Linear Optics Quantum Computation

Produced with pdflatex and xfig

- Optical quantum computation.
- Photon basics.
- On non-linear gates.
- Computation with linear optics.
- Challenges.

E. "Manny" Knill: knill@boulder.nist.gov

Optical Quantum Computing

- Advantages.
  - Easy to observe interference.
  - Room temperature.
  - Fast ops/slow decoherence.
  - Lots of photons.
- Challenges.
  - Mode matching.
  - Non-linear interactions.
  - . . . except photon sources/detectors.
  - Fast feed-forward needed.
  - Low loss optics.
  - Single photon sources.
  - Photon counters.
- Schemes for optical quantum computation.

  With non-linearities:
  - Single photon qubits, Kerr non-linearities. Milburn 1988 [1]
  - Optics and cavity QED. Turchette&*al.* 1995 [2]

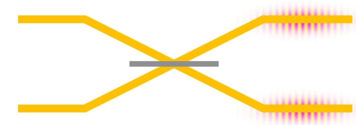
  With exponential resources:
  - Linear optics, one mode per quantum dimension. Cerf&Adami&Kwiat 1998 [3]

  Primarily linear:
  - eLOQC. Knill&Laflamme&Milburn 2000 [4]
  - Linear optics with squeezed state qudits. Gottesman&Kitaev&Preskill 2000 [5]
  - Linear optics with Gaussian state qubits. Ralph&Munro&Milburn 2001 [6, 7]

Photonic Qubit

- Photonic qubit: One photon in a superposition of two modes.

  - Photonic qubits are usually "flying" qubits.
  - Making a superposition state:



Optical Modes

- Mode visualization.

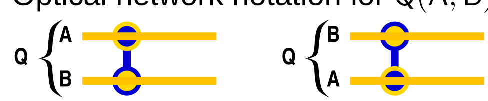
- Mode A can have $0, 1, 2, \ldots$ photons.

  $$|0\rangle_A, |1\rangle_A, |2\rangle_A, \ldots, |n\rangle_A, \ldots$$

  State space: Superpositions $\sum_k \alpha_k |k\rangle_A$.
- Mode A operators:
  $$(\mathbf{a}^\dagger)_A \;:\; |n\rangle_A \;\rightarrow\; \sqrt{n+1}\,|n+1\rangle_A$$
  $$(\mathbf{n})_A \;:\; |n\rangle_A \;\rightarrow\; n\,|n\rangle_A$$
- Bosonic qubit Q(A, B) on modes A, B:

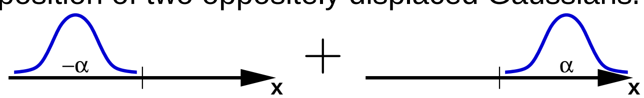
  $$|0\rangle_{Q(A,B)} \leftrightarrow |0\rangle_A|1\rangle_B, \qquad |1\rangle_{Q(A,B)} \leftrightarrow |1\rangle_A|0\rangle_B$$

  Optical network notation for Q(A, B):



Coherent State Qubits

- Coherent state of amplitude α.
  $|\alpha\rangle = \sum_{k=0}^{\infty} e^{-|\alpha|^2/2}\frac{\alpha^k}{\sqrt{k!}}|k\rangle$, characterized by $\mathbf{a}|\alpha\rangle = \alpha|\alpha\rangle$
- Qubit state identification. Let $\alpha \approx 3$.
  $|\mathsf{o}\rangle_{\mathsf{Q}} \mapsto |\alpha\rangle, \qquad |\mathsf{1}\rangle_{\mathsf{Q}} \mapsto |-\alpha\rangle.$
  ...approximately, since $\langle -\alpha|\alpha\rangle = e^{-2|\alpha|^2} \approx 1.5 * 10^{-8}$.
- "Cat" state.
  $\frac{1}{\sqrt{2}}(|\mathsf{o}\rangle_{\mathsf{Q}} + |\mathsf{1}\rangle_{\mathsf{Q}}) \mapsto \frac{1}{\sqrt{2}}(|-\alpha\rangle + |\alpha\rangle)$.
  Superposition of two oppositely displaced Gaussians.



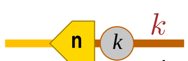
- Scalable q. comp. is possible with cat states, linear optics and photon counters using coherent state qubits.

Ralph&Munro&Milburn 2001 [6] &Gilchrist&Glancy 2003 [7]

Optical Devices for LOQC

- Mode preparations:

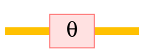
- Photon counter: $|k\rangle$



...can be used for feed-forward.

Passive linear-optics.

- Phase shifter $\mathbf{P}_\theta$: $|k\rangle$



$e^{i\theta k}|k\rangle$

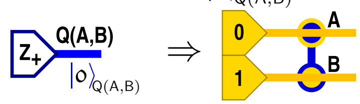
- Beam splitter $\mathbf{B}_{\theta,\phi}$:

$$|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} \rightarrow \cos(\theta)|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + e^{-i\phi}\sin(\theta)|0\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}$$
$$|0\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}} \rightarrow -e^{i\phi}\sin(\theta)|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \cos(\theta)|0\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}$$

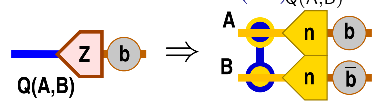
- U is *passive linear* if $U\mathbf{a}^\dagger_{\mathsf{s}}U^\dagger = \sum_r u_{rs}\mathbf{a}^\dagger_{\mathsf{r}}$.
  $U|\mathbf{0}\rangle = |\mathbf{0}\rangle, \quad \hat{u} = (u_{rs})_{rs}$ unitary.

Bosonic Qubit Operations

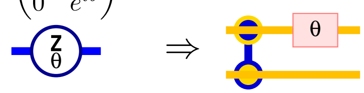
- Preparation of $|\mathsf{o}\rangle_{\mathsf{Q(A,B)}}$:



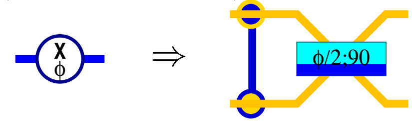
- Measurement of $\langle\sigma_z\rangle_{\mathsf{Q(A,B)}}$:



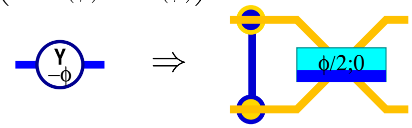
- Z $= \sigma_z$ rotation by θ, $e^{-i\sigma_z\theta/2}$:
  $\propto \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$:



- X $= \sigma_x$ rotation by ϕ, $e^{-i\sigma_x\phi/2}$:
  $\begin{pmatrix} \cos(\phi) & -i\sin(\phi) \\ -i\sin(\phi) & \cos(\phi) \end{pmatrix}$:



- Y $= \sigma_y$ rotation by ϕ, $e^{-i\sigma_y\phi/2}$:
  $\begin{pmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{pmatrix}$:

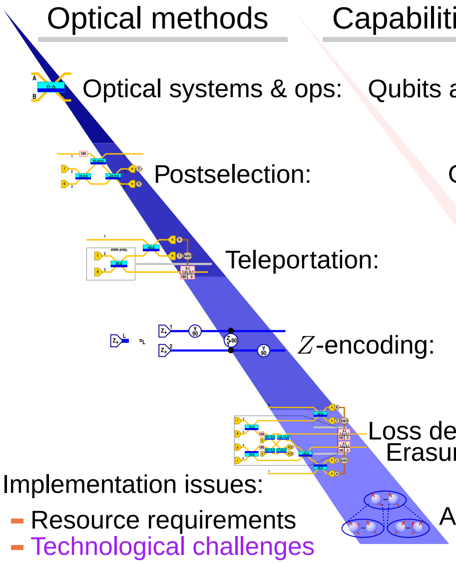


- Still need a "nonlinear" coupling.

Linear Optics No-Go?

- Passive linear optics $\simeq$ classical wave mechanics?
- **Observation.** Coherent state preparations, passive linear optics and photodetectors with feedback are efficiently classically simulatable.
  ...because modes never become correlated.
- **Theorem.** Linear optics and particle detectors for fermions are efficiently classically simulatable.
  Valiant 2001 [8], Terhal&DiVincenzo 2001 [9], Knill 2001 [10]
- **Theorem** Linear optics and measurement of $\hat{x} \propto \mathbf{a}^\dagger + \mathbf{a}$ is efficiently classically simulatable. Bartlett&*al.* 2001 [11]
- **Theorem** Linear optics, single photon sources and photon counters are sufficient for q. comp. Knill&Laflamme&Milburn 2000 [4]

eLOQC Guide

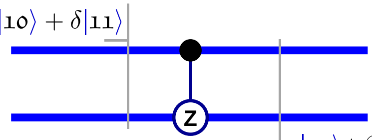


Optical methods	Capabilities realized
Optical systems & ops:	Qubits and one-qubit ops
Postselection:	QC, if lucky
Teleportation:	Standard QC
Z-encoding:	Efficient QC
	Alternatives: Yoran&Reznik 2003 [12], Nielsen 2004 [13]
Loss detection, Erasure codes:	Robuster QC
Accuracy threshold:	Scalable QC

Implementation issues:
- Resource requirements
- Technological challenges

Controlled Sign Flips

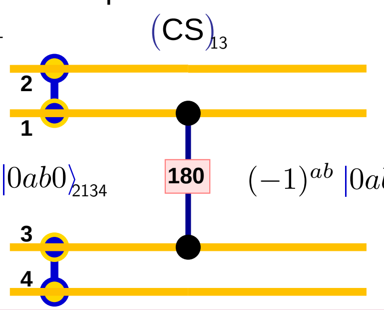
- Sufficient to add **sgn**, the controlled sign flip.



$\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle \rightarrow \alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle-\delta|11\rangle$

- Translation to photonic qubits.

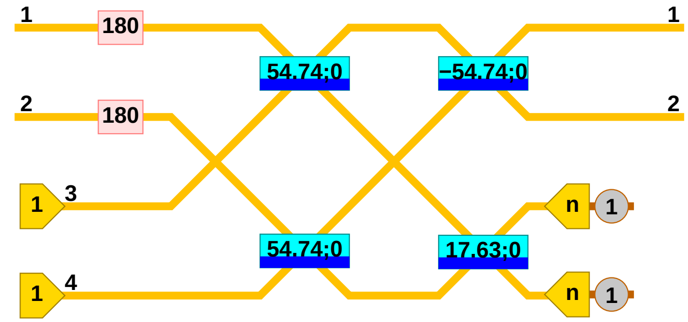
$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
$\|1\rangle$	$\|1\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|0\rangle$



$|0ab0\rangle_{2134} \rightarrow (-1)^{ab}\,|0ab0\rangle_{2134}$ . . . for $a,b=0,1$.

Linear Optical Controlled Sign Flip

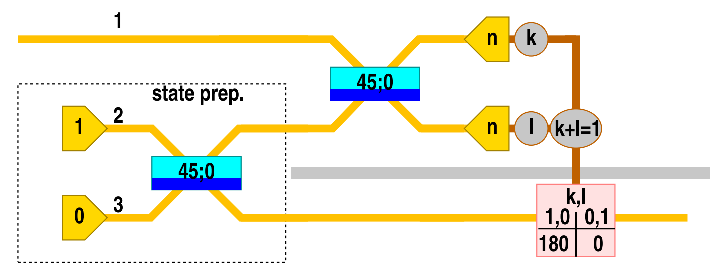
- $|ab\rangle_{12} \rightarrow (-1)^{a\cdot b}|ab\rangle_{12}$ with success probability $1/13.5$:



- Q. comp. with exponentially small probability of success.
- Practical application: State preparation.

One Mode Teleportation

- Teleportation of one mode, success probability $1/2$:



$$(\alpha|0\rangle_1+\beta|1\rangle_1) \longrightarrow$$

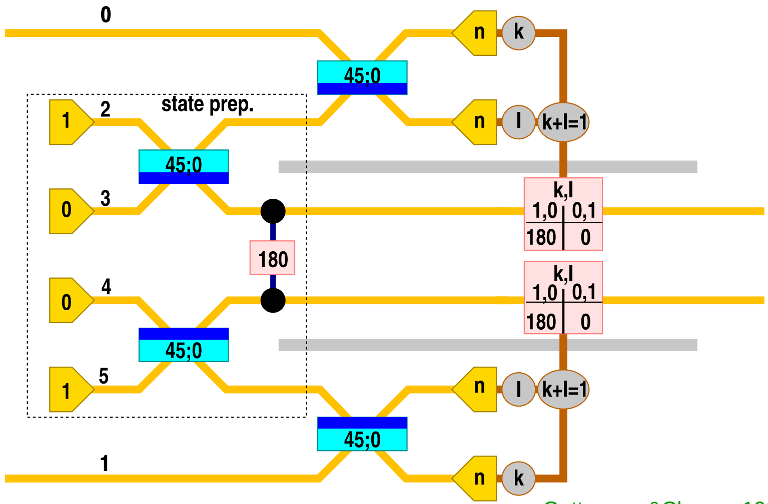
$$(\alpha|0\rangle_1+\beta|1\rangle_1)\frac{1}{\sqrt{2}}(|10\rangle_{23}+|01\rangle_{23})$$

$$=\frac{1}{\sqrt{2}}(\alpha|010\rangle_{123}+\beta|101\rangle_{123}) \longrightarrow \begin{cases}\frac{1}{2}(-\alpha|0\rangle_3-\beta|1\rangle_3) \\ \text{or} \\ \frac{1}{2}(+\alpha|0\rangle_3+\beta|1\rangle_3)\end{cases}$$

$+$ (0 or 2 photons in modes $1,2$)

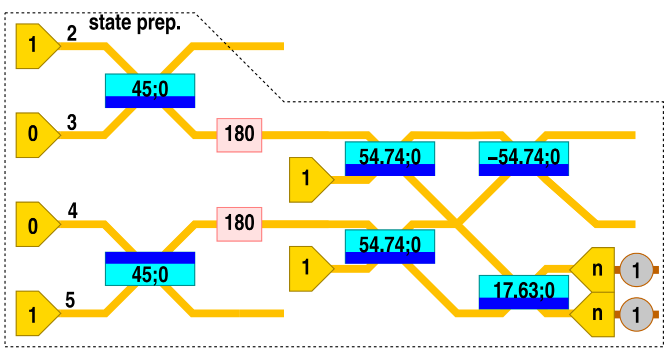
CS by Teleportation

- Implement CS using teleportation, probability of success $1/4$:



State Preparation for CS

- A source of entangled pairs suffices.



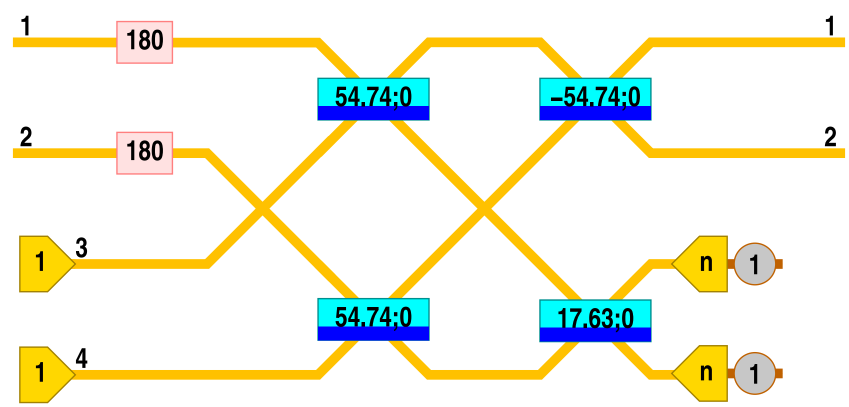
- Or use: Two-mode non-linear sign shift with prob$_{\text{succ}} = 1/13.5$.

Schemes for Improving the Success Probability

- Teleportation with $2n$ ancillas.
  - Generalization of one-mode teleportation.
  - State preparation complexity: Not efficient.
- With Z-measurement error-detecting codes.
  - Rely on failures of CS being unintentional measurement.
  - Logical qubits encoded in n photonic qubits.
  - Efficient.
- Using "linked photon circuits". Yoran&Reznik 2003 [12]
  - Uses one photon per qubit.
  - Effects of CS failures are localized and enabling reconstruction.
  - More efficient.
- Using "cluster states". Nielsen 2004 [13]
  - Uses photonic qubits.
  - Effects of CS failures are localized and enabling reconstruction.
  - Further efficiency improvements.

Back to: Guide

Zooming in on eLOQC

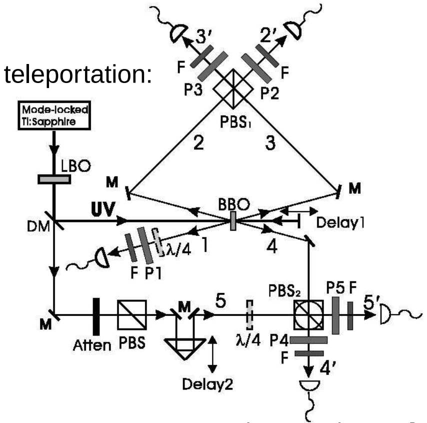


Back to: Guide

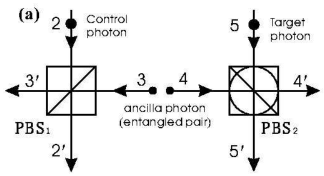
Status of Experimental Efforts

- Current experiments use down-converted photons.
  - Australia: E.g. O'Brien&Pryde&White&Ralph&Branning 2003 [15]
  - US: E.g. Pittman&Jacobs&Franson 2004 [16]
  - Europe/China: E.g. Zhao&Zhang&Chen&Zhang&Du&Yang&Pan 2004 [17]

Cnot for teleportation:



Cnot schematic:



Problem: Low efficiency.

From Zhao& et al. 2004 [17]

Challenges and Device Requirements

- Logical requirements.

Logical teleportation: $\ll 30\%$ detected, $\ll 10\%$ undetected error.
State preparation: $< 100\%$ detected, $\ll 5\%$ undetected error per qubit.

- Device requirement guesses.

	Probably ok?	I'll work harder?
Single photon source:	$> 99.9\%$	$> 90\%$
$0, 1, 2$-photon counter:	$> 99.9\%$	$> 90\%$
Photon loss (≈ 10 devices):	$\ll 1\%$	$< 10\%$
Beamsplitter (δ trans):	$\ll .1\%$	$< 1\%$
Mode matching (overlap^2):	$\ll .1\%$	$< 1\%$

- Further challenges.
  - Feed-forward: Time delay for classical decisions $= \tau_c$.
  - Ph. counter: Detection time $= \tau_p$.
  - Storage: Time to unacceptable loss $= \tau_s$. Mode shape needs to be controlled.
  - Switches: Low loss/fast.

$$\tau_c + \tau_p \ll \tau_s$$

Back to: Guide

Contents

Title: Linear Optics Quantum Computation . . . 0
Optical Quantum Computing . . . 1
Photonic Qubit . . . 2
Optical Modes . . . 3
Coherent State Qubits . . . 4
Optical Devices for LOQC . . . 5
Bosonic Qubit Operations . . . 6
Linear Optics No-Go? . . . 7
eLOQC Guide . . . 8
Controlled Sign Flips . . . 9
Linear Optical Controlled Sign Flip . . . 10
One Mode Teleportation . . . 11
CS by Teleportation . . . 12
State Preparation for CS . . . 13
Schemes for Improving the Success Probability . . . 14
Zooming in on eLOQC . . . 15
Status of Experimental Efforts . . . 16
Challenges and Device Requirements . . . 17
References . . . 19

References

[1] G. J. Milburn. Quantum optical Fredkin gate. *Phys. Rev. Lett.*, 62:2124–2127, 1988.
[2] Q. A. Turchette, C. J. Hood, W. Lange, H. Mabuchi, and H. J. Kimble. Measurement of conditional phase shifts for quantum logic. *Phys. Rev. Lett.*, 75:4710–4713, 1995.
[3] N. J. Cerf, C. Adami, and P. G. Kwiat. Optical simulation of quantum logic. *Phys. Rev. A*, 57:R1477–R1480, 1998.
[4] E. Knill, R. Laflamme, and G. Milburn. A scheme for efficient linear optics quantum computation. *Nature*, 409:46–52, 2001.
[5] D. Gottesman, A. Kitaev, and J. Preskill. Encoding a qudit in an oscillator. *Phys. Rev. A*, 64:012310/1–21, 2001.
[6] T. C. Ralph, W. J. Munro, and G. J. Milburn. Quantum computation with coherent states, linear interactions and superposed resources. quant-ph/0110115, 2001.
[7] T. C. Ralph, A. Gilchrist, G. J. Milburn, W. J. Munro, and S. Glancy. Quantum computation with optical coherent states. *Phys. Rev. A*, 68:042319/1–11, 2003.
[8] L. G. Valiant. Quantum computers that can be simulated classically in polynomial time. In *Proceedings of the 33rd Annual ACM Symposium on the Theory of Computation (STOC'01)*, page ?, El Paso, Texas, 2001. ACM Press.
[9] B. M. Terhal and D. P. DiVincenzo. Classical simulation of noninteracting-fermion quantum circuits. *Phys. Rev. A*, 65:032325/1–10, 2002.
[10] E. Knill. Fermionic linear optics and matchgates. Technical Report LAUR-01-4472, Los Alamos National Laboratory, 2001. quant-ph/0108033.
[11] S. D. Bartlett, B. C. Sanders, S. L. Braunstein, and K. Nemoto. Efficient classical simulation of continuous variable quantum information processes. *Phys. Rev. Lett.*, 88:097904/1–4, 2002. quant-ph/0109047.
[12] N. Yoran and B. Reznik. Deterministic linear optics quantum computation with single photon qubits. *Phys. Rev. Lett.*, 91:037903/1–4, 2003.
[13] M. A. Nielsen. Optical quantum computation using cluster states. quant-ph/0402004, 2004.
[14] D. Gottesman and I. L. Chuang. Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations. *Nature*, 402:390–393, 1999.
[15] J. L. O'Brien, G. J. Pryde, A. G. White, T. C. Ralph, and D. Branning. Demonstration of an all-optical quantum controlled-not gate. *Nature*, 426:264–267, 2003.
[16] T. B. Pittman, B. C. Jacobs, and J. D. Franson. Experimental demonstration of a quantum circuit using linear optics gates. quant-ph/0404059, 2004.
[17] Z. Zhao, A.-N. Zhang, Y.-A. Chen, H. Zhang, J.-F. Du, T. Yang, and J.-W. Pan. Experimental demonstration of a non-destructive controlled-NOT quantum gate for two independent photon-qubits. quant-ph/0404129, 2004.