Interpreting black hole QPOs

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Abstract. In all the microquasars with two hHz QPOs, the ratio of the frequencies is 3:2, supporting our suggestion that a non-linear resonance between two modes of oscillation in the accretion disk plays a role in exciting the observed modulations of the X-ray flux. We discuss the evidence in favor of this interpretation, and we relate the black hole spin to the frequencies expected for various types of resonances that may occur in nearly Keplerian disks in strong gravity. For those microquasars where the mass of the central X-ray source is known, the black hole spin can be deduced from a comparison of the observed and expected frequencies.

INTRODUCTION

Several Galactic low-mass X-ray binaries (LMXBs) exhibit quasi-periodic variability (QPOs) of their X-ray fluxes, with pairs of $\approx 1\,$kHz frequencies typical in the neutron-star sources [1]. Kluźniak and Abramowicz [2, 3] suggested that these twin kHz QPOs are a manifestation of non-linear resonance that can occur between modes of oscillation in an accretion disk in strong field Einstein's gravity, but not in Newton's $1/r$ potential, and pointed out that on this hypothesis the same resonances should be present in black hole systems. Pairs of high frequency QPOs should have been present where only single hHz QPOs had been reported in microquasars. Such pairs of several hHz frequencies have now been reported in four or five black-hole systems, all in a 3:2 ratio [4], substantially strengthening the case for resonance.

Our originally suggested [2] explanation for kHz QPOs in neutron stars was based on these general properties of non-linear resonance, which seemed to us to correspond to the essential features of the observed twin frequency peaks:

1. The frequencies of non-linear oscillations ν depend on amplitude, and for this reason they may be time dependent and may differ from the fixed eigenvalue frequencies $\nu(0)$ of the system, $\nu(t) = \nu(0) + \delta\nu(t)$.

2. Non-linear resonance may occur over a wide frequency range $\delta\nu$.

3. Both (resonant) frequencies increase or decrease "in step" with each other.

4. The eigenfrequencies of resonant modes are approximately in the ratio of small integers, e.g., 2:1.

These ideas initially received a cool reception (see author's note in [3]), because it was not generally appreciated that frequency ratios close to 3:2 actually occur[1] for kHz QPOs in neutron star sources, and no evidence for twin peaks in black hole sources was known at the time.

In addition to the high frequency QPOs, features in the power density spectra can be identified at lower frequencies, and at least one frequency ν_{low} was long known to be correlated with ν_{high}, one of the kHz/hHz frequencies in neutron-star/black-hole systems [6]. Quasi-periodic modulations of the flux (dwarf nova oscillations, DNOs) were first discovered in cataclysmic variables (white dwarfs) and these are analogous in many respects to the QPOs in LMXBs [7]. However, unlike the kHz/hHz QPOs, the highest frequency DNOs do not come in pairs. This is consistent with the idea that the high frequency pairs arise in accretion disks only in strong gravity.

The relativistic resonance model of black hole QPOs is based on fundamental features of strong gravity. Today, it is motivated by observations that sharply illuminate the physical nature of QPOs:

1. The correlation $\nu_{\text{low}} = 0.08\nu_{\text{high}}$ between low and high frequency QPOs in black hole, neutron stars, and white dwarf sources extending over six orders of magnitude [6, 8, 9], proves that in general the QPOs are a hydrodynamic phenomenon, and cannot be attributed to mere kinematic effects, such as Doppler modulation of emission from isolated bright spots. (ν_{low} may be the "ninth wave" [10].)

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[1] Indeed, our paper on this appeared in print only much delayed [5].

2. The frequencies of twin peak hHz QPOs in micro-quasars seem to scale with mass [4], $\nu \sim 1/M$ (Figure 1). If true, this would prove their relativistic origin.

3. In all four microquasars with twin peak hHz QPO pairs, $\nu_{\text{upper}}/\nu_{\text{lower}} = 3/2$ (Table 1).

Although suggestive of a resonance, the ratio 3/2 could also be a signature of overtones (flute modes) [11], or of higher modes of an MHD instability [12] at a 'transition radius' r_* in the innermost part of the disk, which excites quasi periodic oscillations with mode frequencies $\nu \sim n\nu_K(r_*)$ (in contrast with the resonance model, neither of these two models predicts $1/M$ scaling, without making some *ad hoc* assumptions). However, there are additional properties of non-linear resonances which may help in their identification. In a non-linear resonance combination frequencies, e.g., $\nu_{\text{upper}} \pm \nu_{\text{lower}}$, and sub-harmonic frequencies may be present [13], e.g., $\nu_{\text{lower}}/2$.

Our resonance model may also be applied to twin peak QPO sources in neutron stars [13]. In refs. [14, 15], and in these Proceedings [16], we discuss a resonance in an accretion disk or torus excited by an external forcing by a millisecond pulsar. A similar forcing is crucially important in a different, non-relativistic resonance model suggested by Titarchuk [17].

THE ORBITAL AND EPICYCLIC MOTIONS.

Consider a black hole[2] with the mass M_0 and angular momentum J_0. Inside thin, almost Keplerian accretion disks, matter spirals down the central black hole along stream lines that are located almost on the equatorial plane $\theta = \theta_0 = \pi/2$, and that locally differ only slightly from a family of concentric circles $r = r_0 = \text{const}$. The small deviations, $\delta r = r - r_0$, $\delta\theta = \theta - \theta_0$ are governed, with accuracy to linear terms, by

$$\delta\ddot{r} + \omega_r^2\,\delta r = \delta a_r, \quad \delta\ddot{\theta} + \omega_\theta^2\,\delta\theta = \delta a_\theta. \quad (1)$$

Here, the dot denotes a time derivative. For purely Keplerian (free) motion $\delta a_r = 0$, $\delta a_\theta = 0$ and the above equations describe two uncoupled harmonic oscillators with the eigenfrequencies $\omega_\theta \equiv 2\pi\nu_\theta$, $\omega_r \equiv 2\pi\nu_r$ shown together with the Keplerian orbital frequency, $\Omega \equiv 2\pi\nu_K$, in Figure 2 for a non-rotating black hole, and in Figure 3 for a moderately rotating one.

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[2] We rescale mass with $M = GM_0/c^2 = r_G$, angular momentum with $a = J_0 c/(M_0^2 G)$. We use Boyer-Lindquist coordinates, t, r, θ, ϕ, and rescale the radius with $x = r/r_G$.

TABLE 1. The four microquasars in which two hHz QPOs are observed. They all have 3:2 ratio of frequencies. Source of data: [4, 18]

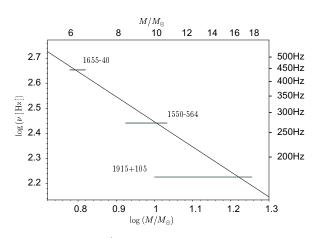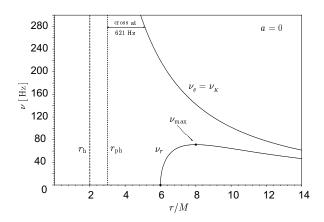| Microquasar	ν_{upper}	ν_{lower}
XTE 1550-564	276	174
GRO 1655-40	450	300
GRS 1915+105	168	113
H 1743-322	240	160

FIGURE 1. The $1/M$ scaling of the pairs of QPOs with the 3:2 frequency ratio [4]. The upper frequency is shown.

In Newton's theory with the $-GM_0/r$ potential, $(2\pi)^{-1}GM_0/r^{3/2} = \nu_K = \nu_r = \nu_\theta$, but in the strong gravity of a rotating black hole, for orbits of the same sense of rotation, $\nu_K > \nu_\theta > \nu_r$. The radial epicyclic frequency ν_r goes to zero at the Innermost Stable Circular Orbit for the Keplerian (free) motion, and has a maximum at a particular circular orbit outside the ISCO, its location depends on the black hole spin [19, 20, 21].

$1/M$ SCALING

Before the RXTE era, Kluźniak, Michelson, Wagoner [22] suggested that the orbital frequency close to the marginally stable orbit may be directly observed as a QPO, once instruments with sufficiently high time resolution are built, and pointed out that the frequency will be inversely proportional to the mass of the compact object. The latter statement applies to any characteristic frequency in general relativity.

For example, note that for black holes, all three orbital frequencies: Keplerian ν_K, radial epicyclic ν_r, and vertical epicyclic ν_θ, also have the general form

$$\nu = f(x,a)\left(\frac{GM_0}{r_G^3}\right)^{1/2}, \quad (2)$$

FIGURE 2. Orbital ν_K, epicyclic radial ν_r, and vertical ν_θ frequencies for a non-rotating black hole (spin $a = 0$) with the mass $M_0 = 10 M_\odot$. Location of the photon radius is marked by r_{ph}, and that of the horizon by r_h. There are no orbits for $r < r_{ph}$.
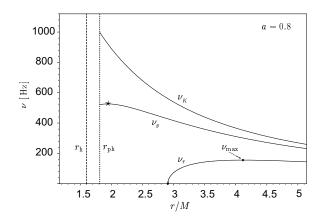

FIGURE 3. The same as in Figure 2, but for a $10 M_\odot$ black hole with a moderately high spin, $a = 0.8$.

with a the dimensionless angular momentum of the black hole, and $f(x, a)$ a dimensionless function, different for each frequency. For all *relativistic* frequencies, $x = x(a)$ is fixed, and then the above formula predicts that frequencies scale as $\nu = (1/M) F(a)$. In particular, each orbital resonance $n : m$ discussed later in this contribution occurs at its own resonance radius $x_{n:m}(a)$, as shown in Figure 13, while at the marginally stable orbit (ISCO) $\nu_r(r_{ms}) = 0$. All of the models discussed below follow the $1/M$ scaling.

NON-RESONANT MODELS

The highest possible orbital frequency

In an accretion disk, matter moves, roughly, on circular orbits in the region $r > r_{in}$ and free-falls in the region

$r < r_{in}$. The transition radius $r = r_{in}$, closely coinciding with the sonic point, is often called the inner radius of the accretion disk. Thin, standard Shakura Sunyaev disks with their high efficiencies have an inner edge located almost exactly at ISCO. In general, the inner edge is located between the ISCO and the marginally bound (RISCO, at r_{mb}) circular orbits [23], depending on the disk efficiency. ADAFs, with their very low efficiencies, have the inner edges almost exactly at RISCO [24]. The same is true for super-Eddington slim [25] and thick [23] disks. For a non-rotating black hole one has $r_{mb} = 4M$, $\nu_K(r_{mb}) = 4037 (M/M_\odot)^{-1}$ [Hz], and $r_{ms} = 6M$, $\nu_K(r_{ms}) = 2197 (M/M_\odot)^{-1}$ [Hz].
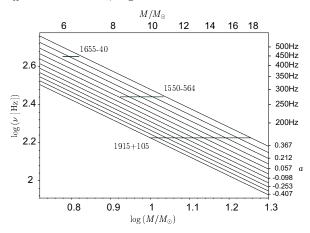

FIGURE 4. ISCO orbital frequencies compared with the observed upper frequency of the hHz pair of QPOs.
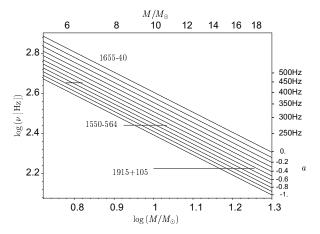

FIGURE 5. RISCO orbital frequencies compared with the observed upper frequency of the hHz pair of QPOs.

Let us ignore for the moment that the frequency pairs are in a 3:2 ratio. Numerous studies have tried to infer the properties of neutron stars on the assumption that the upper kHz QPO frequency is close to the ISCO frequency (e.g., [26]). Identifying RISCO frequencies with the upper frequency of the hHz pairs in microquasars, whose mass is known, would require counter-

rotating disks (Fig. 5). Comparing the same frequency with the one at ISCO leads to sensible but small values of the black hole spin (Fig. 4). In most models, the QPO frequency corresponds to a characteristic frequency not very close to the inner edge of the disk. This leads to substantially higher values of the black hole rotation rate.

The trapped modes

One of the characteristic properties of the oscillations of relativistic disks is the presence of trapped mode oscillations [20, 21, 27, 28, 29, 30, 31]. The physical reason for the trapping is that the radial epicyclic frequency, v_r, is not monotonic but has a maximum value, v_{trapp}, at a radius r_{trap} larger than the ISCO. For the non-rotating black hole hole $r_{\text{trap}} = 8M$ [21]. The g-mode (inertial-gravity) oscillations [30] can be characterized by a restoring force that is typically dominated by the net gravitational-centrifugal force. The axisymmetric ($m = 0$) g-modes are centered at r_{trap}. Non-axisymmetric trapped g-modes with the azimuthal wave-number $m = 1$ have frequencies [28],

$$v \sim v_{\text{K}}(r_{\text{trap}}) \pm v_{\text{trapp}}, \quad \text{and} \quad v \sim v_{\text{K}}(r_{\text{trap}}). \quad (3)$$

In Figure 6 we show the highest frequency connected to these oscillations, $v_{\text{upp}} = v_{\text{K}}(r_{\text{trap}}) + v_{\text{trapp}}$, and compare it with observations.
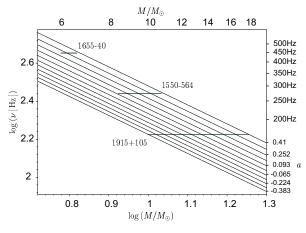

FIGURE 6. Maximum frequency of the trapped, $m = 1$, $g-$mode compared with the upper QPO frequency.

Dragging of inertial frames and the c-mode

The 'corrugation' c-mode [31, 27, 32] is a non-axisymmetric, vertically incompressible wave near the inner edge of the disk that exists only for co-rotating
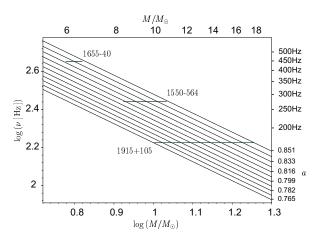

FIGURE 7. Frequency of the c-mode at ISCO compared with the upper QPO frequency.

disks, $a > 0$. It precesses around the angular momentum of the black hole. Its frequency coincides with the Lense-Thirring frequency produced by the dragging of inertial frames. In Figure 7 we compare with observations the highest frequency connected to the c-mode, assuming that the mode locates at ISCO:

$$v_{\text{upp}} = v_{\text{LT}}(r_{\text{ms}}) = \frac{ac}{\pi r_G}\left(\frac{r_G}{r_{\text{ms}}}\right)^3. \quad (4)$$

In reality, the mode is trapped further out in the disk, and correspondingly, the inferred value of black hole spin is higher [32].

NON-LINEAR, RELATIVISTIC ORBITAL RESONANCES

"Keplerian" resonances

It is possible for the radial epicyclic frequency to be in a resonant relation with the orbital frequency, $nv_r = mv_K$, with n, m integer [33]. For example, g-modes have pattern frequency $2\pi v_m = \pm(v_r \pm mv_K)$, and these can be in co-rotation resonance, i.e., with $v_m = v_K$ [34]. The case $v_K/v_r = 3/2$ is excluded by observations [33] (Fig. 8). The remaining possibilities [33] are that the upper frequency is $v_K \pm v_r$, with $v_K/v_r = 2$, or $v_K/v_r = 3$, and $v_{\text{upper}}/v_r = 3/2$ (Figs. 9, 10). However, co-rotation resonance leads to damping, and not excitation, of modes [35].

Another possibility is based on the following idea [36]. When the potential vorticity is conserved, coherent vortices tend to form in pairs with opposite vortices. One may imagine that because the spatial distance between

the two structures, which oscillates with the epicyclic radial frequency, depends on the velocity profile of the disk, i.e., also on the oscillations of orbital velocity, a resonance between these two frequencies is possible.
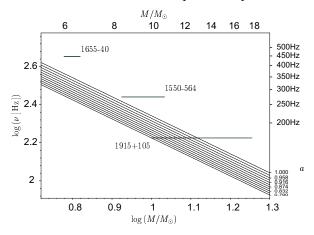

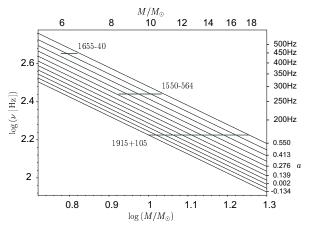**FIGURE 8.** Unsuccessful fit of observations to the Keplerian 3:2 resonance.
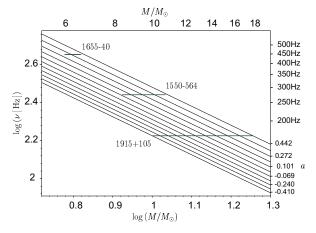

FIGURE 9. Keplerian 3:1 resonance.

FIGURE 10. Keplerian 2:1 resonance.

Epicyclic motions

The effective potential $U(r, \theta; \ell)$ for orbital motion of a particle with a fixed angular momentum $\ell > \ell_{ms}$ has a minimum at $r_0(\ell)$, corresponding to the location of a stable circular orbit. The second order term in its Taylor expansion (for simplicity we write it on the equatorial plane $\theta = \pi/2$) gives the epicyclic frequencies, terms in the next order

$$\frac{1}{2}\left(\frac{\partial^2 U}{\partial r^2}\right)_0 (r - r_0)^2 + \frac{1}{6}\left(\frac{\partial^3 U}{\partial r^3}\right)_0 (r - r_0)^3 + \dots \quad (5)$$

contain higher than quadratic terms, which means that small oscillations around the minimum at $r - r_0$ are described by non-linear differential equations [37, 38]. Non-linear resonances that may be excited in these non-linear oscillations have several characteristic properties that closely resemble those observed in QPOs.

We consider two possibilities.

The forced 3:1 and 2:1 resonances

A direct resonant forcing of vertical oscillations by the radial ones through a pressure coupling, and with $\delta r \sim \cos(\omega_r t)$, is evident in recent numerical simulations of oscillations of a perfect fluid torus [15]. This supports a possible model for the twin peak kHz QPOs: a forced non-linear oscillator,

$$\delta\ddot{\theta} + \omega_\theta^2 \delta\theta + O^2(\delta\theta) = h\cos(\omega_r t), \quad \omega_\theta \approx n\,\omega_r. \quad (6)$$

Obviously, there is no integer value of n such that ω_θ and ω_r could be in the 3:2 ratio. However, non-linear terms alow the presence of *combination frequencies* [37, 38],

$$\omega_- = \omega_\theta - \omega_r, \quad \omega_+ = \omega_\theta + \omega_r. \quad (7)$$

One of these combination frequencies may be in a 3:2 ratio with ω_θ if and only if $n = 2$, or $n = 3$ in this forced resonance. Simple arithmetic shows that in these two cases the observed frequencies $v_{lower} = \omega_{lower}/2\pi$ and $v_{upper} = \omega_{upper}/2\pi$ are uniquely given by,

$$[3:1] \quad \omega_{lower} = \omega_- = 2\,\omega_r, \quad \omega_{upper} = \omega_\theta = 3\,\omega_r, \quad (8)$$

$$[2:1] \quad \omega_{upper} = \omega_+ = 3\,\omega_r, \quad \omega_{lower} = \omega_\theta = 2\omega_r. \quad (9)$$

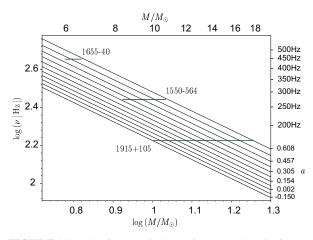We fit observed QPOs to those predicted by the forced epicyclic 3:1 and 2:1 resonances in Figures 11 and 12.

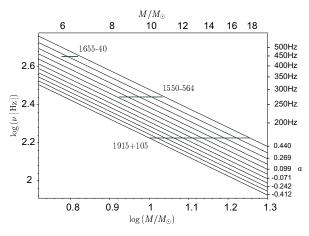**FIGURE 11.** Fit of the predictions of the 3:1 epicyclic forced resonance model to observations.

FIGURE 12. The 2:1 forced epicyclic resonance.

3:2 parametric resonance

We shall start with an argument appealing to physical intuition and showing that the resonance to be discussed now is a very natural, indeed necessary, consequence of strong gravity. In thin disks, random fluctuations have $\delta r \gg \delta\theta$. Thus, $\delta r \delta\theta$ is a first order term in $\delta\theta$ and should be included in the first order equation for vertical oscillations (1). The equation now takes the form,

$$\delta\ddot{\theta} + \omega_\theta^2 [1 + h\,\delta r]\,\delta\theta = \delta a_\theta, \qquad (10)$$

where h is a known constant. The first order equation for δr has the solution $\delta r = A_0 \cos(\omega_r t)$. Inserting this in (10) together with $\delta a_\theta = 0$, one arrives at the Mathieu equation (A_0 is absorbed in h),

$$\delta\ddot{\theta} + \omega_\theta^2 [1 + h\cos(\omega_r t)]\,\delta\theta = 0, \qquad (11)$$

that describes the *parametric resonance*. From the theory of the Mathieu equation one knows that when
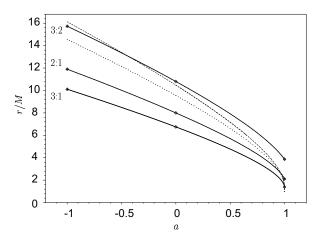

FIGURE 13. The radii of three epicyclic resonances, 3:2, 3:1 and 2:1. Also shown are the location of the centre of largest possible accretion torus (dashed lines), and the location (dotted lines), closer to the horizon, of the maximal locally emitted flux from the standard thin disk.

$$\frac{\omega_r}{\omega_\theta} = \frac{\nu_r}{\nu_\theta} \approx \frac{2}{n}, \quad n = 1, 2, 3..., \qquad (12)$$

the parametric resonance is excited [37, 38]. The resonance is strongest for the smallest possible value of n. Because near black holes $\nu_r < \nu_\theta$, the smallest possible value for resonance is $n = 3$, which means that $2\nu_\theta = 3\nu_r$. This explains [39] the observed 3:2 ratio, if, obviously,

$$\nu_{\text{upper}} = \nu_\theta, \quad \nu_{\text{lower}} = \nu_r. \qquad (13)$$

Parametric resonance of the type discussed above was found in numerical simulations of oscillations in a nearly Keplerian accretion disk by Abramowicz et al. [40] and confirmed by exact analytic solutions [41, 42]. The analytic solution is accurate up to third order terms in δr, $\delta\theta$, and based on the method of multiple scales [38]. Existence of the 3:2 parametric resonance is therefore a mathematical poroperty of thin, nearly Keplerian disks.

It was found that the resonance is excited only in the non-Keplerian case, with some weak forces $\delta a_\theta \neq 0$ and $\delta a_r \neq 0$ present. Their origin is certainly connected to stresses (pressure, magnetic field, viscosity), but the exact details remain to be determined — at present δa_θ and δa_r are not calculated from first principles but described by an ansatz[3]. Of course in real disks neither

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[3] While the lack of a full physical understanding is obviously not satisfactory, the experience tells that such a situation is not uncommon for non-linear systems. Examples are known of mathematically possible resonances causing damage in bridges, areoplane wings etc., for which no specifi c physical excitation mechanism could have been pinned down [38].
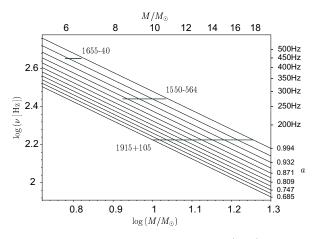
FIGURE 14. Fit of the frequency $\nu_{upp} = \nu_\theta(M, a)$ predicted by the 3:2 parametric resonance model to the frequencies observed in three microquasars with known masses. The spin parameter a is not known from observations, and the lines $a = $ const are calculated from the model. One should notice that the deduced black hole spins are rather high, and fall in a rather narrow range.

$\delta r = A_0 \cos(\omega_r t)$, nor $\delta a_\theta = 0$ exactly, but one may expect that because these equations are approximately obeyed for thin disks, the parametric resonance will also be excited in realistic situations. And this is indeed the case [43].

The parametric resonance occurs at a particular radius $r_{3:2}(a)$, determined by the condition $3\omega_r(r_{3:2}, a) = 2\omega_\theta(r_{3:2}, a)$. We show the function $r_{3:2}(a)$ in Figure 13. In Figure 14 we fit the 3:2 resonance theoretically predicted frequencies to the observational data for the three microquasars with known masses. The scatter for the particular 3:2 resonance is not very large because (Fig. 13) this resonances occurs at $x_{3:2}(a) > 4$, where the influence of a is not dominant.

TABLE 2. Black hole spin in three microquasars calculated by fitting observations to the three resonance models

Microquasar	3:2	2:1	3:1
XTE 1550-564	0.94	0.27	0.46
GRO 1655-40	0.96	0.36	0.55
GRS 1915+105	0.84	0.02	0.23

APPLICATIONS

The $1/M$ scaling of the twin peak QPOs frequencies with the 3:2 ratio, was proposed by Abramowicz, Kluźniak, McClintock & Remillard [44] as a method for estimating black hole masses in AGNs and the recently discovered ultraluminous X-ray sources (ULXs), based on
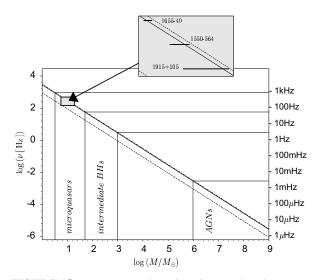

FIGURE 15. The 1/M scaling of the frequencies of 3:2 QPO pairs may be used for mass estimates (from Abramowicz, Kluźniak, McClintock, & Remilard, 2003).

Mirabel's analogy between microquasars in our Galaxy and distant quasars [45]. Indeed, if the analogy is also valid for accretion disk oscillations, then discovering in ULXs the twin peak QPOs frequencies with the 3:2 ratio, would resolve the controversy about their mass: if ULXs black holes have the same masses as microquasars, the frequencies will be $\sim 100\,\text{Hz}$; if ULXs black holes are ~ 1000 times more massive, the frequencies will be $\sim 0.1\,\text{Hz}$ instead.

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