MECHANICAL CRITERIA FOR THE PREPARATION OF FINITE ELEMENT MODELS

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ABSTRACT

The use of CAD in design makes it possible to represent complex components "as manufactured" with a great number of details. A transformation of such models into Finite Element (FE) models often generates a much too large number of elements to be used directly. Therefore, the removal of shape details is required to prepare a FE model. Often, these shape transformations are required to suit the hypotheses of the FE analysis. The user must control this simplification process in order to ensure sufficient accuracy of the FE results.
In this paper, different criteria are studied to evaluate 'a priori' the quality of a shape simplification process like variations in volume, area and center of gravity when the input shape is generated from CAD models. These criteria can be related to different mechanical properties and, according to the simulation objectives, the analyst can assign threshold values to the chosen criteria. These criteria can be evaluated locally over the shape of a component to provide a qualitative analysis of shape changes. Such an approach has been applied in the framework of polyhedral models.

Keywords: shape simplification, mesh, polyhedral model, vertex removal, mechanical criterion, finite element accuracy

1. INTRODUCTION

Expressing hypotheses and simplifying an analysis domain are mandatory for current simulations in the context of FE analyses. Design models are often constructed for the purposes of manufacturing, and therefore contain numerous details that are part of the component "as-manufactured". The adaptation of the geometry for finite elements models is achieved by the elimination of shape details when their presence has no effect or a poor effect on the mechanical behavior while imposing an important local mesh density. Examples of these details include not only fillets, rounds which remove sharp edges for manufacturing purposes, but also detailed entities such as holes, small blocks, etc.

Although modern CAD systems tend to integrate FEA tools in the design environment, generating FE models from design ones remains tedious and lacks of operators to evaluate the impact of modifications with respect to mechanical criteria. This is a major issue for analysis integration into design environment.

Several approaches have been proposed to ease the preparation of the FE models through detail removal operators.

There have been several efforts aimed at removing details after an initial mesh has been generated [1] [2] [3]. In these approaches, mesh elements forming details are removed by performing well-known mesh transitions, e.g. collapsing the faces of a tetrahedron to remove the element.

Sheffer [4] developed an automated scheme for detail removal and geometry clean-up through a concept of virtual topology. The clustering of model faces into regions of restricted curvature and distance deviation is used to generate a new topology of the model more suited to mesh generation while preserving its geometry.

Geometry-based solutions have been proposed in the literature to remove entities such as small features [5] [6], blends [7], bosses, ribs and holes. Key issues uncovered in these approaches are the level of user interaction required and their robustness when applied to models suffering from inconsistencies due to model exchange between CAD and CAE software.

While the Medial Axis Transform (MAT) is a geometric operator, it is a useful operator to identify zones suitable to dimensional reduction [8] or the removal of small shape features which affect only locally the mechanical behaviour [9]. The main limitation of the MAT is also the lack of criterion to evaluate the impact of such shape changes.

Another class of approaches starts with a polyhedral model of the part [10] [11] [12]. In order to adapt the model, adaptation operators modify the object shape. They combine a skin detail removal based on a decimation process and topological detail removal operators. This polyhedral simplification process is monitored by an a priori criterion requiring the user's expertise to set geometric error bounds over the initial polyhedron that reflect the FE map sizes desired.

A major issue still to be addressed is the lack of methods to evaluate the shape changes and their influence on local and global mechanical parameters prior to the simulation. In addition, such methods form a first step to relate simulation hypotheses to shape changes.

The work presented in this paper provides such methods. Based on previous work by Véron [11] and Fine [10], these criteria are developed in the framework of a simplification process based on polyhedral models.

A priori criteria are typically geometric since a finite element simulation is needed to take into account the quantitative effect of boundary conditions. Nevertheless, geometric criteria often provide a good evaluation of the mechanical influence of details.

Thus, a priori mechanical criteria proposed in this work are geometric (e.g. volume, area, center of inertia variations), but their mechanical meaning and their relevance to monitor the shape adaptation process for various analysis is justified.

The main objective of this study is to identify and setup criteria relevant either to drive the simplification process or to validate the simplified model obtained.

To this end, two approaches have been set up to locally evaluate shape changes over simplified models.

This paper is organized as follows. At section 2, different criteria are listed and their relationships with mechanical parameters are addressed. Section 3 presents the existing polyhedral model simplification algorithms for which the mechanical criteria have been designed and the reciprocal images principle used for the cell-based approach. Section 4 presents the two approaches used to compute the mechanical criteria. Section 5 presents examples and results obtained with the implementation of the proposed approaches for mechanical criteria.

2. SHAPE ADAPTATION CRITERIA

The simplification of a geometrical model is monitored by many mechanical criteria. The goal of these mechanical criteria is to evaluate the influence of shape changes on a mechanical analysis.

2.1 Existing mechanical criteria

Former work on mechanical criteria for FEA model simplification is partly based on a bounded error criterion proposed by Véron [11] and Fine [10].

In this approach, the geometric transformations are monitored by an error zone concept and an inheritance mechanism. During the initialization step of the simplification process, a spherical error zone is assigned to each vertex of the input polyhedron. This set of spheres define a discrete envelope around the input polyhedron where the simplified polyhedron must lie (see figure 1 and 2).

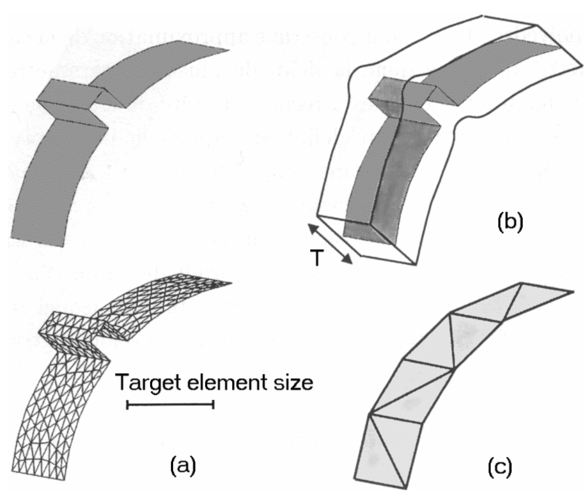

Figure 1: Discrete envelope criterion concept proposed by Véron [11] (a) initial polyhedron, (b) discrete envelope, (c) simplified polyhedron, (d) initial polyhedron, (e) set of spheres defining the discrete envelope around the polyhedron.

The radius of the error spheres can be set up using values specified through two different ways. The first

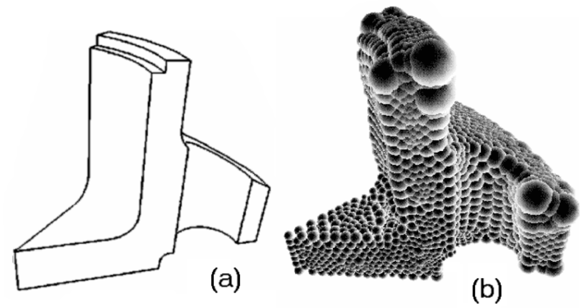

Figure 2: Discrete envelope criterion concept proposed by Véron [11] (a) initial polyhedron, (b) set of spheres defining the discrete envelope around the polyhedron.

criterion is an *interactive a priori criterion*: the radius of the spheres is set by the user and attached to different areas of the object. The radii values can be assigned proportionally to the FE map of sizes desired using interpolation functions between key vertices. This approach contributes to the effective characterization of shape details for a given analysis. The second way is an *automatic a posteriori criterion*: the radii of the spheres are automatically assigned. In this case, the size of the error zones reflect the size of the finite elements required to match the analysis accuracy specified by the user. The sphere sizes can be defined using a strain energy error estimator based on a previous analysis to provide a new model for better FEA results. The simplification process is inserted into a simplification FEA computation loop as shown on figure 3.

In the following subsections, the simulation preparation process is strictly restricted to an a priori approach.

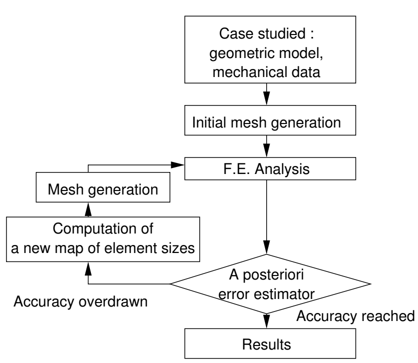

Figure 3: A posteriori simplification process proposed by Véron and Fine [10]

Though this approach helps to characterize shape details efficiently, it defines 'bounds' of shape changes but it is unable to give complementary feed back to the analyst when the shape changes lie in between the bounds. In addition, this approach is essentially related to the FE discretization of the domain whereas other mechanical concepts could be addressed.

2.2 Criteria for analysis

During the simulation preparation process, these mechanical criteria aim at (see figure 4):

a bounding the shape variation with the thresholds set by the analyst,

b validating the simplified model obtained by visualizing the map of changes of mechanical properties. This type of results is shown in most figures of section 5.

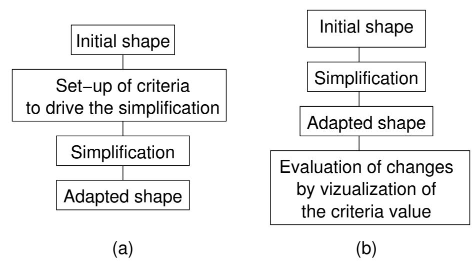

Figure 4: Combination of different criteria classes in the simplification: (a) Criteria used to constrain the simplification (b) Criteria used to validate the shape adaptation process.

However, the two distinct uses of these criteria according to figure 4 can be combined since they are complementary. Configuration *a* can be expressed either at a macro-scale, i.e. over the whole object, or at a meso-scale, i.e. a sub-domain of the object, as a preparation constraint. Configuration *b* can be used for shape changes inspection at macro, meso and micro-scales, i.e. at the size of the smallest possible entity of the geometric model.

The various criteria relating geometric quantities to mechanical parameters can be enumerated as follows:

Volume variation criterion: The amount of variation in mass between CAD and FEA models can be a parameter that the analyst needs to evaluate in a dynamic analysis case such as modal analysis or in a thermal analysis case (preservation of the thermal capacity for a non stationary thermal transfer study). Therefore, the analyst may need to specify at a meso-scale the maximum mass variation in order to ensure that the model simplification has no significant effect on the analysis results. Besides verifying local mass changes, the volume criterion can be derived to construct a stiffness criterion which evaluates the influence of local volume differences over the stiffness of the part.

Center of gravity and inertia: During the simplification of a geometric model, each transformation on curved areas introduce a displacement of the center of gravity and a variation in inertia moments. Then, the analyst may need to avoid transformations that generate a displacement of center of gravity over a prescribed threshold distance, or a variation in some inertia matrix values over a given value.

Such a criterion is meaningful both for volumes, free form surfaces, as well as planar sections when considering beam-shaped components.

Area variation criterion: In several cases, the variation in area of a surface is an interesting criterion to monitor the simplification process.

Shape adaptation of surface models based on shell or plate type elements lead to area variation equivalent to volume variation for a 3D model. Another example is the preservation of the area of the surface loaded by a pressure boundary condition or a heat flux which should remain nearly constant during the simplification process. Otherwise, there would be differences between the fields of the initial and adapted parts [10] (see figure 5).

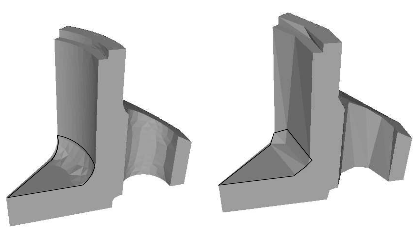

Figure 5: Variation in pressure area during a simplification process.

Besides preserving the area of loaded surfaces of a part, the area variation of a shell model is equivalent to volume changes for solid models. Indeed, when the surface of a shell model is smoothed, the variation in area influences its mass and its stiffness.

Line variation criterion: The length of a line loaded with a linear boundary condition is an important property in some cases too. This criterion has not been implemented in this work yet, but the reciprocal images mechanism presented at section 3.3 can be used to easily implement this criterion.

However, beam structures are frequent in FE models and this criterion is similar to volume and surface variations for higher dimension models.

Remarks:

- the advantage of a mechanical criterion over a pure geometrical one depends on the load case, the type of analysis and finally on the simulation hypotheses.
- the formulation of criteria is closely related to the hypotheses formulated by the analyst (for example, the analyst assumes the FE model is not correct if it varies in volume of more than 2% from the design model).
- these criteria can evaluate shape changes on different scales: macro-scale (the whole domain), meso-scale (a sub-domain), micro-scale (smallest geometric element of comparison between models).

The above taxonomy of criteria, though it has been stated as prescriptions for the shape adaptation process, can be also considered as a set of inspection criteria after a shape simplification criterion. In addition, it should be understood that all these criteria can be combined together and added to geometric deviation criteria (see section 2.1). The shape adaptation process being stopped by either of these criteria.

3. THE POLYHEDRAL SIMPLIFICATION PROCESS

The taxonomy described in the previous section is intrinsic to a shape, i.e. it is independent of the geometric model used to describe the object. Here, polyhedral models have been considered for the following reasons:

- they produce a simplified shape locally comparable to the initial polyhedron using either reciprocal images (presented in 3.3) or transfer of information (presented in 4.2). Indeed, comparing B-Rep models is much more complex due to the unavailability of a common parametrization between the initial and simplified B-Rep models as well as the difficulty to perform shape changes on such models.
- they can handle any class of details with its generality and its independence with respect to the topological structure of the object,
- treat efficiently large and complex models: this process has been successfully applied to an industrial CAD model of the full Airbus A380 aircraft cockpit, and has led to a time reduction of four months to one month for the preparation of a thermal model.

Though the taxonomy of criteria can be related to various classes of shape adaptation operators, i.e. boundary modifications or so-called skin operators, topology modification operators, the present work focuses only on shape changes through skin operators.

The adaptation of the shape is applied to an intermediate polyhedral model generated from a CAD model and is based on the identification of geometric areas whose removal or modification enables the simplification of the simulation model (decrease of the number of vertices), without affecting the simulation results. The zones considered as "details" are therefore zones where the discretisation generates a large number of nodes which are unnecessary to ensure acceptable simulation results. It is assumed here that the input polyhedron has a discretization compatible with the requirements of the map of FE sizes expressed by the analyst, i.e. in curved areas the edge length is smaller than the target FE edge length and in directions of null curvatures the edge length can be greater than the target FE size. Such a configuration preserves the consistency with the FE map of sizes criterion described at section 2.1, enables the reduction of the model complexity while avoiding adverse effects on the target FE mesh, as pointed out by Owen [12].

3.1 Overview of the skin operator

The simplification process of a polyhedral model on which the proposed mechanical criteria is briefly reviewed hereunder. The input data of the simplification process is a polyhedral model of the part. The simplification process is based on an iterative vertex removal algorithm. First, the edges and vertices of the initial polyhedral model are classified in accordance to their topological information. This classification is necessary to apply the appropriate selection criterion and vertex removal operators to each class of vertices: boundary entities, surface entities, unremovable entities, ...

Then, the simplification treatment is initialized. A spherical error zone is assigned to each vertex of the input model. The radius of these spheres can be set up using an a priori or a posteriori mechanical criterion (see section 2.1). At each face an inheritance process of error zones is performed to monitor the shape restoration during the simplification process. As illustrated in figure 1, the spherical error zone criterion aim at simplifying the geometry according to the element size chosen by the analyst, using his a priori expertise of the simulation needs. The mechanical criteria proposed here act in this part of the algorithm. Therefore, local volume variation, local area variation, center of gravity deviation, ..., are monitored and used as a priori criteria at this stage of the algorithm. Maximum allowed values of volume variation, area variation, or center of gravity deviation can be assigned interactively to a set of faces of the initial polyhedral model. Once the simplification criteria has been set, the simplification process starts, and a loop is executed until no more candidate vertex can be removed.

At each iteration of the process, a vertex removal operator is applied to create a new local geometry from the contour polygon of this vertex (see fig. 6). This new set of faces covering the contour polygon is defined by the remeshing scheme selected. The set of faces around the candidate vertex is called the star-shaped polyhedron. Each criterion assigned by the user is then applied to determine whether or not the vertex can be removed. If the mechanical properties of the initial model are correctly restored, the faces of the star-shaped polyhedron are replaced by the appropriate remeshing created by the vertex-removal operator as shown on figure 6.

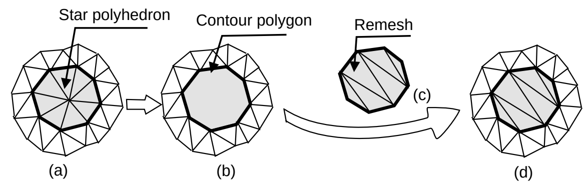

Figure 6: Vertex removal operator: (a) star-shaped polyhedron around the candidate vertex and initial polyhedron, (b) hole created for star polyhedron removal and its corresponding contour polygon, (c) candidate remeshing of contour polygon created, (d) the remeshing and the final polyhedron.

The remeshing scheme of the star polyhedron is processed by optimising element shape criteria. The iterative application of the vertex removal operator tends to smooth the global shape of the polyhedron, and then, tends to remove small and curved shape details which are irrelevant details as shown on figure 7.

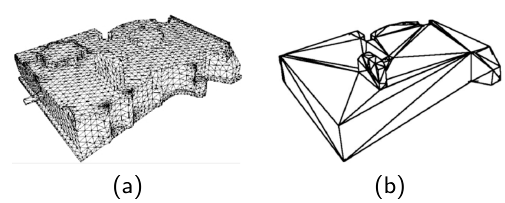

Figure 7: Solid model containing many skin details (a) initial polyhedron, (b) coarse model obtained using the iterative vertex removal operator.

The simplified shape obtained from this process forms now the basis of a FE mesh generation process. However, under appropriate criteria (size, form, ...), this process can produce directly the target FE mesh [13]. The influence of this operator has been evaluated from the stress point of view and has produced satisfactory results when comparing the results obtained from the initial geometry and the simplified geometry generated from an enveloppe produced by an a posteriori error estimator [14].

3.2 Structure of a geometric criterion

All the criteria listed previously (see section 2.2) need now to be evaluated using the vertices, edges and faces of the polyhedral model. Hence, the local geometric quantities required are reduced to a face, an edge or a vertex.

The evaluation of changes at the local scale is necessarily based on a representation of some geometric elements of the initial shape and their image on the simplified one, i.e. a mapping.

In our work, the method of comparison between polyhedrons characterizes the two approaches proposed in this work: while the propagation approach is based on the differences between the star polyhedron and its associated remeshing at each step of the skin detail removal operator, the cell-based approach relies on the difference between the remeshing and its "mapping" on the initial polyhedron.

Many configurations arise when one try to calculate the geometric projection of one polyhedron onto another as illustrated in figures 8 and 10.

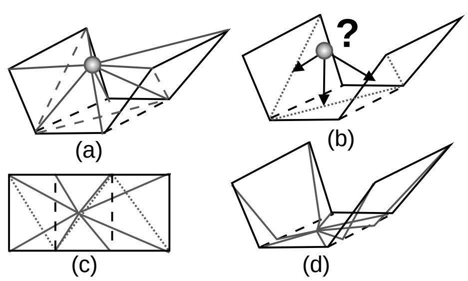

Figure 8: (a) the star polyhedron and its remeshing after the vertex removal, (b) non-uniqueness of the direction of projection to construct the image of the removed vertex on the remeshing, (c) construction of a planar configuration of the remeshing by unfolding the remeshed faces around their shared edge ⇒ no need for projection, (d) image of the star polyhedron on the remeshing obtained with the reciprocal images scheme.

Therefore, the cell-based criteria need a mapping between the initial polyhedron and the final one (and this mapping should be one to one mapping and always existing, as well as geometrically faithful to ensure the robustness of the process).

The quality of the images plays an important role in the cell-based approach. For example, any shift or distortion of the image will produce erroneous results on criteria. Mapping and bijection characterize aspects related to parametrization of triangulations [15] [16] [17] [18].

The propagation approach transfers values of criteria from entities at iteration i of the corresponding star polyhedron to the ones of the remeshing scheme (iteration $(i+1)$) and calculates the variation of criteria caused by the vertex removal operation to update the value assigned to the new faces.

3.3 The reciprocal images

In this work, the comparison operators between an initial polyhedron ($\mathrm{P_i}$) and a final polyhedron ($\mathrm{P_f}$) are based on a one to one mapping function providing for any point N of $\mathrm{P_i}$ its position on $\mathrm{P_f}$.

This one to one mapping function is based on reciprocal images.

The image of a polyhedron ($\mathrm{P_i}$) on another one derived from vertex removal operations is built with the "mapping" of its vertices and its edges on the target polyhedron ($\mathrm{P_f}$). To be more precise, the removed vertex is relocated on the remeshing according to some barycentric coordinates on the contour polygon. Though such a mapping plays an important role to provide good results, it is not possible to describe further its principle within this paper because of the lack of space. However, it is a principle similar to a mesh parameterization though it enables a fully automatic behaviour requiring no user input and capable of coping with any topological configuration, either manifold or not.

We call reciprocal images the image of $\mathrm{P_i}$ on $\mathrm{P_f}$, and the image of $\mathrm{P_f}$ on $\mathrm{P_i}$ (see figure 9).

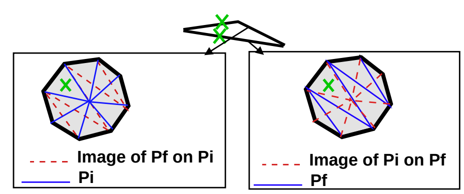

Figure 9: Reciprocal images.

In order to understand the problems coming from the use of a geometrical projection of a node N onto the remeshing, figures 8 and 10 illustrate typical configurations. Indeed, in the general case, there can exist 0, 1, or many images of N on $\mathrm{P_f}$. This configuration is the one which can be encountered at every step of the polyhedral simplification, when the vertex of the star polyhedron is removed and its contour is remeshed.

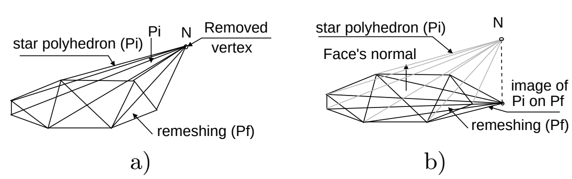

Figure 10: The geometric projection problem: the projection of the removed vertex is localized outside of the remeshing.

Using the above principle of reciprocal images during the skin detail removal process provides an appropriate and robust mapping for transferring criteria values during the shape changes.

4. APPROACHES FOR CRITERIA EVALUATION

All mechanical criteria presented in section 2 are founded on local variation in volume, area, center of gravity displacement, and inertia moments of the polyhedral model.

This section presents two different approaches for the calculation of the basic quantities related to these criteria:

- The first approach, called *cell-based approach*, is an exact calculation of the local value of the criteria since it relies on an exact representation of the local difference between the two polyhedrons. To achieve the exact representation of the local difference, this approach is based on reciprocal images presented in section 3.3 which are geometrically faithful and don't 'slide' on the target polyhedron. Currently, the computation cost of the reciprocal images is about four times the vertex-removal operator cost.
- The second approach called *propagation approach* is based on the information propagation through either faces or nodes during the vertex removal process, without use of the reciprocal images. This approach is faster and can be used to handle polyhedrons which contain a large number of faces.

Both approaches are suited to inspection purposes since their use is rather qualitative. Though the first approach is more suitable since there is no approximation, both approaches are equivalent from the prescriptive point of view since they evaluate different values summing up to the same global quantity over the polyhedron.

4.1 Cell-based approach

The principle of this method is to construct locally on the polyhedron P_i (or P_f) the solid model of the space between itself and the other polyhedron P_f (or P_i) as illustrated in figures 10, 11, and 12.

In order to clearly express the algorithms, some terminology and symbols are given at this point:

- **F_i and F_f** symbolize a face of the initial polyhedron P_i and a face of the final polyhedron P_f respectively. The image of F_i on P_f lies partly in F_f and reciprocally the image of F_f on P_i lies partly in F_i.
- A **cell** is defined by the image of F_f on F_i and by the image of F_i on F_f. We call $\mathbf{C_i}$ the polygonal shape representing the image of F_i on F_f and $\mathbf{C_f}$ the image of F_i on F_f as shown in figure 11.

A cell is the image of a face of P_i (or P_f) on a face of P_f (or P_i). The figure 11 shows the construction of a cell.

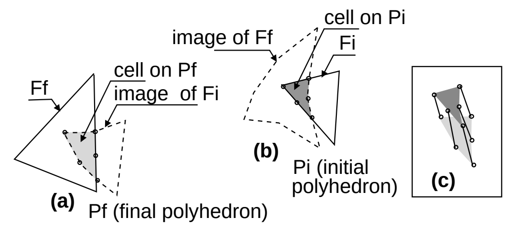

Figure 11: Illustration of a cell: **(a)** image of a face of P_i on P_f,**(b)** image of a face P_f on P_i, **(c)** construction of a cell from both images.

The cell concept represents geometrically and accurately the elementary variations resulting from the simplification process. The set of cells having an image on a face of P_f represents the total variation between this face and P_i. Therefore, the set of solid volumes of cells having an image on a face of P_f represents the total volume difference between this face and P_i. This property is illustrated on figure 12.

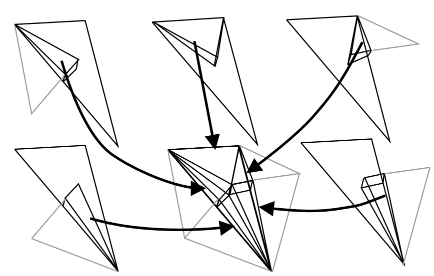

Figure 12: Set of cells having an image on a face of P_f: the assembly of the set represents the volume variation lying on the face of P_f.

4.1.1 Algorithm for cells construction

Each cell is represented by the following data structure:

- The identifiers of its associated faces F_f and F_i,
- Its images C_f and C_i on P_f and P_i respectively.

Cells are constructed from reciprocal images using the following algorithm:

for each face F_i of P_i **do**
  Get the three oriented edges of F_i

for each face F_f of P_f where F_i has an image on F_f **do**
  for each edge of F_i **do**
    Get the polyline representing the image of this edge on F_f
    Get the polyline representing the portion of the edge having an image on F_f
  end for
  Build the data structure which represents a cell
end for
end for

Finally, the data structure helps finding the set of cells lying on a face as shown in the adjacency graph in figure 13.

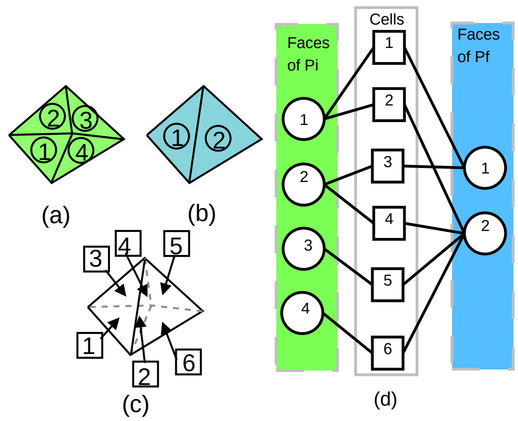

Figure 13: Cells-faces adjacency graph (a) star polyhedron on P_i (b) remeshing on P_f (c) cells lying between the remeshing and the star polyhedron, (d) adjacency graph of cells, faces of P_i and faces of P_f.

4.1.2 Volume of a cell

The volume of a cell is calculated by tessellating the volume between its images on P_i and P_f as shown in figure 14. The volume of an oriented triangulated closed surface can be efficiently calculated using the following sum:

$$V = \frac{1}{6}\sum_{i=1}^{N}(z_0^i + z_1^i + z_2^i)(x_0^i(y_1^i - y_2^i) + x_1^i(y_2^i - y_0^i) + x_2^i(y_0^i - y_1^i))$$

where $i = 1 \ldots N$ is the index of the faces forming the tessellation, x_j^i, y_j^i, z_j^i are the 3D coordinates of the j^{th} vertex of the i^{th} face.

The images of the cell C_i and C_f on P_i and P_f respectively, are two 2D-polygons lying in the plane of F_i and F_f respectively. The cell is tesselated by constructing triangles between the two polygons (these triangles are called lateral faces) and inside the two polygons (these triangles are called interior faces) as shown on figure 14 using the following algorithm:

Set N = number of points of each cell polygon
construct Gi: an interior point of the polygon of F_i visible from any of its polygon vertices
construct Gf: a point inside the polygon of F_f visible from any of its polygon vertices
for i = 1 to N **do**
  j = (i+1) modulo N (j is the index of the neighbour vertex of i)
  construct lateral faces:
  construct face (C_i(i), C_i(j), C_f(j));
  construct face (C_f(j), C_f(i), C_i(j));
  tessellate C_f*:*
  construct face (Gf, C_f(i), C_f(j));
  tessellate C_i*:*
  construct face (Gi, C_i(j), C_i(i));
end for

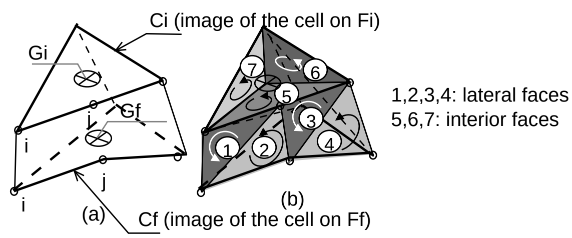

Figure 14: (a) Volume of the cell, (b) Tessellation of the volume of the cell.

4.1.3 Center of gravity of a cell

The volume of cell is a local variation in mass of the final model P_f which moves its center of gravity compared to the initial model P_i. The center of gravity of the simplified model is then moved with the contribution of the whole set of volume cells. The x coordinate of the center of gravity is obtained by the integral $1/V \int_{\Omega} x dv$. During a vertex removal operation, the volume subtracted (Ω_2) to the volume of P_i (Ω_1) gives the volume of $P_f(\Omega_3)$. Then one can write:

$$x_{G1} = \frac{1}{V_1}\int_{\Omega_1} x dv = \frac{1}{V_1}\int_{\Omega_2} x dv + \frac{1}{V_1}\int_{\Omega_3} x dv = \frac{V_2}{V_1} \times x_{G2} + \frac{V_3}{V_1} \times x_{G3}$$

where x_{G1}, x_{G2}, x_{G3} is the position along the $\vec{x}$ axis of the center of gravity of Ω_{G1}, Ω_{G2} and Ω_{G3} respectively, and V_1, V_2, V_3 is the volume of Ω_{G1}, Ω_{G2} and Ω_{G3} respectively.

$$\frac{V_3}{V_1}(x_{G1} - x_{G3}) = \frac{V_2}{V_1}x_{G2} + \frac{V_3 - V_1}{V_1}x_{G1}$$

$$x_{G1} - x_{G3} = \frac{V_2}{V_3}x_{G2} + \frac{V_3 - V_1}{V_3}x_{G1} = \frac{V_2}{V_3}(x_{G2} - x_{G1})$$

the displacement of the center of gravity is:

$$x_{G1} - x_{G3} = V_2/V_3(x_{G2} - x_{G1})$$

where the index 2 corresponds to the subtracted solid volume, the index 1 to $\mathrm{P_i}$, and the index 3 to $\mathrm{P_f}$.

Thus, one needs to calculate the position of the center of gravity for $\mathrm{P_i}$ and, for the subtracted volume, to obtain the position of the center of gravity of $\mathrm{P_f}$. The position of the center of gravity of a tessellated closed surface can be efficiently calculated using the following sum:

$$\begin{aligned} x_G = & \frac{1}{24V} \sum_{i=1}^{N} \left({x_0^i}^2 + {x_1^i}^2 + {x_2^i}^2 + x_0^i x_1^i + x_1^i x_2^i + x_2^i x_0^i \right) \\ & \times \left((y_1^i - y_0^i)(z_2^i - z_0^i) - (y_2^i - y_0^i)(z_1^i - z_0^i) \right) \end{aligned}$$

where $i = 1 \ldots N$ is the index of the faces defining the tessellation, x_j^i, y_j^i, z_j^i are the 3D coordinates of the j^{th} vertex of the i^{th} face.

The tessellation used for the center of gravity calculation is the one introduced in paragraph 4.1.2 and in figure 14.

4.1.4 Area of a cell

The area of a cell enables the calculation of the local expansion of the surface. Then, the area of $\mathrm{C_i}$ and $\mathrm{C_f}$ are calculated and the area expansion ratio $\frac{S_f - S_i}{S_f}$ can be used to control the simplification.

The area of $\mathrm{C_i}$ and $\mathrm{C_f}$ is calculated by tessellating the polygons with interior faces using the principle of paragraph 4.1.2 shown in figure 14. The area of both cell images is then calculated by summing up the triangle areas.

4.2 Information propagation based approach

The simplification is obtained by the iterative vertex removal operation. At every step of this process, density informations can be assigned to the faces or vertices and transfered by inheritance during the simplification process.

This information transfer approach provides an approximative value of the initial information.

4.2.1 Propagation through vertices

This propagation method consists in assigning informations of local area and volume variations to the vertices of the polyhedron. At each step of the process, the informations assigned to the vertex removed are inherited by the vertices of the contour of the star polyhedron.

For every vertex removal operation, the algorithm used is:

1. calculation of the volume subtracted between the star polyhedron and its remeshing as shown in figure 15. Because the faces of this volume are triangular, the calculation of the volume does not need a tessellation of the volume, and the volume is directly obtained using formula 4.1.2.

2. calculation of the area of the star polyhedron and of its remeshing.

3. inheritance from the information attached to the removed vertex to the vertices of the remeshing contour. Area and volume information are updated by weighting the value of the removed vertex proportionally to the inverse of the distance between this vertex and one of the contour vertex:

$$\Delta V_i = \frac{\frac{1}{d_i}}{\sum_{j=1}^{N} \frac{1}{d_j}} \times (\Delta V_{star\ poly.} + \Delta V_{removed\ vertex})$$

   where d_i is the distance between the removed vertex and the contour vertex, and ΔV_i is the value to be added to the information of the i^{th} contour vertex.

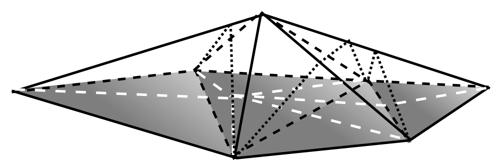

Figure 15: Volume variation produced by the vertex removal operation

4.2.2 Propagation through faces

This propagation scheme is similar to the propagation through vertices presented in 4.2.1. At each step of this process, density informations can be assigned to the faces and inherited from the N initial faces, i.e. the star polyhedron faces, to the $(N-2)$ final faces, i.e. the remeshing faces, according to their area.

For every vertex removal operation, the algorithm used is:

1. calculation of the volume subtracted between the star polyhedron and the remeshing as shown in figure 15.

2. calculation of the areas of the star polyhedron and of the remeshing.

3. the total volume and area variations are distributed on each face of the remeshing proportionally to their areas, with the sum of:

(a) the area and volume variation informations inherited from the faces of the star polyhedron,

(b) and the area and volume variations caused by the vertex removal operation.

This sum writes:

$$\begin{aligned}\Delta V_{final} &= \sum_{i=1}^{N} \delta V_i + \Delta V_{star} \\ &= \sum_{j=1}^{N-2} \delta V_j + \Delta V_{star} \\ \text{where } \delta V_j &= \frac{S_j}{S_{remesh}} \Delta V_{remesh}\end{aligned}$$

where δV_i is the information inherited from the star polyhedron, i is the index of the faces of the star polyhedron, and j is the index of the faces of the remeshing. Similarly, it comes for the area variation:

$$\begin{aligned}\Delta S_{final} &= \sum_{i=1}^{N} (\alpha_i S_i) + \Delta S_{star} \\ &= \sum_{j=1}^{N-2} (\alpha_j S_j) + \Delta S_{star} \\ \text{where } \alpha_j &= \frac{S_j}{S_{remesh}} \Delta S_{remesh}.\end{aligned}$$

5. EXAMPLES AND RESULTS

Model	Faces	Edges	Vertices
Bracket	1824	2736	908
Sub-domain	4302	6453	2750
Head	1374	2061	689

Figure 16: Characteristics of the test models

The algorithms were implemented as part of the Simpoly[1] software. The implementation of the reciprocal images and cell-based approaches for our a priori criteria was used to evaluate the simplification results and to monitor the simplification process.

The results are illustrated below through several examples representing different shapes.

The criteria aim at verifying whether a simplification is acceptable or not according to the threshold value set by the analyst. However, this configuration of prescriptive use of the criteria is difficult to illustrate graphically. Hence, the proposed illustration of the criteria are based on their inspection usage, i.e. after a simplification operation, the analyst is able to evaluate how the volume or area or center of gravity location of the simplified model has evolved.

[1] this software consists in a polyhedral approach for analysis model preparation, featuring conformity set-up, skin and topological detail removal, idealization.

The local values of criteria are updated at each step of the iterative vertex removal process. A remeshing of the star polyhedron contour is rejected if some of the previous criteria listed violates a threshold value.

The computation of criteria with the cell-based approach provides robust and accurate results. This quality is mainly due to the fact that the calculation is based on an exact geometric representation of the local differences between the initial polyhedron and the simplified one.

However, the cell-based approach remains slower than the information propagation-based approach because of the computation cost of reciprocal images, of the cell construction and criteria evaluation. Since the reciprocal images are modified only over the star polyhedron of the vertex removed, the only informations to update at each step of the vertex removal operator are the cells adjacent to this star polyhedron. Then, the differences of volume or area lying on a set of the initial polyhedron faces can be efficiently monitored to satisfy a simplification value set by the analyst.

Processing large models (number of faces greater than 10^5) is often needed, and the computation cost of reciprocal images is about four times the sole vertex removal one. The latter being around 25% slower than the commercial decimation software.

The results obtained with the information propagation approaches highlight an approximation of the map of densities of volume and area variations. The results obtained with the propagation through nodes (fig. 17, 18, 19), and with the propagation through faces (fig. 19) are compared to the results obtained with the cell-based approach. The comparison of these results clearly highlights the approximation effects of the propagation through vertices or faces. These approximations create a smoothing effect which can lead to inappropriate interpretation of the simplification operation. Because in both categories of approaches (cell-based and information propagation) the cumulative values sum up to the same value prescribed by the user, the information propagation approach seems more suited to prescriptive uses only of the criteria since their computation time is better than the cell-based approach. However, if the criteria are used both in prescriptive and inspection configurations, the cell-based approach should be preferred because of its better restitution of the simplification effects.

The models in figure 17 are colorized with the densities of volume differences computed for each face of the final polyhedron. One can see that the curved surfaces have the highest differences. The differences are positive in the convex regions where some material has been removed (yellow and red colored areas), and negative in the concave regions where some material has been added (blue colored areas).

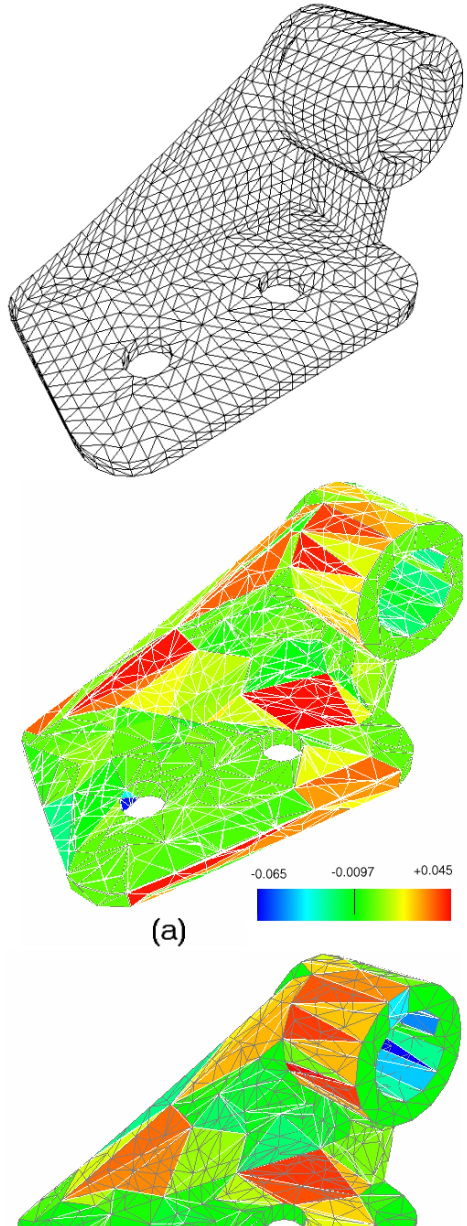

Figure 17: Model of a bracket: (a) Map of volume variation obtained with the cell-based approach, (b) Map of volume variation obtained with the propagation through faces approach.

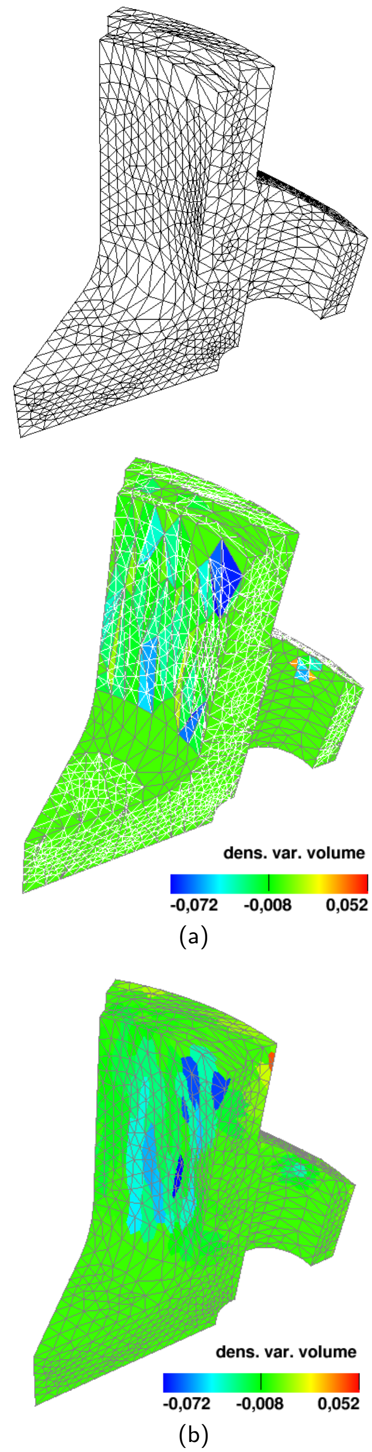

Figure 18: Model of 'sub-domain' colored with the densities of volume variations (a) cell-based approach, (b) propagation through vertices approach.

On this bracket (fig. 19), the results of volume variations obtained by propagation through vertices highlight quasi null values in the neighbourhood of the two holes (see figure 19 (c)). These values should be more visible as depicted on the solution obtained with the cell-based approach in figure 19 (a)). The approach of propagation through faces provide better results in the neighbourhood of the holes as shown in figure 19(b).

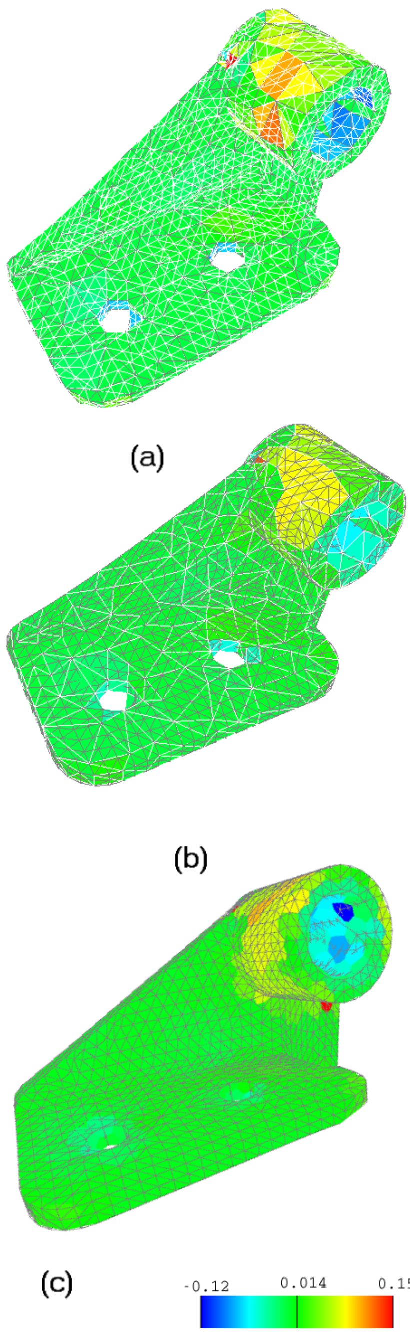

Figure 19: Map of volume variation on the bracket model (a) cell-based approach, (b) propagation through faces approach, (c) propagation through vertices approach.

Another criterion can be illustrated using the following example to illustrate how they can be related to beam-shaped components through sections. Here, the criterion set up characterizes the area variation of beam sections. The sections are defined through reference planes and interpolation criteria between them. The figure 20 illustrates the variation of contours during the simplification process: contracted and expanded sections are colored in blue and red respectively. Here, the principle is based on the calculation of the intersection between each plane defining a section and the faces of the polyhedral model.

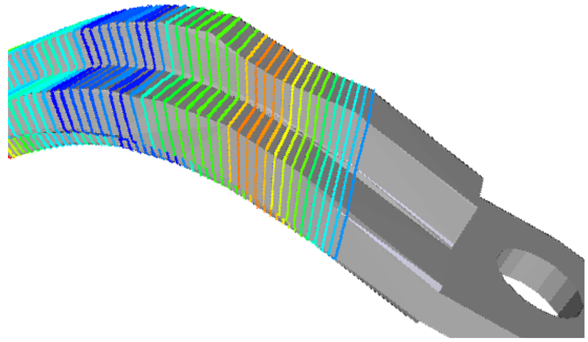

Figure 20: Principle of the section area variation criterion.

The figure 21 shows the contraction of many faces on the final polyhedron compared to the initial polyhedron.

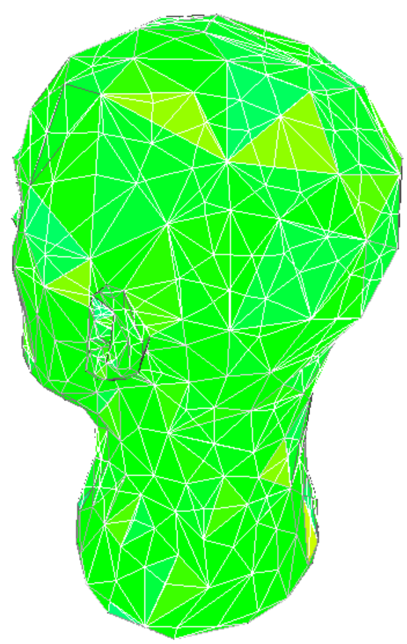

Figure 21: Vizualization of the surfacic densities of variation in area $\frac{S_{final}-S_{init.}}{S_{final}}$ on a human head model obtained with the cell-based approach.

6. CONCLUSION AND FUTURE WORK

In the context of FE model preparation, this work has proposed a set of criteria for monitoring the shape adaptation process of design models. These criteria are linked to mechanical properties such as mass, area, stiffness, and have been expressed for a polyhedral representation of the structure.

Two classes of approach have been used to assess these criteria: the most rigorous one based on reciprocal images and an approximate one. The methods are especially well-suited to constrain the polyhedral model simplification locally for the needs of the analysis application. They are designed to be used simultaneously with the discrete envelope criterion proposed by Véron [11] and Fine [10]. Currently, these methods enable us to:

- simplify a model while preserving its physical properties and avoiding transformations which imply a variation in some physical property greater than the threshold set by the analyst,
- visualize and evaluate the physical properties preservation at the end of a simplification operation before starting the FE mesh generation phase.

Among the two propagation schemes tested for the propagation approach, the propagation through faces is the best suited to reliably spread the criteria values over the component.

The cell-based approach gives accurate results of the criteria values but is slower than the propagation schemes. Mixing both approaches over the shape of the object according to user prescriptions can improve further the performance of these criteria.

The ability of criteria to preserve mechanical properties relevant for some analysis has been discussed and evaluated but work on other criteria are planed to complete the simplification process. Future work will focus on the completeness of these criteria and their combination strategies. Complementary a priori criteria like shape of sections, stiffness, will be developed to lead to a performant solution for conversion from design models to structural analysis models. Ongoing work focuses on transfering CAD data conveying semantics about a model to the polyhedral model, thus leading to the notion of simplification features to incorporate further constraints.

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