SU(3) gauge theory at finite temperature in 2+1 dimensions

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I. Comparison between $2+1$ and $3+1$ dimensional gauge theories

II. Perturbation theory vs dimensional reduction

III. Screening masses

IV. Thermodynamics

V. Conclusions

SU(3) Gauge Theories in d + 1 dimensions

Differences

2 + 1	**3 + 1**
$Dim(g^2) = 1$	$Dim(g^2) = 0$
Superrenormalizable	**Renormalizable**
classically: $V(r) \sim \log r$	$V(r) \sim -\frac{1}{r}$
1 gluon/colour	**2 gluons/colour**

"There is a serious risk that working in 2 + 1 dimensions you are wasting your time ...
Nevertheless ..."

R. Feynman (1981)

Similarities

2 + 1	**3 + 1**
$g_3^2(l) \equiv lg^2 \xrightarrow{l\to 0} 0$	$g_4^2(l) \simeq \frac{-c}{\log \Lambda l} \xrightarrow{l\to 0} 0$
$g_3^2(l) \xrightarrow{l\to\infty} \infty$	$g_4^2(l) \xrightarrow{l\to\infty} \infty$
$m_i = c_i g^2;\ m_{GB} \geq 4.4T_c$	$m_i = c_i \Lambda;\ m_{GB} \geq 5.9T_c$
MC at $T = 0$: $V(r) \simeq \sigma_0 r$**,** r **large**	$V(r) \simeq \sigma_0 r$**,** r **large**
MC at $T > 0$: **second order transition at $T = T_c$,** $\sigma \to 0$	**weakly first order at $T = T_c$,** $\sigma \to c \ll \sigma_0$
univ. class **2d 3 state Potts**	**3d 3 state Potts**

Legeland, BP

$T > T_c$

Gluon Plasma

2 + 1 **3 + 1**

$$g_3^2(T) \simeq \frac{g^2}{T} \xrightarrow{T\to\infty} 0$$

$$g_4^2(T) \simeq \frac{c}{\log T/\Lambda} \xrightarrow{T\to\infty} 0$$

But IR div

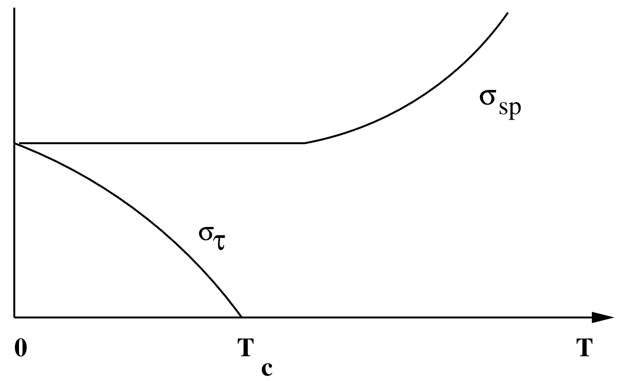
$$f_N(T) \sim g^{2N} \prod_{i=1}^{N+1} \left(T \sum_{n_i} \int d^d k_i \right) \prod_{j=1}^{2N} \left(\bar{q}_j^2 + (2\pi l_j T)^2 + \mu^2 \right)^{-1}$$

all $n_i = 0$ $N \geq 1$

all $n_i = 0$ $N \geq 3$

Non-perturbative spacelike string tension

$$\sigma_{sp}(T) \sim g^2 T$$



$$\sigma_{sp}(T) \sim T^2$$

Selfconsistent perturbation theory in 2 + 1 dimensions (SCPT)

D'Hoker, 1981

- **Choose the class of "static gauges"**

$$\partial_0 A_0(x) = 0$$

- $$L(\bar{x}) = Tr\, e^{i\,\frac{g}{T}\,A_0\,(\bar{x})}$$

- **Add and subtract explicit mass term**

$$\tfrac{1}{2} m_g^2\, Tr(A_0(\bar{x}))^2$$

- **propagator in static Feynman gauge**

$$D_{00}^{ab}(0,\bar{p}) = \frac{\delta_{ab}}{\bar{p}^2+m_g^2} \qquad D_{ij}^{ab}(p_0 \neq 0,\bar{p}) = \delta_{ab}\frac{\delta_{ij}+p_i p_j/p_0^2}{p_o^2+\bar{p}^2}$$

$$D_{ij}^{ab}(0,\bar{p}) = \delta_{ab}\frac{\delta_{ij}}{\bar{p}^2}$$

- **No further infrared divergences; choose e.g.** $\partial_0 A_0 = 0$ $\int_0^{1/T} d\tau\, A_2(\tau,\bar{x}) = 0$ **(static axial gauge) Then in** A_1 **static propagator one should take the principal value.**

Dimensional reduction $2+1 \longrightarrow 2$

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- **Choose the class of static gauges**

$$\partial_0 A_0(x) = 0$$

- **Integrate perturbatively over non-static modes (no infrared div.!)**

- **Effective 2d adjoint Higgs model for static modes**

$$S = \int d^2 x Tr \left\{ \frac{1}{2} F_{ij}^2 + (D_i \phi)^2 - \frac{3g^2 T}{2\pi} \left[\frac{5}{2} \log 2 + 1 - \log aT \right] \phi^2 + \frac{g^4}{32\pi} \phi^4 \right\}$$

- **R-symmetry** $\phi \longrightarrow -\phi$

- **2 dimensional adjoint Higgs model not solved analytically**

- **2 dimensional pure SU(3) gauge theory trivially solvable**

(Gross, Witten)

Lattice simulation:

two parameters $\beta = \frac{6}{ag^2}; \quad N_\tau = \frac{1}{aT}$

D = 2

$$S = \sum_x \Big\{ \beta N_\tau \left(1 - \tfrac{1}{3} Re\, Tr\, U_p\right) + Tr\left[U(\bar{x}; i)\phi(\bar{x} + \bar{i})U(\bar{x}, i)^{-1} - \phi(\bar{x})\right]^2$$
$$- \tfrac{9}{\pi N_\tau \beta}\left[\tfrac{5}{2}\log 2 - 1 + \log N_\tau\right] Tr\,\phi^2(\bar{x}) + \tfrac{9}{8\pi\beta^2} Tr\,\phi^4(\bar{x}) \Big\}$$

- Z_3 **symmetry of (2 + 1)D theory explicitly broken**

cf. Bialas, Morel, B.P. (2005)

Screening mass

$$\langle Re(L(p_1=0,x_2)L^+(p_1=0,0))\rangle \sim \langle L\rangle^2 + e^{-m|x_2|}$$

lowest order perturbation theory $m=2m_g$ **for** $ReL(\bar{x})$ $m=3m_g$ **for** $ImL(\bar{x})$ $(mod(Z_3))$

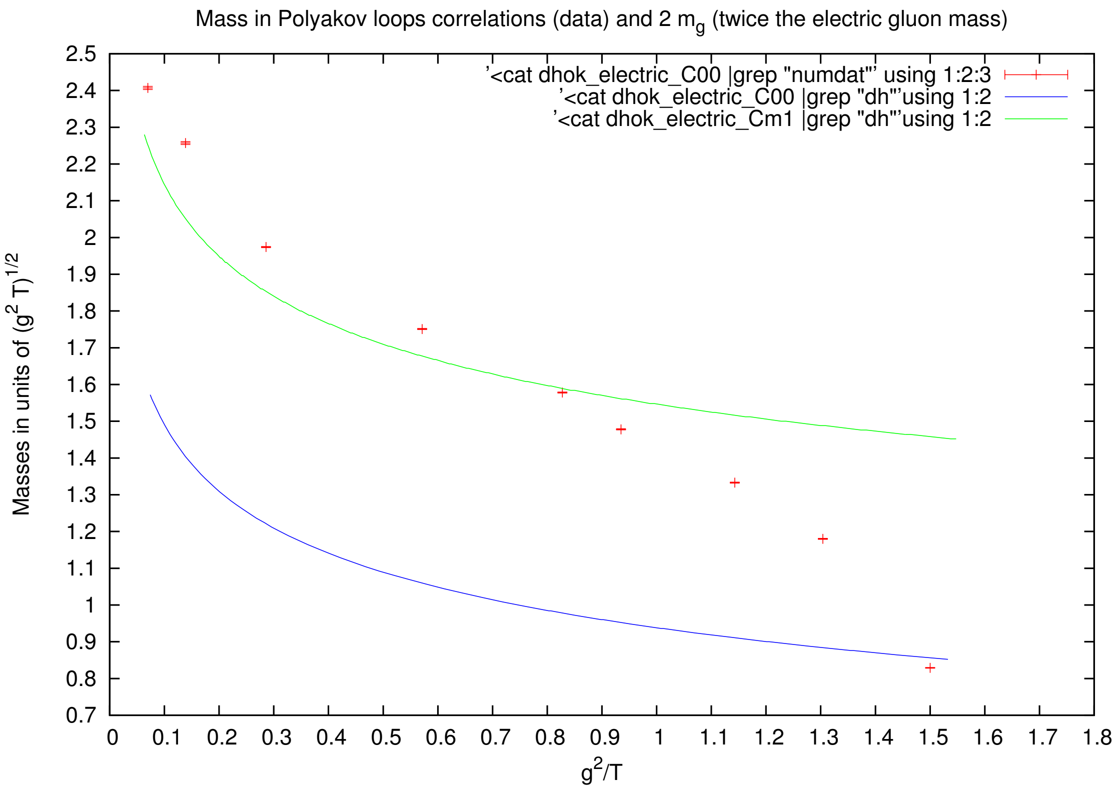
SCPT:

$$\frac{m_g^2}{T^2} \sim \frac{g^2}{T}\left(a\log\frac{T}{m_g}+b+O\left(1/\log\frac{T}{m_g}\right)\right)$$

scales $\begin{cases} T \\ g_3(T)T=\sqrt{\frac{g^2}{T}}T \ll T \quad \text{for } T \text{ large} \end{cases}$ **note:** $\frac{g^2}{T_c}=1.81(2)$ **Legeland, B.P.**

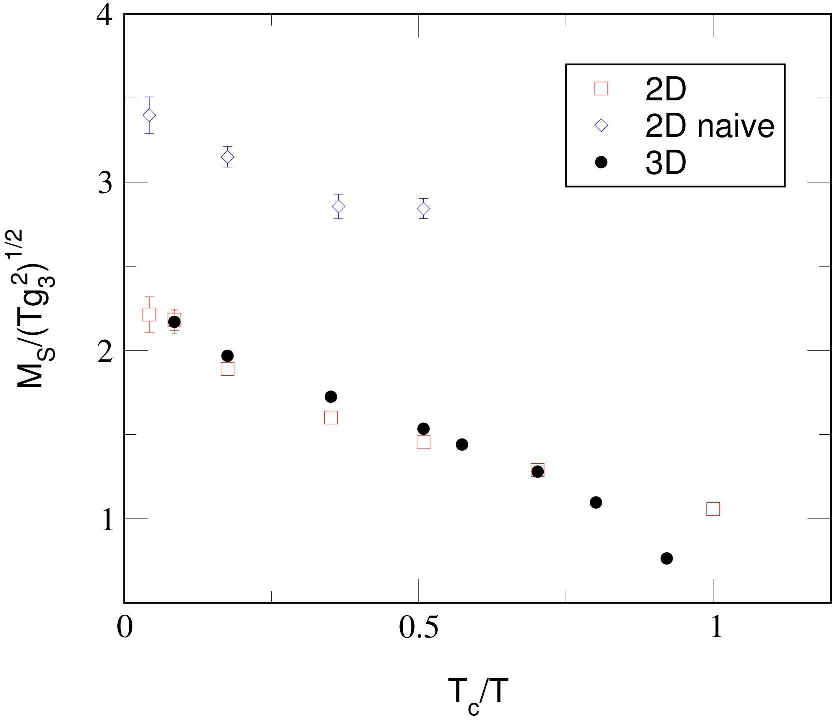
Comparison with the MC data

SCPT: $$\frac{m_g^2}{g^2 T} = \frac{3}{2\pi}\left(\log(\frac{T}{m_g}) + C\right) + O\left(\frac{1}{\log T/m_g}\right); \qquad C = -1$$



$C = +1$ **!**

Dimensional reduction



$m/\sqrt{g^2T}$ vs T_c/T

In the reduced model the two exchanged states corresponding to $Re\,L(\bar{x})$ and $Im\,L(\bar{x})$ are simple poles, not 2 gluon and 3 gluon cuts respectively

Thermodynamics in 2 + 1 dimensions

Lattice integral method:

$$S_0(\beta) = 3\langle P_0(\beta, U)\rangle_0 \ ; \ \ S_T(\beta) = \langle 2P_\tau(\beta, U) + P_\sigma(\beta, U)\rangle_T$$

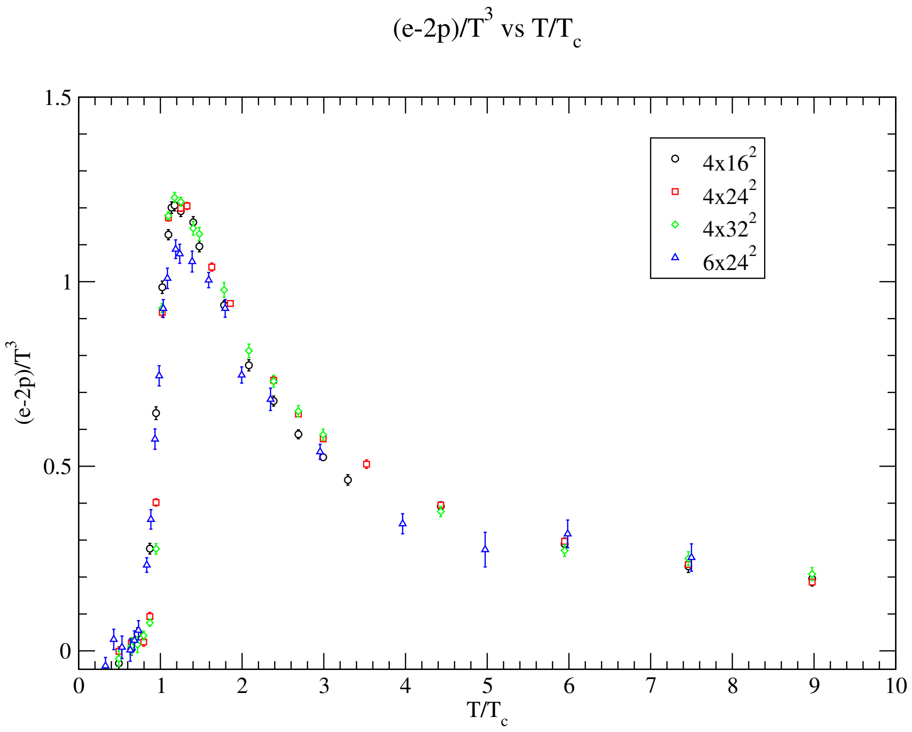
$\beta = 3.3 N_\tau \ \ T/T_c + 1.5$**;** **from** $T_c(\beta)$ Legeland, B.P.

$$(\epsilon - 2p)/T^3 = N_\tau^3 T \, \frac{d\beta}{dT} \, [S_0(\beta) - S_T(\beta)]$$

$$P/T^3 = -f/T^3 = N_\tau^3 \ \int^\beta [S_0(\beta') - S_T(\beta')] d\beta' + \mathrm{const}$$

Trace anomaly

2 + 1



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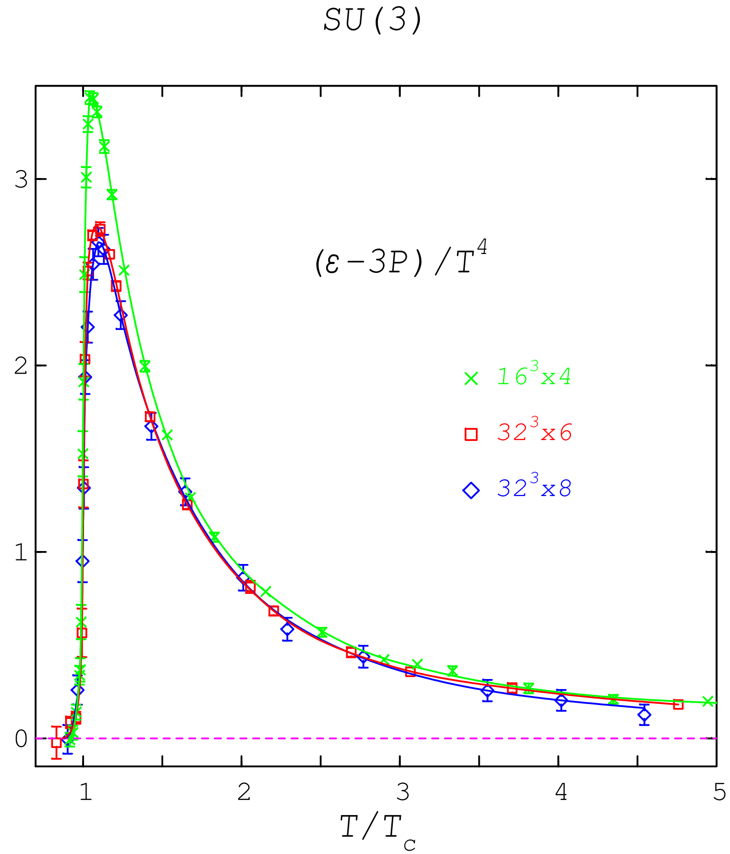
3 + 1

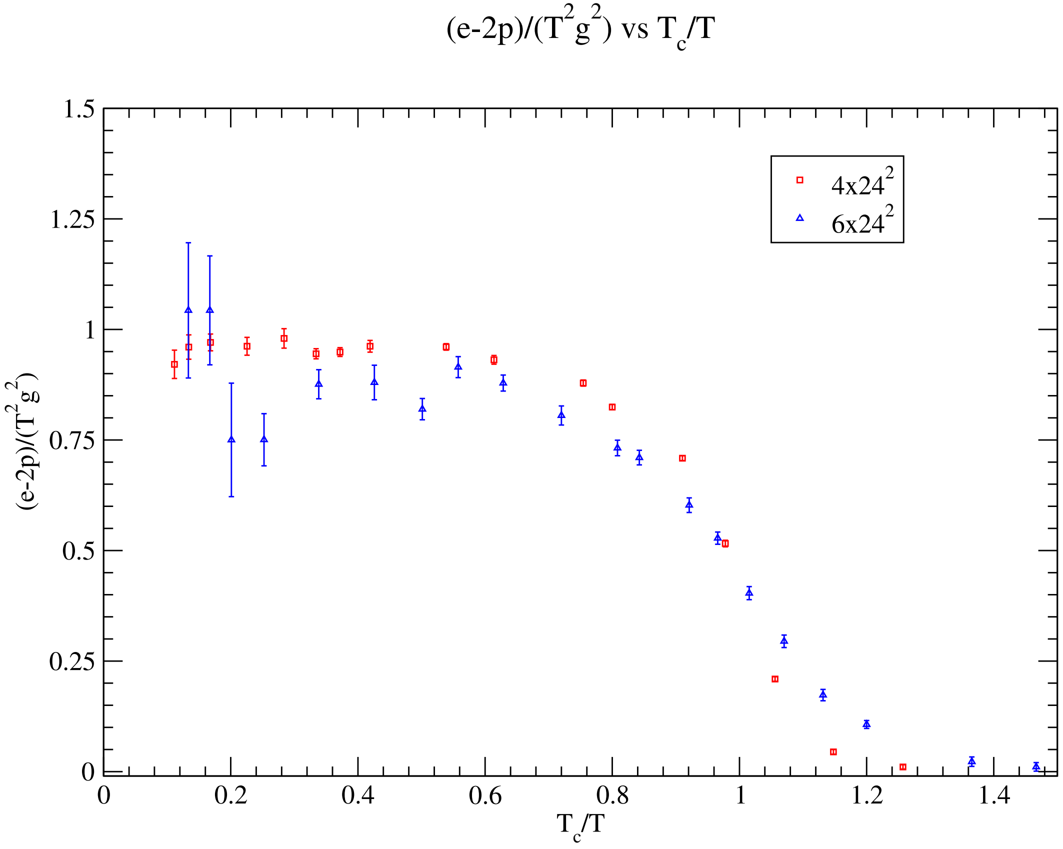


Boyd et al.

$(\epsilon - 2p)/T^3 = 0$ **for free massless gluons in 2 + 1 dimensions**

Expect

$$(\epsilon - 2p)/T^3 \simeq A\,\frac{g^2}{T}\,f(\log T/g^2)$$

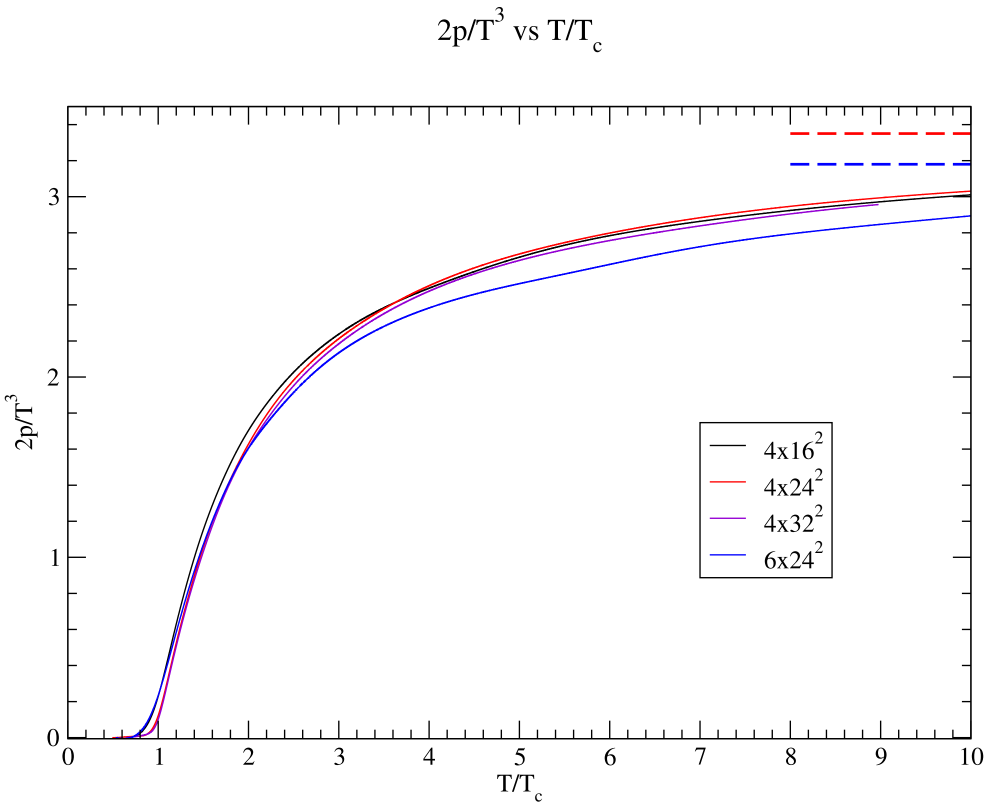


Pressure

Free massless gluons in 2 + 1 dimensions:

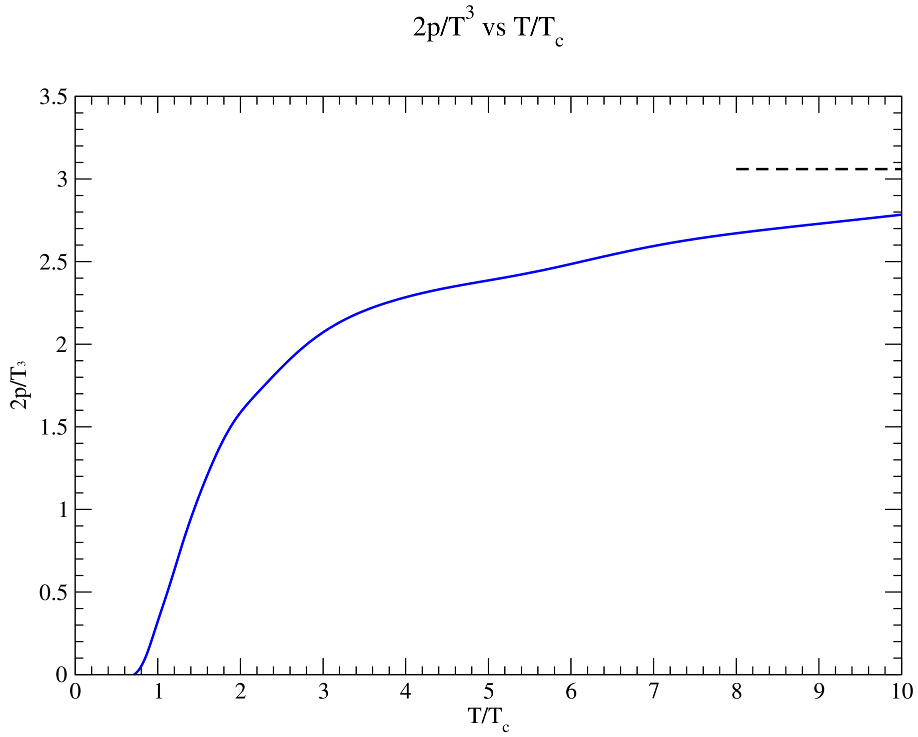
$$\frac{2p}{T^3} = 3.06\left(1 + \frac{1.51}{N_\tau^2}\dots\right)$$

MC data:

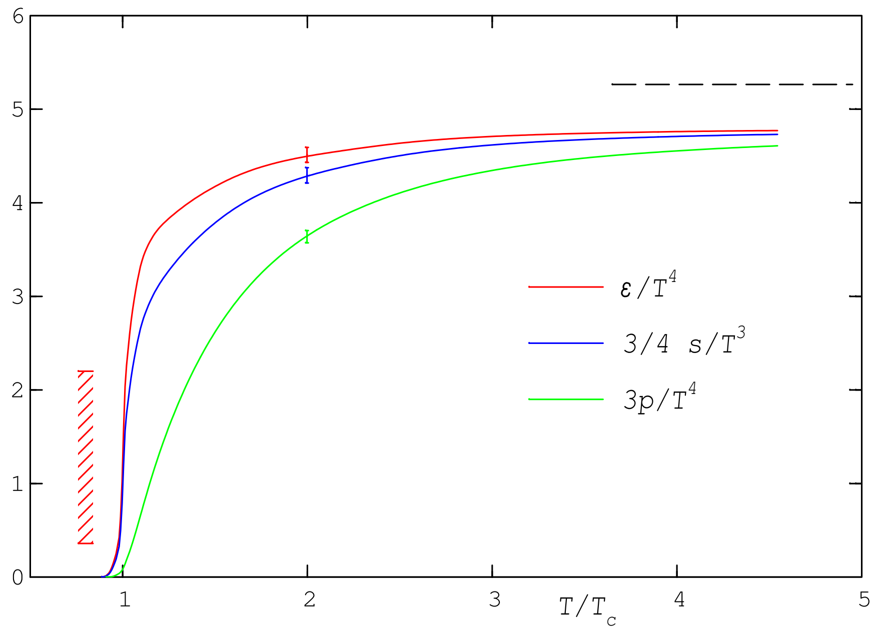


Thermodynamics, continuum extrapolation

2 + 1 preliminary



3 + 1



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Conclusions:

I SU(3) gauge theories in 2 + 1 and 3 + 1 dimensions have similar qualitative behaviour also for $T > T_c$

In 2 + 1 dimensions:

II Perturbation theory has two scales: T **and** $\sqrt{g^2T} \ll T$ **for large** T

III Screening masses are well described by dimensional reduction for $T \geq 1.5T_c$

IV Description of thermodynamics by dimensional reduction in progress