BioStat Bootstrapping for Estimating Uncertainty in Commercial Landings at Age

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Introduction

The program BioStat has been used for a number of years at the NEFSC to estimate commercial landings at age using landed weight by market category along with length and age samples collected by port samplers. The estimation process is quite straightforward and relies upon sufficient port sampling to characterize the landings by length and the associated age composition at length. However, until now the uncertainty of these estimates has not been characterized due to the complexity of the bookkeeping inherent in the BioStat calculations. Bootstrapping has been incorporated in the software to allow estimation of the uncertainty of the landings at age as well as the correlation among estimates at age. This working paper describes the bootstrapping process within BioStat and describes a few of the ways this information can be used by stock assessment scientists.

Methods

There are four basic types of information used in the BioStat calculations: 1) landings in weight, 2) length-weight relationship, 3) length samples, and 4) age at length samples. These data are usually collected in blocks consisting of time periods, for example quarters or half-years, and market categories, for example small, medium, and large fish, and sometimes also by region when growth differs. Within a given block, the length samples are summed and the expected weight calculated using the length-weight equation. Dividing the landings by the expected weight produces an expansion factor that is multiplied by the summed length frequency to produce the number of fish at length in that block. The age samples are summed for that time period, but over all market categories, to estimate the proportions of fish at age for each length. If there are lengths with no age data, then the analyst must supply an age distribution for that length. Multiplying the numbers at length by the proportions at age and summing over lengths produces the number of fish at age for that block. Other characteristics of that block, such as average length and weight at age are also calculated. Remaining market categories are then examined using possibly different time periods due to availability of samples. Final estimates of landings at age are sums over all the estimated market categories. Market categories with only small amounts of landings and no length samples can be prorated to the total distribution automatically, if desired.

The new bootstrapping feature of BioStat attempts to replicate the original sampling process as much as possible to derive estimates of variance for the landings at age and covariance of estimates among ages. Two potential sources of uncertainty that are not addressed in the bootstrapping are the landings by market category and the length-weight relationship. These are both assumed known without error for the time period considered, for example, quarter 1 or half 2. Inclusion of uncertainty in these parameters would increase the overall uncertainty and could impact the correlations among estimates of landings at age. However, these sources of uncertainty are probably minor in most cases relative to the uncertainty in length at age and age at length from the port samples as estimated by bootstrapping.

The bootstrapping procedure attempts to mimic the actual sampling by linking the length at age samples. Consider one block containing L length samples. Each length sample contains a length distribution. The age sampling is usually conducted on a number of ages per length interval basis, although not all length samples have age samples collected and occasionally there are age samples collected without corresponding length samples. The bootstrapping process begins by randomly selecting with replacement the length samples L times. The next level of bootstrapping is to resample with replacement the length distributions within each length sample. The final level of bootstrapping is to randomly select ages at length for each of the bootstrapped length samples that had an associated age sample. These age samples are limited to the number of ages at length in the original age sample, but if fewer fish at that length occur in the bootstrap, then only this number will occur in the age bootstrap. After all three levels of bootstrapping have been completed for all market categories, the process of estimating landings at age proceeds in the same manner as the point estimate, with the exception that blocks with lengths but no associated ages are not filled by hand but rather by using the original age-length key for that block. This can occur because of the bootstrap process not selecting length samples with ages associated with specific lengths or due to the "orphan effect" of the original sample containing age samples without length samples. This method of filling the holes will reduce the overall amount of uncertainty in the estimates, but the number of times this occurs is tracked and can be used as a diagnostic for insufficient samples for bootstrapping. The distribution of landings at age from the bootstraps is then used directly to estimate the variance at age and covariance among ages of the estimates.

Diagnostics for the bootstrapping process include the proportion of times ages had to be filled as mention above, along with a comparison of the point estimates with the mean of the bootstrapped landings at age estimates and coefficients of variation for landings at age. The comparison of the point estimate and the mean is a measure of bias. If this difference is large, say greater than 5% for ages with reasonable numbers of fish estimated, then this is an indication that the original samples are insufficient for bootstrapping. This could be caused by too few samples, too much variability within samples, too many unlinked length at age samples, or some other problem with the collection of the original data. This means that the point estimate may be of questionable quality as well. The coefficients of variation by age, as well as the covariance matrix, can be used as an indicator of sufficiency of sampling when deciding a plus group or for modifying future sample collection requests. Often the youngest or oldest ages have quite high CVs and may be positively correlated. Georges Bank haddock in years 2000 through 2003 was used as an example for the BioStat bootstrapping process using both quarterly time steps and half-year time steps to demonstrate the diagnostic ability of the bootstrapping procedure.

Results and Discussion

Comparison of the landings at age point estimates with the means from 1000 bootstraps for both quarterly and half-year time steps show low bias in general (Tables 1 and 2). The point estimates from the quarter and half-year time steps also showed high agreement in general (Table 3). The two time steps also showed strong agreement in the proportion of lengths that required filling for age distribution from the original data, a measure of the sufficiency of the age samples (Table 4). In general, the half-year time steps have lower bias than the quarter time steps. This occurs because the same length-weight equation was used in quarters 1 and 2 and in quarters 3 and 4 and the length and age samples did not vary appreciably by quarter, meaning a net result of higher sample size and thus lower bias when combined at half-years instead of quarters. If the length-weight equation did change between quarters or the length distributions or distributions of ages at length changed by quarter, then a larger difference between the two time periods would be expected. In general, larger time periods with higher sample sizes are recommended as long as the length-weight equations do not change and there is no evidence of growth during the shorter time periods.

The coefficient of variation (CV) in bootstrapped mean landings at age were rather large for both the youngest and oldest age classes (age 2 and the 9+ group). For the remaining age classes, the CV was generally around 5-20% (Table 5). These results were consistent between the quarterly and half-year time step, with very little difference between the two analyses. For the years examined, the correlation matrix showed very little correlation between landings at age, with most correlations between 0 and -0.1 (Table 6, Fig. 1). In a few instances, there were positive correlations, often occurring between the older age classes. Stronger negative correlations tended to occur among ages 4-6. Again, this result was consistent for both quarterly and half-year time steps.

These patterns in correlations can be explained both mechanistically and biologically. The positive correlations seen at older ages is induced partly by the bootstrap sampling procedure and partly from the fact that those age classes are asymptoting in length. There are generally few samples at large lengths, and the associated age can span several age classes. Therefore, the augmentation of landings among the older age classes is positively correlated with whether or not a larger length sample is selected in the bootstrap procedure. Length samples at the smallest sizes can also be scarce, but may not necessarily induce positive correlations because growth is more rapid and lengths would not be spread across many ages, as it is for the largest lengths. Instead, the scarcity of smaller lengths, and the bootstrapping procedure which may or may not select them at any given iteration, tends to produce very wide ranges of estimated landings, as is seen in the estimated CVs (Table 5) as well as in the density plots (Fig. 2). The negative correlations for the fish in the midrange of lengths and ages, where there is often the highest sampling, occurs because the bootstrapped length frequencies are relatively stable and the bootstrapped age samples allocate them proportionally among relatively few ages. Thus, an increase in one age means a decrease in the adjacent ages. Fish behavior such as schooling by size, management measures such as minimum size limits, and year class strength and growth pattern, in combination with the bootstrap sampling procedure, are all factors which contribute to the correlations between the estimated landings at age.

The positive correlations among some ages conflicts with the common assumption of a multinomial distribution where all ages are negatively correlated. Many forward projecting models make this assumption when comparing estimates of observed and predicted catch at age. Some very preliminary work comparing the multinomial assumption with using the covariance matrix directly in the calculation of the likelihood produced estimates of F that could be quite different. More testing is needed though before any definitive results can be concluded regarding the appropriateness of the error structure for forward projecting models. If the multinomial assumption must be made, different plus groups could be explored to determine which most closely replicates the multinomial distribution with all negative correlations. Alternatively, the bootstrapped landings at age could be incorporated into VPA using each bootstrap as a realization of the landings at age.

For comparison with the model-based estimates of variance of landings at age presented in Tim Miller's working paper, witch flounder landings data in 2003 were bootstrapped 1000 times and the coefficients of variation at age (Table 7) and correlation matrix (Table 8) were computed. The trends in landings at age and coefficient of variation at age are quite similar to those from the analytic method (Figure 3) and the correlation matrix agrees in general, although there are some differences. It cannot be determined from the results alone which approach is more correct. There is a speed difference between the two methods though, with bootstrapping taking minutes and the model-based solutions taking hours to compute. This makes estimations with different plus groups much easier with the bootstrapping approach than the model-based approach.

The use of a plus group is common in many stock assessment models. The age at which to plus the catch can be based on the bootstrapping results by examining the trends in coefficient of variation over age. When the landings for older ages are compressed into a plus group, the coefficient of variation for the plus group age is almost always lower than all of the coefficients of variation for the individual ages, due to the increased sample size for the plus group. Conversely, the requests to port samplers could be increased for market categories containing larger fish if it was desired to have an older plus group.

Conclusions

The addition of bootstrapping to the BioStat program offers an easy method to estimate the uncertainty of commercial landings at age. These uncertainty estimates can be used to help guide the determination of a plus group, inform port sampling requests, and provide input to statistical catch at age models. It is recommended that bootstrapping be performed regularly as part of the estimation of landings at age.

Table 1. Comparison of Georges Bank haddock landings at age from the point estimate and the mean of 1000 bootstraps for years 2000 through 2003 using quarterly time steps. Highlighted cells denote percent difference of more than 5%.

Year	Source	age2	age3	age4	age5	age6	age7	age8	age9+	total
2000	point	81237	175918	333282	328216	309828	155841	75899	29667	1489888
	boot	83356	178963	326136	329596	313554	156826	74262	29972	1492665
	% diff	2.6	1.7	-2.1	0.4	1.2	0.6	-2.2	1.0	0.2
2001	point	70349	648290	412383	449273	360585	220329	129751	80860	2371819
	boot	70895	659324	412370	466666	355559	215341	121422	80975	2382552
	% diff	0.8	1.7	0.0	3.9	-1.4	-2.3	-6.4	0.1	0.5
2002	point	3342	100506	1234897	517611	436281	277458	194453	270754	3035302
	boot	2557	99891	1269466	509792	431189	282375	188913	264911	3049094
	% diff	-23.5	-0.6	2.8	-1.5	-1.2	1.8	-2.8	-2.2	0.5
2003	point	1019	164578	209960	1441384	263780	331775	136897	239484	2788876
	boot	887	175945	206098	1444833	260650	333039	135133	237101	2793686
	% diff	-13.0	6.9	-1.8	0.2	-1.2	0.4	-1.3	-1.0	0.2

Table 2. Comparison of Georges Bank haddock landings at age from the point estimate and the mean of 1000 bootstraps for years 2000 through 2003 using half-year time steps. Highlighted cells denote percent difference of more than 5%.

Year	Source	age2	age3	age4	age5	age6	age7	age8	age9+	total
2000	point	78827	167555	316663	337828	322566	161583	68071	35223	1488317
	boot	81156	168470	312523	335392	322981	164126	71086	34835	1490569
	% diff	3.0	0.5	-1.3	-0.7	0.1	1.6	4.4	-1.1	0.2
2001	point	70598	628534	406950	450190	371833	230629	136837	89174	2384745
	boot	70899	637175	405335	461197	367193	227223	130234	90424	2389680
	% diff	0.4	1.4	-0.4	2.4	-1.2	-1.5	-4.8	1.4	0.2
2002	point	2145	96824	1252626	542783	438461	277055	199371	271153	3080418
	boot	2016	95315	1268238	542625	435730	281142	194445	267252	3086763
	% diff	-6.0	-1.6	1.2	0.0	-0.6	1.5	-2.5	-1.4	0.2
2003	point	1426	168871	213394	1467495	260863	342311	147093	248758	2850211
	boot	1257	173757	208628	1472002	258834	343014	145982	247855	2851329
	% diff	-11.9	2.9	-2.2	0.3	-0.8	0.2	-0.8	-0.4	0.0

Table 3. Comparison of Georges Bank haddock landings at age from the point estimates of quarterly time steps and half-year time steps for years 2000 through 2003. Highlighted cells denote percent difference of more than 5%.

Year	Source	age2	age3	age4	age5	age6	age7	age8	age9+	total
2000	half-year	78827	167555	316663	337828	322566	161583	68071	35223	1488317
	quarter	81237	175918	333282	328216	309828	155841	75899	29667	1489888
	% diff	3.1	5.0	5.2	-2.8	-3.9	-3.6	11.5	-15.8	0.1
2001	half-year	70598	628534	406950	450190	371833	230629	136837	89174	2384745
	quarter	70349	648290	412383	449273	360585	220329	129751	80860	2371819
	% diff	-0.4	3.1	1.3	-0.2	-3.0	-4.5	-5.2	-9.3	-0.5
2002	half-year	2145	96824	1252626	542783	438461	277055	199371	271153	3080418
	quarter	3342	100506	1234897	517611	436281	277458	194453	270754	3035302
	% diff	55.8	3.8	-1.4	-4.6	-0.5	0.1	-2.5	-0.1	-1.5
2003	half-year	1426	168871	213394	1467495	260863	342311	147093	248758	2850211
	quarter	1019	164578	209960	1441384	263780	331775	136897	239484	2788876
	% diff	-28.5	-2.5	-1.6	-1.8	1.1	-3.1	-6.9	-3.7	-2.2

Table 4. Proportion of lengths that required filling of age distribution from the original data by year and time period used within year for Georges Bank haddock. Median, 5%, and 95% refer to the distribution of proportions from the 1000 bootstraps.

Year	Period	median	5%	95%
1999	quarterly	5%	2%	10%
	semi	4%	0%	10%
2000	quarterly	4%	0%	6%
	semi	5%	0%	7%
2001	quarterly	1%	0%	4%
	semi	2%	0%	4%
2002	quarterly	0%	0%	1%
	semi	0%	0%	0%
2003	quarterly	3%	1%	5%
	semi	4%	0%	7%

Table 5. Coefficients of variation at age for Georges Bank haddock in years 2000-2003 using half-year time steps from 1000 bootstraps.

Year	age2	age3	age4	age5	age6	age7	age8	age9+
2000	0.25	0.13	0.13	0.11	0.10	0.16	0.23	0.40
2001	0.37	0.10	0.10	0.10	0.08	0.10	0.15	0.19
2002	1.36	0.30	0.09	0.10	0.12	0.13	0.15	0.19
2003	1.31	0.26	0.17	0.05	0.13	0.09	0.13	0.13

Table 6. Correlation matrix of landings at age for Georges Bank haddock in 2003 using half-year time steps from 1000 bootstraps. Highlighted cells had correlations of 0.25 or greater.

Age	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
2	1.00	0.04	-0.12	-0.03	0.05	-0.06	0.08	-0.01
3	0.04	1.00	0.04	-0.38	-0.11	-0.01	0.04	-0.03
4	-0.12	0.04	1.00	-0.25	-0.09	-0.05	-0.05	-0.06
5	-0.03	-0.38	-0.25	1.00	-0.16	-0.33	-0.19	-0.42
6	0.05	-0.11	-0.09	-0.16	1.00	-0.16	-0.27	-0.06
7	-0.06	-0.01	-0.05	-0.33	-0.16	1.00	-0.04	-0.02
8	0.08	0.04	-0.05	-0.19	-0.27	-0.04	1.00	0.04
9	-0.01	-0.03	-0.06	-0.42	-0.06	-0.02	0.04	1.00

Table 7. Point estimates, bootstrap means, percentage difference between the point estimate and bootstrap mean, standard deviation and coefficient of variation from 1000 bootstraps for witch flounder in 2003.

Age	Point Est	Boot Mean	% diff	Std Dev	Coeff. Variation
4	57533	51103	-11.2	29921	59%
5	548442	558256	1.8	113680	20%
6	1213878	1221826	0.7	165188	14%
7	1732633	1752644	1.2	158337	9%
8	1527581	1516227	-0.7	152649	10%
9	742558	729196	-1.8	106946	15%
10	437585	437312	-0.1	75343	17%
11	145794	146997	0.8	32348	22%
12	64170	65242	1.7	16967	26%
13	28686	28728	0.1	8452	29%
14	73407	71300	-2.9	15546	22%
15	2694	2799	3.9	1155	41%
16	4004	4077	1.8	1689	41%
17	1910	1836	-3.9	1302	71%
18	12800	12728	-0.6	3944	31%
19	270	249	-7.8	182	73%
20	391	432	10.5	268	62%
21	215	212	-1.4	195	92%
22	2387	2490	4.3	1063	43%
23	0	0	0.0	0	
24	18	16	-11.1	20	124%
25	31	29	-6.5	43	147%
26	101	119	17.8	164	138%
27	122	119	-2.5	178	149%
28	44	32	-27.3	37	117%
29	110	140	27.3	174	125%
30	513	558	8.8	542	97%
31	149	178	19.5	240	135%
32	0	0	0.0	0	
33	18	16	-11.1	22	140%

Table 8. Correlation matrix for witch flounder in 2003 from 1000 bootstraps.

Age	4	5	6	7	8	9	10	11	12	13	14	15	16	17
4	1.00	0.07	0.07	0.00	0.00	-0.16	0.01	-0.03	-0.08	-0.11	-0.06	-0.05	-0.03	-0.01
5	0.07	1.00	0.22	0.08	-0.23	-0.37	-0.10	-0.12	-0.28	-0.01	-0.08	0.01	0.00	-0.01
6	0.07	0.22	1.00	0.02	-0.32	-0.36	-0.30	-0.16	-0.01	-0.27	-0.14	-0.04	-0.02	-0.05
7	0.00	0.08	0.02	1.00	-0.33	-0.46	-0.34	-0.18	-0.01	0.00	-0.16	0.00	-0.06	0.03
8	0.00	-0.23	-0.32	-0.33	1.00	-0.08	-0.13	0.24	-0.17	-0.07	-0.02	0.02	-0.02	0.02
9	-0.16	-0.37	-0.36	-0.46	-0.08	1.00	0.25	-0.05	0.14	0.07	0.17	0.01	0.06	-0.03
10	0.01	-0.10	-0.30	-0.34	-0.13	0.25	1.00	-0.14	-0.04	0.27	-0.07	-0.10	0.05	-0.10
11	-0.03	-0.12	-0.16	-0.18	0.24	-0.05	-0.14	1.00	0.03	-0.06	0.26	-0.01	0.03	0.05
12	-0.08	-0.28	-0.01	-0.01	-0.17	0.14	-0.04	0.03	1.00	0.04	0.14	-0.07	-0.07	-0.13
13	-0.11	-0.01	-0.27	0.00	-0.07	0.07	0.27	-0.06	0.04	1.00	0.03	-0.02	0.16	-0.08
14	-0.06	-0.08	-0.14	-0.16	-0.02	0.17	-0.07	0.26	0.14	0.03	1.00	0.09	0.09	0.19
15	-0.05	0.01	-0.04	0.00	0.02	0.01	-0.10	-0.01	-0.07	-0.02	0.09	1.00	0.05	0.31
16	-0.03	0.00	-0.02	-0.06	-0.02	0.06	0.05	0.03	-0.07	0.16	0.09	0.05	1.00	0.08
17	-0.01	-0.01	-0.05	0.03	0.02	-0.03	-0.10	0.05	-0.13	-0.08	0.19	0.31	0.08	1.00
18	0.00	-0.05	-0.04	0.01	0.01	-0.01	-0.09	0.02	-0.12	-0.08	0.29	0.32	0.04	0.56
19	-0.01	-0.04	-0.04	-0.04	0.04	0.04	0.04	0.02	0.00	0.10	0.02	-0.09	0.31	-0.07
20	-0.04	-0.02	0.02	0.00	-0.04	0.03	0.01	-0.01	0.06	0.03	0.06	-0.02	-0.08	0.03
21	-0.02	-0.04	0.04	0.06	0.01	-0.06	0.00	0.04	-0.04	-0.02	-0.01	0.00	0.00	-0.02
22	0.02	0.01	0.08	0.03	0.00	-0.08	0.00	-0.03	0.05	0.04	-0.07	0.02	-0.06	-0.09
24	-0.02	-0.02	0.01	0.03	-0.04	0.02	-0.03	-0.02	0.06	0.01	0.04	-0.01	-0.03	-0.02
25	0.04	-0.05	-0.04	-0.02	0.06	0.04	0.01	0.03	0.00	0.02	0.03	-0.06	-0.04	-0.03
26	0.02	-0.03	0.02	-0.01	0.03	-0.02	0.02	0.02	0.02	-0.02	0.01	0.10	-0.03	0.02
27	-0.05	0.01	-0.01	0.02	0.03	-0.02	-0.03	0.01	-0.01	-0.06	-0.02	0.00	-0.09	0.14
28	-0.01	0.00	-0.05	0.01	0.03	0.03	-0.01	0.02	0.01	0.01	0.02	0.00	-0.04	0.04
29	-0.03	0.02	0.01	0.01	0.02	-0.02	-0.01	-0.01	-0.01	0.03	-0.04	0.04	-0.04	0.00
30	0.00	-0.02	-0.05	-0.01	0.03	0.03	-0.01	0.02	-0.02	0.00	0.01	0.02	-0.04	0.01
31	0.00	0.01	0.00	0.02	0.00	-0.01	0.02	-0.01	0.02	0.01	0.01	-0.06	-0.02	0.02
33	0.00	0.01	-0.02	0.04	0.00	-0.01	-0.02	-0.02	0.00	-0.06	-0.01	-0.04	-0.01	-0.01

Table 8. continued

Age	18	19	20	21	22	24	25	26	27	28	29	30	31	33
4	0.00	-0.01	-0.04	-0.02	0.02	-0.02	0.04	0.02	-0.05	-0.01	-0.03	0.00	0.00	0.00
5	-0.05	-0.04	-0.02	-0.04	0.01	-0.02	-0.05	-0.03	0.01	0.00	0.02	-0.02	0.01	0.01
6	-0.04	-0.04	0.02	0.04	0.08	0.01	-0.04	0.02	-0.01	-0.05	0.01	-0.05	0.00	-0.02
7	0.01	-0.04	0.00	0.06	0.03	0.03	-0.02	-0.01	0.02	0.01	0.01	-0.01	0.02	0.04
8	0.01	0.04	-0.04	0.01	0.00	-0.04	0.06	0.03	0.03	0.03	0.02	0.03	0.00	0.00
9	-0.01	0.04	0.03	-0.06	-0.08	0.02	0.04	-0.02	-0.02	0.03	-0.02	0.03	-0.01	-0.01
10	-0.09	0.04	0.01	0.00	0.00	-0.03	0.01	0.02	-0.03	-0.01	-0.01	-0.01	0.02	-0.02
11	0.02	0.02	-0.01	0.04	-0.03	-0.02	0.03	0.02	0.01	0.02	-0.01	0.02	-0.01	-0.02
12	-0.12	0.00	0.06	-0.04	0.05	0.06	0.00	0.02	-0.01	0.01	-0.01	-0.02	0.02	0.00
13	-0.08	0.10	0.03	-0.02	0.04	0.01	0.02	-0.02	-0.06	0.01	0.03	0.00	0.01	-0.06
14	0.29	0.02	0.06	-0.01	-0.07	0.04	0.03	0.01	-0.02	0.02	-0.04	0.01	0.01	-0.01
15	0.32	-0.09	-0.02	0.00	0.02	-0.01	-0.06	0.10	0.00	0.00	0.04	0.02	-0.06	-0.04
16	0.04	0.31	-0.08	0.00	-0.06	-0.03	-0.04	-0.03	-0.09	-0.04	-0.04	-0.04	-0.02	-0.01
17	0.56	-0.07	0.03	-0.02	-0.09	-0.02	-0.03	0.02	0.14	0.04	0.00	0.01	0.02	-0.01
18	1.00	-0.14	0.05	-0.01	-0.09	0.01	-0.03	-0.02	-0.02	-0.02	0.00	0.02	-0.01	0.00
19	-0.14	1.00	0.04	0.00	-0.11	-0.06	-0.02	0.06	-0.05	-0.07	-0.03	-0.05	0.02	-0.07
20	0.05	0.04	1.00	-0.06	-0.06	-0.06	-0.04	-0.05	0.03	-0.06	0.19	0.06	-0.01	0.00
21	-0.01	0.00	-0.06	1.00	0.00	0.04	-0.02	0.01	0.02	-0.07	-0.04	-0.02	0.01	0.01
22	-0.09	-0.11	-0.06	0.00	1.00	0.03	-0.01	-0.08	-0.04	0.03	-0.05	0.03	0.17	0.04
24	0.01	-0.06	-0.06	0.04	0.03	1.00	-0.08	0.04	0.01	-0.04	-0.01	0.01	-0.06	0.40
25	-0.03	-0.02	-0.04	-0.02	-0.01	-0.08	1.00	0.00	-0.06	0.42	-0.02	-0.03	0.12	-0.11
26	-0.02	0.06	-0.05	0.01	-0.08	0.04	0.00	1.00	-0.02	-0.04	-0.02	-0.04	0.01	-0.03
27	-0.02	-0.05	0.03	0.02	-0.04	0.01	-0.06	-0.02	1.00	-0.03	-0.02	-0.03	-0.05	0.03
28	-0.02	-0.07	-0.06	-0.07	0.03	-0.04	0.42	-0.04	-0.03	1.00	-0.04	-0.02	0.15	-0.07
29	0.00	-0.03	0.19	-0.04	-0.05	-0.01	-0.02	-0.02	-0.02	-0.04	1.00	0.08	-0.02	0.02
30	0.02	-0.05	0.06	-0.02	0.03	0.01	-0.03	-0.04	-0.03	-0.02	0.08	1.00	0.08	0.01
31	-0.01	0.02	-0.01	0.01	0.17	-0.06	0.12	0.01	-0.05	0.15	-0.02	0.08	1.00	-0.06
33	0.00	-0.07	0.00	0.01	0.04	0.40	-0.11	-0.03	0.03	-0.07	0.02	0.01	-0.06	1.00

Figure 1. Scatterplots of estimated landings at age for Georges Bank Haddock in year 2003, using half year time steps from 1000 bootstraps.

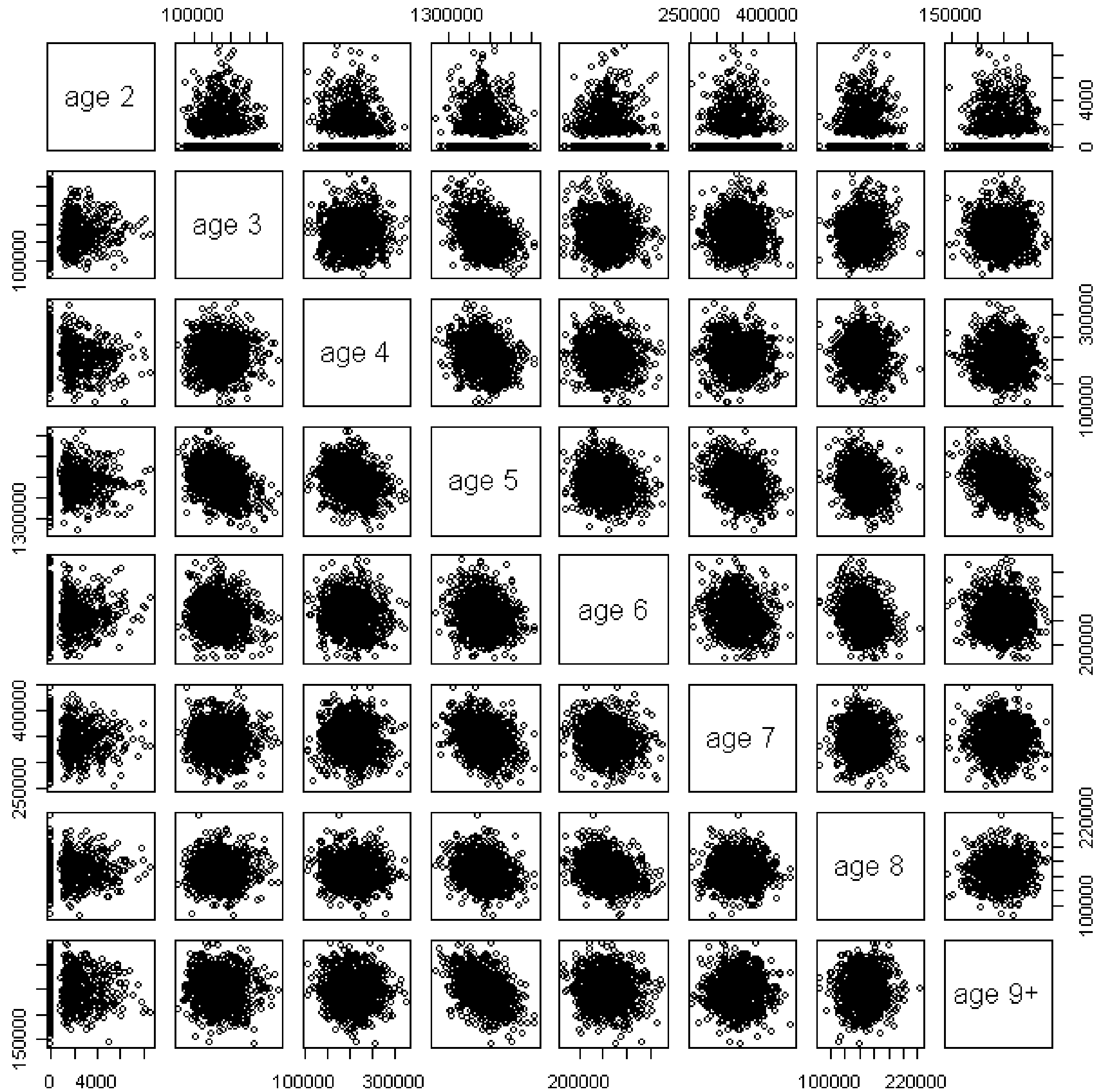

Figure 2. Densities of landings at age for 1000 bootstrap iterations for Georges Bank haddock in year 2003 from half-year time steps.

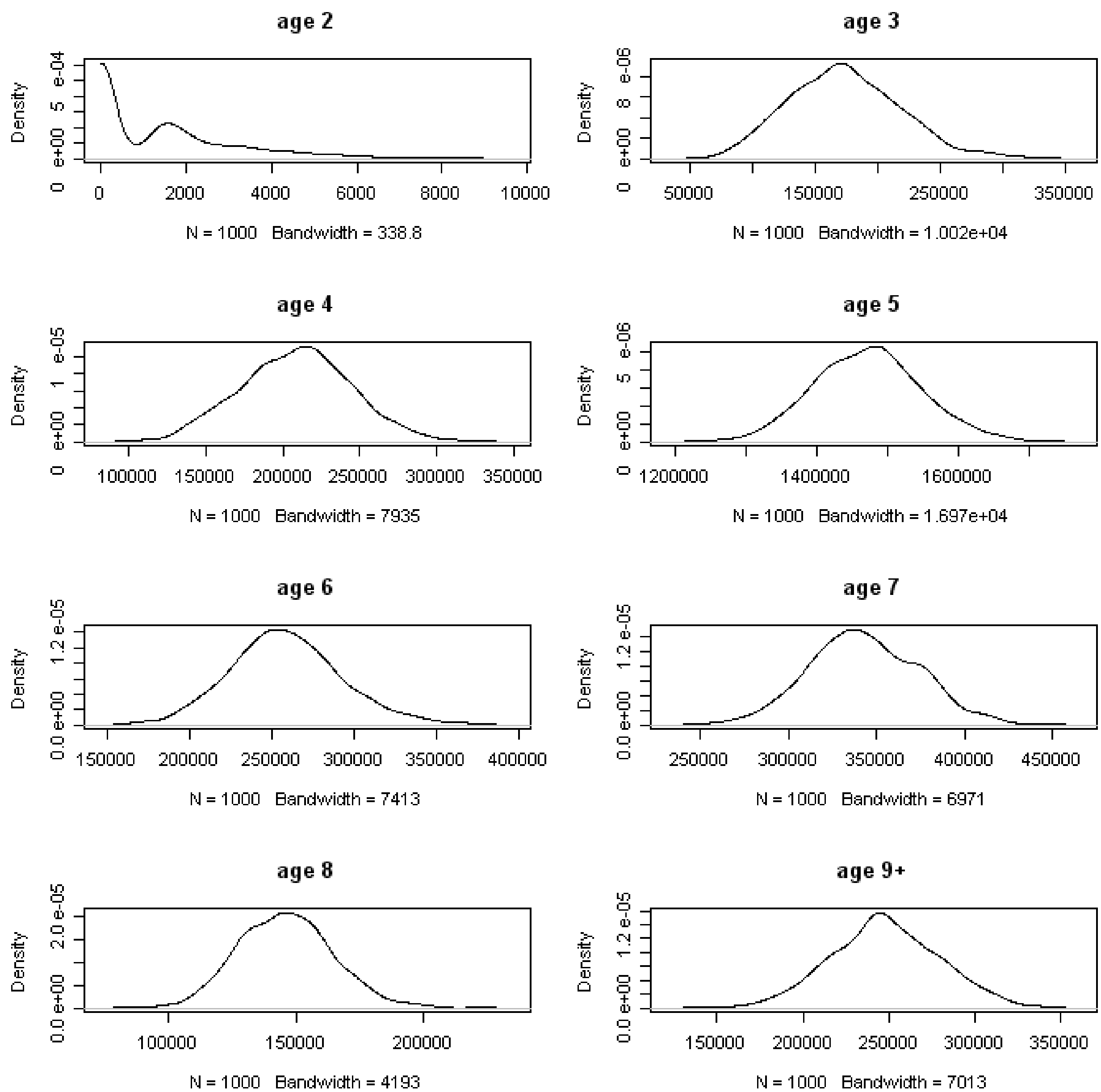

Figure 3. Comparison of BioStat and model-based estimates of landings at age and associated coefficients of variation for witch flounder in 2003.