firm leverage, while firm fixed effects explain 54 percent. Several industry equilibrium models of capital structure have been developed that predict heterogeneity in the leverage choices of firms. These models have been motivated by considerations of oligopoly (Maksimovic (1988)), operating leverage (Maksimovic and Zechner (1991)), growth rate (Maksimovic et al (1999)), agency cost

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(Williams (1995)), and demand elasticity and incumbent/entrant status (Fries, Miller and Perraudin (1997)).

Another aspect of industry equilibrium is taken up in Shleifer and Vishny (1992). Suppose a firm defaults on its debt, and creditors seek to liquidate its assets. If the assets are suited to a particular purpose, the only users that put a high value on them are other firms in the industry. However, if these firms are also in financial distress, debt overhang will prevent them from paying the full intrinsic value of the assets. The costs of bankruptcy will be higher if other firms in the industry are financially constrained. Ex-ante, a firm may choose a level of debt that is otherwise lower-than-optimal, just to avoid the costly situation of liquidating when others in the industry are also in distress. This presents an interaction effect among the capital structures of firms in an industry; a firm responds to the higher leverage of others by decreasing its own leverage. Shleifer and Vishny stop short of a full equilibrium model, though. Their model consists of two firms, one of which is known ex-ante to be hit harder by an industry downturn than the other. Thus, one firm plans for the contingency of liquidating, and the other for acquiring. The result is heterogeneity in leverage choice, but the equilibrium choices of the firms depend upon their status in the downturn.

My paper develops a model of equilibrium based on the bankruptcy costs proposed by Shleifer and Vishny, but uses firms that are ex-ante identical, and endogenizes the order in which they default. In choosing leverage, a firm is also choosing whether it will default early in a downturn, positioning itself to be acquired, or later in a downturn, positioning itself as a potential acquirer. In the first case, the firm's cost of debt depends on the value creditors can recover in default, which is decreasing in the leverage of industry peers. In the second case, the firm's cost of debt depends on whether the firm makes acquisitions. An acquisition will increase equity's stake in the firm, acting as a buffer against the acquirer's default. The likelihood of making an acquisition is increasing in the leverage of industry peers, which lowers the cost of the firm's debt. The resulting equilibrium consists of firms that are highly levered and quick to default, and others that are well capitalized and in a position to make acquisitions in times of adversity. This explains how heterogeneity in leverage can arise from considerations of bankruptcy cost.

The model also makes some interesting predictions for the relationship between the leverage of incumbents and entrants. The relationship turns out to be non-monotonic. If the incumbent is well capitalized, the entrant chooses higher leverage, and positions itself to be acquired. In this case, the leverages are negatively related, as Shleifer and

Vishny would predict. If the incumbent is financially distressed, the entrant chooses lower leverage, assuming the position of acquirer. Contrary to Shleifer and Vishny, in this case the leverages are positively related. The more indebted the incumbent, the sooner it will default, and the more likely the entrant is to make an acquisition. This lowers the cost of the acquirer's debt, increasing its optimal leverage. It is logical that any investment financed partly with equity will act as a buffer against default. Most investments, however, occur when conditions in the industry are favorable. In this case, the buffering effect is of little value, since bankruptcy is remote anyway. When acquiring a bankrupt competitor, the investment occurs when industry conditions are depressed. In this case, the buffering effect is valuable because the acquirer is near default itself. The strategy of the entrant is also influenced by industry factors, such as volatility, technological and operational uniformity across firms, and the ability of outsiders to run the firms (or employ their assets) efficiently.

My model is based on Leland's (1994) model of optimal capital structure with endogenous default. Leland has a firm choose its debt level by weighing the benefits of tax advantages against the costs of default and liquidation. The higher these costs, the less debt a firm will use. This model can also be used to find the amount a firm's equity holder's would be willing to bid to acquire another firm, given the acquirer's level of debt. The result is debt overhang: the higher the acquirer's leverage, the less it will bid. This bid translates into the liquidation value a firm in default can expect to receive from its industry peers (I use the term "liquidate" to refer to any selling of the firm, not only in piecemeal fashion). Ex-ante, this liquidation cost feeds back into Leland's model of optimal capital structure, relating the leverage of one firm to the leverage choice of another. This step yields some intermediate results regarding debt overhang. The bid for acquisition can be computed under alternative assumptions of how the acquisition is financed (equity only, new senior debt, debt secured by the acquired assets, etc). In line with the findings of Stulz and Johnson (1985), the ability to issue debt secured by the acquired assets significantly attenuates the effect of debt overhang. In the presence of taxes, it also allows the acquirer to optimize its capital structure with respect to the post-acquisition level of cash flow. For both reasons, issuing secured debt results in higher bids. Surprisingly, the model also reveals that under certain assumptions, the larger the liquidating firm, the closer the bid will be to the firm's intrinsic value.

Leland's endogenous default model has inspired many extensions, covering short-term debt (Leland and Toft (1996)), agency cost (Mauer and Ott (2000)), debt renegotiation (Christensen (2000)), bankruptcy procedures (Francois and Morellec (2002), asset substitution (Morellec (2001)), macroeconomic conditions (Hackbarth et al (2006)), and more. Liquidation value enters into all these models, but it is always treated exogenously. My paper contributes to this line of research by endogenizing it in a way that is consistent with these models. It also contributes to Shleifer and Vishny's work by exploring their concepts in continuous time, allowing us to observe the effects of insider debt overhang on the yield spread, debt value, equity value, and firm value. Also, while the use of debt is motivated by the need to constrain overinvestment in their model, debt is used for tax advantages in mine. Finally, their model makes the counterfactual assumption that firms in default automatically liquidate. I develop a model where debt holders can wait for industry conditions to improve in order to liquidate at a higher price. This has a major effect on the results.

Several empirical studies have tested Shleifer and Vishny's hypothesis that financial distress in the industry reduces liquidation value. The findings strongly support this hypothesis. Pulvino (1998) finds that commercial aircraft are sold well below intrinsic value in times of industry distress. Asquith, Gertner and Scharfstein (1994) find that firms are less likely to sell assets to avoid bankruptcy when the industry is distressed. Sibilkov (2006) finds that leverage of industry peers is positively related to asset liquidity. In a study of U.S. bankruptcies from 1982 to 1999, Acharya, Bharath and Srinivasan (2005) conclude that creditors recover less if the industry is distressed, after controlling for the lower economic value of the firm's assets. They also find that the effects of industry distress are greater when assets are less re-deployable, there are fewer firms in the industry, and the remaining firms are cash constrained. These findings provide motivation for a model in which firms' leverage choices are inter-related by considerations of bankruptcy cost.

Section I presents Leland's model of a firm's optimal capital structure with exogenous liquidation value. It then shows how the model can be used to obtain the bid to acquire a liquidating firm, subject to debt overhang. Section II connects the bid to the liquidation value of the firm, thereby endogenizing liquidation value. A model of industry equilibrium is developed under the simplifying assumption that firms automatically liquidate upon default. Empirical studies find that this assumption does not hold in reality. For instance, Acharya et al find that bankrupt firms in distressed industries are very unlikely to liquidate or be acquired. Maksimovic and Phillips (1998) use plant-level data to find that industry conditions are important to the asset sales and closure conditions of bankrupt firms. To capture this feature, Section III extends the model by giving debt holders the option to liquidate following default. Section IV concludes.

I. Capital Structure and Bids

A. A model of the firm

We start by laying out the model developed by Leland in a way that fits the needs of this study. Firm i has a production flow of one unit of output, yielding a pre-tax cash flow P_t that evolves according to

$$dP_t/P_t = \mu dt + \sigma dW_t \tag{1}$$

where W is a standard Brownian motion, and μ and σ are constant.

The firm is financed with equity and a single class of perpetual debt. The advantage of using debt is that the interest payments are tax deductible. The disadvantage, as we will see, is that debt increases the risk of costly bankruptcy. Firm i's debt holders receive a constant coupon payment C_i (this is the combined payment to all debt holders). Equity holders receive the residual cash flow, after debt and tax payments. If the firm were unlevered, all cash flow would be taxed at rate π, leaving $(1-\pi)P_t$ for equity holders. In the levered case, tax is levied on earnings net of debt payments, leaving $(1-\pi)(P_t - C_i)$ for equity holders. If P_t falls below C_i, equity holders must contribute capital to make up the difference. If cash flow reaches a low enough level, equity holders will choose to let the firm default rather than contribute any more capital. This default trigger is increasing in the level of debt. Denote it $P^*(C_i)$. If the firm defaults, control passes to debt holders, who liquidate the firm. Assuming priority of claims is fully respected, debt holders receive the full liquidation value, and equity receives nothing.

Debt level C_i is chosen to maximize firm value when the firm is established, and cannot be altered later. If P_t subsequently rises, it would be optimal to issue more debt to take advantage of tax benefits. If P_t falls, debt should be reduced to avoid default. Leland demonstrates that although these adjustments maximize firm value, debt holders will reject the former while equity holders will reject the latter. Any issuance of debt decreases the value of debt. Any incremental buyback of debt reduces the value of equity (though a large buyback may increase equity value). It may benefit both parties to negotiate for lower debt payments if the firm is near default, but it is assumed the holdout problem (Bolton and Scharfstein (1994)) prevents this (at the end of Section II we consider the implications of relaxing some of these restrictions). Thus, the goal of a firm entering the industry at time t is to choose the optimal C_i relative to P_t.

For ease of notation, P_t and P will be used interchangeably. Let r be the constant risk-free rate, r_P the required rate of return on an asset that pays P, and $\delta = r_P - \mu$. Equity value, $E(P, C_i)$, is determined by

$$0 = \frac{1}{2}\sigma^2 P^2 E_{PP} + (r - \delta) P E_P - rE + (1 - \pi)(P - C_i) \tag{2}$$

and the boundary conditions

$$\begin{aligned} \lim_{P \to \infty} E(P, C_i) &= (1 - \pi)\left(\frac{P}{\delta} - \frac{C_i}{r}\right) && \text{(3a)} \\ E(P_i^*, C_i) &= 0 && \text{(3b)} \\ E_P(P_i^*, C_i) &= 0 && \text{(3c)} \end{aligned}$$

The first condition follows since the present value of the option to default goes to zero as P increases, in which case equity is worth the present discounted value of future cash flow. The second and third are the value-matching and smooth-pasting conditions associated with equity's option to default. The general solution to (2) is

$$E(P, C_i) = (1 - \pi)\left(\frac{P}{\delta} - \frac{C_i}{r}\right) + a_1 P^{\alpha} + a_2 P^{-\beta}$$

where α and β are, respectively, the positive root and the absolute value of the negative root, of $\sigma^2\lambda(\lambda - 1)/2 + (r - \delta)\lambda - r = 0$. Condition (3a) implies that $a_1 = 0$. Conditions (3b) and (3c) determine $a_2 = P_i^{*\beta+1}(1 - \pi)/(\beta\delta)$ with default trigger

$$P_i^*(C_i) = KC_i \tag{4}$$

where $K = \beta\delta/(r(\beta + 1)) > 0$. Then

$$E(P, C_i) = (1 - \pi)\left[\frac{P}{\delta} - \frac{C_i}{r} + \frac{P_i^{*\beta+1}}{\beta\delta} P^{-\beta}\right] \tag{5}$$

Note that $P_i^*(C_i)$ is increasing in C_i. The higher the coupon payment, the sooner equity holders will default. Equity value is decreasing and concave in C_i, and increasing and convex in P. The first two terms in the brackets make up the present value of cash flow to equity in the absence of default. The third term is the value of the option to default. An important property of the default option is that its value is increasing in C_i and decreasing in P. In other words, the default option is more valuable the more highly levered the firm.

Now consider debt value, $D(P, C_i)$. Debt holders receive coupon payment C_i until the firm defaults at P_i^*, at which time they receive liquidation value $L(P_i^*)$. Debt value is determined by

$$0 = \frac{1}{2}\sigma^2 P^2 D_{PP} + (r-\delta)PD_P - rD + C_i \tag{6}$$

and the boundary conditions

$$\begin{aligned} \lim_{P\to\infty} D(P, C_i) &= C_i/r & \text{(7a)} \\ D(P_i^*, C_i) &= L(P_i^*) & \text{(7b)} \end{aligned}$$

The general solution to (6) is $D(P, C_i) = C_i/r + b_1 P^{\alpha} + b_2 P^{-\beta}$. The boundary conditions determine $b_1 = 0$ and $b_2 = (L_i(P_i^*) - C_i/r)P_i^{*\beta}$, yielding

$$D(P, C_i) = \frac{C_i}{r} + \left[L(P_i^*) - \frac{C_i}{r}\right]\left(\frac{P_i^*}{P}\right)^{\beta} \tag{8}$$

The first term on the right-hand side is the value of default-free debt. The rest subtracts from this the present value of loss in default. Debt value is increasing and concave in P, since the probability of default is decreasing in P. Note that debt value depends on liquidation value, while equity value does not. This is a direct result of the assumption of absolute priority of claims in bankruptcy.

Firm value $V(P, C_i)$ equals the sum of equity and debt: $V(P, C_i) = E(P, C_i) + D(P, C_i)$. Leland analyzes the capital structure that maximizes firm value in the case where liquidation value is a fraction θ of unlevered firm value: $L(P) = \theta(1-\pi)P/\delta$. The coupon payment that maximizes firm value at time t is

$$C_i^{Max}(P_t) = \arg\max_{C_i} V(P, C_i) = \frac{P_t}{K}\left(\frac{\pi}{\pi + \beta(\theta(\pi - 1) + 1)}\right)^{1/\beta} \tag{9}$$

The optimal coupon payment balances the benefit of tax deduction against the higher probability of bankruptcy. Note that $C_i^{Max}(P)$ is increasing in P. When P is higher, the probability of bankruptcy for any given coupon payment is more remote. $C_i^{Max}(P)$ is increasing in the tax rate π, which is expected, since the motivation for debt is the tax benefit. $C_i^{Max}(P)$ is increasing in the parameter for liquidation value, θ. As the cost of bankruptcy decreases, there is less to be lost by the higher probability of bankruptcy associated with higher coupon payments. And, as other theories suggest, optimal debt is decreasing in the volatility of cash flow, σ. From $C_i^{Max}(P)$, optimal leverage is $D(P, C_i^{Max})/V(P, C_i^{Max})$.

B. Bidding for acquisitions

Imagine an industry that includes a number of firms like the one described above. We have seen that a firm's default trigger is positively related to its level of debt. In a downturn, a firm with more debt will default first. How much will another firm bid to acquire it? Consider a firm that earns a pre-tax cash flow of P_t, and is considering acquiring a bankrupt competitor that has cash flow γP_t. The constant γ could represent the difference in the size of the firms. Alternatively, the competitor could have cash flow P_t, but the acquirer may only be able to extract $\gamma < 1$ through lack of familiarity with operations. The merged firm will have cash flow $(1+\gamma)P_t$. Applying the procedure laid out in (2)-(5) gives the equity value of the merged firm

$$E_m(P, C_i) = (1-\pi)\left[\frac{(1+\gamma)P}{\delta} - \frac{C_i}{r} + \frac{(1+\gamma)P_{mi}^{*\beta+1}}{\beta\delta}P^{-\beta}\right] \tag{10}$$

where the default trigger for the merged firm is

$$P_{mi}^{*} = KC_i/(1+\gamma) \tag{11}$$

All-equity finance

Suppose this acquisition can be financed only through equity, not the issuance of new debt. This would be the case if lenders could not verify the value of the asset being acquired, or if the covenant on the original debt forbade issuing any more. We will relax this constraint shortly. Assume also that the liquidator has all the bargaining power. Then the firm bids the entire increase in equity value:

$$\text{Bid}(P, C_i) = E_m(P, C_i) - E(P, C_i) = (1-\pi)\left[\frac{\gamma P}{\delta} + \frac{(1+\gamma)P_{mi}^{*\beta+1} - P_i^{*\beta+1}}{\beta\delta}P^{-\beta}\right] \tag{12}$$

This reveals some important properties of the bid. First, the bid is decreasing in the acquirer's indebtedness:

$$\frac{\partial \text{Bid}}{\partial C_i} = \frac{1-\pi}{r}(P_{mi}^{*\beta} - P_i^{*\beta})P^{-\beta} < 0 \tag{13}$$

which follows since $P_{mi}^{*} < P_i^{*}$. To understand this, note from (2) and (10) that acquisition has two effects on equity: it increases cash flow, but in doing so, decreases the value of equity's option to default. With the default trigger reduced from P_i^{*} to

P^*_{mi}, the option is farther from exercise, and is worth less. In (12), the first term in brackets, $\gamma P/\delta$, is the present value of the increase in cash flow. The second term is the decrease in the value of the option to default. $\partial \text{Bid}/\partial C_i$ measures the effect of C_i on the decrease in default option value, which is greater when the firm has more debt. In other words, the more highly levered the firm is before acquisition, the more valuable is the option to default, and the more it declines through an increase in cash flow. Thus, less acquisition value accrues to equity holders, and the lower their bid. Correspondingly, when the firm is more highly levered, more of the gain from acquisition goes to debt holders. Acquisition moves the firm farther from default, thereby increasing the value of debt. This is equivalent to debt overhang as characterized by Myers (1977).

Second, the bid is increasing and convex in acquisition value:

$$\frac{\partial^2 \text{Bid}}{\partial \gamma^2} = \frac{1-\pi}{r} \frac{\beta C}{(1+\gamma)^2} P^{*\beta}_{mi} P^{-\beta} > 0 \tag{14}$$

When a merger causes the cash flow of the acquirer to increase, at first a large share of the gain goes to debt holders. The larger the increase, the more secure is the debt, and each incremental increase in cash flow accrues less to debt holders and more to equity holders. Thus, the larger the acquisition, the more equity holders will bid per dollar of cash flow being acquired.

Debt and equity finance

Generally, acquisitions are financed with both equity and new secured debt. Assume the new debt, like the original debt, is also a perpetuity, with coupon payment C_{ni}. New debt could either have the same standing as the original debt, receiving a proportional share of the liquidation value of all firm assets in default, or it could be secured by the newly acquired assets, while the original debt is secured by the original assets. We will make the latter assumption. This is the case when debt is secured by specific assets, or when the acquired firm is set up as a separate operating subsidiary with its own debt. Again applying (2)-(5), the post-acquisition value of equity is

$$E_m(P, C_i, C_{ni}) = (1-\pi) \left[\frac{(1+\gamma)P}{\delta} - \frac{C_i + C_{ni}}{r} + \frac{(1+\gamma)P^{*\beta+1}_{mi}}{\beta\delta} P^{-\beta} \right] \tag{15}$$

where

$$P^*_{mi} = K(C_i + C_{ni})/(1+\gamma) \tag{16}$$

(8) gives the value of new secured debt:

$$D_n(P, C_i, C_{ni}) = \frac{C_{ni}}{r} + \left[L_n(P^*_{mi}) - \frac{C_{ni}}{r}\right]\left(\frac{P^*_{mi}}{P}\right)^{\beta} \tag{17}$$

where $L_n(P^*_{mi})$ is the liquidation value of the assets securing the new debt (the assets of the acquired firm). Again assuming the liquidator has all the bargaining power, the acquirer bids

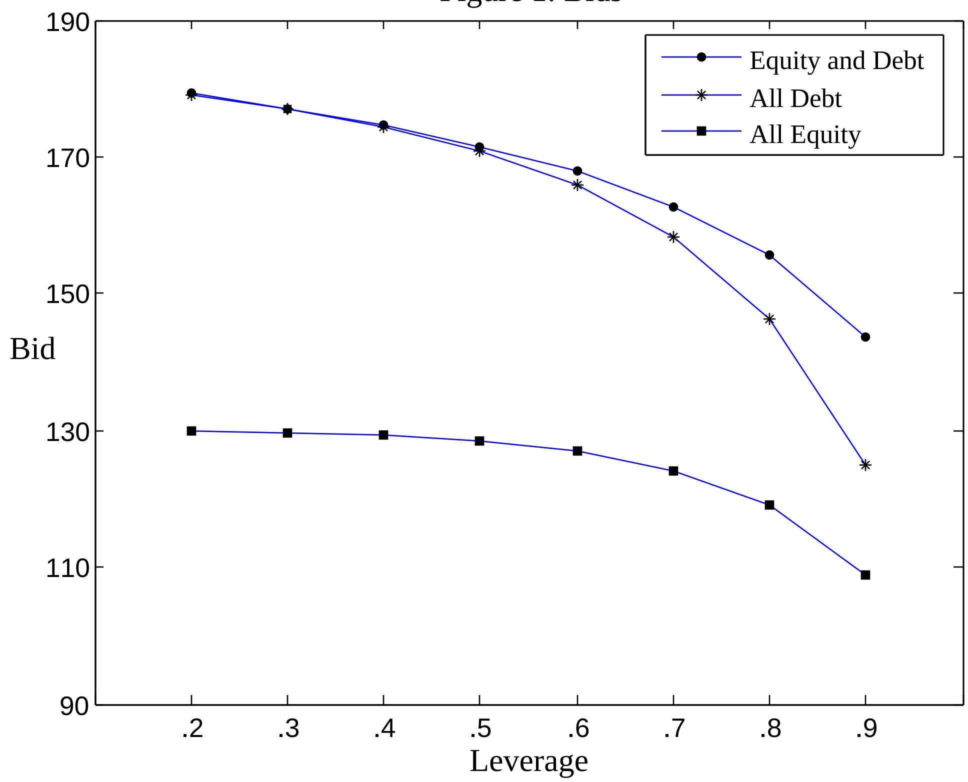
$$\text{Bid}(P, C_i) = \max_{C_{ni}}\{E_m(P, C_i, C_{ni}) - E(P, C_i) + D_n(P, C_i, C_{ni})\} \tag{18}$$

There is no closed-form solution to this bid, but the Envelope Theorem confirms that this bid shares properties (13) and (14). It was already stated that a firm's original debt holders would not agree to an increase in debt. This provided the rationale for assuming C_i is fixed. As we will see in Section II, debt issued to fund acquisition will not meet objection from original debt holders, as the increase is cash flow raises debt value.

Figure 1 plots all-equity and debt-and-equity bids as a function of the acquirer's leverage. Assume the parameters $P = 10$, $\gamma = 1$, $\theta = .5$, $\pi = .35$, $r = .05$, $\delta = .05$, and $\sigma = .15$. Unless noted, these parameter values will be used in all examples throughout the paper. C_i is adjusted so that the firm has a given leverage, $D(P, C_i)/V(P, C_i)$. Then for that value of C_i, $\text{Bid}(P, C_i)$ is computed for all-equity and debt and equity financing.

Consistent with debt overhang, both bids are decreasing in firm leverage. However, removing the restriction on issuing new debt increases the bid for two reasons. First, as demonstrated by Stulz and Johnson, funding an investment with secured debt attenuates the effect of debt overhang by passing more of the gain from investment to equity holders and less to the original debt holders. While payments on the new debt decrease cash flow to equity, the effect is partially offset because the new debt reduces the loss in value of equity's option to default that occurs from acquisition. Comparing (16) with (11), one can see that the post-acquisition default trigger is higher when new debt is issued, and hence the option to default retains more value. Second, in the presence of taxes, new debt allows the firm to re-optimize its leverage with respect to post-merger cash flow. Note that as leverage approaches zero, the all-equity bid approaches 130. This is just the unlevered, after-tax value of the acquired cash flow, $(1 - \pi)\gamma P/d$. When new debt can be issued, the bid can exceed this amount, since it allows the firm to take

Figure 1: Bids



advantage of the tax benefits of debt.[1]

An all-debt bid is also graphed. The bid equals the value of new debt secured by the acquired assets:

$$\text{Bid}(P, C_i) = \max_{C_{ni}} D_n(P, C_i, C_{ni})$$

subject to $E_m(P, C_i, C_{ni}) \geq E(P, C_i)$. In other words, this is the amount of new debt issued subject to the constraint that the acquisition not reduce the value of equity (otherwise equity holders would not make the bid). As leverage decreases and debt overhang goes away, this bid does not reach the same limit as in the all-equity case, because the new debt provides tax benefits for the additional income. At higher leverage, the bid is greatly reduced by debt overhang. The combined debt payments bring the firm close to default, rapidly cutting into debt value.

II. Automatic Liquidation in Default

Now we interpret the bid as the liquidation value of the bankrupt firm, and relate this to optimal capital structure. The model developed in this section rests on the assumption that when equity holders default, debt holders liquidate immediately, regardless of the value they obtain. This assumption is relaxed in Section III.

A. Liquidation value

The model developed here is based on the world described by Shleifer and Vishny. A liquidating firm can be acquired either by industry insiders or by outsiders, often referred to as "deep pockets." Outsiders will not be willing to pay full value, either because they are unable to run the business properly, or because asymmetric information prevents them from properly appraising the firm. Insiders are typically willing to pay more,

[1] Recall that one premise of the model is that, once chosen, the firm cannot adjust its level of debt. By allowing new debt issue during acquisition, we have made an exception to this rule. This introduces into the model the possibly undesirable feature that, through making an acquisition, the acquirer not only re-optimizes debt level with respect to the new cash flow, but has the chance to re-optimize with respect to existing cash flow. However, this feature of the model is not as arbitrary as it sounds. Leland and others argue that one or more classes of claimants will resist adjustments to the capital structure. Acquisitions and other investments that allow for the issuance of new debt may be unique opportunites to break this deadlock. Even with new debt issuance, existing debt holders are made better off. In any event, acquisition does not offer such an opportunity in the context of this model. Liquidation (and acquisition) only occur when P_t is low; in this case, the acquirer could only increase its pre-acquisition value by reducing debt, while making the acquisition will only increase debt.

P	Rate of cash flow from operation
D	Debt value
C_i	Coupon payment on debt
D_n	Value of debt secured by acquired assets
C_{ni}	Coupon payment on debt secured by acquired assets
E	Equity value (pre-acquisition)
E_m	Equity value (post-acquisition)
V	Firm value
V_L	Value of firm as liquidator
V_A	Value of firm as acquirer
V_S	Value of firm if default is simultaneous
P_i^*	Default trigger of Firm i
P_{mi}^*	Default trigger of merged firm when Firm i was the acquirer
$\text{Bid}_{i,Out}$	Outsider's bid on Firm i
$\text{Bid}_{i,In}$	Insider's bid on Firm i
π	Tax rate
γ	Percent of cash flow acquirer extracts from acquired firm
θ	Percent of intrinsic unlevered value bid by outsider
$L(P)$	Liquidation value

Table 1: Notation

since they can extract more value from the acquired assets. This model is governed by a few assumptions:

A1 Outsiders bid a constant fraction $\theta \in (0,1)$ of the unlevered value of the liquidating firm's cash flow.

A2 If the liquidating firm is acquired by an insider, the acquirer receives a fraction $\theta < \gamma \leq 1$ of the cash flow of the liquidating firm.

A3 Liquidation occurs immediately upon default.

A4 The debt holders of the liquidating firm hold all the bargaining power. They receive the maximum the insider or outsider is willing to pay, whichever is greater.

A5 The acquirer can fund the acquisition by issuing both equity and debt secured by the acquired firm.

A1 formalizes Shleifer and Vishny's hypothesis that outsiders will not bid full value for the firm. The result is that the outsider bid is

$$\text{Bid}_{Out}(P) = \theta(1-\pi)\frac{P}{\delta}$$

A2 allows for the possibility that industry insiders may also not be able to fully capture the cash flow of the acquired firm, either because of unfamiliarity with the operations or decreasing returns to scale. Alternatively, they may suffer asymmetric information in valuing the firm. Assuming $\gamma > \theta$ implies that these problems are less severe for the insider than the outsider. One could imagine $\gamma > 1$ if the merger increased the acquirer's market power. We will ignore this issue here. A3 is used to establish base-case results here, and is relaxed in Section III. A4 is made for simplicity; it may be reasonable if liquidation is subject to approval by numerous debt holders, who present a holdout problem to the acquirer. A5 puts no restrictions on acquisition finance, representing an upper bound on insider bids. Implications of changing A4 and A5 are discussed at the end of the section.

Consider an industry with two firms, Firm 1 and Firm 2. The firms each produce a single unit flow of a commodity, each earning P_t. Suppose $C_1 > C_2$, so from (4), $P_1^* > P_2^*$. This means that Firm 1 will default before Firm 2, and Firm 2 can bid to acquire it. When cash flow falls to P_1^*, Firm 1 defaults and is liquidated. It is bid on by Firm 2 (the insider) and by outsiders. The insider bid can be modeled as in (18), so that Firm 2's bid for Firm 1 is

$$\mathrm{Bid}_{1,In}(P_1^*, C_2) = \max_{C_{ni}} \{E_m(P_1^*, C_2, C_{n2}) - E(P_1^*, C_2) + D_n(P_1^*, C_2, C_{n1})\}$$

Note the argument P equals P_1^* at the time Firm 1 defaults and liquidates. Firm 2's post-merger equity value is given by (15):

$$E_m(P, C_2, C_{n2}) = (1-\pi)\left[\frac{(1+\gamma)P}{\delta} - \frac{C_2 + C_{n2}}{r} + \frac{(1+\gamma)P_{m2}^{*\beta+1}}{\beta\delta}P^{-\beta}\right]$$

where $P_{m2}^* = K(C_2 + C_{n2})/(1+\gamma)$. New debt value is given by (17):

$$D_n(P, C_2, C_{n2}) = \frac{C_{n2}}{r} + \left[\mathrm{Bid}_{1,Out}(P_{m2}^*) - \frac{C_{n2}}{r}\right]\left(\frac{P_{m2}^*}{P}\right)^{\beta}$$

After the merger, Firm 2 is the only firm left in the industry. Thus, when the merged firm defaults, its assets will be acquired by an outsider. $\mathrm{Bid}_{1,Out}$ is what holders of the debt secured by Firm 1's assets receive upon default of the merged firm.

If Firm 2's indebtedness is great enough, outsiders will win the bid. From (13), we know this occurs when C_2 is high. The insider bid depends on the value of P at the time of the acquisition, P_1^*, which in turn is a function of C_1. Define $\overline{C_2}(C_1)$ as the value of C_2 such that if $C_2 > \overline{C_2}(C_1)$, then $\text{Bid}_{1,Out}(P_1^*) > \text{Bid}_{1,In}(P_1^*, C_2)$. $\overline{C_2}(C_1)$ can only be solved numerically. The liquidation value of Firm 1 is

$$\begin{aligned} L(P_1^*, C_2) &= \text{Bid}_{1,In}(P_1^*, C_2) \text{ if } C_2 \leq \overline{C_2}(C_1) \\ &= \text{Bid}_{1,Out}(P_1^*) \qquad \text{otherwise} \end{aligned}$$

Plugging $L(P_1^*, C_2)$ into (8) yields Firm 1 debt value before default, $D(P, C_1, C_2)$, where the coupon payment of the other inside firm (C_2) is added as an argument to reflect its effect on liquidation value. In this way, the indebtedness of insiders affects the cost of debt for other firms in the industry.

To observe this effect, let the industry be governed by the parameters from the previous example, and set $\gamma = .8$. With these values, either firm would default immediately ($P_i^* = 10$) if its coupon payment exceeds 16. For any coupon payment lower than that, the yield spread on Firm 1's debt equals $C_1/D(P, C_1, C_2) - r$.[2] Figure 2 plots the yield spread as a function of C_1. Of course, as C_1 approaches 16, the probability of default increases, driving up the yield spread. The spread is also affected by C_2. If Firm 2 has higher leverage, $\text{Bid}_{1,In}(P_1^*, C_2)$ is diminished, and the liquidation value of Firm 1 is lower.[3]

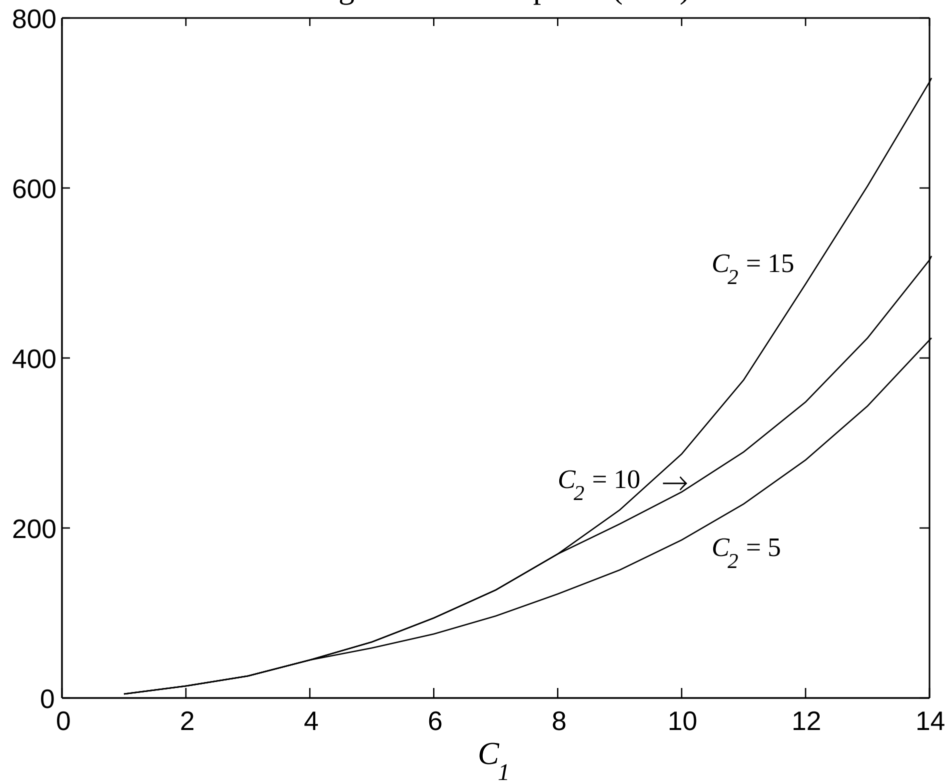
B. Firm value

The prospects of defaulting and liquidating, or acquiring, affect firm value and capital structure choices ex-ante. A firm knows at the time that it chooses its capital structure whether it will default before or after some other inside firm. Consider again the two firm industry. Suppose Firm 2 has debt level C_2. Firm 1 knows that Firm 2 will default at $P_2^* = KC_2$. Firm 1 knows that in choosing C_1, it is also choosing its own default trigger, $P_1^* = KC_1$. If $C_1 = C_2$, then $P_1^* = P_2^*$, and both firms default at the same time. If C_1 is any lower, Firm 2 will default first, and be acquired either by Firm 1 or an outsider. If C_1 is any higher, Firm 1 will default first, and be acquired by Firm 2 or an outsider. The firm also knows the bids it will receive when it liquidates, and

[2] Here it is assumed for simplicity that Firm 1 cannot acquire Firm 2 if Firm 2 defaults, but Firm 2 can bid on Firm 1. This could arise if Firm 1 is a privately held firm with limited access to funds with which to make acquisitions. The more general case is the subject of the rest of this section.

[3] Figure 2 reveals that the yield spread is independent of C_2 at lower levels of C_1. This reflects regions of C_1 where the outsider wins the bid. Recall from (14) that the insider bid per dollar of acquired cash flow is smaller when the amount of acquired cash flow is small. If C_1 is low, then P_1^* is low, and the cash flow of Firm 1 at default is small. Although the outsider wins the bid when C_1 is low, the yield spread is low because the event of default is remote.

Figure 2: Yield Spread (BPS)



by whom it will be acquired. For instance, if Firm 1 will default before Firm 2, and if $C_2 \leq \overline{C}_2(C_1)$, then Firm 1 knows that $\text{Bid}_{1,In}(P_1^*, C_2) >\text{Bid}_{1,Out}(P_1^*)$, and it will be acquired by Firm 2.

This leaves several possible outcomes to keep track of. A simple observation will make the analysis easier: over a broad range of reasonable parameter values, the outsider never actually wins the bid, unless both firms default at the same time, or one of the insiders has already defaulted and there is no insider left to make acquisitions. Numerical solutions indicate that for reasonable parameter values the region over which the outsider wins the bid for Firm 2 has no intersection with the region over which Firm 2 defaults first. This means that when Firm 2 defaults, Firm 1 will win the bid. Likewise, when Firm 1 defaults, Firm 2 will win the bid. The analysis proceeds using only parameter values for which this holds, and assumes the first firm to default is acquired by the remaining insider.

Now the formula for firm value depends only on the order of default, with three possibilities: the firm defaults first (and is acquired by the insider), the firms default simultaneously (and are both acquired by outsiders), or the firm defaults second (and is the acquirer). These future events affect both the default trigger, and the payout debt holders receive upon default. The value of debt in each case can be found by applying the appropriate default trigger and liquidation value to (8). Firm 1 debt value is

$$D_L(P, C_1, C_2) = \frac{C_1}{r} + \left[\text{Bid}_{1,In}(P_1^*, C_2) - \frac{C_1}{r}\right]\left(\frac{P_1^*}{P}\right)^\beta \quad \text{for } P_1^* > P_2^* \tag{19a}$$

$$D_S(P, C_1, C_2) = \frac{C_1}{r} + \left[\text{Bid}_{1,Out}(P_1^*) - \frac{C_1}{r}\right]\left(\frac{P_1^*}{P}\right)^\beta \quad \text{for } P_1^* = P_2^* \tag{19b}$$

$$D_A(P, C_1, C_2) = \frac{C_1}{r} + \left[\text{Bid}_{1,Out}(P_{m1}^*) - \frac{C_1}{r}\right]\left(\frac{P_{m1}^*}{P}\right)^\beta \quad \text{for } P_1^* < P_2^* \tag{19c}$$

D_L applies to the case where Firm 1 defaults first (and is the liquidator). Firm 1 debt holders receive $\text{Bid}_{1,In}(P_1^*, C_2)$ in default. The lower C_2 is, the higher the bid, and the higher the value of Firm 1 debt before default. D_S applies where the firms default simultaneously and are each acquired by outsiders. Upon default, Firm 1 receives $\text{Bid}_{1,Out}(P_1^*)$. D_A applies where Firm 2 defaults first, and Firm 1 acquires it. From (16), the resulting merged firm will have default trigger $P_{m1}^* = K(C_1 + C_{n1})/(1 + \gamma)$. In this case, Firm 1 debt holders know from the beginning that Firm 1 will default at P_{m1}^*.[4]

[4]Each firm also knows how much new debt, C_{si}, will be issued for the aquisition.

After Firm 2 has been acquired, there are no other insiders left, so an outsider acquires the merged firm when it defaults. Firm 1's original debt holders are only secured by the original cash flow of Firm 1, and thus receive $\text{Bid}_{1,Out}(P^*_{m1})$.

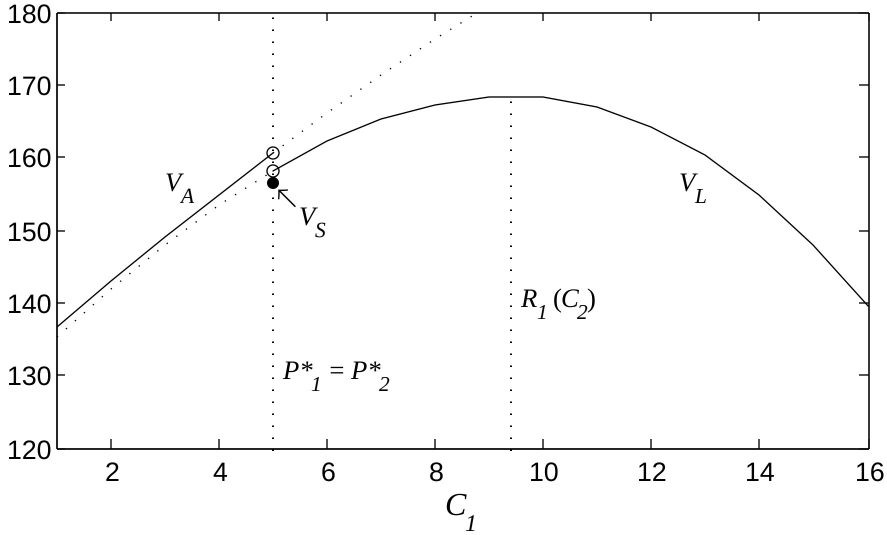
Comparing D_L and D_A, one can see how playing the acquirer can benefit debt holders. If $P^*_{m1} < P^*_1$, the second term on the right-hand side, measuring bankruptcy cost, is lower. From (4) and (16), this will be true if $C_{n1} < \gamma C_1$. Numerical solutions reveal that this is true when C_{n1} is chosen according to (18) over the range of reasonable parameter values. In making the acquisition, equity holders increase their cash flow. This delays their decision to default, offering protection for debt holders.

Though debt value depends on the order of default, equity value does not. The pre-merger equity value of Firm 1, $E(P, C_1)$, is given by (5) regardless of liquidation value or the order of default. We have assumed that the liquidating firm has all the bargaining power, and that equity holders bid the full amount by which acquisition increases equity value. Thus, while Firm 1 equity value increases from $E(P, C_1)$ to $E_{m1}(P, C_1)$, this amount is bid for the acquisition, leaving the ex-ante value of equity unchanged.

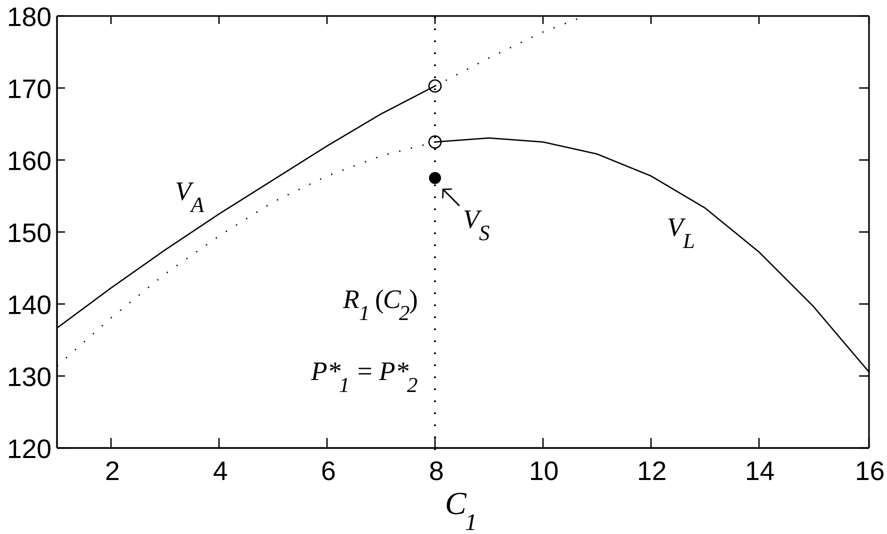
Using the relevant formula for debt value, Firm 1 value equals $V(P, C_1, C_2) = E(P, C_1) + D(P, C_1, C_2)$. Corresponding to the notation for debt value, let $V_L(P, C_1, C_2)$ denote Firm 1 value when it is the liquidator, $V_S(P, C_1, C_2)$ when default is simultaneous, and $V_A(P, C_1, C_2)$ when Firm 1 is the acquirer. These formulas apply to Firm 2 as well; when denoting Firm 2, the subscripts on the arguments will be switched. For instance, $V_L(P, C_2, C_1)$ denotes Firm 2 value when it defaults first.

The optimal choice of C_1 can be thought of as a response function to C_2. Let $C_2 = 5$ and again suppose $\gamma = .8$. The first plot in Figure 3 graphs $V_L(P, C_1, C_2)$, $V_S(P, C_1, C_2)$, and $V_A(P, C_1, C_2)$ as functions of C_1. If $C_1 = 5$, default is simultaneous and Firm 1 value is given by V_S. For $C_1 > 5$, Firm 1 value is V_L. For $C_1 < 5$, Firm 2 defaults first and V_A applies. Thus, maximizing Firm 1 value is a matter of maximizing V_L, V_S, and V_A over the permissible range of C_1, and then choosing the maximum of the three. As might be expected, V_L is first increasing, and then decreasing in C_1. For low levels of debt, the possibility of bankruptcy is remote, and the tax advantages of debt outweigh expected bankruptcy costs. At higher levels of debt, bankruptcy becomes more probable. As we saw, this is reflected in the yield curve. However, V_A is increasing in C_1 over a much greater range. If Firm 1 debt holders know that Firm 2 will default and be acquired by Firm 1, they know Firm 1's default trigger will fall in the future, guarding them against default. This makes debt finance very cheap for Firm 1. However, if Firm 1 issues too much debt ($C_1 > 5$), debt holders will know Firm 1 will default first and

Figure 3: Firm Value (C_2=5)



Firm Value (C_2=8)



the acquisition is never going to occur; hence V_A no longer defines Firm 1 value. It is clear from Figure 3 that if Firm 1 is going to limit debt issuance and play the role of the acquirer, it will maximize firm value by issuing as much as possible ($C_1 = C_2 - \epsilon$). With these parameters, Firm 1 value is maximized at $C_1 = 9.5$, whereby Firm 1 defaults before Firm 2. This strategy yields a firm value of 168.5, as compared to 160.5 for choosing $C_1 = 5 - \epsilon$ and acquiring Firm 2, and 157.0 for choosing $C_1 = 5$ and defaulting simultaneously.

How does C_2 affect this decision? The second plot in Figure 3 graphs Firm 1 values for $C_2 = 8$. The higher leverage of Firm 2 causes Firm 1 liquidation value to be lower, shifting V_L down. Since Firm 1 faces a higher yield spread, V_L is maximized at a lower debt level ($C_1 = 9.0$). The relationship of C_2 to V_A is more subtle. A greater C_2 raises P_2^*, and hence the value of Firm 2 when it defaults. In maximizing the acquisition bid, Firm 1 issues more new debt than if P_2^* were lower. This increase in new debt raises the post-merger default trigger P_{m1}^* (see (16)), thereby limiting the gains reaped by original Firm 1 debt holders.[5] However, by increasing P_2^*, the line $P_1^* = P_2^*$ shifts to the right, allowing Firm 1 to issue more debt while still being the acquirer. The net result is to increase the maximum value of V_A. The result is that Firm 1 maximizes value by setting $C_1 = 8 - \epsilon$, playing the acquirer and attaining firm value 170.2, as opposed to 163.0 by playing the liquidator, and 158.1 for simultaneous default. Notice that in this example, Firm 1 achieves greater value when $C_2 = 8$ than when $C_2 = 5$. In other words, Firm 1 value is higher when Firm 2 is more leveraged.

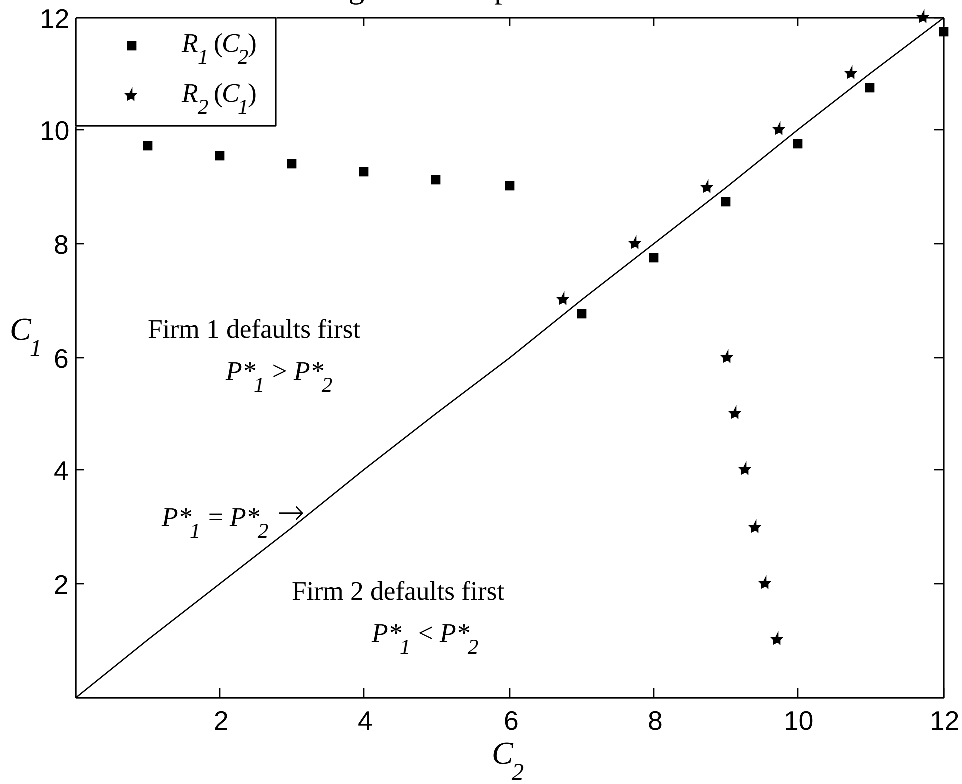
C. Capital structure

For a more complete picture of financing choices, define $R_1(C_2)$ as the choice of C_1 that maximizes the value of Firm 1 for a given C_2. Figure 4 graphs $R_1(C_2)$. For $C_2 \leq 6$, Firm 1's choice of C_1 is greater than C_2, implying that Firm 1 defaults first. This is in accord with the results from Figure 3. As C_2 increases from zero to 6, Firm 1's optimal debt level decreases, as the yield spread on its debt increases. For $C_2 \geq 7$, Firm 1 chooses to play the acquirer, setting C_1 as high as possible while still convincing its debt holders that it will make an acquisition. Firm 2's strategy, $R_2(C_1)$, is symmetrical to that of Firm 1.

It is apparent from the points plotted in Figure 4 that there is no Nash Equilibrium. Firm 2 never chooses C_2 low enough to make Firm 1 default first. For the values of C_2 actually chosen, Firm 1 chooses C_1 just low enough that Firm 1 will be the acquirer.

[5] If the acquisition were all-equity financed, V_A would be independent of C_2.

Figure 4: Response Functions

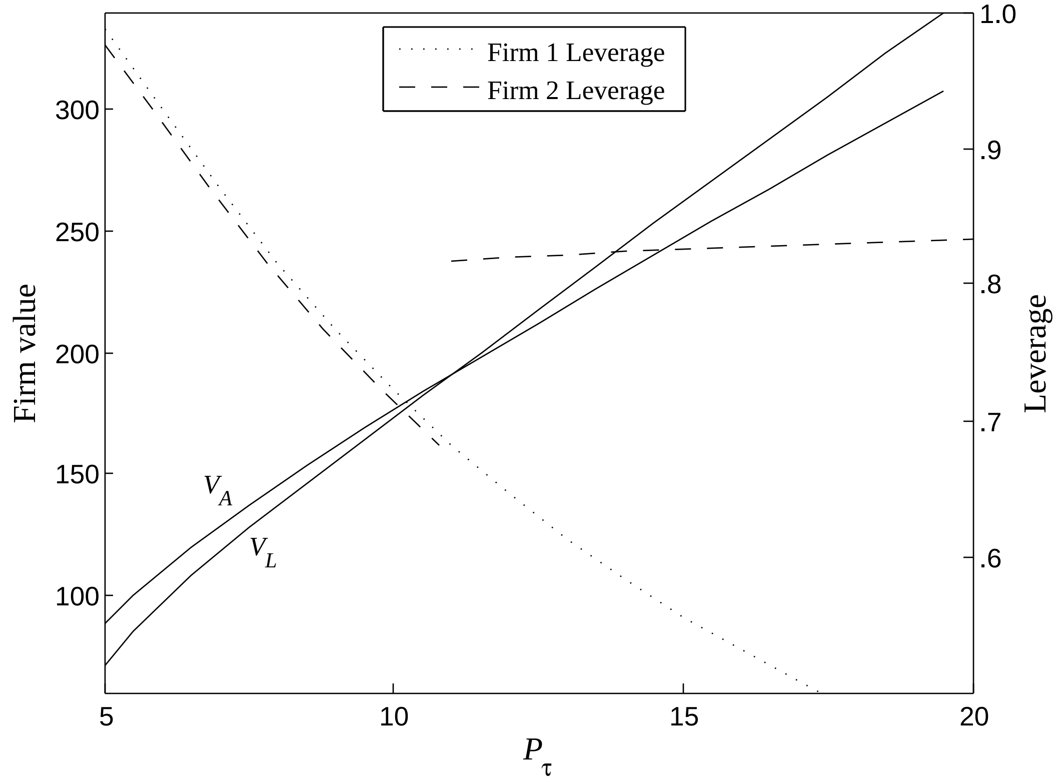


However, Firm 2 responds to this by choosing C_2 just low enough that Firm 2 will be the acquirer. Repeating the experiment for different parameters reveals that the same outcome holds over all reasonable values.

Analyzing the sequential entry of firms is more fruitful. Suppose Firm 1 is the first to enter an industry, entering at time 0. Firm 2 enters at some time $\tau > 0$. Let $P_0 = 10$. For simplicity, assume that when Firm 1 enters it does not anticipate any future entrants, choosing its capital structure as if the only bidders upon default are outsiders. According to (9), $C_1(P_0) = 7.4$. How does Firm 2 choose $C_2(P_\tau)$? This depends on the relationship between P_0 and P_τ. Figure 5 graphs the value of Firm 2 under the strategies of playing the liquidator and playing the acquirer. If P_τ is low compared to P_0 it is better off playing the acquirer. In this case, Firm 1 has very high leverage at time τ. It will not be willing to bid much for Firm 2 if Firm 2 defaults. Also, Firm 1 will be near default itself. If Firm 2 was to set C_2 so as to default first, it would have to set it very high compared to P_τ. Firm 2 would be highly levered, with low liquidation value. This is clearly a bad strategy. It would be better to take on slightly less debt than Firm 1, thereby signalling to debt holders that it will default after Firm 1, and acquire Firm 1 when Firm 1 defaults. On the other hand, if P_τ is much greater than P_0, Firm 1 will have low leverage at time τ. It will bid high if Firm 2 defaults. In order for Firm 2 to play the acquirer, defaulting after Firm 1, it must take on very low leverage, thereby foregoing the valuable tax advantages of debt. It is better off being more highly levered. It can secure debt at a reasonable price because in the event of default, liquidation value will be high. Note that over this range of P_τ Firm 2's leverage increases modestly in P_τ, as Firm 1 becomes less levered and offers a higher bid. The result is that the entrant's leverage choice is modestly decreasing in the leverage of the incumbent when the incumbent's leverage is low, and rapidly increasing in the incumbent's leverage when the incumbent's leverage is high.

The intuitive prediction of capital structure in industry equilibrium is that optimal firm leverage is decreasing in the leverage of insiders. If Firm 1 defaults and Firm 2 is highly levered, debt holders will receive a lower bid. Ex ante, debt is more expensive and Firm 1 uses less. Here we find the more dramatic relationship running the other way. If competitors have sufficiently high leverage, a firm can signal that it will buy them by issuing slightly lower debt. The cushion this provides against default lowers the cost of debt so much that the firm uses as much as it can (while still playing the acquirer). Of course, if incumbents are highly levered because the industry is in a downturn, investment costs may make entry unprofitable even though the potential to

Figure 5: Sequential Entry



acquire firms exists. Industry equilibrium and sequential entry will be explored in much greater detail after the more generalized model is developed in the next section.

Are the results of the model robust to alternative assumptions? We have used an industry with only two firms. Consider a model with multiple inside bidders. Bids are decreasing in a firm's debt level. In order to win the bid, a firm must have a debt level not only lower than the target of the acquisition, but also lower than other insiders. This places a higher hurdle on choosing the acquirer strategy, but does not alter the general result. For instance, if all incumbents were highly levered, the entrant would still play the acquirer. How would the results change if the liquidator did not have all the bargaining power? Clearly, the value of the liquidating firm would fall, and that of the acquirer would rise. This would only increase the value of playing the acquirer. It is assumed that cash flow is perfectly correlated across firms. If not, it is possible that an idiosyncratic shock will lead to default before any acquisitions are made to buffer against it. While having lower debt payments than other firms would not guarantee protection for debt holders in this case, a correlation would still exist.

It is also assumed that capital structure cannot be altered. If not for the holdout problem, debt and equity could both benefit from a renegotiation of debt if the firm is near default. Christensen et al. examine this possibility in the Leland framework. The ability to extract concessions from debt holders depends on the liquidation value they receive in default. This in turn depends on the leverage of the acquirer. The higher the acquirer's leverage, the more power the liquidator's equity holders have to renegotiate, and the higher the liquidator's cost of capital. This reinforces the negative relationship between the entrant's leverage and that of the incumbent if the entrant is going to play the liquidator. However, the effect on V_A and V_L is not obvious. The ability to renegotiate alters the cost of debt, optimal leverage, and the timing of default. The implications for industry equilibrium deserve further study.

Also troublesome is the assumption that liquidation is automatic upon default. If Firm 1 debt holders can wait for a higher bid, the link between default and acquisition breaks down. This opens up the possibility that Firm 2, as a would-be acquirer, might default before Firm 1 debt holders choose to liquidate. This causes a break in the relationship between debt levels and the order of default. As already noted, empirical studies find that it is common for debt holders of bankrupt firms to wait out a downturn. Wruck (1990) and White (1996) characterize two types of bankrupt firms: those that are economically inefficient, and those that are financially distressed. The latter type, they argue, should recover. In the model developed in Section II, all firms have the same cash flow. Default has nothing to do with economic efficiency, stemming only from financial distress. Bris, Welch and Zhu (2003) demonstrate that it is optimal for creditors of such firms to wait to liquidate. In this sense, the model of automatic liquidation is too good to be true from the acquirer's standpoint. Acquisition does not automatically follow from default of industry peers.

III. Debt Holder-run Firms

Suppose debt holders can operate the firm after default, earning cash flow $\eta(1-\pi)P_t$, where $\eta < \gamma$. This means the remaining insider is a more efficient operator than the debt holders, but debt holders might not sell the firm immediately. They have the option to liquidate, and may wait for cash flow to recover before exercising it. Liquidation value rises with cash flow both because the intrinsic value of the firm is higher, and the acquirer's debt overhang is less severe. This option to wait increases the value of the firm's debt, both before and after default. It also allows the possibility that the would-be acquirer may itself default before making the acquisition, altering the value of its own debt. This reduces the value of playing the acquirer while raising the value of the liquidator.

All the other assumptions of Section II still hold, except that now there is no outside bidder. If a firm defaults and no insider is left to acquire it, the debt holders operate it forever. This simplifies the new model greatly, with little effect on the results. In fact, if we assume that debt holders can only extract as much cash from the firm as outsiders ($\eta = \theta$), the model yields the same results with or without outside bidders (this will be noted below). This assumption seems reasonable, since creditors generally have little knowledge of their borrower's business, and face agency costs in hiring management.

A. Firm value

To obtain formulas for firm value in this model, suppose Firm 1 defaults at $P_1^* > P_2^*$. There is a liquidation trigger $P_1^{**} \geq P_1^*$ at which debt holders accept the acquirer's bid. The first task is to find $D_L(P, C_1, C_2)$, the value of Firm 1's debt prior to default, and the liquidation trigger P_1^{**}.

Let $D_{Ld}(P, C_1, C_2)$ denote the value of Firm 1's debt between the time Firm 1 defaults and the time debt holders liquidate the firm. This follows

$$0 = \frac{1}{2}\sigma^2 P^2 D_{PP} + (r-\delta)PD_P - rD + \eta(1-\pi)P$$

with boundary conditions

$$D_{Ld}(0, C_1, C_2) = 0 \tag{20a}$$

$$D_{Ld}(P_1^{**}, C_1, C_2) = \text{Bid}_{1,In}(P_1^{**}, C_2) \tag{20b}$$

$$\partial D_{Ld}(P_1^{**}, C_1, C_2)/\partial P = \partial \text{Bid}_{1,In}(P_1^{**}, C_2)/\partial P \tag{20c}$$

(20b) and (20c) are the value-matching and smooth-pasting conditions corresponding to the option to liquidate. This yields

$$D_{Ld}(P, C_1, C_2) = \frac{\eta(1-\pi)P}{\delta} + \left[\text{Bid}_{1,In}(P_1^{**}, C_2) - \frac{\eta(1-\pi)P_1^{**}}{\delta}\right]\left(\frac{P}{P_1^{**}}\right)^{\alpha} \tag{21}$$

To obtain $D_L(P, C_1, C_2)$, replace $L(P_1^*)$ in (8) with $D_{Ld}(P_1^*, C_1, C_2)$:

$$D_L(P, C_1, C_2) = \frac{C_1}{r} + \left[D_{Ld}(P_1^*, C_1, C_2) - \frac{C_1}{r}\right]\left(\frac{P_1^*}{P}\right)^{\beta} \tag{22}$$

Since there is no closed-form solution for $\text{Bid}_{1,In}$, there is no closed-form solution for $D_L(P, C_1, C_2)$ or P_1^{**}. Intuitively, the higher C_2, the lower Firm 2's bid, and the longer debt holders will wait to liquidate. This means a higher P_1^{**}. This in turn decreases the value of Firm 1's debt both before and after default. Both the liquidation trigger and debt value are increasing in η, as operating in default is a more viable alternative. If η is high, a firm is more sheltered from the debt overhang of other firms. It will be important later to note that P_1^{**} is independent of C_1. Since the firm has already defaulted on the coupon payments, they do not enter the decision of when to liquidate.

Equity value is unchanged from the analysis of Section II, so Firm 1 value, $V_L(P, C_1, C_2) = E(P, C_1) + D_L(P, C_1, C_2)$, is given by (5) and (22).

The ability of debt holders to wait before liquidating also affects the value of the potential acquirer. With liquidation trigger $P_1^{**} > P_1^*$, there are two possible scenarios following Firm 1's default: 1) P rises to P_1^{**} and Firm 2 acquires Firm 1 before Firm 2 defaults, 2) P falls to P_2^* first, and Firm 2 defaults without having made the acquisition. In the latter case, the default of Firm 1 plays no role in postponing the default of Firm 2. Thus, the potential to acquire Firm 1 provides only the possibility of protection for Firm 2 debt holders, not a guarantee.

The task now is to find $D_A(P_1^*, C_2, C_1)$, Firm 2 debt value prior to the default of Firm 1. Consider Firm 2 debt value if Firm 1 has already defaulted and been acquired. The default trigger of the merged firm is P_{m2}^*. When Firm 2 defaults, debt holders operate

Firm 2's original assets forever, which has present value $\eta(1-\pi)P^*_{m2}/\delta$. Substituting this as the "liquidation value" in (8) yields debt value at the time of the acquisition:

$$\frac{C_2}{r}+\left[\frac{\eta(1-\pi)P^*_{m2}}{\delta}-\frac{C_2}{r}\right]\left(\frac{P^*_{m2}}{P}\right)^{\beta} \tag{23}$$

If Firm 2 defaults before acquiring Firm 1, Firm 2 debt holders operate Firm 2 forever, which has value $\eta(1-\pi)P^*_2/\delta$ at the time of default.[6]

Now consider Firm 2 debt value after Firm 1 defaults, but before either Firm 1 is acquired or Firm 2 defaults. Denote this $D_{Ad}(P, C_2, C_1)$, which follows the process (6), subject to boundary conditions

$$\begin{aligned} D_{Ad}(P^{**}_1, C_2, C_1) &= \frac{C_2}{r}+\left[\frac{\eta(1-\pi)P^*_{m2}}{\delta}-\frac{C_2}{r}\right]\left(\frac{P^*_{m2}}{P^{**}_1}\right)^{\beta} \\ D_{Ad}(P^*_2, C_2, C_1) &= \eta(1-\pi)P^*_2/\delta \end{aligned}$$

The general solution is $D_{Ad}(P, C_2, C_1) = C_2/r + e_1P^{\alpha} + e_2P^{-\beta}$. The boundary conditions determine

$$\begin{aligned} e_1 &= \left\{\left[\frac{\eta(1-\pi)P^*_{m2}}{\delta}-\frac{C_2}{r}\right]\left(\frac{P^*_{m2}}{P^{**}_1}\right)^{\beta}-\left[\frac{\eta(1-\pi)P^*_2}{\delta}-\frac{C_2}{r}\right]\left(\frac{P^*_2}{P^{**}_1}\right)^{\beta}\right\}Q^{-1} \\ e_2 &= \left[\frac{\eta(1-\pi)P^*_2}{\delta}-\frac{C_2}{r}\right]P^{*\beta}_2 - e_1P^{*\alpha+\beta}_2 \end{aligned}$$

where $Q = P^{**\alpha}_1 - P^{*\alpha}_2\left(P^*_2/P^{**}_1\right)^{\beta}$.

The value of Firm 2 debt before Firm 1 defaults follows (6), subject to boundary conditions

$$\begin{aligned} \lim_{P\to\infty} D_A(P, C_2, C_1) &= C_2/r \\ D_A(P^*_1, C_2, C_1) &= D_{Ad}(P^*_1, C_2, C_1) \end{aligned}$$

This yields

$$D_A(P, C_2, C_1) = \frac{C_2}{r}+\left[D_{Ad}(P^*_1, C_2, C_1)-\frac{C_2}{r}\right]\left(\frac{P^*_1}{P}\right)^{\beta} \tag{24}$$

This has no closed-form solution, but numerical solutions verify some important properties. First, $D_A(P, C_2, C_1)$ is increasing in C_1, as this makes acquisition more

[6] If there were an outside bidder, Firm 2 debt holders would have the option of waiting to liquidate. This adds another layer to the computation of Firm 2 debt value. If $\eta = \theta$, the value of operating in default equals the bid of the outsider, leaving the results unchanged.

likely before Firm 2 defaults itself. P_1^* is increasing in C_1, which decreases the distance between P_1^* and P_1^{**} while increasing the distance between P_1^* and P_2^* (recall that P_1^{**} is independent of C_1). This increases the probability that P will reach Firm 1's liquidation trigger before reaching Firm 2's default trigger. Second, $D_A(P, C_2, C_1)$ is decreasing in η, as this increases P_1^{**} and decreases the probability of acquisition. Third, $D_A(P, C_2, C_1)$ is increasing in γ, as this increases Firm 2's bid, and the probability of acquisition.

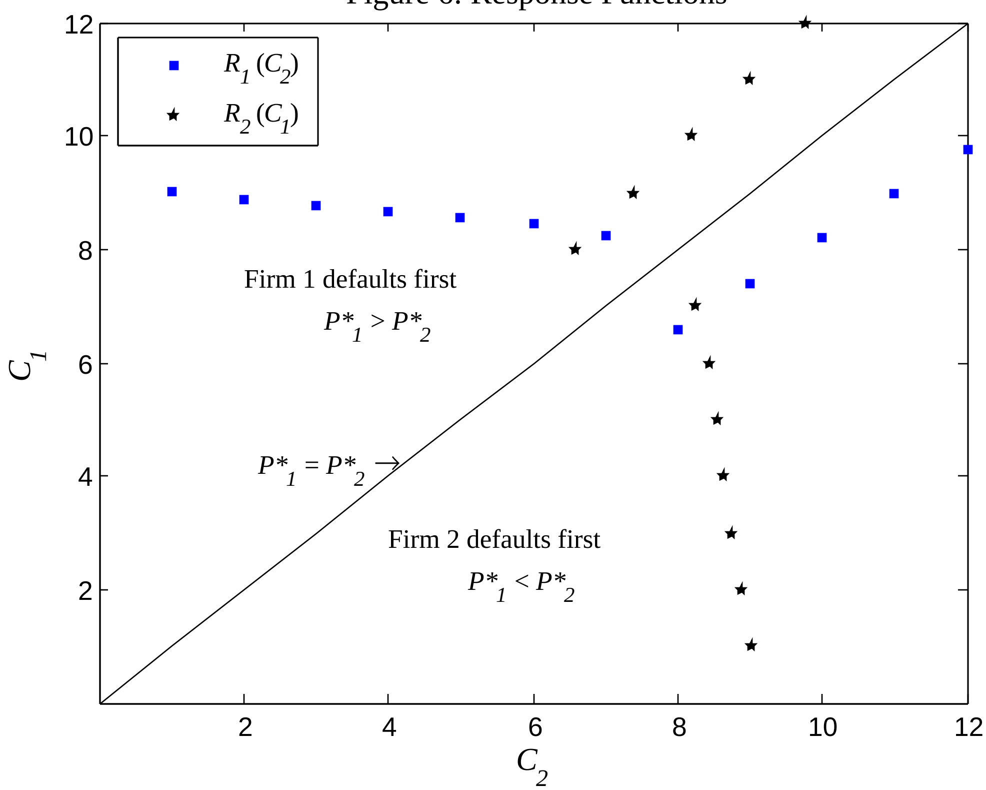
As in Section II, by assuming the liquidator has all the bargaining power, equity gains nothing by the acquisition. Firm 2 value is $V_A(P, C_2, C_1) = E(P, C_2) + D_A(P, C_2, C_1)$, given by (5) and (24).

B. Capital structure

As before, maximizing firm value proceeds by maximizing V_A and V_L over the relevant ranges of coupon payments, and then choosing the maximum of the two strategies. Figure 6 plots the response functions of each firm for $\eta = .5$ and $\gamma = .7$. $R_1(C_2)$ is clearly decreasing in C_2 over the region in which Firm 1 defaults first. Even with the ability of debt holders to operate the firm in default, the effect of Firm 2's leverage on acquisition bids still raises the cost of debt to Firm 1. For instance, $R_1(3) = 8.9$ while $R_1(7) = 8.26$. In leverage terms, if Firm 2 leverage is .38, Firm 1's optimal leverage is .82. If Firm 2 leverage is .71, Firm 1 leverage is .78. This relationship is more sensitive when η is low, as debt holders rely more on Firm 2's bid to recoup losses in default, raising the cost of Firm 1's debt. The relationship is more sensitive for higher values of σ. With a greater chance of default, recovery in default weighs more heavily in the cost of debt.

In contrast to the results in Section II, the values of $R_1(C_2)$ and $R_2(C_1)$ plotted in Figure 6 suggest that a Nash equilibrium does exist, with $(C_1^{NE}, C_2^{NE}) = (8.3, 6.8)$. This corresponds to leverages of .78 and .69. Firm 1 defaults first and Firm 2 is the potential acquirer. Since the firms are identical, an equilibrium also exists in which Firm 2 defaults first. Assume for purposes of discussion the equilibrium in which Firm 1 is the first to default. Why does an equilibrium exist in this case and not in the automatic liquidation model? The ability of debt holders to wait before liquidating lowers the cost of debt to the firm defaulting first, while increasing it for the firm defaulting second. V_L rises while V_A falls, and defaulting first maximizes Firm 1 value over a greater range of C_2. Note from Figure 6 that when Firm 2 is second to default, it does not use as much debt as possible; C_2 is significantly lower than C_1. Since debt is now more costly, Firm 2 uses less, shifting the plotted points to the left, away from the default order boundary.

Figure 6: Response Functions



At this lower level of C_2, Firm 1 value is now maximized by being the first to default. In this equilibrium, two identical firms have quite different leverages. This may help explain why a large degree of heterogeneity is observed in the capital structure of firms in the same industry.

Numerical solutions also suggest that if η is increased, the equilibrium debt level of Firm 1 rises while that of Firm 2 falls. This follows since Firm 1 debt holders earn more and wait for a higher bid in default, while those of Firm 2 face a greater risk that the acquisition will never be made. An increase in γ raises the equilibrium debt level of both firms, since Firm 1 debt holders receive a higher bid, while Firm 2 debt holders benefit from the greater probability that an acquisition is made. This suggests another reason why older, more established industries should have higher leverage than new, high-tech ones. In older industries, technology is more uniform across firms, and operations are understood by a greater number of people. One firm can take over the operations of another without much loss in productivity. Newer industries are often associated with a high degree of idiosyncratic risk, which suggests the use of low leverage for two reasons: 1) the risk of bankruptcy costs is greater, 2) the asymmetry of information between insiders (equity holders) and outsiders (debt holders) is greater. Here we see that even when every firm in the industry receives the same cash flow (no idiosyncratic shocks), and the cash flow process is observed by both equity and debt holders, leverage should still be lower if there is great asymmetric information regarding the firm's technology and operations. This will raise bankruptcy costs by preventing either debt holders or acquirers, or both, from being fully productive operators. For some parameter values, a Nash equilibrium does not exist. If γ is too great compared to η, V_L falls in relation to V_A, with a result similar to that in Section II.

Returning to the relationship between incumbent and entrant, consider a firm (the incumbent) entering at time 0 and choosing debt level C_I to maximize firm value given cash flow P_0. At time τ, an entrant chooses debt level C_E to maximize firm value given P_τ and C_I. As a result of the entrant's choice, the two firms have leverages Lev_I and Lev_E, respectively, based on their debt levels, P_τ, and the order in which they default. Suppose $P_\tau = 10$. Figure 7 displays the entrant's choice of C_E for a given C_I, and the resulting leverages of the two firms. The graphs reveal whether an entrant will position itself to potentially acquire, or be acquired by, the incumbent. They also show how sensitive the entrant's optimal leverage is to that of the incumbent.

Graphs (a) and (b) demonstrate the effect of cash flow volatility by plotting C_E and Lev_E for $\sigma = .15$ and $\sigma = .25$ (with $\gamma = .7$ and $\eta = .5$). The solid line in Graph

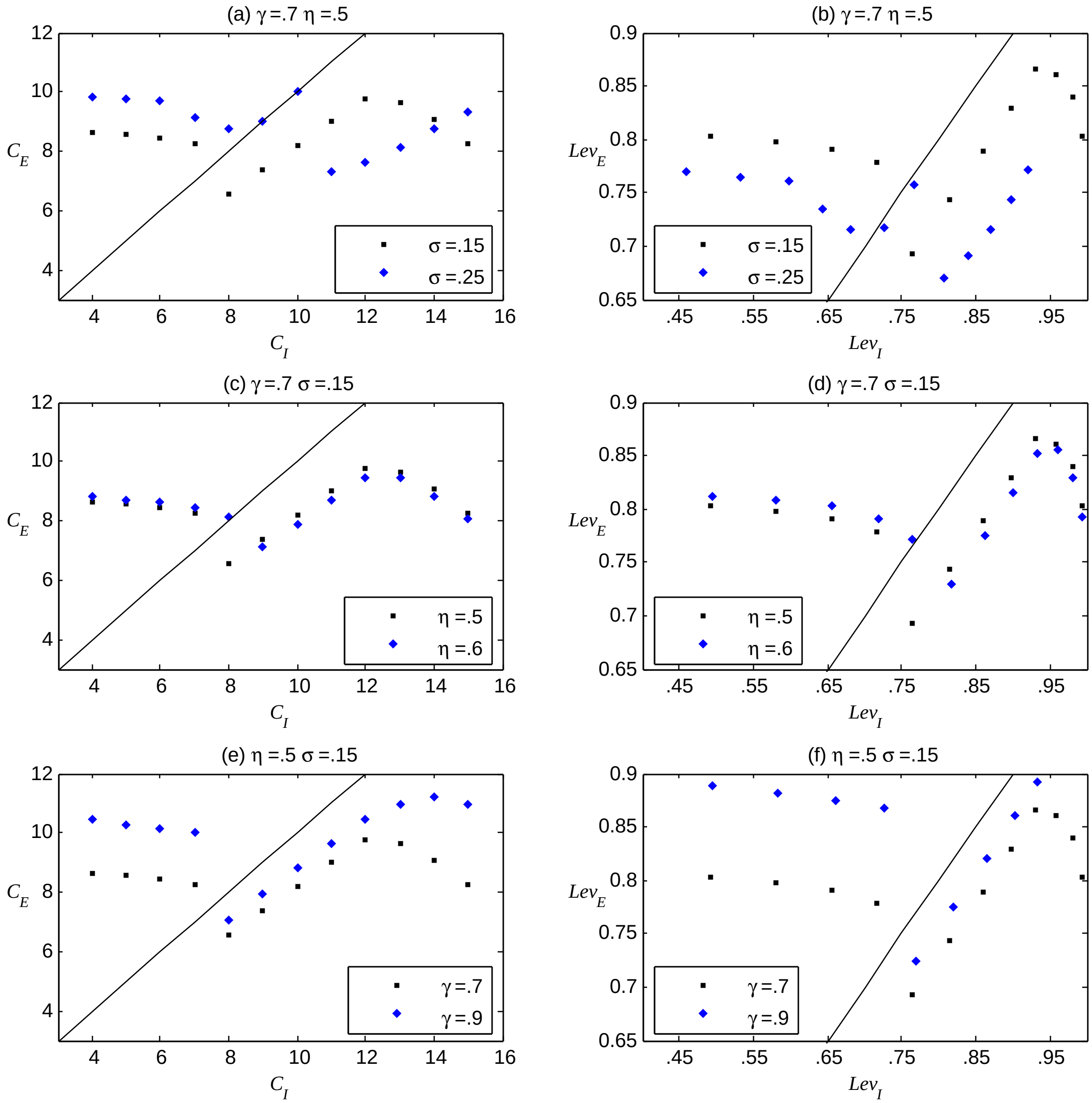
(a) marks $C_E = C_I$, and reveals the order of default. The solid line in Graph (b) marks $Lev_E = Lev_I$. This does not exactly correspond to the order of default, since the formulas for firm value, and leverage, vary with the order of default. It is simply for comparing the leverages. First, note that higher volatility decreases Lev_E for almost every value of Lev_I. This is simply because the probability of default is greater, and the cost of debt higher. Note that for $\sigma = .25$, Lev_E is more sensitive to Lev_I when the entrant is first to default. Since there is a higher probability of default, the higher loss in default owing to the lower bid makes a greater impact on the cost of the entrant's debt. Finally, the entrant in the more volatile industry is quicker to switch to the acquirer strategy as Lev_I increases. For $\sigma = .25$, the entrant plays the acquirer when $C_I \geq 9$, or $Lev_I \geq .72$. For $\sigma = .15$ the entrant still plays the liquidator at those values. This is because volatility increases default probability, raising V_A compared to V_L.

Graph (c) and (d) display the effects of debt holders' ability to operate the firm in default, plotting C_E and Lev_E for $\eta = .5$ and $\eta = .6$ ($\sigma = .15$, $\gamma = .7$). For higher η, optimal Lev_E is higher when the entrant is first to default, and lower when it defaults second. Since debt holders can better operate the firm, recovery in default is higher, and the cost of debt lower when the firm is first to default. For the same reason, debt holders of the defaulted firm will wait for a higher bid, decreasing the chance that the would-be acquirer actually makes an acquisition, raising its cost of debt. It is also clear that the entrant is quicker to switch to playing the acquirer as Lev_I increases. This is because the ability of debt holders to operate reduces V_A compared to V_L.

Graphs (e) and (f) demonstrate the effect of the insider's ability to operate a competitor's firm, plotting C_E and Lev_E for $\gamma = .7$ and $\gamma = .9$ ($\sigma = .15$, $\eta = .5$). The greater is γ, the greater is optimal leverage both when playing the liquidator and the acquirer. The debt holders of the first firm to default have the benefit of a higher bid, while those of the acquirer benefit from a greater probability of acquisition.

It may be tempting to draw conclusions from these graphs about how these parameters affect heterogeneity among firms in an industry. For instance, Graph (b) shows that in volatile industries Lev_I and Lev_E will be closer when Lev_I is low. Since entry generally occurs when P is high and Lev_I low, one may deduce that firms in less volatile industries should have more heterogeneity in their leverage. However, entry occurs only when P is sufficiently high. According to standard models of entry under uncertainty, the threshold for P is higher when volatility is higher. This means Lev_I is lower at entry if the industry is volatile, and Lev_E higher. This increases heterogeneity in volatile industries compared to less volatile ones. Thus, it is not clear how volatility affects

Figure 7: Leverage in Sequential Entry



heterogeneity. To explore these issues, the model could be extended to endogenize the entry decision.

IV. Conclusions

This paper has investigated the effects of debt overhang on acquisition bids for bankrupt firms. It has derived the implications for the cost of debt, both for the liquidator and the acquirer, and used these in a model of industry equilibrium where the order of default is chosen endogenously. The main findings are:

1. Acquisition bids are decreasing in the leverage of the acquirer. The degree of this debt overhang effect depends on how the acquisition is financed. The bid for all-equity finance has a low upper bound. The bid for all-debt finance can be much higher if the firm is less leveraged, but is severely diminished by debt overhang. Bids financed by debt and equity are highest, but still subject to debt overhang.

2. The leverage of a potential acquirer can significantly increase the cost of debt for a potential liquidator. A firm can reduce its cost of debt by being better capitalized than industry peers, thereby signaling that it will make acquisitions in an industry downturn. This can reduce the acquirer's cost of debt by offering a cushion against default. The cost of debt can be decreasing in the leverage of the potential liquidator.

3. The response function of a firm to the leverage of an industry peer is non-monotonic. If the peer has low leverage, it is optimal to use higher leverage and play the liquidator. If the peer has high leverage, it is optimal to use somewhat lower leverage and play the acquirer. Over the lower range of peer leverage the response function is decreasing. Over the higher range it is increasing. This is in contrast to Shleifer and Vishny, who find only a negative relationship.

4. Industry equilibrium involves heterogeneity in capital structure, consisting both of firms that are highly levered to take advantage of tax benefits, and those that are well capitalized and will acquire the others in times of adversity.

These results add to those of several recent studies by suggesting another channel by which heterogeneity in capital structure exists in industry equilibrium. Although the implications for the cross-section of leverage within an industry may not be distinguishable from those of other studies, this model has unique implications for the entrant/incumbent relationship. This highlights the importance of taking a "dynamic" rather than cross-sectional approach to empirical work, in which capital structure choices at time of entry are observed as a response to conditions in the industry.

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