ESTIMATING CAPITAL EFFICIENCY SCHEDULES WITHIN PRODUCTION FUNCTIONS*

By

Mark E. Doms**

CES 92-4 May 1992

Abstract

The appropriate method for aggregating capital goods across vintages to produce a single capital stock measure has long been a contentious issue, and the literature covering this topic is quite extensive. This paper presents a methodology that estimates efficiency schedules within a production function, allowing the data to reveal how the efficiency of capital goods evolve as they age. Specifically we insert a parameterized investment stream into the position of a capital variable in a production function, and then estimate the parameters of the production function simultaneously with the parameters of the investment stream. Plant level panel data for a select group of steel plants employing a common technology are used to estimate the model. Our primary finding is that when using a simple Cobb Douglas production function, the estimated efficiency schedules appear to follow a geometric pattern, which is consistent with the estimates of economic depreciation of Hulten and Wykoff (1981). Results from more flexible functional forms produced much less precise and unreliable estimates.

Keywords: capital stock, efficiency schedule, investment

*This research could not have been conducted without the generous support of Robert McGuckin, Chief, Center for Economic Studies. I would like to thank Tim Dunne, Martin Bailey, Sang Nguyen, and Tom Holmes for their many helpful comments. Any opinions, findings, or conclusions expressed here are those of the author and do not necessarily reflect the views of the Census Bureau.

**Center for Economic Studies, U.S. Bureau of the Census

I. Introduction

The appropriate method for aggregating capital goods across vintages into a single measure of capital stock has long been a contentious issue, and the literature covering this topic is extensive.[1] The works of Leontief (1947), Fisher (1965) and Diewert (1980) explore the theoretical conditions under which an aggregate capital stock exists and when a capital stock may be expressed as a weighted sum across all vintages of capital in use. These weights reflect the relative efficiency of capital as it ages, and the series of these weights is referred to as an efficiency schedule. The empirical literature on capital efficiency schedules includes studies that estimate economic depreciation for specific classes of capital goods [e.g. Hall (1971) and Hulten and Wykoff (1981)], dynamic factor demands that explicitly estimate a geometric capital efficiency rate [e.g. Epstein and Denny (1980), Kim (1988), and Prucha and Nadiri (1990)], and models that measure the effects of a distributed lag of investments on current profit [e.g. Pakes and Griliches (1983)].[2]

This paper presents a methodology that estimates capital efficiency schedules by inserting a parameterized investment stream into the position of a capital variable in a production function. The parameters of the production function are then simultaneously estimated with the parameters of the investment stream. In order to perform this exercise, data on inputs, output, and previous investments for a group of manufacturing plants that employ a common technology is required. To meet this need, the model is estimated

---

[1] See Hulten (1989) and Jorgenson (1989) for summaries.

[2] Obtaining measures of the decline in the productive ability is difficult because the flow of capital services are not directly observable because most capital is owner used, and the rents that accrue from the capital are internalized by the owner. What is observed is a stream of capital investments, so what must be done is to convert these observed investment streams into a capital stock measure, where the capital stock is proportional to the capital services.

using a plant level panel dataset from the Census Bureau's Longitudinal Research Database (LRD). The panel data used in this study provides two means of identifying how efficiency of capital evolves as it ages. First, in a cross section, plants are heterogenous with respect to their capital age distributions. Ceteris paribus, if two plants differ only by the age of their equipment, then the plant with the older equipment will not be capable of producing as much as the plant with newer capital. Second, the panel dataset allows us to observe the levels of output and other inputs while the capital of a plant ages.

We view this study as an exploratory exercise, testing whether the rich panel data the LRD provides can indeed be used to estimate the impacts of investment on production. In particular, efficiency schedules are estimated for a group of steel making plants that employ electric arc furnace technology, also known as mini-mills. These plants are ideal to study capital efficiency in that they possess a common technology and produce similar outputs. Unlike integrated steel plants, much of the capital stock in these plants was purchased in the 1970's and 1980's, coinciding with the coverage of the LRD.

This paper tests several hypotheses regarding efficiency schedules and fundamental questions regarding capital accumulation. For instance, we test whether the efficiency of capital goods initially increase, due to learning or other phenomena. Pakes and Griliches (1983) find that when profits are regressed on previous investments, investments made 3-4 years prior have a larger impact than more recent investments. This result is also consistent with a long time to build story [Kydland and Prescott (1982)] in which capital purchased at time t may not be productive until time t+1 or t+2 when the entire project is complete.

Once the capital becomes productive, we estimate how quickly the efficiency of capital deteriorates as it ages. Jorgenson (1989) reviews some of the differing opinions regarding the form of efficiency schedules. A point of contention is whether most production machinery is able to perform close to original standards over time before rapidly deteriorating [e.g. one-hoss shay], or whether machines become steadily more inefficient [e.g. geometric]. These issues are not only important for obtaining accurate productivity measures, but also provide information for a wide range of dynamic models that contain capital accumulation equations. For instance, one of the more frequently used capital accumulation equations is $K_t=(1-8)K_{t-1}+I_{t-1}$, where K_t is capital at time t, I is investment, and 8 is the geometric rate of decay. One of our goals to test whether the data support this frequently used model.

Empirically the most popular method for obtaining estimates of efficiency schedules has been to examine prices of used capital goods. Studies of this type collect price and age for specific asset types from second hand markets[3]. By examining how prices of used assets vary by age, economic depreciation, and hence, the decline in capital efficiency can be deduced [see Jorgenson (1973)]. As an example, consider a light bulb with an expected life of two years. After one year, the light bulb still illuminates the same amount of light as it did initially, it has not lost efficiency in a productive sense. However, the light bulb has economically depreciated since the expected remaining lifetime of the light bulb has diminished by one half. In the special case where the loss of efficiency is geometric, then the rates of economic depreciation and the loss in efficiency are equivalent.

---

[3] To date, Hulten and Wykoff (1981) have performed the most thorough and comprehensive used asset price study, examining depreciation patterns for 22 classes of producer durables and 10 classes of structures. The question arises whether the type of assets that participate in second hand markets are representative of assets in place. In particular, this paper uses plant level data on U.S. steel mills that employ electric arc furnace technology. A large portion of the production machinery in these plants is large, rather bulky, and infrequently traded in second hand markets. If large machinery is under represented in used asset markets and large machinery depreciates at different rates than smaller machinery, then using depreciation rates based on used asset price studies may be inappropriate.

Another approach for inferring physical deterioration is to estimate a geometric rate of deterioration within dynamic factor demand models. A large share of these studies are based on Malinvaud (1953). In these models, firms take into account the adverse effects that current production has on machine life. The more produced, the sooner machines will have to be replaced. Examples of empirical applications include Epstein and Denny (1980), Kim (1988) and Prucha and Nadiri (1990). These models possess the attractive feature of modeling the firm's intertemporal optimization problem. A drawback of these models is that they assume geometric decay for tractability, while this is an assumption we test. Additionally, the dynamic element in these models rely on quadratic adjustment costs of capital, an assumption that has come under increasing criticism.[4]

Our primary finding is that when using a Cobb Douglas production function, the estimated efficiency schedules follow a geometric pattern, with efficiency declining by 7-9% per year. These estimated geometric rates of deterioration are consistent with Hulten and Wykoff estimates of economic depreciation. Additionally, after the first year, the data do not support initial increases in the efficiency schedule. The model is also estimated using more flexible forms, including other production functions and dynamic factor demand models. However, the estimates from these models are extremely sensitive to model specification.

Do these results imply that capital goods in mini-mills physically deteriorate at 7-9% a year? There is no doubt that older machines are more likely to suffer break downs more frequently than newer machines. However, the trade literature seems to suggest that the primary capital goods in this industry require maintenance proportionate to use, and there are instances of

---

[4] Rothschild (1973) and Bertola and Cabellero (1990) provide criticisms of quadratic cost of adjustment models. Doms and Dunne (1992) have shown that the observed plant level investment behavior appears to be inconsistent with the standard quadratic cost of adjustment framework.

furnaces that were built in the 1940's still in use in the 1980's[5]. One possible explanation for this is that older equipment may not be used as frequently as newer equipment. For instance, many plants possess more than one furnace. These plants are more likely to use the newer and more efficient furnaces before having to use older furnaces. This results in older equipment being idled more often than newer equipment, and consequently older equipment may appear to be less productive than newer equipment.

The paper proceeds with a description of the models to be estimated in section II. Section III describes the data used in estimation while section IV discusses the results. In section V, several extensions to the basic models are explored. First, the estimation technique is modified to exploit the panel nature of the data. Second, a test is derived to test whether there errors in the investment deflators are non-exponential. Finally, sensitivity analysis is performed by allowing the functional form of the production function to be more flexible. The last section summarizes and concludes the paper.

II. Model

This section presents the procedure used to estimate capital efficiency schedules. We begin by decomposing the effects of time, vintage, and age on the efficiency of a capital good. The capital stock at time t, K_t, is a constructed as a weighted sum of previous investments:

$$(2.1) \quad K_t = \sum_{J=0}^{\infty} *_{t,t-J} I_{t,t-J}$$

[5] The primary pieces of capital equipment in mini-mills are the electric arc furnaces, casters, and rollers. Publications such as *33 Metal Producing U.S. Steel Industry Data Handbook 1989* present age distributions for these pieces of equipment.

where $I_{t,t-J}$ is the nominal investment made at time t-J that has not been retired by time t.[6] The weights, $*_{t,t-J}$, measure such effects as inflation, deterioration, and embodied technical change, and these weights transform the nominal investments into common capital units.

Hall (1968) decomposes * into three independent components; functions of age, vintage, and time.

(2.2) $$*_{t,t-J} = d(t)b(t-J)M(J)$$

The first element of *, d(t), often referred to disembodied technical change, affects the productive ability of all inputs in production, not just capital.[7] The second component of *, b(t-J), varies by the vintage of the capital. Initially we assume that the vintage component equals the investment deflator series. This assumes that the investment deflator series accurately captures the effects of inflation and embodied technical change. Under this scenario, we need only estimate d and M. Section V examines the consequences and results from allowing b to deviate from the investment deflator series.

The remaining term in *, M(J), is usually assumed to be a decreasing function of age; as a piece of capital ages, it becomes less productive.[8] We refer to M(J) as the efficiency schedule and several popular forms for efficiency schedules have been posited in the capital measurement literature. The form of M(J) varies by the characteristics of the capital goods under consideration. For example, light bulbs exhibit one hoss shay deterioration;

---

[6] In this paper we focus on constructing capital stocks for production machinery. Capital machinery is a heterogeneous mixture of different types of capital goods, but our data does not break out the expenditures by type of capital good.

[7] d(t) may be a result of Hicks neutral technical change, or Harrod technical change. Given the production model that will be estimated, it will not be necessary to distinguish between these.

[8] It may be argued that age in and of itself has little effect on deterioration, but usage does. In this case, age acts as a proxy for usage.

light bulbs retain the same luminescence over time until they burn out. On the other hand, dry ice experiences geometric deterioration; the amount of dry ice that evaporates is proportional to the amount remaining. Unfortunately the efficiency patterns for production machinery are unknown, and we would like the data to tell us which pattern is correct.

Three functional forms of M(J) are discussed and used in estimation, each with its own advantages and disadvantages. These parameterizations cover a vast array of possible efficiency patterns. The first efficiency model is the geometric. Due to its simplicity and special characteristics, the geometric model is one of the more widely used in applied and theoretical modeling,

$$(2.3) \quad M(J) = (1 - 8)^{J}.$$

This model only has one parameter to be estimated, 8. Hulten and Wykoff (1981) compare the results from a geometric model to a more flexible and parameterized form, the Box Cox transformation. The Box Cox model possesses the ability to produce both concave and convex efficiency patterns, including one hoss shay, geometric, and linear,

$$(2.4) \quad M^{*}(J) = 8_1 + 8_2 J^{*},$$

$$\text{where} \quad J^{*} = \frac{J^{8_3} - 1}{8_3}, \qquad M^{*}(J) = \frac{M^{8_4}(J) - 1}{8_4}.$$

Although the Box Cox has the flexibility to be concave or convex, it, like the geometric, does have the disadvantage of being monotonic. A priori, there are several reasons to expect M to be non monotonic. For instance, learning how to optimally use machinery may take time, resulting in older machines being relatively being more productive than newer machines. Investment timing provides another reason for M to initially increase. An investment made at time t may be part of a project not completed until time t+2. The productive ability of capital purchased at t increases upon project completion at time t+2.

A functional form that possesses the ability to be non-monotonic is the polynomial:

$$(2.5) \quad M(J) = 8_0 + \sum_{s=1}^{g} 8_s J^s$$

The polynomial has g+1 parameters. A drawback to the polynomial is the difficulty it has in fitting curves that contain flat regions. If capital equipment initially deteriorates slowly, or reaches a plateau, then the polynomial may not fare so well.

The geometric, Box Cox, and polynomial functions encompass the commonly assumed forms of efficiency. Our goal is to estimate these three models and let the data reveal which model is the most appropriate. This goal is accomplished by substituting (2.1) and (2.2) into the capital variable in a production function, and then estimating the parameters of the production function simultaneously with the parameters of M and d. For a Cobb Douglas production in log form, the estimated model becomes:

$$(2.6) \quad Q_{it} = \$_0 + X_{it}\$_x + \$_k \ln\left(\sum_{J=0}^{t-1} b(t-J)M(J)I^{i}_{t,t-J} \right) + \sum_{j=1}^{T-1} (_j Y_j + e_{it} \,,$$

where the i subscripts and superscripts denote plant i, Q_t is log of output, X_t is a k vector of log inputs and other variables, and the ß's are appropriately dimensioned parameter vectors. An i.i.d error term, e_{it}, is appended, where e_{it} ~ $N(0,F^2)$. The disembodied technical change component, $ß_k ln(d(t))$, is changed to a series of time dummies, $Y_1,..,Y_{T-1}$.

We estimate (2.6) using nonlinear least squares. Before presenting the estimation results, we briefly discuss the data used.

III. Data

In order to estimate a production function (2.6), data from production units employing a common technology is needed. Additionally, a near complete time series of investment for each unit is required. To meet these stringent data requirements, we use a panel dataset consisting of annual observations on individual raw steel producing plants that employ electric arc furnaces (EAFs) as their sole source of raw steel making capacity. Most of the data items used in the estimation come from confidential, establishment level data at the U.S. Census Bureau. Data on inputs and investment come from the Longitudinal Research Database (LRD), while output data are taken from the Current Industrial Reports (CIR). The LRD contains establishment level data on employment, inventories, outputs, inputs, investments, retirements, and the book value of capital. The LRD contains the 1963, 1967, 1972, 1977, and 1982 Census of Manufacturers and the Annual Survey of Manufacturers from 1973 through 1986. Except for 1963 and 1967, annual investments and retirements before 1972 are not available. Appendix A discusses the procedures used to estimate investments made prior to 1972.

The CIR augments the LRD with seven digit output detail. Additionally, the CIR identifies the raw steel making technology employed at the plant level. The CIR data are used to construct output measures and to identify plants that use EAF technology. The CIR and LRD overlap between 1978 and 1986, and these are the data used in estimation.

Table III.1 presents sample summary statistics. Notice that the sample size begins to tail off after the 1982 steel depression. This is due to several factors. First, 1983 is the end of an ASM panel, and several plants failed to make the 1984 panel. Second, although mini-mills have received much attention due to their success against large integrated steel mills, many mini-mills have gone bankrupted and closed. Finally, other observations are dropped that contained erroneous or largely imputed data. These observations tended to be at the beginning and end of a plant's life. This sample attrition results in declining industry coverage, as measured by the total sample EAF output compared to total industry EAF output.

Table III.1
Sample Summary Statistics

Year	N	Average Eaf Output	% of All EAF Output	Capacity Utilization
1978	49	376	57	.90
1979	50	414	61	.95
1980	50	377	60	.90
1981	53	403	62	.91
1982	50	287	62	.68
1983	46	312	54	.75
1984	39	391	49	.86
1985	40	376	50	.86
1986	35	433	50	.89

N=number of observations   Average EAF Output=1000's of tons of raw steel produced
% of All EAF OUTPUT=sample EAF output/total industry EAF output
Capacity Utilization=raw steel output/rated raw steel capability

IV. Results

This section presents and discusses the parameter estimates of (2.6). Estimates from the geometric, Box Cox, and polynomial models are presented and compared. Before preceding with the results, we address several points concerning the implementation of the model: the treatment of current year investment, and other variables included.

The effect of current year investment on current year production is ambiguous. Current year investment does increase the current year capital stock. However, the productivity of capital purchased in year t depends on what time of year the investment is installed, and this information is unavailable. New investment may also enter the production function outside of the capital variable. Cost of adjustment models postulate that current investment diminishes current output. Conversely, Olley and Pakes (1991) suggest that current year investment is positively correlated with omitted, plant specific factors, such as managerial ability. The results presented in this section are based on excluding current year investment from the capital stock: the hypothesis that the coefficient for current year investment is zero, H_o: $M(1)=0$, could never be rejected. However, current year investment as a ratio to total capital appears in the model.

In preliminary work, many other variables that may affect production were included. These variables included ownership changes, unionization, plant age, output mix, and location information. The inclusion of these variables had little impact on the remaining parameters in the model. The last point concerns the whether capital stock serves as an adequate proxy for capital service. The traditional solution is to assume that capital stock is proportional to service, and this proportion is constant over time and across plants. To assume otherwise requires plant/year measures of capacity utilization. A plant specific capacity utilization measure is constructed by using the ratio of CIR data on raw steel production to private estimates of raw steel making capability. When measures of capacity utilization are included in the model, the capital parameter, $ß_K$, usually increases, but the parameters for the efficiency schedules remain unchanged. The results present here do not make an adjustment for capacity utilization.

Figure IV.1 presents the estimates of M in addition to a baseline efficiency schedule. This baseline model constructs capital by using economic depreciation rates from Hulten and Wykoff (1981) and the 1977 Capital Flows Table (CFT). The CFT presents distributions of asset purchases by I-O industry group. For each of these asset categories Hulten and Wykoff have estimated geometric economic depreciation rates. Using the CFT distribution, an average geometric depreciation rate of 8=.09 is derived.[9]

The most striking feature of figure IV.1 is the amazing similarity between the three estimated efficiency schedules. The polynomial and Box-Cox possess the ability to deviate greatly from the geometric, but the best model fits are obtained with geometric-like patterns. The figure also includes a 90% confidence interval generated from the polynomial model. This confidence interval envelopes the other efficiency schedules while excluding the baseline case. The schedules begin deviating at age 26, when the estimates begin getting severely less precise.[10]

These estimates are based on a value added definition of output [i.e. value of shipments-cost of materials]. When (2.6) is estimated with materials as a separate factor, the estimated geometric rate of deterioration increases to 8=.09, nearly identical to the baseline case. Again, the polynomial and Box Cox models nearly replicate the geometric results.

---

[9] This figure is based on a correction for retirements, since data used in estimation does make an adjustment for retirements. Based on a conversation with Charles Hulten, the average depreciation rate is multiplied by two thirds.

[10] Less than 2% of all deflated investment in this sample is greater than 25 years old.

The similarity between the efficiency schedules is perhaps the strongest result in this paper. Using used asset price data, Hulten and Wykoff (1981) find a similar result in that the Box-Cox results resembles a geometric pattern. The geometric decline in efficiency found in this paper supports the

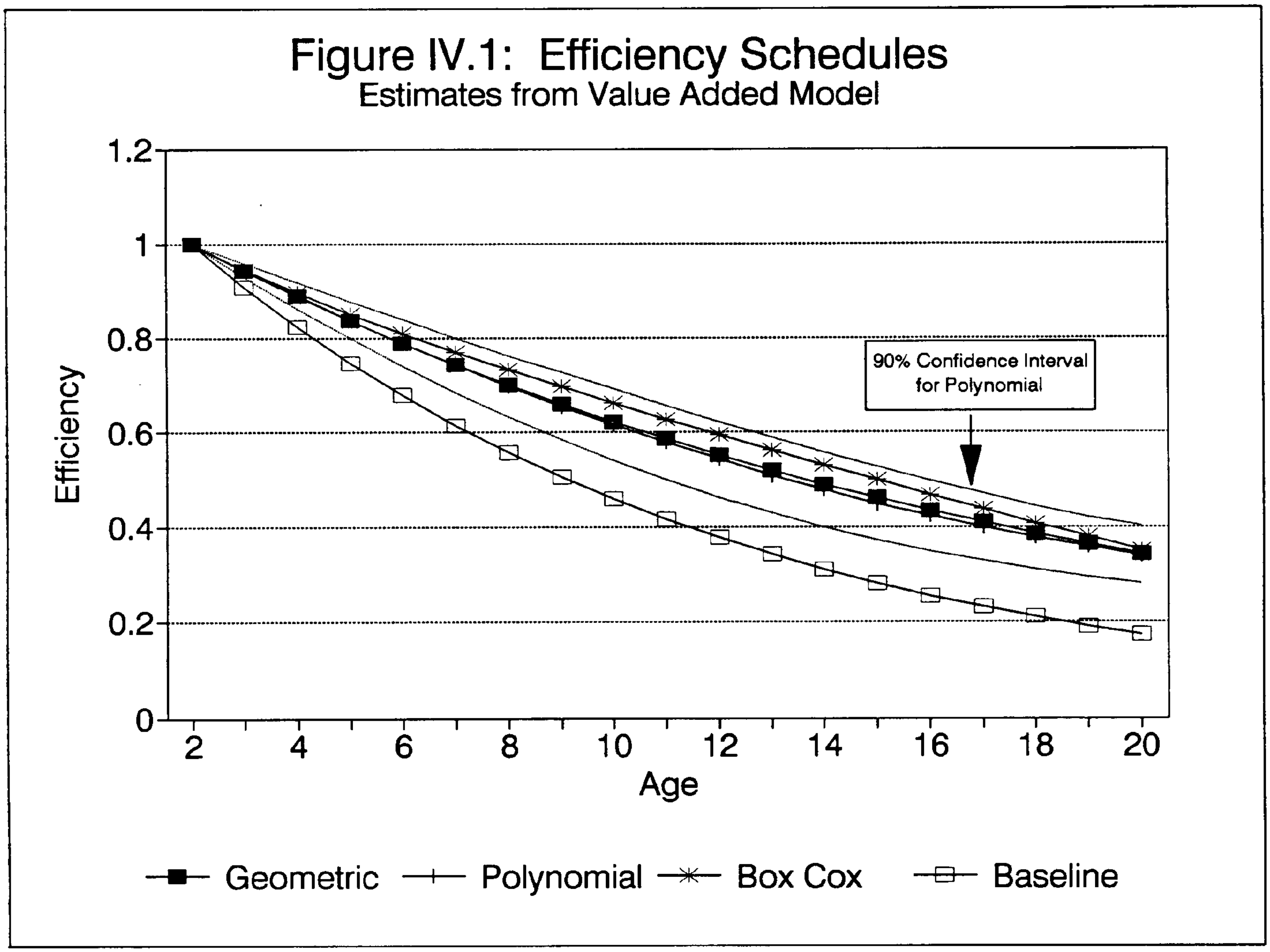
Figure IV.1: Efficiency Schedules
Estimates from Value Added Model
1.2
1
0.8
0.6
0.4
0.2
0
Efficiency
2
4
6
8
10
12
14
16
18
20
Age
90% Confidence Interval
for Polynomial
Geometric
Polynomial
Box Cox
Baseline

Table IV.1
NLS Estimates of 2.6, Cobb-Douglas Production Function
(standard errors in parentheses)

Dependent Variable=log(shipments-materials)

	Baseline	Geometric	Box Cox	Polynomial†
Intercept	3.84	3.74	3.69	3.73
	(.150)	(.156)	(.182)	(.534)
K	.324	.350	.352	.350
	(.0356)	(.0344)	(.0383)	(.0399)
L	.366	.333	.332	.334
	(.0358)	(.0300)	(.0301)	(.0488)
E	.193	.188	.189	.189
	(.0360)	(.0360)	(.0360)	(.0347)
DK	-.176	.0742	.0905	.0716
	(.338)	(.344)	(.345)	(.593)
DK^2	.380	.101	.0953	.0999
	(.673)	(.681)	(.682)	(1.10)
DL	-.333	-.3412	-.352	-.347
	(.0908)	(.0907)	(.0909)	(.0373)
DL^2	-6.3E-4	-6.6E-4	-6.7E-4	-6.6E-4
	(1.7E-4)	(1.7E-4)	(1.7E-4)	(.0919)
Y79	.0697	.0679	.0681	.0677
	(.0543)	(.0540)	(.0541)	(.0541)
Y80	.0805	.0770	.0778	.0765
(.0550)	(.0548)	(.0549)	(.0549)	
Y81	.0362	.0293	.0298	.287
(.0543)	(.0541)	(.0542)	(.0542)	
Y82	-.0647	-.0768	-.0759	-.0775
(.0590)	(.0589)	(.0591)	(.0591)	
Y83	-.145	-.152	-.150	-.153
(.0579)	(.0577)	(.0580)	(.0580)	
Y84	-.0258	-.0388	-.0380	-.0401
(.0578)	(.0573)	(.0575)	(.0575)	
Y85	.00160	-.0150	-.0148	-.165
(.0599)	(.0601)	(.0601)	(.0601)	
Y86	.0561	.0374	.0374	.0356
(.0608)	(.0611)	(.0613)	(.0611)	
R^2	.90	.90	.90	.90
SSE	33.4	33.0	33.0	33.0

Number of observations=458
Notes: K=log of capital L=log of labor, 1000's of production hours
E=log of electricity, 1,000,000's of kilowatts
DK=current real investment as a % of total real investment
DL=% change in total employment, Y79-Y86=year dummies, SSE=sum of squared residuals

† These are the results from a second order polynomial. Higher order polynomials produced only slightly smaller SSE's.

geometric pattern of economic depreciation found by Hulten and Wykoff since physical deterioration and economic depreciation coincide when deterioration occurs geometrically [see Jorgenson (1973)].

In a variety of ways we test whether the deterioration schedule should initially be convex, not concave, as in Pakes and Griliches (1983). Specifically, for each model we test whether M remains constant or increases during the first 2-3 years by allowing the efficiency schedule to deviate from its functional form. In each case, the data did not support deviations from the geometric pattern.

The actual parameter estimates and standard errors are presented in Table IV.1. Given that the three parameterizations of M yielded nearly identical patterns, the magnitude and variance of parameter estimates show little variation across the models. The greatest variation in the table is between the baseline model and the other models. The baseline capital coefficient is slightly lower, as is the overall model fit. Current investment as a fraction of capital is uniformly insignificant, but the change in labor is generally largely negative, perhaps supporting large costs of adjustment for labor. The only statistically significant time dummy is for 1983, a year in which the steel industry was only beginning to recover from its 1982 depression.

V. Model Extensions

In this section we extend our analysis in three directions. First, the estimation technique of Cornwall, Schmidt, and Sickles (CSS) (1990) is extended to a nonlinear framework to exploit the panel nature of the data. Second, we allow the vintage component, b(t-J), to deviate from the deflator series. The model no longer is identified, however, a test is devised as to whether the errors in the deflator series are not exponential. Lastly, the Cobb Douglas assumption is relaxed, and the results from more flexible functional forms are discussed.

1. Panel Data

The omission of unobserved, plant level production factors when estimating production functions can yield biased parameter estimates. Fixed effects models assume that these unobserved, plant specific factors vary by plant, but do not vary over time. CSS present a model that extends the panel data literature by allowing not only the intercepts to vary across plants, but also other slope coefficients. Their paper, which estimates productivity for eight airlines, allows the efficiency time patterns for each airline to follow a unique time path. Their approach does not model why firm level efficiency evolves, it only allows for the evolution to occur, and it allows the examination of that evolution. We extend (2.6) to include plant specific functions of time.

$$(5.1) \qquad Q_{it} = \$_0 + X_{it}\$_x + \$_k \ln\left(\sum_{J=0}^{t-1} b(t-J)M(J)I^{i}_{t,t-J}\right) + \sum_{j=1}^{T-S} (_j Y_j + \sum_{j=0}^{S} t^{j}"_{ij} + e_{it}$$

The plant specific coefficients, $"_{ij}$, enter as a polynomial function of time. For identification, the number of time dummies is reduced by S.

For all values of S, equation (5.1) is nonlinear in variables and parameters, and is estimated by nonlinear least squares (NLS). When S > 0, the model is transformed to purge the plant specific component. For simplicity, collapse (5.1) into matrix notation,

$$(5.2) \qquad Q^{*} = X^{*}\#_x + \#_k \ln(K^{*}) + TS" + TD(+ e ,$$

where TS is the block diagonal time series matrix, TD is the time dummy matrix, and K^* is a vector containing the weighted investment stream. Let Q=diag(TS), let $P=Q(Q'Q)^{-1}Q'$, and let M=I-P . Multiplying both sides of (5.2) by M eliminates TS. On this transformed data, nonlinear least squares can be applied to obtain consistent estimates of ß, (, ", and M. When S=1, the data is mean differenced as in traditional fixed effects models.[11]

We find that the estimates for the geometric deterioration function, when S=1 and S=2, closely resemble those when S=0, although the standard errors increase significantly: 8=.08 when S=1 and S=2, with t-statistics less than 1. For the Box Cox and polynomial functions, the model has a difficult time converging when S>0. In the case of the polynomial, the deterioration estimates become more volatile for J>10, and statistically insignificant.

2. Errors in Investment Deflators

The models and results presented so far assume that the investment deflators accurately measure changes in technology and price levels. Controversy has arisen over whether investment deflators and price deflators for capital goods in general, are properly constructed [see Gordon (1989) and Tripplett (1989) for examples]. In this section we discuss the consequences and results when this assumption is relaxed.

Hall (1968) demonstrates that when b is allowed to vary the three components of * are no longer uniquely identified.

$$(5.3) \quad *_{t,t-J} = d(t)b(t-J)M(J) = d^*(t)b^*(t-J)M^*(J)$$

$$\text{with } d^*(t) = R^{-t}d(t)$$
$$b^*(t-J) = R^{t-J}b(t-J)$$
$$M^*(J) = R^{J}M(J), \quad \text{for } R>0$$

[11] For a more thorough description of this procedure, see Cornwall, Schmidt, and Sickles (1990).

Let b be the correct deflator series. Suppose we use an investment deflator series, b^u, that understates the effect of technical progress, that is, $b^u <$ b. When the error in the b^u series increases exponentially over vintages, then this is equivalent to (5.3) with R>1 and $b^u=b^*$. Under this special scenario, estimates of d and M will be biased; $\hat{d}=d^*$ and $\hat{M}=M^*$. However, since the product of these biased parameters replicate the true *, the rest of the parameters in the production function will be unaffected.

Hall proves that the identification issue raised in (5.3) has a unique form: the same * may be achieved by erroneous b, d, and M only when the errors in these measures are exponential. If the errors in b are not exponential, then there do not exist M and d that produce the correct *. Under this scenario, estimates of d and M will be biased in an unpredictable manner. Also, given the inability the parameters to replicate the true *, the parameter estimates of ß are also likely to be biased.

The enhanced model allows for non exponential errors in investment deflators by bisecting b, the correct deflator, into a product of the investment deflators and an error index.

$$(5.4) \quad b(t-J) = b^{e}(t-J)b^{D}(t-J)$$

where $b^D(t\text{-}J)$ is the investment deflator for vintage t-J, and

$$(5.5) \quad b^{e} = 1 + \sum_{j=1}^{i} (_{j}(t-J)^{j}$$

If there are no errors in b^D, or the errors in the deflator are exponential, then the addition of (5.4) will not provide any improvement in data fit since the basic model suffices in producing the true *. In the case where the errors are not exponential, adding (5.4) does allow the true * to be derived. However, the enhanced model will not allow M, d, and b, to be separately identified as shown in (5.3), the enhanced model only provides the ability to replicate the true *.

We test the restrictions imposed by the basic model, ($_i$=0 $\forall$ i by using a likelihood ratio test. The identification problem in (5.3) implies that there is no loss of predictive ability when one of the indices is restricted, and a restriction must be imposed for the model to converge. In practice we impose a restriction on the disembodied technical change component, d(1978)=d(1986). The enhanced models did not provide a statistically better fit than the basic models, implying that if there are errors in the investment deflators, these errors may be exponential. Given the identification problems, this is the strongest test we can devise.

This result may or may not be surprising for the electric steel mill industry. The basic steel technology has remained unchanged, however, large technical improvements are frequently reported in trade journals: the steel making process is more computerized, larger transformers are more energy efficient, water cooled panels save energy and extend refractory life. The question still remains whether investment deflators truly capture the full impact of these changes.

3. Other Models Tested

It is fair to criticize the results presented so far on several grounds. Perhaps the greatest fault lies in model choice, the Cobb Douglas production function. The functional form is simplistic and restrictive. A more flexible production function, the translog, which the Cobb Douglas is a special case, is also estimated. We find that the estimated efficiency schedules from the translog are very sensitive to the precise specification. We also estimate cost and factor demand equations with the assumption that capital is fixed in the short run. Again, like the translog production function, the estimates for 8 proved sensitive to model specification, the results being extremely unrobust. Specifically, these models are sensitive to which equations are included in estimation, how time is interacted with other inputs, and whether translog or generalized leontief functional forms are used. Further, cross equation parameter restrictions rarely hold statistically. Our results suggest that although flexible functions are theoretically attractive, empirically they produce unreliable estimates.

VI. Conclusions

This paper investigates how the relative efficiency of capital varies by age. This exercise is useful given the vital role capital plays in production, and hence the analysis productivity and economic growth. To estimate efficiency schedules, this study employs a straight-forward methodology: insert a parameterized investment stream for a capital variable in a production function, and then estimate the parameters of the production function simultaneously with the parameters of the investment stream. This study exploits a rich panel data set that contains input, output, and investment information for a group of steel producing plants that use the same technology.

Our results show that reasonable and fairly precise estimates of efficiency schedules are generated from simple production models, the Cobb Douglas. More elaborate production and factor demand models produce much less precise estimates. However, for the simple production models several interesting results emerge. The relative efficiency of capital appears to deteriorate at approximately a geometric rate. This is similar to results from studies that examine the dual to this problem of examining prices of used assets. Although the estimated geometric rate depends on the functional form of the production function, the estimates are similar to those of Hulten and Wykoff (1981).

This paper has demonstrated that plant level panel data provide a new source of information on capital accumulation in the manufacturing sector, and an important tool in testing fundamental hypotheses regarding the productive capability of capital as it ages. A natural course for future research would be to expand this analysis to other industries, since the characteristics of capital differ markedly across industries.

Appendix A: Presample Investment Estimation

As mentioned in the data section, continuous annual investment data do not exist prior to 1972. However, for those plants that started before 1972, the 1972 book value of capital, BV_{72}, is observed, and BV_{72} is the sum of all net investments made on or before 1972. The problem is how to appropriately distribute BV_{72} over the presample period. In this section, two complementary models of presample gross investment and retirement are presented.

The first method exploits observed investment behavior of young plants in the insample period. These plants display a pattern of investing heavily in their first two years followed by a substantial reduction during the next two years. This witnessed investment pattern is used to impute the first four or five gross investments made by plants born after 1958.

The second method imputes gross investments and retirements for older plants. More specifically, gross investments made between the 1963, 1967 and 1972 CM's are modeled as a function of plant age, gross industry investment, and the initial and ending book values of capital. The change in the book value of capital between two censuses equals the sum of gross investments less retirements made during the interval. Once gross investments are estimated, then retirements can be calculated.

Both methods rely heavily on a plant's start date, t_0. The start dates for the plants used in this study were ascertained from various trade journals. Other studies using the LRD may obtain some birth data that were collected in the 1975 and 1981 ASM's. Because the birth data may not be available in for every plant, the following discussion is general in that it assumes that birth information for one reason or another is not available. However, when the birth information is available a plant's start date is known, as is the case for all plants in this paper, then the estimation procedure is easily amended.

The first step for both imputation procedures is to classify each plant born before 1972 into one of three categories. The classification is determined by when a plant is first observed in a CM.

GROUP	First CM	Start Date
1	1972	$1967 < t_0 < 1972$
2	1967	$1963 < t_0 \leq 1967$
3	1963	$t_0 \leq 1963$

The birth year for plants in groups 1 & 2 fall within an interval ranging from the census of first appearance to the previous available census. We make no lower bound assumptions concerning plants that start before 1963.

Imputation Method #1

The first imputation method is used in imputing ($I_{72,67}$, .., $I_{72,71}$) for group 1 and ($I_{67,63}$, .., $I_{67,66}$) for group 2. Based on the observation that young plants rarely retire any equipment in the first 5 years of operation, we assume no retirements are made in the first four or five years of a plant.

There are several reasons to expect why investment patterns for new plants in the U.S. electric steel mill industry follow a similar pattern; large initial investments followed by smaller investments. A plant in this industry is initially built with a certain scale and changing this scale requires relatively large expenditures. If managers initially do not know their true costs or market demand, they may wait several periods until making large investments. Another reason for this pattern could be that a new plant is likely to embody the latest technology. When there is technical progress, the marginal benefit of investing into a new technology increases with time. Initially after the plant commences operation, this marginal benefit will be small.

To examine the initial investment patterns of new plants, we compute the real net investment distribution for plants that started after 1972. Table A.1 presents means and standard deviations for these vectors. These data indicate that, on average, the initial two investments in a plant are large relative to consequent investments. Another interesting phenomenon that occurs is the large investments that occur again when plants are 5 years old. Portions of the standard deviations in columns 1 and 2 is attributable to the timing of plant construction. For several plants, the initial investment is small, beginning towards the later half of the year. The investment made in the second period for these plants will tend to be quite large. In contrast, for plants that begin construction early in the year, the second period investment will be small. Associated with the construction timing issue is when the LRD picks up these plants. If the LRD picks up births with a time lag, then an investment stream will be lumped together into the first period.

Table A.1

Mean Real Net Investment Distributions by Plant Age
(standard deviations in parentheses)

		Year of Operation					
		1	2	3	4	5	N
Age	2	.834	.167	-	-	-	16
		(.290)	(.290)				
	3	.724	.226	.0498	-	-	14
		(.326)	(.316)	(.0892)			
	4	.657	.238	.0533	.0518	-	12
		(.344)	(.319)	(.0843)	(.0744)		
	5	.453	.236	.0529	.0515	.206	12
		(.329)	(.306)	(.0709)	(.0325)	(.251)	

N = number of plants

Using investment patterns of plants born after 1972 as a basis of imputing presample investment patterns has limited use. The LRD can only calculate real net investment distributions to a maximum of 15 years. Notice that the number of plants contributing to estimating the table A.1 distributions decreases from 16 to 12. Investment spans greater than 5 years are not presented due to small samples and high variances.

We assume the probability of birth is uniformly distributed between 1968 and 1971 for group 1, and between 1963 and 1967 for group 2. A mean estimated real investment distribution not conditional on plant age is calculated for the two periods. Using these distribution and BV_{72} we estimate the 1967-1971 investments for group 1, and using BV_{67} we estimate the 1963-1966 investments for group 2.

Imputation Method #2

The first method of imputation takes advantage of the special behavior of young plants. Now we present the methodology used for older plants. We use plant specification information observed in the 1963, 1967, and 1972 CM's to infer annual investments and retirements.

Consider the following example. Suppose BV_{72}=200 and BV_{67}=100. An infinite number of linear combinations of gross investments and retirements could have occurred in the 1972-1967 period. At the one extreme, gross investments for the period could have been 100 with no retirements. Suppose we assumed total investments totaled 150. Then simultaneously we are assuming retirements=50.

Using insample data we estimate the relationship between changes in book value and gross industry investment for 4 and 5 year periods. We then apply the parameter estimates from the insample estimation, ß, to the following model:

(A.1) $(I_{71}, \ldots, I_{68}) = g(BV_{72}, BV_{67}, GI_{67,..72}, ß)$.

where GI represents gross industry investment. A.1 is estimated using a linear SUR model to minimize the variance of ß. The book values and gross industry investment are fully interacted. Estimates of ß are used to impute the 1964-1966 and 1968-1971 gross investments. Retirements are calculated by looking at total gross estimated investments and the change in book values of capital.

References

Barnett, D. and R. Crandall (1986), Up From the Ashes: The Rise of the Steel MiniMill in the United States, Washington, DC, The Brookings Institution.

Bertola, G. and R. Caballero (1990), "Kinked Adjustment Costs and Aggregate Dynamics" in NBER Macroeconomics Annual 1990, O. J. Blanchard and S. Fischer eds.

Bureau of Labor Statistics (1983), Trends in Multifactor Productivity, 1948-1981, Bulletin 2178, Washington, DC, U.S. Department of Labor.

Delong, B. and L. Summers (1991), "Equipment Investment and Economic Growth," Quarterly Journal of Economics, CVI:445-502.

Denison, E. (1985), Trends in American Economic Growth, 1929-1982, Washington, DC, The Brookings Institution.

Diewert, E. (1980), "Aggregation Problems in the Measurement of Capital," in The Measurement of Capital, D. Usher editor.

Doms, M. and T. Dunne (1992), "Lumpy Investment Patterns in U.S. Manufacturing Plants," mimeo.

Epstein, L. and M. Denny (1980), "Endogenous Capital Utilization in a Short Run Production Model: Theory and an Empirical Application," Journal of Econometrics, 12:189-207.

Fisher, F. (1965), "Embodied Technical Change and the Existence of an Aggregate Capital Stock," Review of Economic Studies, 32:263-288.

Gordon, R. J. (1989), The Measurement of Durable Goods Prices, Chicago: University of Chicago Press.

Hall, R. E. (1968), "Technical Change and Capital from the Point of the Dual," Review of Economic Studies, 35:35-46.

Hall, R. E. (1971), "The Measurement of Quality Change from Vintage Price Data," in Zvi Griliches, ed. Price Indexes and Quality Change, Cambridge, MA: Harvard University Press.

Hulten, C. (1988), "The Measurement of Capital Input: Part I," presented at the Conference on Research on Income and Wealth, May 1988.

Hulten, C. and F. Wykoff (1981), "The Measurement of Economic Depreciation," in Depreciation, Inflation, and the Taxation of Income from Capital, Washington: The Urban Institute Press.

Jorgenson, D.W. (1989), "Capital as a Factor of Production", in Technology and Capital Formation, D.W. Jorgenson editor, Cambridge MA: MIT Press.

Kim, M. (1988), "The Structure of Technology with Endogenous Capital Utilization," International Economic Review, 1:111-130.

Kydland, F. and E. Prescott (1982), "Time to Build and Aggregate Fluctuations", Econometrica, 50:1345-1370.

Leontief, W. W. (1947), "Introduction to a Theory of the Internal Structure of Functional Relationships," Econometrica, 15:361-373.

Malinvaud, E. (1953), "Capital Accumulation and Efficient Allocation of Resources," Econometrica, 21:233-269.

Olley, S. and A. Pakes (1991), "Dynamic Behavioral Responses in Longitudinal Data Sets: Productivity in Telecommunications Equipment Industry," working paper.

Pakes, A. and Zvi Griliches (1984), "Estimating Distributed Lags in Short Panels with an Application to the Specification of Depreciation Patterns and Capital Stock Constructs," Review of Economic Studies, 243-262.

Penton Publishing (1989), 33 Metal Producing: U.S. Steel Industry Data Handbook 1989, 26:5.

Prucha, I and Nadiri, M. (1991), "Endogenous Capital Utilization and Productivity Measurement in Dynamic Factor Demand Models: Theory and and Application to the U.S. Electrical Machinery Industry," preliminary draft.

Rothschild, M. (1973), "On the Cost of Adjustment," Quarterly Journal of Economics, 605-622.

Triplett, J. E. (1989), "Price and Technological Change in a Capital Good: A survey of Research on Computers," in Technology and Capital Formation, D. Jorgenson and R. Landau eds, Cambridge MA, The MIT Press, 1989.

Usher, D., editor(1980), The Measurement of Capital, Chicago, IL, The University of Chicago Press, 1980.