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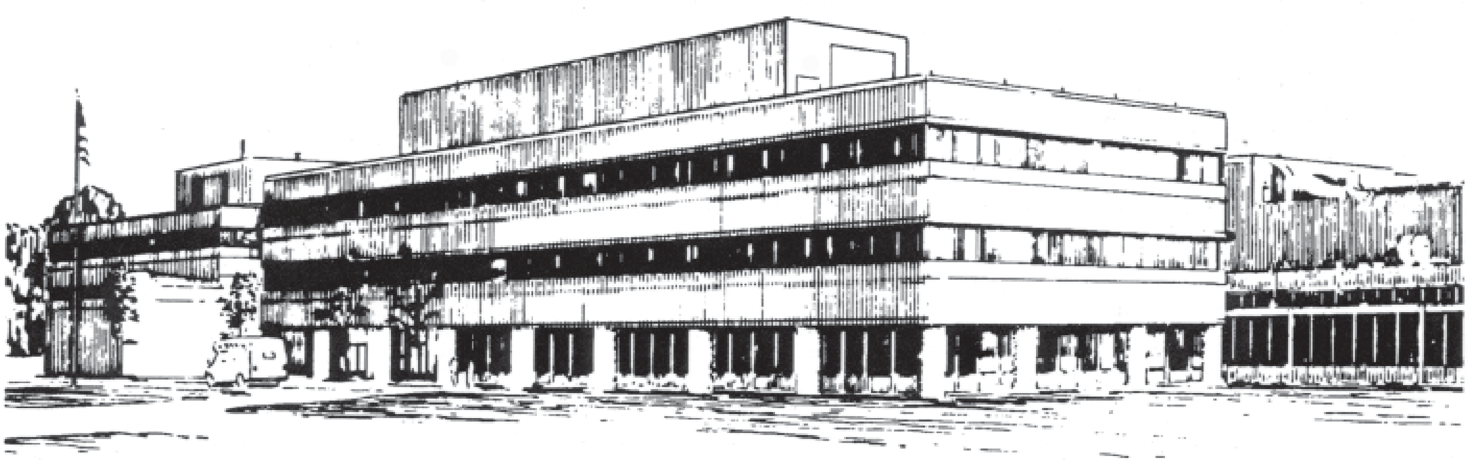

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Infernal fishbone mode

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Abstract

A new kind of fishbone instability associated with circulating energetic ions is predicted. The considered instability is essentially the energetic particle mode; it is characterized by $m/n \neq 1$ (m and n are the poloidal and toroidal mode numbers, respectively); the mode is localized inside the flux surface where the safety factor (q) is $q_* = m/n$, its amplitude being maximum near q_*; the instability arises in plasmas with small shear inside the q_* surface and $q(0) > 1$. A possibility to explain recent experimental observations of the $m = 2$ fishbone oscillations accompanied by strong changes of the neutron emission during tangential neutral beam injection in the National Spherical Torus Experiment [M.Ono *et al.*, Nucl. Fusion **40**, 557 (2000)] is shown.

The fishbone instability belongs to most interesting and important phenomena in toroidal plasmas with energetic ions.[1−4] It can be very dangerous, leading to expulsion of a considerable fraction (up to 50%) of energetic ions.[5] But, on the other hand, sometimes it is useful, being a trigger of formation of the internal transport barrier.[6] Typically, the fishbone instability is associated with an $m = n = 1$ perturbation (m and n are the poloidal and toroidal mode numbers, respectively) in the plasma core. Theory predicts that features of the fishbone mode in spherical tori (ST) differs from those in tokamaks. In particular, the high β (the ratio of plasma pressure to the magnetic field pressure) tends to stabilize the known fishbone instabilities (which can be associated with trapped or circulating energetic ions)[7,8] but, on the other hand, a new new kind of the fishbone mode ("bounce-frequency fishbones") can arise in moderate-β shots of STs.[9] A theory developed in Ref.[9] seems to explain recent experimental observations of the $m = 2$ fishbones in the NSTX spherical torus, shot #106218.[9] The mentioned theory suggests that the instability is caused by the trapped energetic ion population. The latter appears in the NSTX shot #106218 mainly due to pitch-angle scattering of partly slowed down injected ions having the energy $\mathcal{E} \ll \mathcal{E}_0$, where $\mathcal{E}_0 = 80keV$ is the injection energy. The theoretical conclusion in Ref.[9] that the energy of particles driving the instability is relatively small is consistent with the fact that fishbones only weakly affect the neutron yield in the mentioned shot (the neutron production in NSTX is predominantly from beam-plasma reactions). However, recently results of other observations of $m = 2$ fishbones in NSTX were reported, showing strong influence of the fishbone instability on the neutron yield.[10] This indicate that particles with the energy $\mathcal{E} \sim 80keV$ are involved to the process, which motivated the present work. In this Letter, we show that energetic circulating particles can result in a new $m \neq 1$ instability in plasmas with a wide shearless plasma core characterized by the safety factor, q, close to but less than m/n. The physics of the considered instability differs from the known circulating-particle-induced instability of the $m = n = 1$ mode[11,12,8] because of different radial mode structure. We will refer to a new instability as ”infernal fishbones” because the infernal instability is known to be driven by pressure gradient in low-shear plasmas (in our case the instability is driven by the pressure gradient of the energetic ions, although the velocity anisotropy of the energetic ions can be a destabilizing factor, too).[13–15]

We proceed from the following vorticity equation:[16]

$$\vec{B}\cdot\nabla\left[\frac{1}{B^2}\nabla_\perp^2(\vec{A}\cdot\vec{B})\right]-\frac{i\omega c}{v_A^2}\nabla_\perp^2\Phi+\frac{4\pi}{B^2}\vec{\kappa}\times\vec{B}\cdot\nabla_\perp(2\delta p_c+\delta p_{\|\alpha}+\delta p_{\perp\alpha})=0, \quad (1)$$

where B is the magnetic field strength, $\vec{A}$ and Φ are the mode vector potential and electrostatic potential, respectively, $\vec{\kappa}=\vec{b}\cdot\nabla\vec{b}$ with $\vec{b}=\vec{B}/B$ is the magnetic field line curvature, v_A is the Alfvén velocity, δp is the perturbed pressure, the subscript c is relevant to the bulk plasma, and subscript α labels the energetic ion magnitudes, $\nabla_\perp=\nabla-\vec{b}(\vec{b}\cdot\nabla)$. Using a standard technique, Eq. (1) can be reduced to a single second order differential equation for the $m>1$ radial displacement given by

$$\frac{1}{r}\frac{d}{dr}r^3\left[\frac{\omega^2}{v_A^2}(1+2q^2)-k_\|^2\right]\frac{d\xi}{dr}-(m^2-1)\left[\frac{\omega^2}{v_A^2}(1+2q^2)-k_\|^2\right]\xi$$
$$=-[\hat{g}(r,\xi)+\hat{h}(r,\xi)], \quad (2)$$

where $k_\|\equiv(m-nq)/qR\simeq const$ for $r<r_0$ with $q(r_0)=m/n, m-nq\ll m$, the term $(1+2q^2)$ is an enhancement of the inertia due to toroidal effects[17], $\hat{g}(r,\xi)$ represents the bulk plasma fluid-potential energy including the stabilizing Mercier term, the destabilizing contribution of toroidal coupling[15] and the beam fluid contribution, and $\hat{h}(r,\xi)$ represents the kinetic contribution given by[18]

$$\hat{h}(r,\xi)=\frac{2mr}{B_0^2}\int_{-\pi}^{\pi}\exp\left(i\omega t+in\phi-im\theta\right)\vec{b_0}\times\vec{\kappa}\cdot\nabla(\delta p_{\perp\alpha}^k+\delta p_{\|\alpha}^k)d\theta, \quad (3)$$

$$\begin{pmatrix}\delta p_{\perp\alpha}^k\\ \delta p_{\|\alpha}^k\end{pmatrix}\equiv im_\alpha^2\int d^3v\begin{pmatrix}\frac{v_\perp^2}{2}\\ v_\|^2\end{pmatrix}\left[(\omega-n\omega_{*\alpha})\frac{\partial F_j}{\partial\mathcal{E}}\int_{-\infty}^{t}\left(\frac{v_\perp^2}{2}+v_\|^2\right)\vec{\kappa}\cdot\vec{\xi_\perp}dt'\right], \quad (4)$$

where $\omega_{*\alpha}\equiv(\partial F_\alpha/\partial P_\phi)(\partial F_\alpha/\partial\mathcal{E})^{-1}$. Treating the right-hand side (RHS) of Eq. (2) as a perturbation and assuming that $k_\|(r)v_A\simeq const$ in the shear-free core we obtain for the lowest order eigenmode in the ”outer” region, $r<r_0$, as

$$\xi_0(r) = \xi_{00}\left(\frac{r}{r_0}\right)^{m-1} H(r_0 - r). \tag{5}$$

For $r > r_0$ the mode amplitude should decrease rapidly (on the scale $x \equiv (r - r_0)/r_0 \sim \omega/\omega_A \ll 1$ with $\omega_A = v_A/R$) in order to minimize the continuum damping at the local Alfvén resonance $\omega = k_\parallel v_A/(1+2q^2)$. An approximate eigenmode equation in the large-shear layer around $r = r_0$ can be obtained retaining only the higher order derivative term in Eq. (2):

$$\frac{d}{dr}\left\{\left[\frac{\omega^2}{v_A^2}(1+2q^2) - k_\parallel^2\right]\right\}\frac{d\xi}{dr} = 0. \tag{6}$$

Integrating Eq. (6) once with the integration constant chosen to match with the solution given by Eq. (5) outside the layer, we obtain $d\xi/dr$. The latter we put in the following expression for the sum of the kinetic and potential energies in the layer, I:[19]

$$I = -2\pi^2\frac{R_0}{m^2}n_0 m_i\int_{layer} r^3\left(\frac{d\xi}{dr}\right)^2[\omega^2(1+2q^2) - k_\parallel^2 v_A^2]dr, \tag{7}$$

where $n_0 m_i$ is the plasma mass density. As a result, we have:

$$I = \pi r_0^2\frac{B_0^2}{2R_0}\xi_{00}^2\left\{\int_{-\infty}^{\infty}\left[\left(-i\frac{\omega}{\omega_A}\right)^2\frac{1+2q^2}{m^2} + \frac{s_0^2x^2}{q^2}\right]^{-1}dx\right\}^{-1} =$$
$$-i\frac{\omega}{v_A}\frac{B_0^2}{2}r_0^2\xi_{00}^2\frac{s_0}{m^2}\sqrt{n^2+2m^2}, \tag{8}$$

where s_0 is the magnetic shear at the $q_* = m/n$ surface. Summing Eq. (8) with potential energy in the small-shear region calculated using eigenfunction given by Eq. (5) results in the following generic eigenvalue equation:

$$-i\frac{\omega}{\hat{\omega}_A} + \delta\hat{W}_f + \delta\hat{W}_k = 0, \tag{9}$$

where

$$\hat{\omega}_A = \frac{v_A}{Rs_0}\frac{m^2}{\sqrt{n^2+2m^2}}, \tag{10}$$

$$\delta\hat{W}_k \equiv \frac{R}{r_0^2\xi_{00}^2B_0^2}\int\vec{\xi}_\perp^*\cdot\nabla\delta\Pi_\alpha^k d^3r$$
$$= -\frac{2\pi^2 R m_\alpha}{\omega_{c\alpha}r_0^2\xi_{00}^2B_0^2}\sum_\sigma\int v^3dv\int dP_\phi\int d\Lambda\tau_b\frac{\partial F_\alpha}{\partial\mathcal{E}}\frac{\omega - n\omega_{*\alpha}}{\omega - k_\parallel v_\parallel}$$
$$\times\left|\left\langle\left(\frac{v_\perp^2}{2} + v_\parallel^2\right)\vec{\xi}_0\cdot\vec{\kappa}\exp\left[i(\omega - k_\parallel v_\parallel)t\right]\right\rangle\right|^2, \tag{11}$$

$\delta\Pi_\alpha^k = \delta p_{\perp\alpha}^k \hat{I} + (\delta p_{\|\alpha}^k - \delta p_{\perp\alpha}^k)\vec{b}\vec{b}$ is the pressure tensor with $\delta p_{\|,\perp\alpha}^k$ given by Eq. (4), $\sigma \equiv v_\| / |v_\||, \Lambda \equiv \mu B_0/\mathcal{E}, \tau_b$ is the particle transit time, $\hat{I}$ is the identity tensor and $\langle ... \rangle$ denotes the orbit averaging. Note that when obtaining Eq. (11), we have used the relation $[\nabla \cdot (\vec{b}\vec{b})]_\perp = \vec{\kappa}$ and an expression for the phase space volume element[20]

$$d^3rd^3v = \sum_\sigma \frac{2\pi^2 v^3}{m_\alpha \omega_{c\alpha}} d\tau dv d\Lambda dP_\phi, \tag{12}$$

where τ is the time along the orbit, and P_ϕ is the canonical angular momentum. Below we omit the terms odd in θ in $\vec{\xi} \cdot \vec{\kappa}$ in the integrand of Eq. (11), which yields :

$$\vec{\xi_0} \cdot \vec{\kappa} = -\frac{\xi_{00}}{R_0} \left(\frac{r(t)}{r_0} \right)^{m-1} H(r_0 - r(t)) \cos[\theta(t)] \exp\left[i(m\theta(t) - n\phi(t) - \omega t)\right], \tag{13}$$

where

$$r(t) = \bar{r} - \Delta_\alpha \cos[\theta(t)], \quad \Delta_\alpha = \frac{q(\bar{r})}{v_\| \omega_{c\alpha}} \left(\frac{v_\perp^2}{2} + v_\|^2 \right),$$
$$\theta(t) = \frac{v_\|}{q(\bar{r})R} t, \quad \phi(t) = \frac{v_\|}{R} t, \tag{14}$$

$\omega_{c\alpha}$ is the fast ion gyrofrequency, and $\bar{r}$ is a constant of motion $[\bar{r} = r(\theta = \pi/2)]$.

The considered mode does not have the Alfvén resonance in the small-shear core. Nevertheless, Cherenkov resonance with circulating particles in this region is possible when the energetic ions are highly super-Alfvénic (which is the case in NSTX experiments with neutral beam injection).

Below we assume that the energetic-ion population consists of well-circulating particles with the equilibrium distribution function given by

$$F_\alpha = \frac{\sqrt{2} m_\alpha^{3/2}}{\pi \varepsilon_\alpha} p_\alpha(\bar{r}) H(\mathcal{E}_\alpha - \mathcal{E}) \mathcal{E}^{-3/2} \delta(\Lambda), \tag{15}$$

where $p_\alpha(\bar{r})$ is approximately the beam particle pressure, $H(x)$ is the Heavyside function. Substituting Eqs. (13)-(15) into Eq. (11), performing orbit averaging and velocity space integration, we finally obtain a dispersion relation in the form :

$$-i\frac{\Omega}{\Omega_A} + \delta\hat{W}_f + C\left[1 + \frac{5}{4}\Omega + \frac{5}{3}\Omega^2 + \frac{5}{2}\Omega^3 + 5\Omega^4 + 5\Omega^5 \ln\left(1 - \frac{1}{\Omega}\right)\right] = 0, \tag{16}$$

where $\Omega \equiv \omega/\bar{k}_\parallel v_\alpha, \Omega_A \equiv \hat{\omega}_A/\bar{k}_\parallel v_\alpha$, $\bar{k}_\parallel$ is some average value of $k_\parallel(r)$ in the small-shear core,

$$C = -\frac{m^4(m-1)^2}{5\pi n^3}\frac{R\rho_\alpha^3}{r_0^4}\int_0^{r_0}\left(\frac{r}{r_0}\right)^{2(m-2)}\frac{d\beta_\alpha}{dr}dr, \tag{17}$$

ρ_α is the Larmor radius at injection energy, and $\omega \ll \omega_{*\alpha}$ has been assumed. Note that, in contrast with a theory of the $m = n = 1$ fishbone instability,[12] we neglected a contribution from particles crossing the q_* surface, assuming poor population of energetic ions at this surface. In our case of $m/n \neq 1$, the fast ion drive is dominated by the particles deposited in the shear-free core due to radial variation of a poloidal electric field of the mode, whereas almost no energy exhange occurs between these particles lying inside the $q = 1$ surface and the $m = n = 1$ rigid shift perturbation.

Let us consider a specific example relevant to NSTX observations of the $m/n = 2/1$ fishbone instability strongly affecting the neutron yield. We take $s_0 = 0.6$, $\beta_\alpha(r) = \beta_{\alpha 0}(1 - r^2/a^2)^\nu$ with $\nu \gg 1$, in which case $\int_0^{r_0}(d\beta_\alpha/dr)dr \simeq -\beta_{\alpha 0}$, and $\rho_\alpha = 20cm, r_0 = 40cm, R = 100cm$, so that $R\rho_\alpha^3/r_0^4 = 0.31$. Then, assuming that in the absence of the energetic ions a plasma is marginally stable ($\delta\hat{W}_f \simeq 0$), we obtain from Eq. (16):

$$\Omega \simeq 0.93, \quad \beta_{\alpha 0}^{cr} \simeq 3.6\%, \tag{18}$$

where Ω and $\beta_{\alpha 0}^{cr}$ are the frequency and the threshold on-axis fast ion beta at the mode onset, respectively. For the shot with $q_0 = 1.7$, we obtain $f \simeq 44.4kHz$ assuming $\overline{k_\parallel}R \simeq 0.1$, which compares favorably with observed initial fishbone frequency in the plasma frame $f \simeq 45kHz$ and volume averaged beam ion beta $\langle\beta_\alpha\rangle \simeq 2\%$.

In summary, we have shown for the first time that the $m/n > 1$ mode in plasmas with monotonic $q(r) > 1$ can be destabilized by the interaction with energetic circulating ions. The istability is an energetic particle mode, the Cherenkov resonance being responsible for the interaction between the energetic ions and the perturbation. In contrast to the conventional $m = n = 1$ circulating-ion-driven fishbone instability, the considered instability is caused mainly by particles with orbits inside the flux surface with the radius r_0 where the safety factor is $q_* = m/n$ (rather than by the particles crossing the q_* surface). The strong energy exchange between these particles and the perturbation takes place due to finite orbit width of the energetic ions and a specific radial mode structure, $\delta E_\theta \propto r^{m-1}$ at $r < r_0$. Both the mode frequency and critical fast ion pressure are in reasonable agreement with experimental observations of bursting $m = 2$ fishbone oscillations accompanied by strong changes of the neutron yield in the NSTX spherical torus.

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