

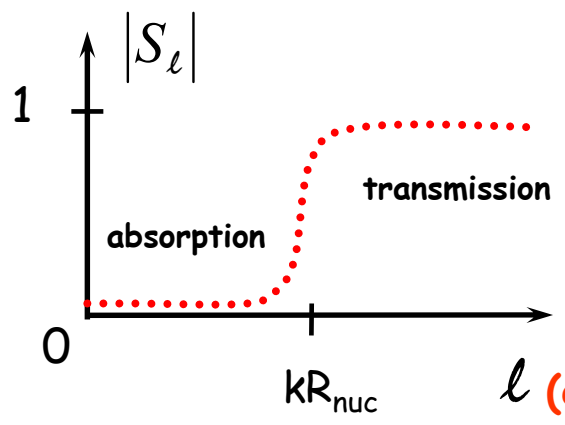
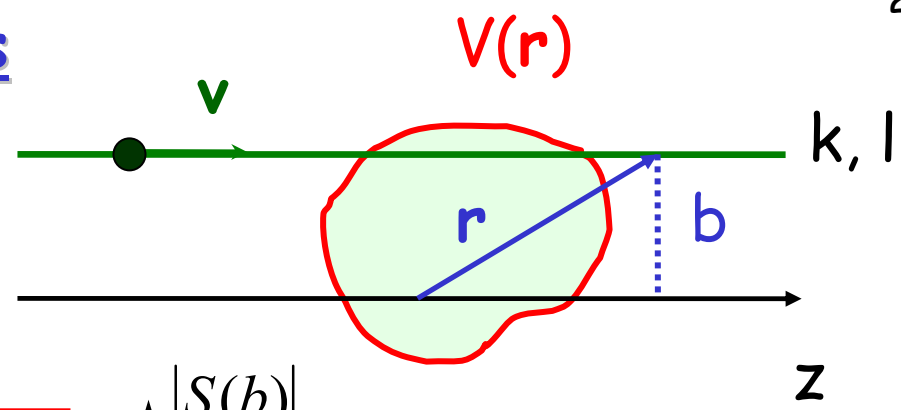
Nuclear Spectroscopy with Knockout Reactions

Carlos Bertulani
University of Tennessee

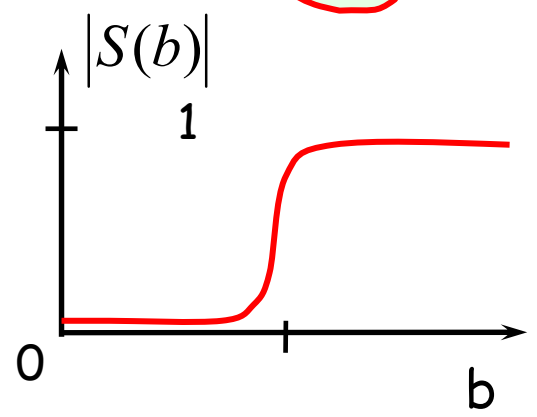
Alexandra Gade
Michigan State University

Basic Theory: Eikonal waves

$$\Psi^{(+)}(\mathbf{r})\Psi^{(-)*}(\mathbf{r}) = S(\mathbf{b}) e^{i\mathbf{q}\cdot\mathbf{r}}$$



$$S(\mathbf{b}) = e^{i\chi(\mathbf{b})}$$



"t-rho-rho" approximation

Ray, PRC 20 (1979) 1957

Hussein, Rego, C.B., Phys. Rep. 5 (1991) 279

$$\chi_{AB}^{(N)}(b) = \frac{1}{k_{nn}} \int_0^\infty dq q \tilde{\rho}_A(q) \tilde{\rho}_B(q) f_{nn}(q) J_0(qb)$$

$$f_{nn}(q) = \frac{k_{nn}}{4\pi} \sigma_{nn} (i + \alpha_{nn}) e^{-\beta_{nn} q^2}$$

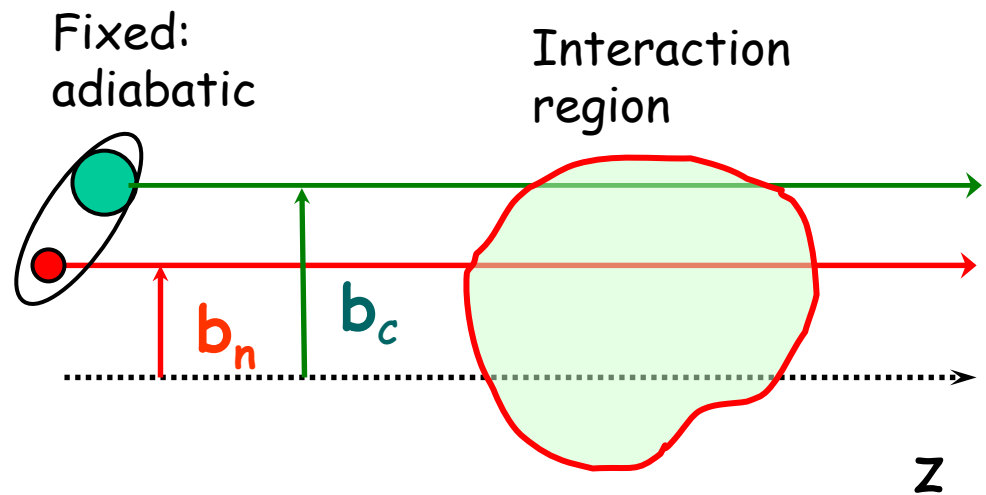
(from nn scattering)

**Glauber,
Nobel Prize
2006**

**Arizona
2006**



Elastic scattering



Best possible wfs:
(Spectroscopy)

$$S_{elast}(\mathbf{b}) = \langle \phi_0 | S_C(\mathbf{b}_C) S_n(\mathbf{b}_n) | \phi_0 \rangle$$

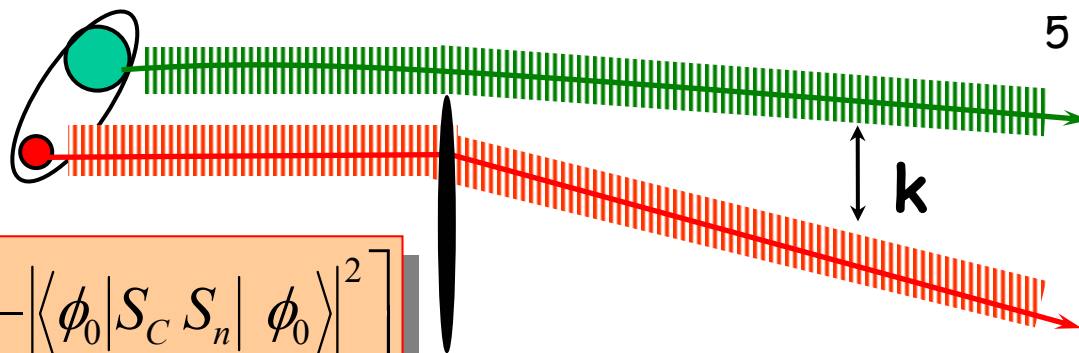
Survival amplitude
for projectile at impact
parameter b

Survival amplitudes
for particles C and n at impact
parameters b_C and b_n

(Dynamics)

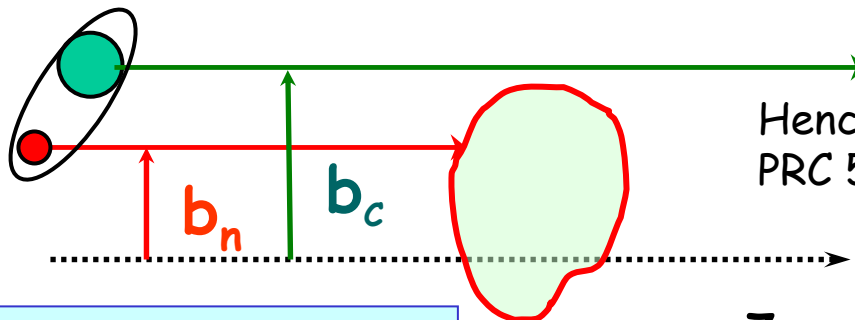
Hencken, Bertsch, Esbensen, PRC 54, 3043 (1996)

Diffraction Dissociation



$$\sigma_{dif.dis.}(\mathbf{b}) = \int d\mathbf{b} \left[\langle \phi_0 | |S_C S_n|^2 | \phi_0 \rangle - \left| \langle \phi_0 | S_C S_n | \phi_0 \rangle \right|^2 \right]$$

Stripping

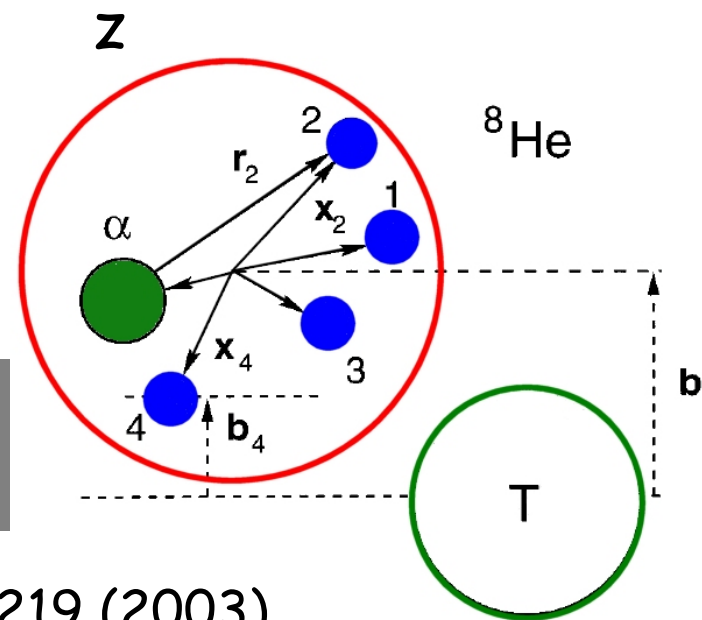


Hencken, Bertsch, Esbensen,
PRC 54, 3043 (1996)

$$\sigma_{strip} = \int d\mathbf{b}_n d\mathbf{b}_c \langle \phi_0 | |S_C|^2 (1 - |S_n|^2) | \phi_0 \rangle^2$$

Composite particles

$$\sigma_{strip} = \int d\mathbf{b}_1 \dots d\mathbf{b}_N \prod_{j \text{ survive}} |S_j(\mathbf{b}_j)|^2 \prod_{k \text{ absorbed}} (1 - |S_k(\mathbf{b}_k)|^2)$$



Applications: Hansen & Tostevin, ARNPS 53, 219 (2003)

Momentum Distributions

Serber model:
PR 72 (1947) 1008

$$S_C(\mathbf{b}_C) \approx 1$$



$$\frac{d\sigma_{strip}}{d^3k_C} = C_{geometry} \left| \tilde{\psi}_{l_0 m_0}(\mathbf{k}_C) \right|^2$$

Tanihata (1985): ^{11}Li

Momentum distributions in reactions with radioactive beams

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Physics Department, University of Wisconsin, Madison, Wisconsin 53706

(Received 2 September 1992)

We investigate the longitudinal and transverse momentum distributions of charged fragments originating from reactions with radioactive, neutron-rich beams. It is shown that the width of the narrow peak of the longitudinal momentum distribution is insensitive to the details of the collision and the size of the target nucleus. In contrast, the width of the peripheral region from which transversely moving particles originate is significantly narrowed via absorption of the outgoing neutrons. This diffractively broadens the width of their transverse momentum distribution, in a manner which depends on details of the collision, and so makes the transverse distribution less reliable than the longitudinal one for measuring the size of the original neutron halo.

PACS number(s): 25.60.+v, 25.70.Mn

$$\frac{d\sigma}{dp_z}$$

better structure probe

Momentum Distributions: (a) Stripping

$$\frac{d\sigma_{strip}}{d^3k_C} = \frac{1}{(2\pi)^3} \frac{1}{(2l_0+1)} \sum_{m_0} \int d^2b_n \left[1 - |S_n(\mathbf{b}_n)|^2 \right] \left| \int d^3r e^{-i\mathbf{k}_C \cdot \mathbf{r}} S_C(\mathbf{b}_C) \psi_{l_0, m_0}(\mathbf{r}) \right|^2$$

(b) Diffraction dissociation

$$\frac{d\sigma_{dif. dis.}}{d^2K_{\perp} d^3k} = \frac{1}{(2\pi)^5} \frac{1}{(2l_0+1)} \sum_{m_0} \left| \int d^3r d^2b e^{-i\mathbf{K}_{\perp} \cdot \mathbf{b}} \psi_{\mathbf{k}}^*(\mathbf{r}) S_C(\mathbf{b}_C) S_n(\mathbf{b}_n) \psi_{l_0, m_0}(\mathbf{r}) \right|^2$$

Open theoretical issues:

- a) "In medium" modification of σ_{NN} C.B., JPG 27 (2001) L67
- b) Final state interactions Tostevin et al., PRC 66 (2002) 024607
C.B., Hansen, PRC 70 (2004) 034609
- c) Diffraction dissociation C.B., NPA 767 (2006) 155
- d) Connection with DWBA C.B., NPA 767 (2006) 155

Computer Code: MOMDIS

C.B., Gade, CPC 17 (2006) 372

Download from <http://www.cpc.cs.qub.ac.uk/>

Catalogue id: ADXZ

Calculates:

- a) Total stripping + diff. dissociation X-sections
- b) Momentum distributions
- c) Elastic scattering
- d) Coulomb effects

Based on: C.B., Hansen, PRC C70, 034609 (2004)

$$S(b) = \sum_{j=1}^N \alpha_j e^{-b^2 / \beta_j^2}, \quad \beta_j = \frac{R}{j}$$



$$\underbrace{\iiint \cdots \iiint}_{\text{lots}} \cdots \Rightarrow \underbrace{\sum \cdots \sum}_{\text{few}} \cdots$$

S-matrices expanded on Gaussians

lots

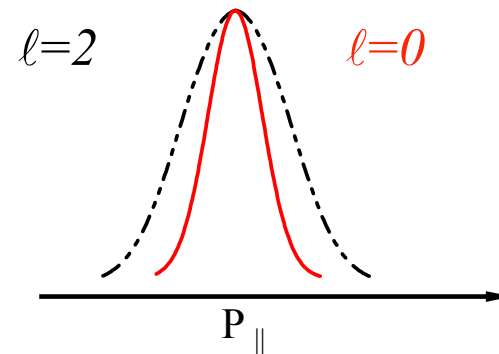
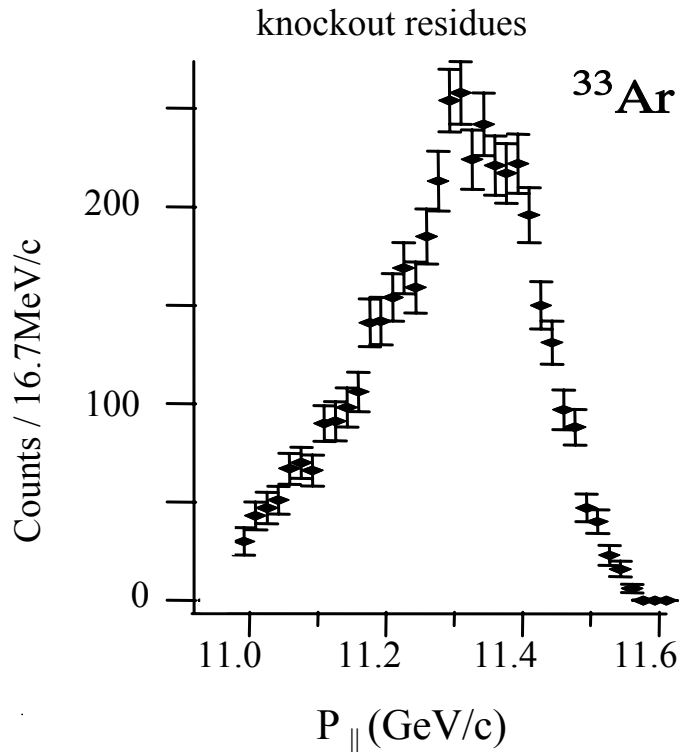
few

Nuclear spectroscopy

$$\sigma(nI^\pi) = \sum_j \overset{\text{spectroscopic factor}}{C^2 S(j, nI^\pi)} \overset{\text{single-particle cross section}}{\sigma_{sp}(j, S_n)}$$

nuclear structure information
reaction process

$$\sigma_{sp}(j, S_n) = \sigma_{sp}^{strip}(j, S_n) + \sigma_{sp}^{diffr}(j, S_n)$$



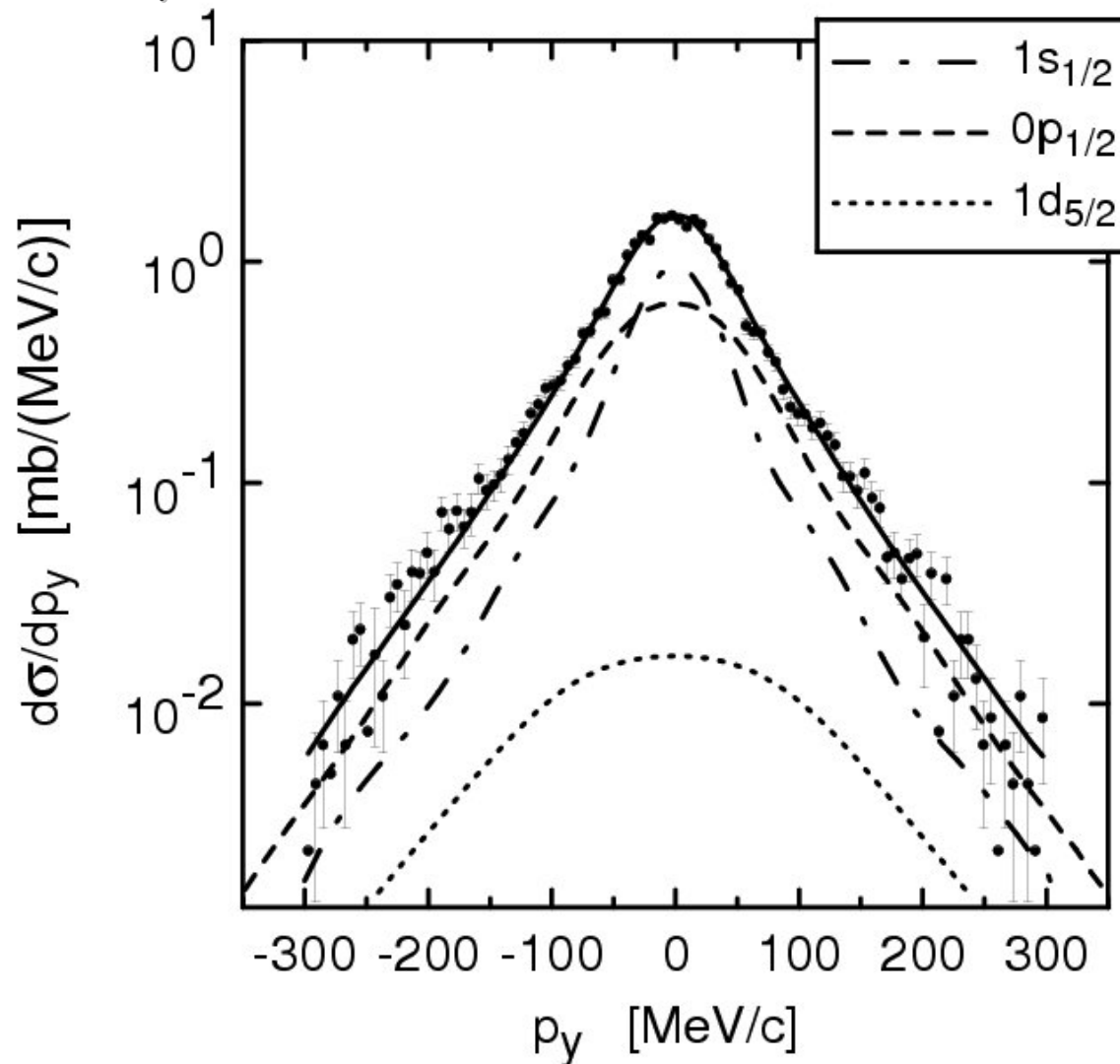
l -value assigned in comparison to model calculations

Hansen, PRL 77, 1016 (1996)

C.B., Hansen, PRC 70, 034609 (2004)

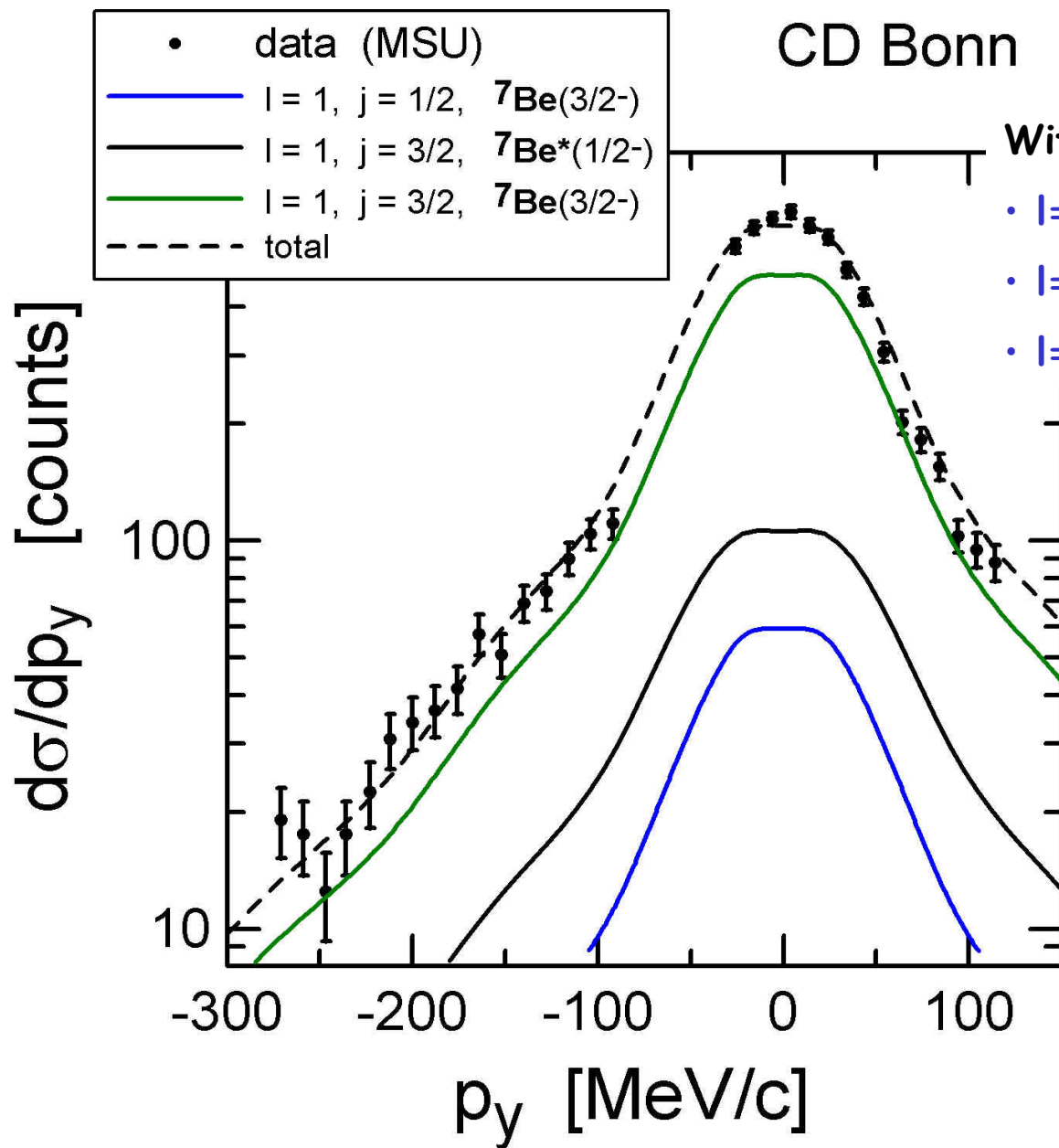
Applications: Testing nuclear models

(a) Three-body Model: C.B., Hansen, PRC C70, 034609 (2004)



Data: H.Simon et al., PRL 83,496 (1999)

(b) No-Core Shell Model

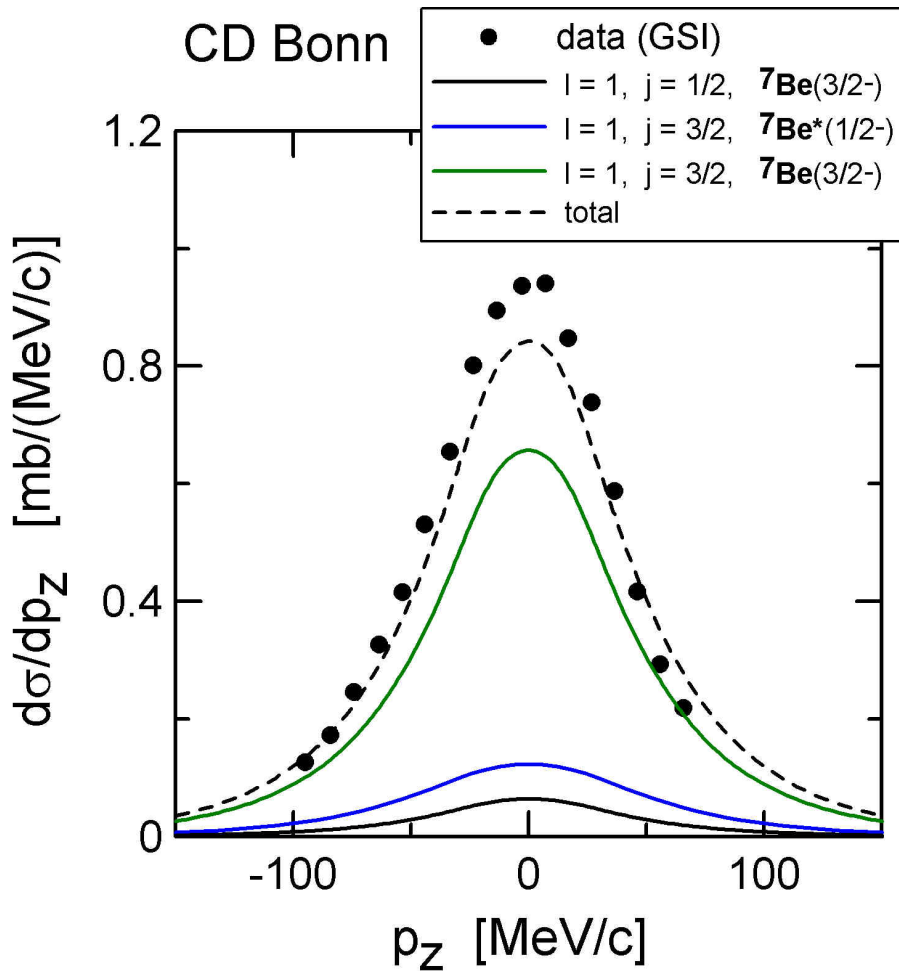


With NCSM wavefunctions ($10 \hbar\omega$)

- $l=1, j=1/2, I_{{}^7\text{Be}}=3/2$: $C^2S=0.085$
- $l=1, j=3/2, I_{{}^7\text{Be}}=1/2$: $C^2S=0.280$
- $l=1, j=3/2, I_{{}^7\text{Be}}=3/2$: $C^2S=0.958$

Navratil, C.B., Caurier,
 PLB 634 (2006) 191
 PRC 73 (2006) 065801

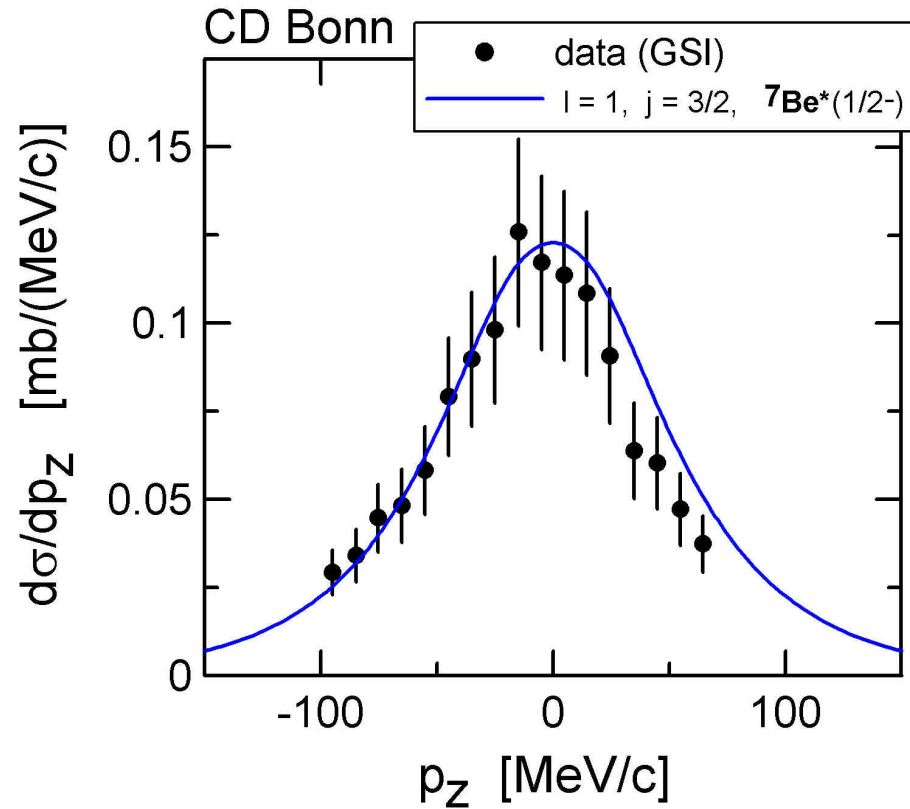
^8B (938 MeV/nucleon) + ^{12}C \rightarrow ^7Be + X



$\sigma_{\text{str}} = 99.39$ mb

exp = 94 ± 9 mb

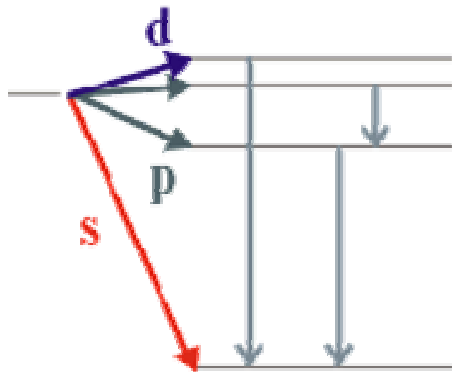
^8B (938 MeV/nucleon) + ^{12}C \rightarrow ^7Be + γ + X



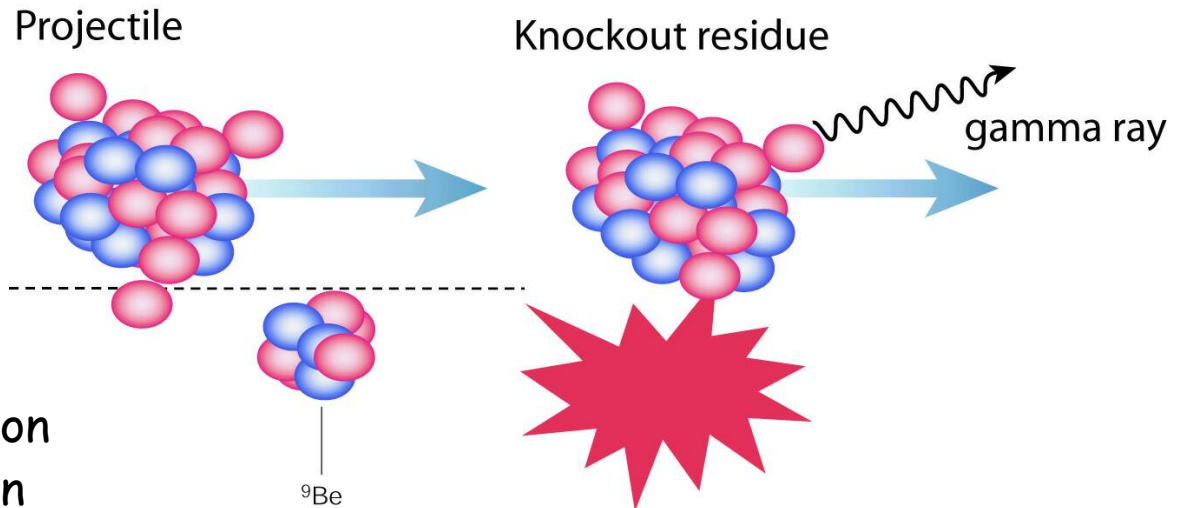
$\sigma_{\text{str}} = 15.31$ mb

exp = 12 ± 3 mb

DATA: Cortina-Gil et al, 2002



residue moment distribution
 → l -value of knocked-out n



Spectroscopic Factors

determined from the population of the residue with $A-1$

- Fast exotic beams allow for
 - thick secondary targets (100-1000 thicker than at low energy)
 - event-by-event identification
- Example
 - $\sigma = 100$ mbarn
 - $N_T = 10^{22} \text{ cm}^{-2}$
 - $N_B = 30$ Hz
 - $N_R = 2600/\text{day}$

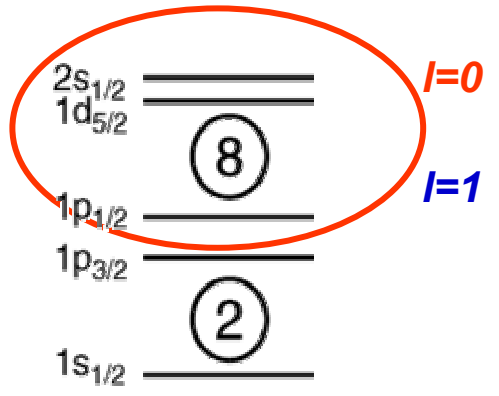
Applications: Detailed nuclear spectroscopy

Selected examples:

- a) The breakdown and persistence of shell gaps
- b) Evolution of nuclear structure approaching the "Island of Inversion"
- c) Reduced spectroscopic strength and proton neutron asymmetry at the Fermi surface

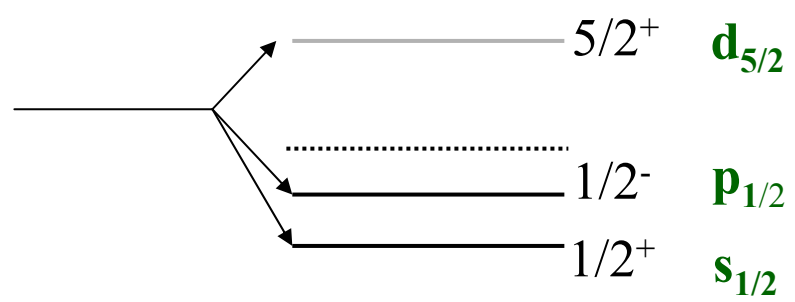
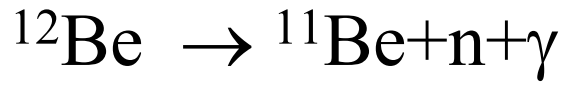
A classic example: N=8 in ¹²Be

Single particle states in normal order:

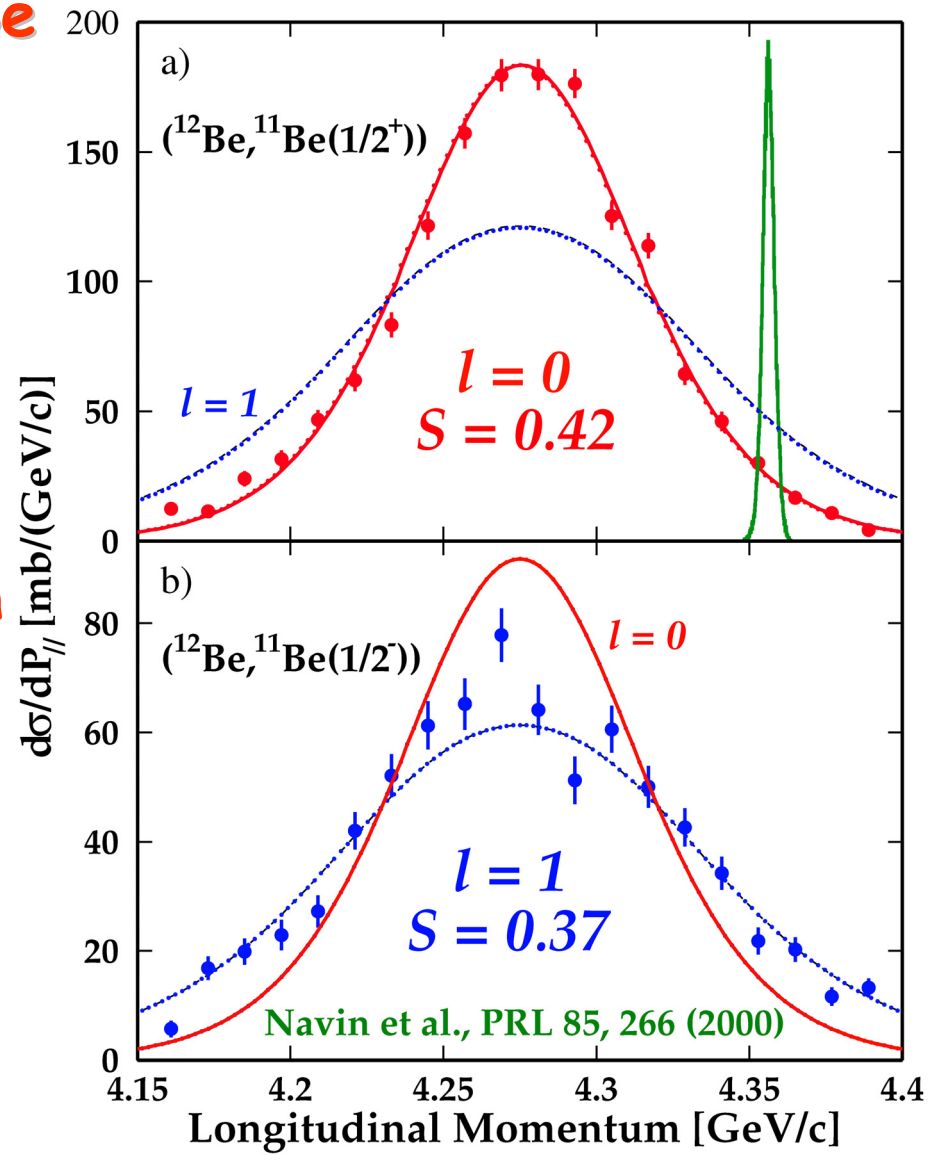


But there is a strong l=0 contribution

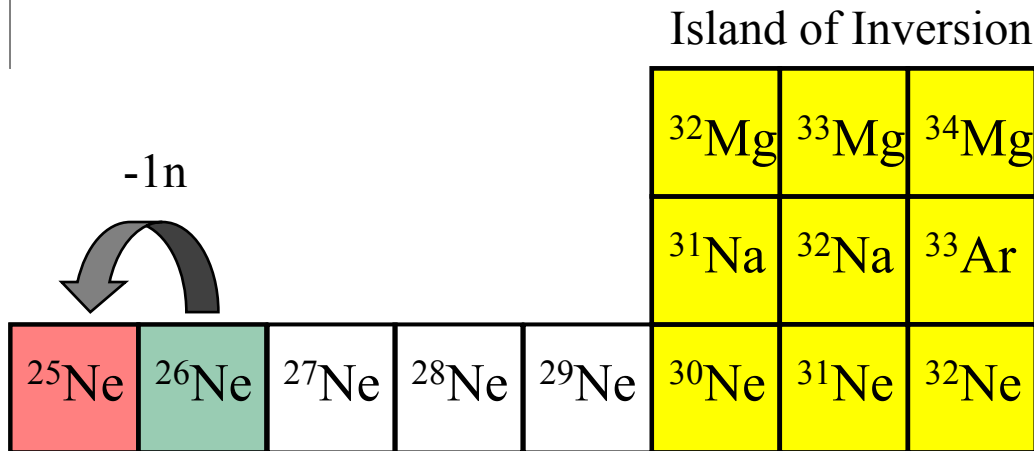
J^π	E_{ex} (MeV)	σ_{exp} (mb)
$1/2^+$	0	32.0(47)
$1/2^-$	0.32	17.5(26)



The $s_{1/2}$ orbit lowered below the N=8 gap and became an intruder



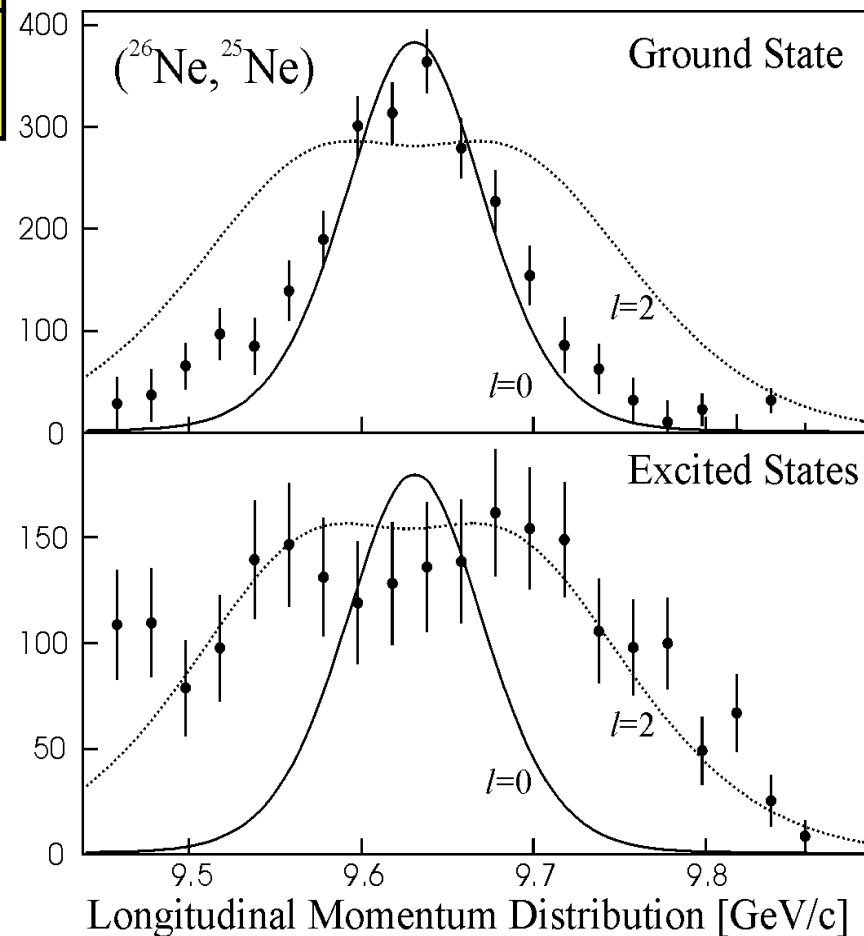
Approaching the island of inversion in the Ne chain I



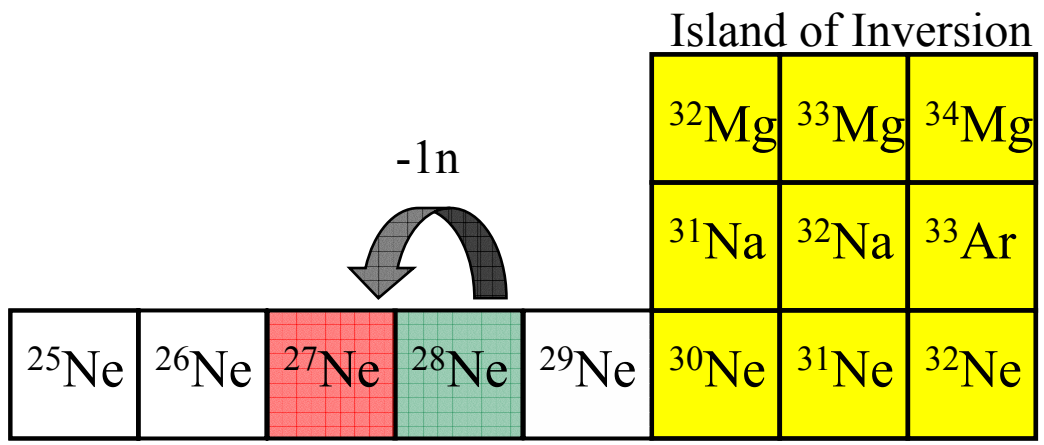
- Strongest feeding to $1/2^+$ ground state and to $5/2^+$ states
- Weaker feeding to first $3/2^+$ state

Terry et al., PLB 86 (2006).

Consistent with enhanced $N=16$ sub-shell gap, no evidence for intruder states

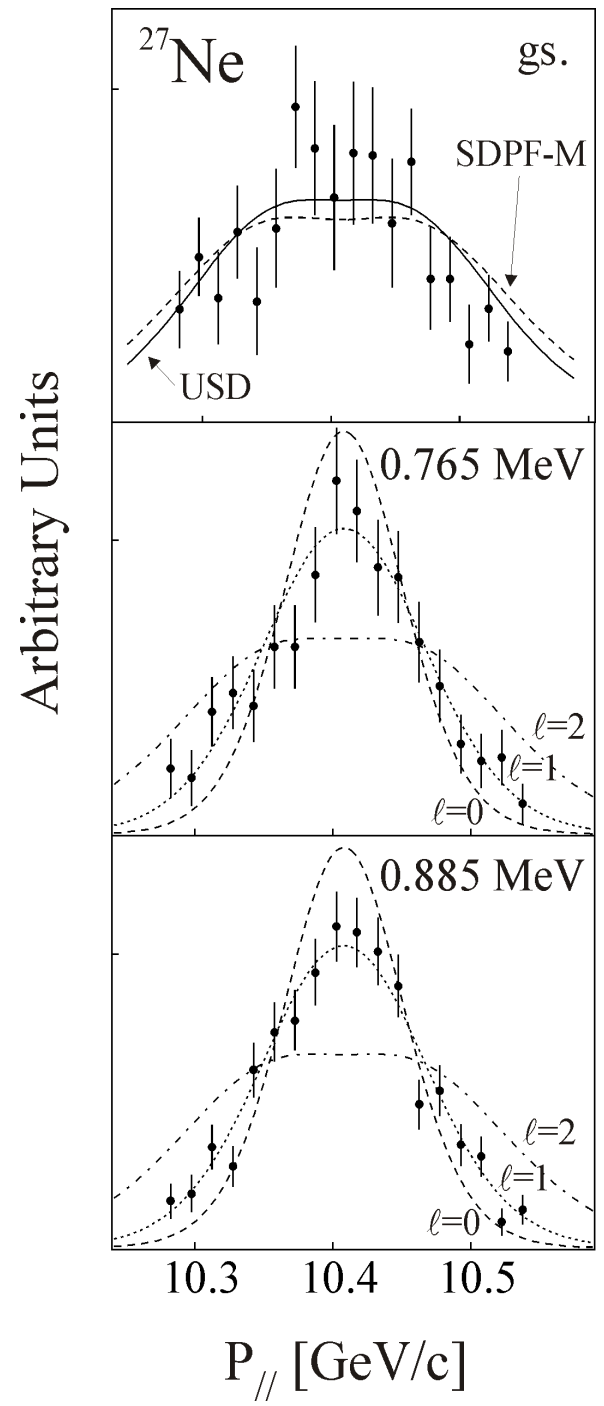


Approaching the Island of Inversion in the Ne chain II



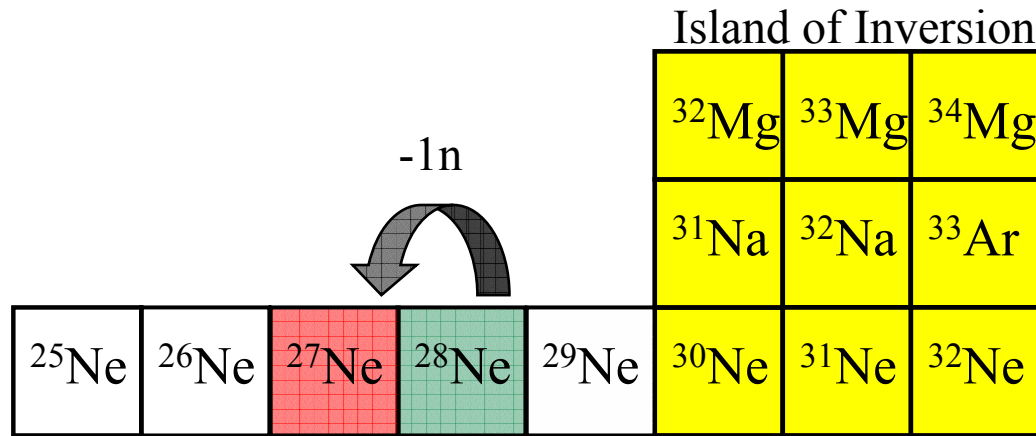
- 1- Observed three gamma-ray transitions; level scheme confirmed by γ - γ coincidences.
- 2- Momentum distributions extracted for 0.765- and 0.885-MeV transitions; both show $\ell < 2$, but low statistics make $\ell=0$ and $\ell=1$ difficult to distinguish

Terry et al., PLB 86 (2006).

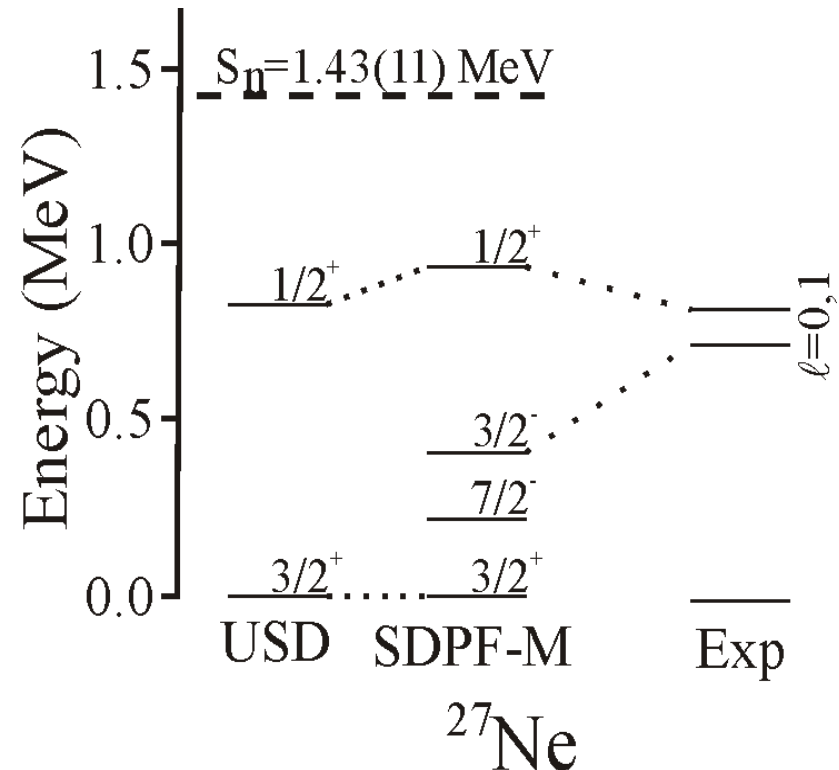
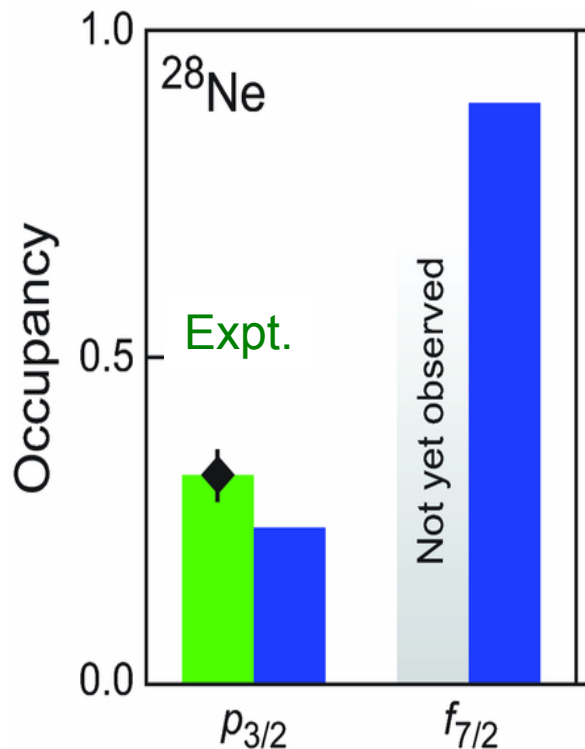


Approaching the Island of Inversion in the Ne chain II

Terry et al., PLB 86 (2006).

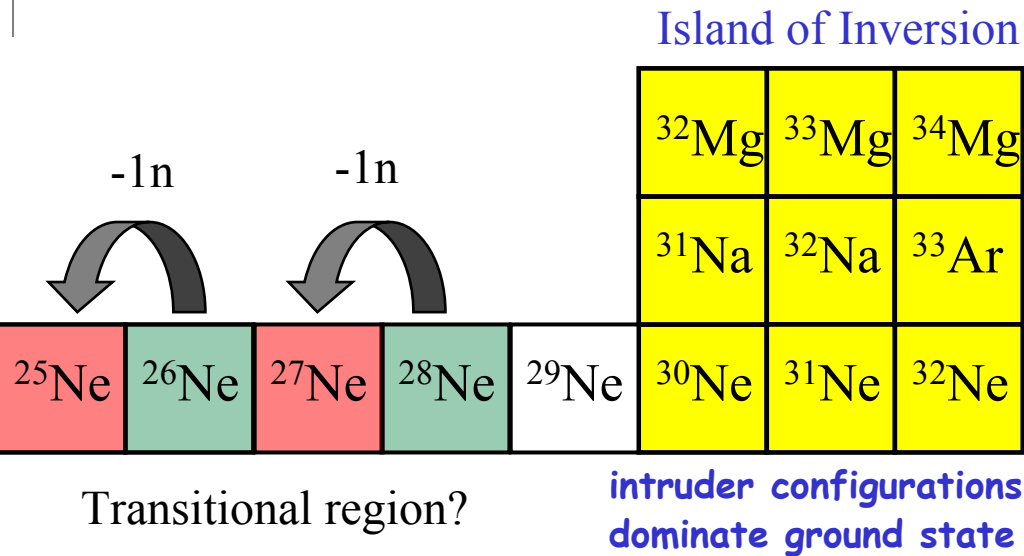


SDPF-M SM



- ▷ Additional excited state indicates intruder states at low excitation energy, likely $p_{3/2}$
- ▷ ℓ -value and spectroscopic strength inconsistent with SDPF-M prediction

A gradual transition into the "Island"



- γ -ray spectroscopy in coincidence with ^{25}Ne , ^{27}Ne knockout residues \rightarrow intruder state in ^{27}Ne but not in ^{25}Ne
 - Parallel momentum distributions: intruder state in ^{27}Ne consistent with $3/2^-$ assignment as expected from theory (SDPF-M shell model, Tokyo)
- \rightarrow Intruder states come down in energy

Terry et al., PLB 86 (2006)

^{25}Ne

no low-lying intruder states

^{26}Ne

no intruder configurations in gs

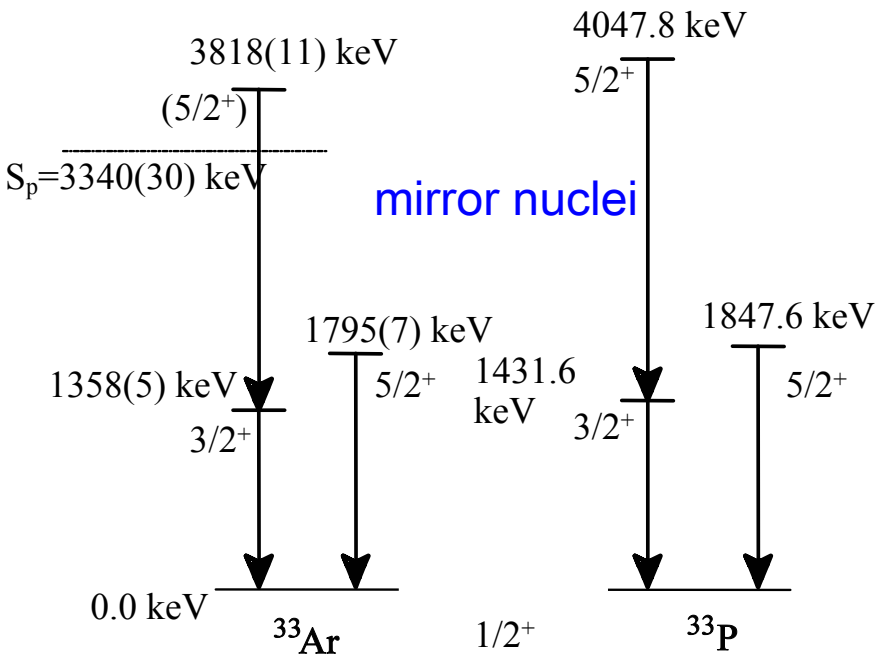
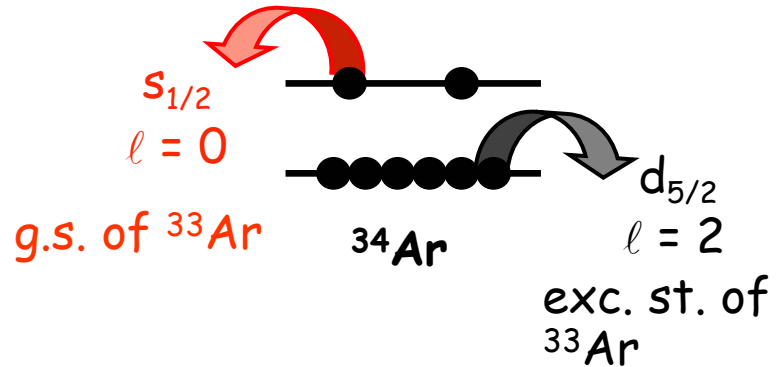
^{27}Ne

low-lying intruder state (~ 800 keV)

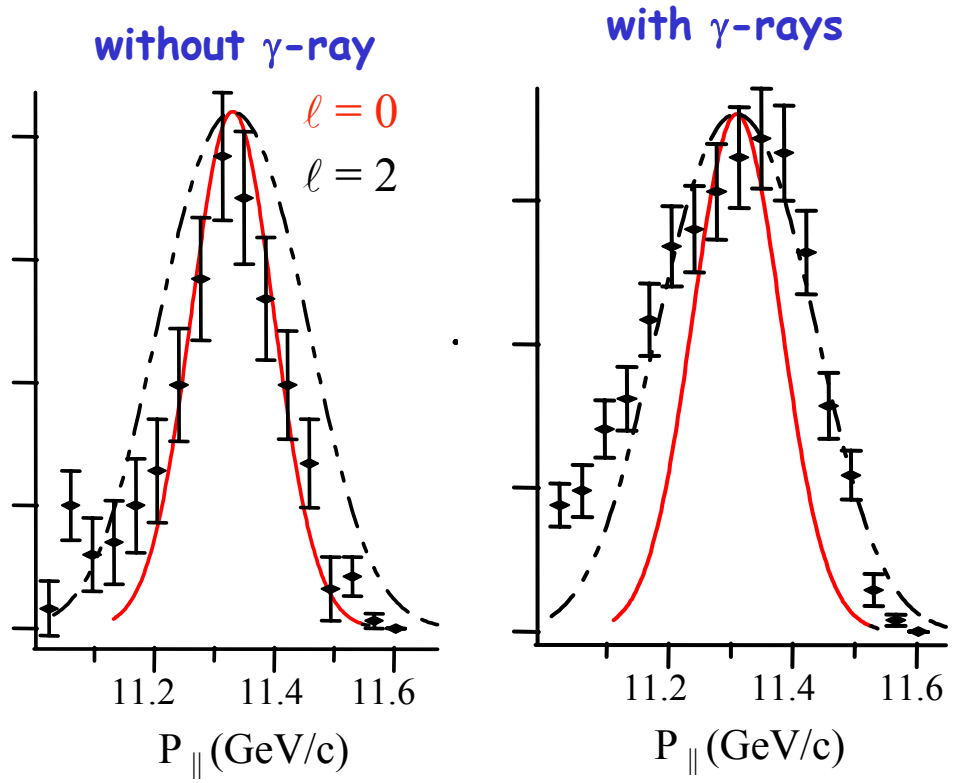
^{28}Ne

$np_{3/2}$ intruder configuration in gs

${}^9\text{Be}({}^{34}\text{Ar}, {}^{33}\text{Ar}+\gamma)\text{X}$



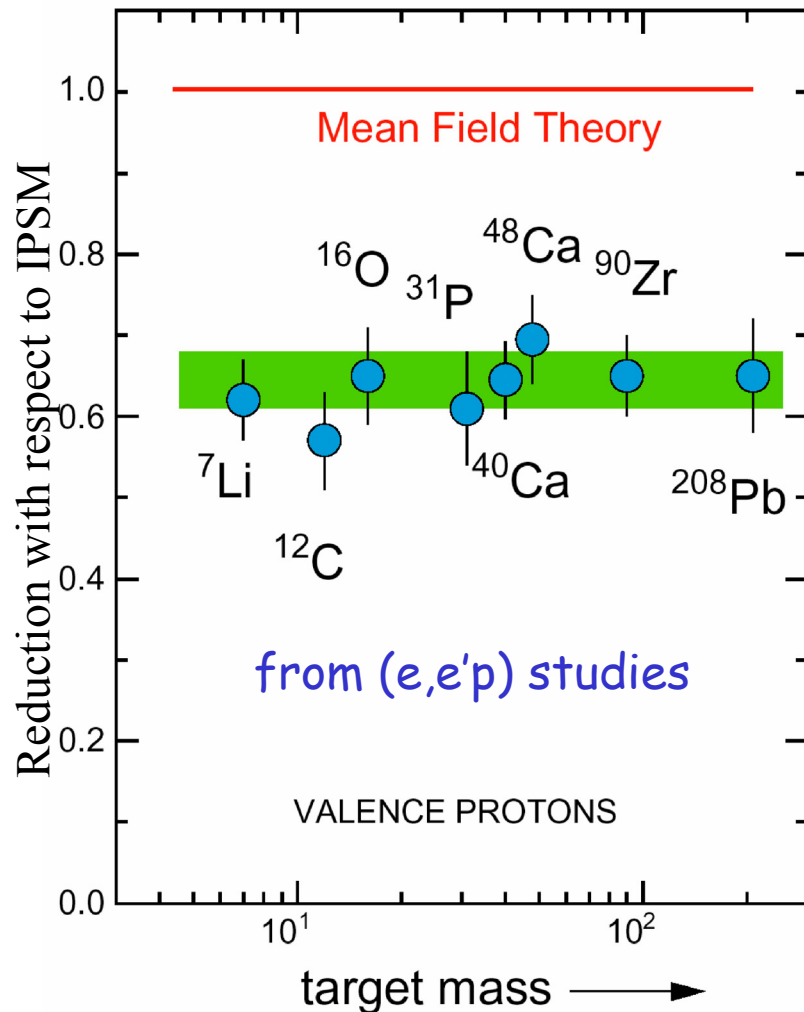
${}^{33}\text{Ar}$ level scheme confirmed in Clement et al, PRL 92, 172502 (2004)



Gade et al, PRC 69, 034311 (2004)

Correlation effects - stable nuclei

Figure from W. Dickhoff et al.



Reduction factor with respect to
the shell model $R_s = C^2 S_{\text{exp}} / C^2 S_{\text{th}}$

The nuclear shell model pictures deeply-bound states as fully occupied by nucleons. At and above the Fermi sea, configuration mixing leads to occupancies that gradually decrease to zero.

Correlation effects (short-range, soft-core, long-range and coupling to vibrational excitations) are beyond the effective interactions employed in the shell model and mean-field approaches. The picture will be modified depending on the strength of the correlation.

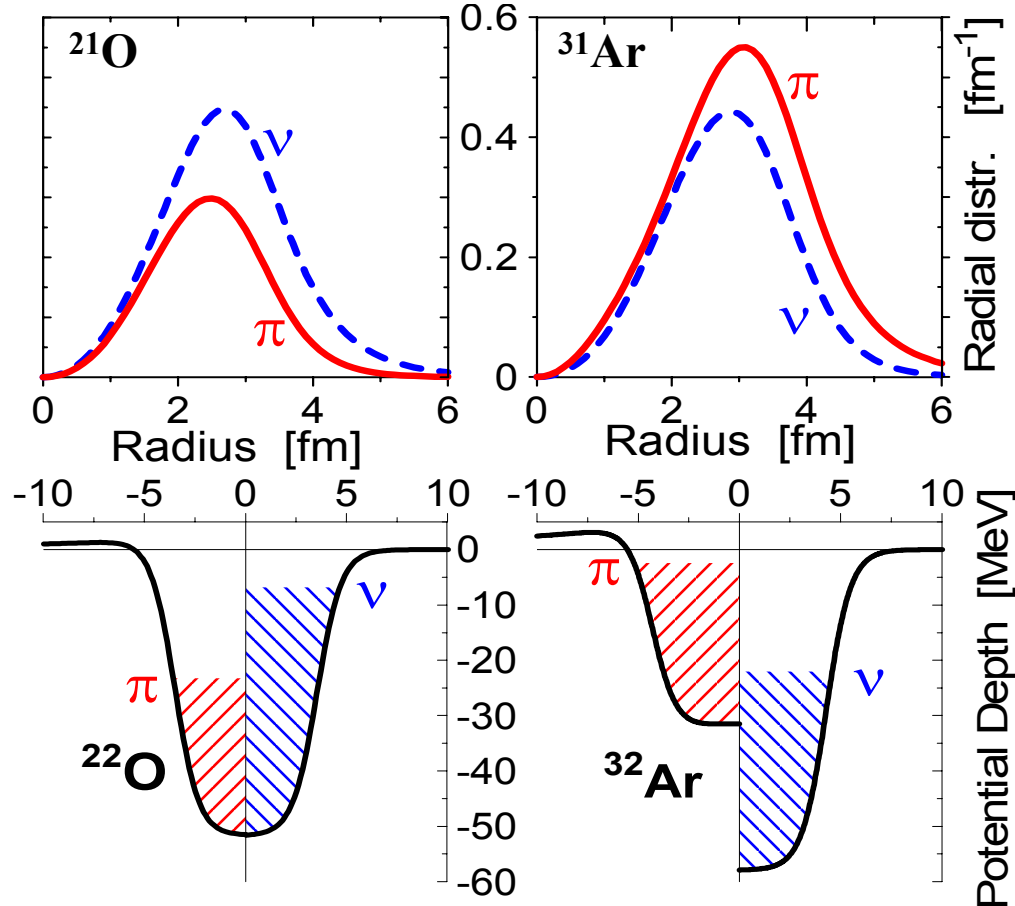
Pandharipande *et al*, *Rev. Mod. Phys.* 69, 981 (1997)

Dickhoff, Barbieri, *Prog. Nucl. Part. Sci.* 52, 377 (2004)

Correlation effects - Weakly and deeply bound systems

neutron skin

proton skin



What is the situation beyond the valley of β stability?

At rare-isotope accelerators:

Very deeply and weakly bound exotic nuclei become accessible

Experimental approach to assess the occupation of single-particle orbits in exotic nuclei:
1-nucleon removal reactions

Measured spectroscopic factor C^2S relates to the occupation number of the orbit involved

${}^9\text{Be}({}^{15}\text{C}, {}^{14}\text{C})\text{X}$ - A weakly-bound neutron system

Experiment:

$\sigma_{\text{inc}} = 140.2(46) \text{ mb}$

$b(\text{g.s.}) = 71.8(24) \%$
resulting partial cross
section

$\sigma(0^+) = 100.8(44) \text{ mb}$
gives with

$\sigma_{\text{sp}} = 99.4 \text{ mb}$
 $C^2S = 1.01(4)(5)$

Terry et al., PRC 69, 054306 (2004).



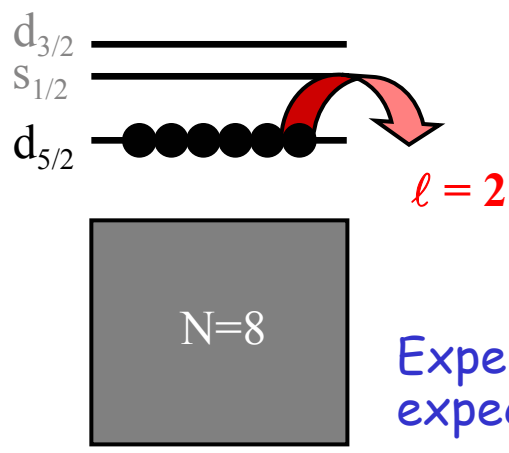
$$S_n({}^{15}\text{C}) = 1.2181(8) \text{ MeV}$$

...	${}^{14}\text{C}$	${}^{15}\text{C}$	${}^{16}\text{C}$...
...	${}^{13}\text{B}$	${}^{14}\text{B}$	${}^{15}\text{B}$...
...	${}^{12}\text{Be}$		${}^{14}\text{Be}$	neutron drip line

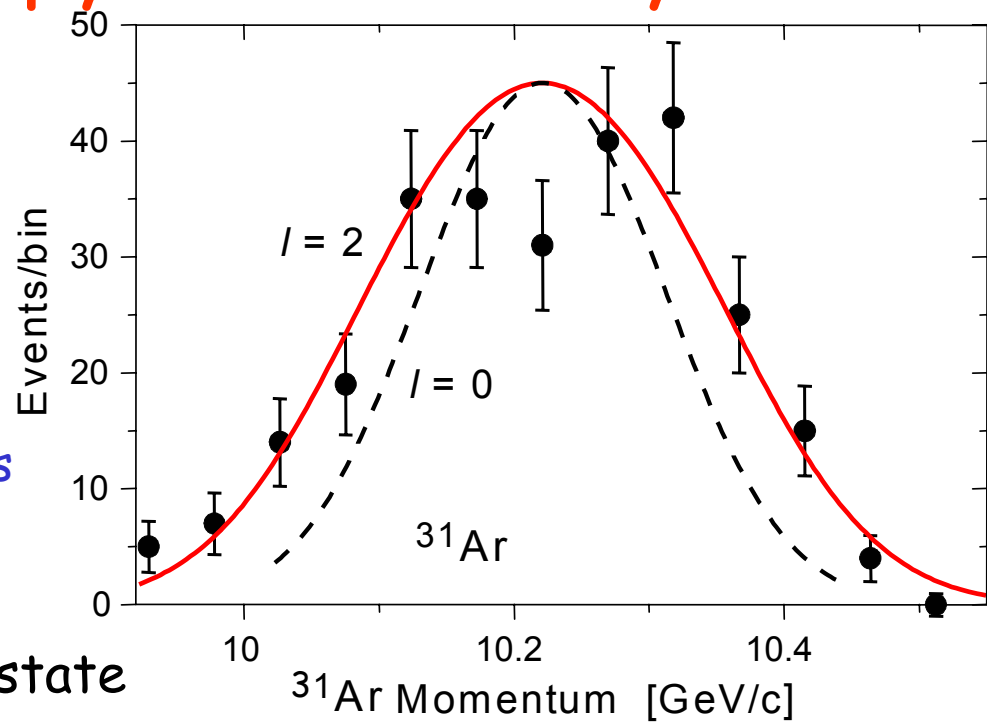
USD shell model (B.A. Brown)
 $C^2S = 1.128$

Reduction of the experimental vs. theoretical
spectroscopic factor: $R_s = 0.90(4)(5)$

${}^9\text{Be}({}^{32}\text{Ar}, {}^{31}\text{Ar})\text{X}$ - A deeply-bound neutron system



Experiment confirms expected value $l=2$



Only bound state in ${}^{31}\text{Ar}$: ground state

one-neutron knockout

				${}^{35}\text{K}$
proton drip line	${}^{31}\text{Ar}$	${}^{32}\text{Ar}$	${}^{33}\text{Ar}$	${}^{34}\text{Ar}$
		${}^{31}\text{Cl}$	${}^{32}\text{Cl}$	${}^{33}\text{Cl}$

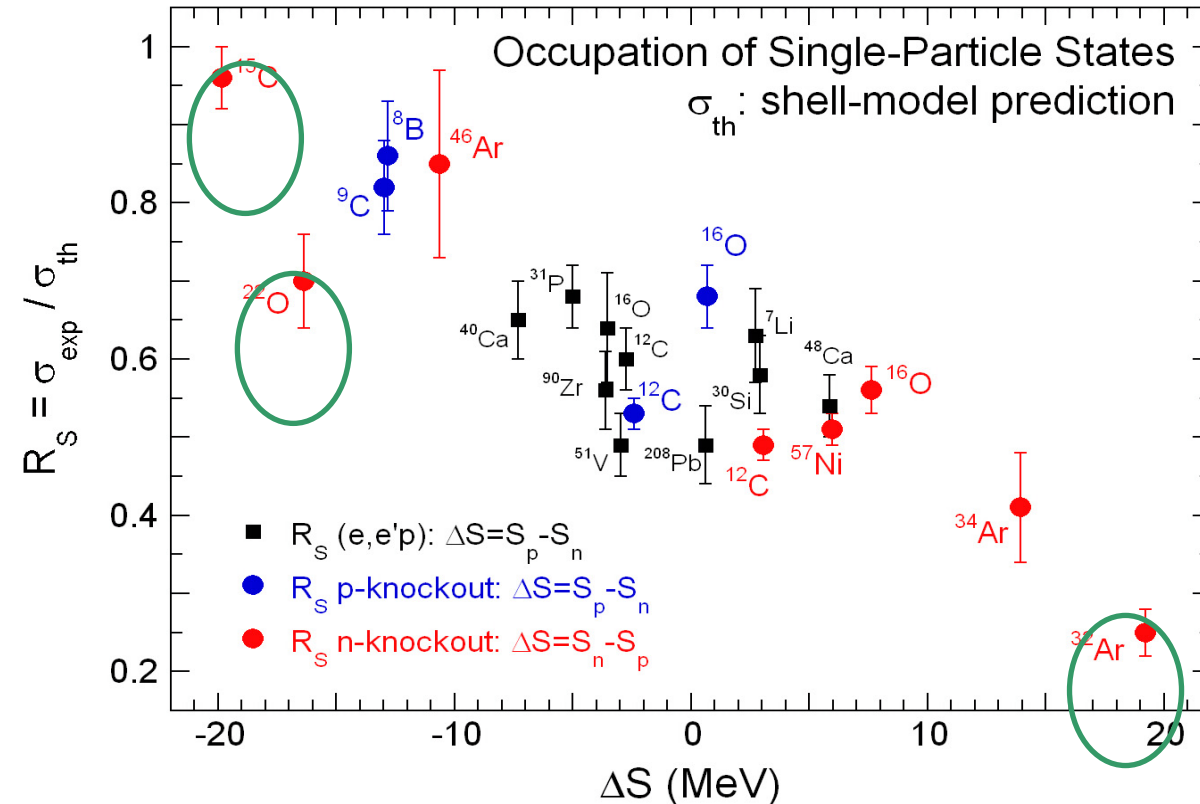
$S_n = 21.99(5) \text{ MeV}$

$\sigma_{\text{exp}} = 10.4(13) \text{ mb}$
 $\rightarrow C^2S = 1.00(13)$

USD shell model:
 $C^2S = 4.12$ (B.A. Brown)

Reduction of the experimental vs. theoretical spectroscopic factor: $R_s = 0.24(3)(4)$

Reduction of spectroscopic strength



^{32}Ar and ^{22}O have the same neutron number but the reduction R_s is very different

Gade et al., PRL 93, 042501 (2004)

$^{15}\text{C}-1n$: precision measurement showing less reduction for weakly bound systems

Terry et al., PRC 69, 054306 (2004)

Consistent with a DOM study of elastic scattering:
 Charity, Sobotka and Dickhoff, PRL 97 (2006) 162503

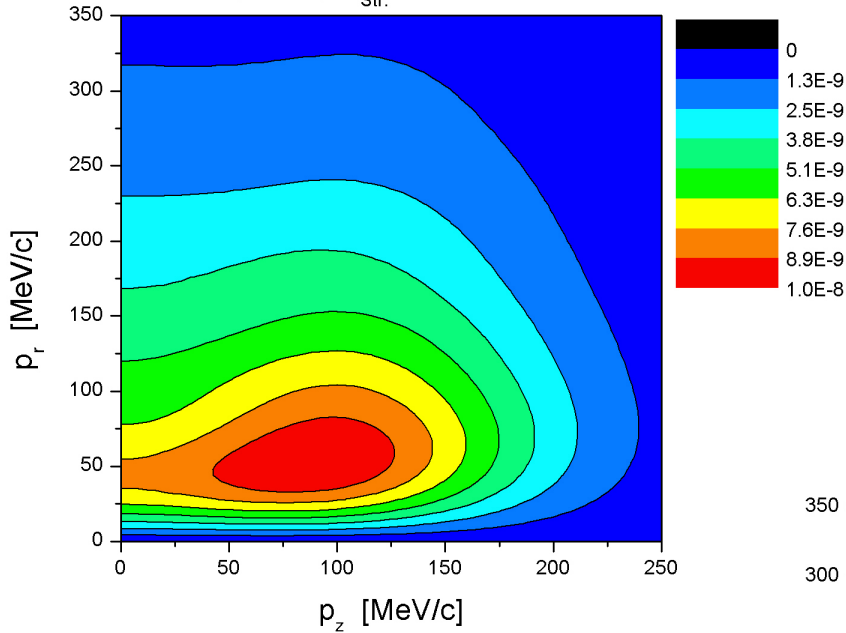
Consistent with transfer:
 Lee, Tostevin et al., PRC 73 (2006) 044608

Determination of the occupancies provides information on the presence of correlation effects beyond effective-interaction theory

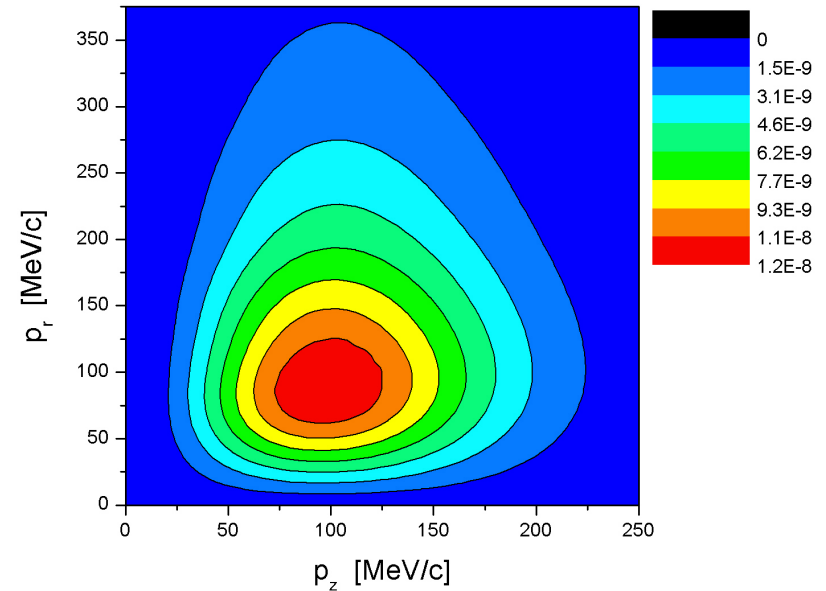
Future: Alignment experiments

C.B., Hansen, PRC C70, 034609 (2004)

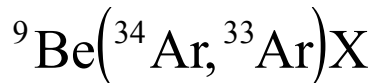
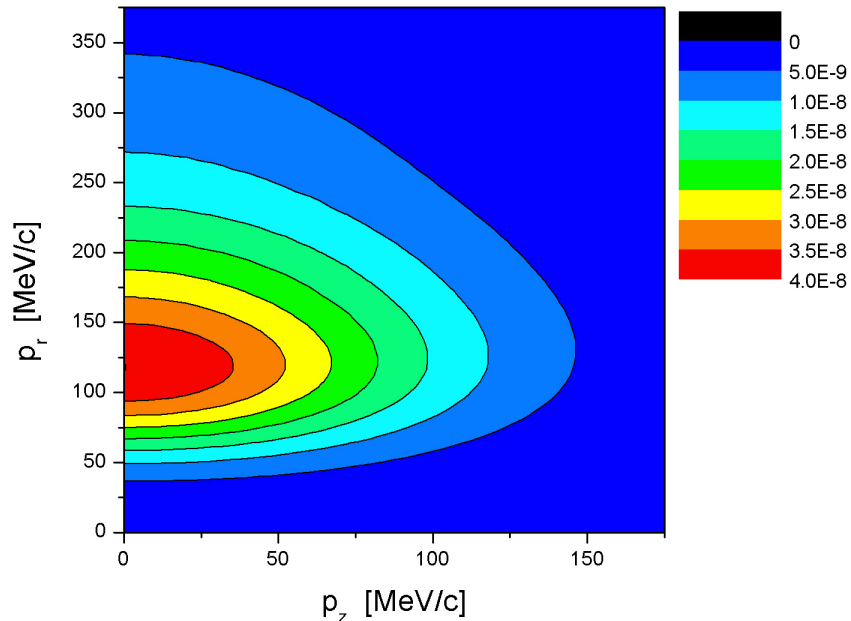
^{34}Ar (68.2 MeV/nucleon)
 $l=2, m=0, \sigma_{\text{Str.}} = 1.175 \text{ mb}$



^{34}Ar (68.2 MeV/nucleon)
 $l=2, m=1, \sigma_{\text{Str.}} = 2.296 \text{ mb}$



^{34}Ar (68.2 MeV/nucleon)
 $l=2, m=2, \sigma_{\text{Str.}} = 4.715 \text{ mb}$

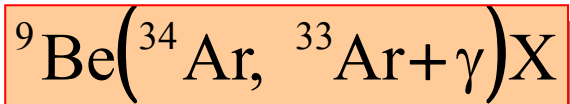


One neutron-removal

68.2 MeV/nucleon

Od5/2 neutron, $S_n = 18.43 \text{ MeV}$

TMD x LMD: Alignment - γ -ray angular distribution



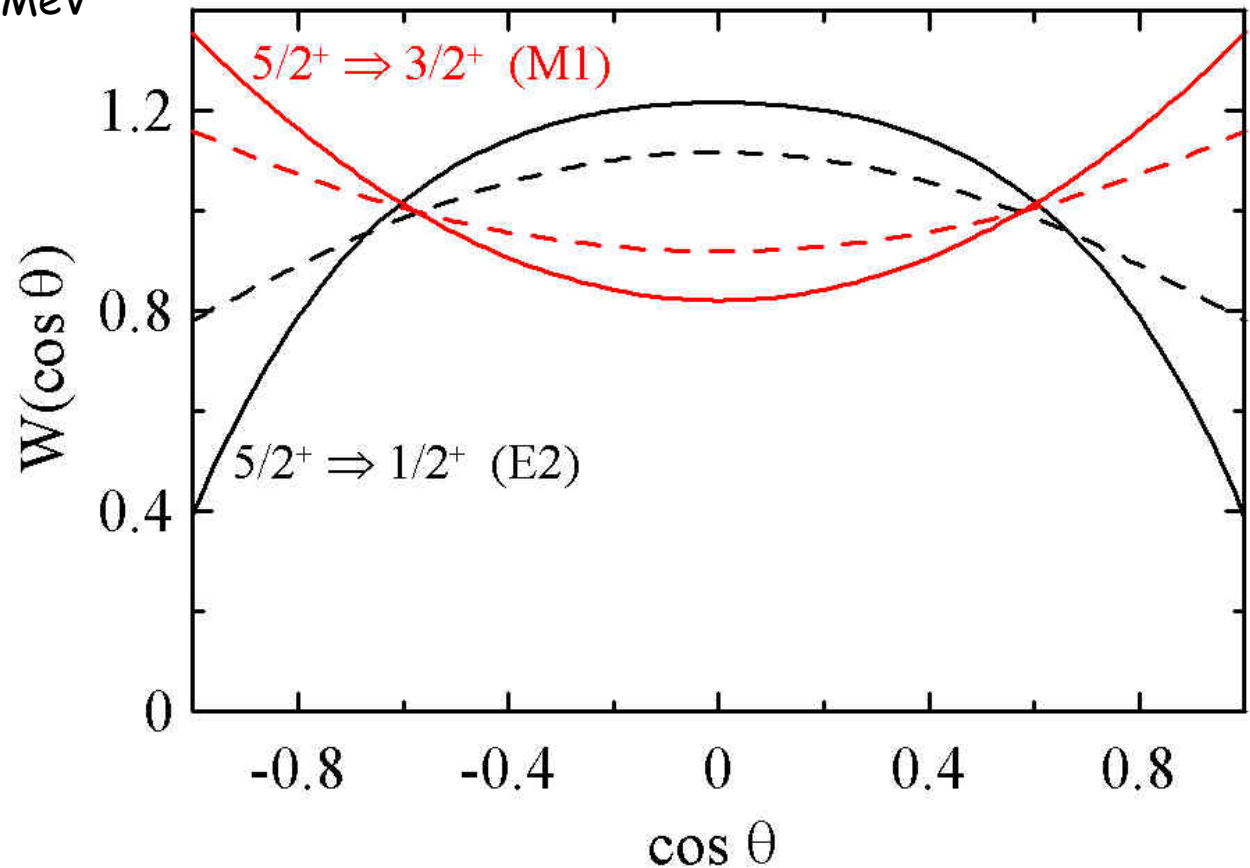
One neutron-removal

68.2 MeV/nucleon

0d5/2 neutron, $S_n = 18.43$ MeV

${}^{34}\text{Ar}$

Angular distributions, center-of-mass system
dashed: all, full drawn: 13% cut around $M=2$



Brief summary and outlook

A few selected examples:

- a) Track the evolution of nuclear structure in the vicinity of new and conventional magic numbers by wave-function spectroscopy with knockout reactions
- b) Precision measurement of spectroscopic factors to compare to ab-initio calculations
- c) Study the effect of the continuum (Gamov shell model)
- d) Many more exciting opportunities lie ahead... too many to list