web page: http://w3.pppl.gov/~zakharov

# Calibrating JET for equilibrium reconstruction[1]

## *(iron core & eddy currents effects)*

*Leonid E. Zakharov*[1]

[1] *Princeton Plasma Physics Laboratory, MS-27 P.O. Box 451, Princeton NJ 08543-0451*

JET Task Force-D meeting,

October 27, 2005, Culham, UK

[1] **This work is supported by US DoE contract No. DE–AC020–76–CHO–3073.**

# Abstract

*The calibration technique for JET tokamak is presented. It targets elimination of uncertainties in magnetic signals due to the presence of the iron core and due to eddy currents in passive conductors.*

*The correlation matrix between sensors located outside and inside the vacuum vessel is introduced in order to determine the parasitic* $n \neq 0$ *perturbation in magnetic fields generated by the iron core.*

*The time dependent matrix of response functions is introduced in order to eliminate the* $n \neq 0$ *perturbation generated by the eddy currents.*

*While both elements can be determined using only the calibration shots (without the plasma), they allow to pre-process magnetic signals of plasma discharges for further use in the equilibrium reconstruction codes.*

*The calibration technique is planned to be implemented in JET using the existing experience with the similar approach developed for CDX-U tokamaks and with numerical code Cbc2e.*

# Contents

**Equilibrium reconstruction $\simeq$ GSh equation+magnetic diagnostics**

**The problem is to find a right hand side of the GSh equation**

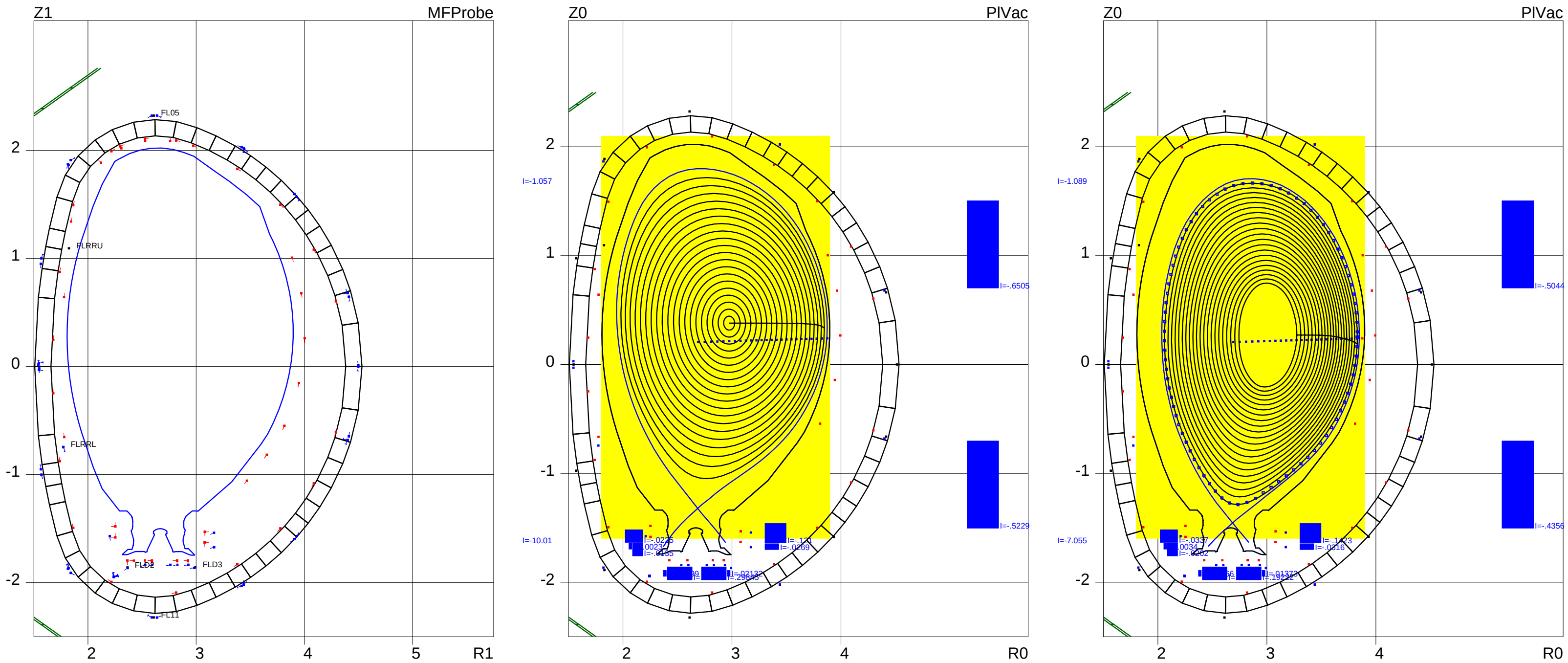
$$\Delta^*\bar{\Psi} \equiv \frac{\partial^2\bar{\Psi}}{\partial r^2} - \frac{1}{r}\frac{\partial\bar{\Psi}}{\partial r} + \frac{\partial^2\bar{\Psi}}{\partial z^2} = -J(r,\bar{\Psi}),$$
$$J(r,\bar{\Psi}) \equiv T(\bar{\Psi}) + r^2 P(\bar{\Psi}), \quad T \equiv \bar{F}\frac{d\bar{F}}{d\bar{\Psi}}, \quad P \equiv \frac{d\bar{p}}{d\bar{\Psi}}r^2, \tag{1.1}$$

**which leads to a solution consistent with magnetic measurements.**

$$J(r,\bar{\Psi}) \equiv J^{pl}(r,\bar{\Psi}) + \sum_{k=0}^{k<K} J_k^{pl} j^k(r,\bar{\Psi}). \tag{1.2}$$

**Axisymmetry is assumed in the GSh equation**

## Given $J(r,\bar{\Psi})$, measurements of $\bar{\Psi}(l)$ are sufficient for GSh Eq.




6 flux, 18(+12) saddles loops, 43(104) magnetic probes on JET

## Adjustable parameters $J_k^{pl}$ are introduced using expansion

$$J(r,\bar{\Psi}) \equiv J^{pl}(r,\bar{\Psi}) + \sum_{k=0}^{k<K} J_k^{pl} j^k(r,\bar{\Psi}). \tag{1.3}$$

**Measurements of $B_i$ is used to limit the freedom in $J(r,\bar{\Psi})$**

## Equilibrium reconstruction is based on axisymmetric fields

### Appropriate notations

$$f(r,z,\varphi,t) \equiv f_{\sqcup} \quad \text{for arbitrary function,}$$
$$f_{\sqcup} = f_0 + f_{\sim} \quad \text{averaged and oscillatory parts,}$$
$$f_0 = f_0(r,z,t) \equiv \frac{1}{2\pi}\oint f d\varphi \quad \text{for averaged part,} \tag{1.4}$$
$$f_{\sim} = f_{\sim}(r,z,\varphi,t) \equiv f_{\sqcup} - f_0 \quad \text{for oscillatory part,}$$
$$\oint f_{\sim} d\varphi = 0$$

### Flux loops in tokamaks are measuring just the averaged component of vector potential $\vec{A}$

$$\bar{\Psi} = (rA_{\varphi})_0 \tag{1.5}$$

### Measurements of $B$ are local and contain both components

$$B = B_{\sqcup} = B_0 + B_{\sim} \tag{1.6}$$

**Only $B_0$ is appropriate for equilibrium reconstruction**

## Calibrations shots are performed without plasma

**Axisymmetric solution of equation**

$$\Delta^* \bar{\Psi} = 0 \tag{1.7}$$

**is uniquely determined by $\bar{\Psi}(l)$ measurements.**

**In its turn, the solution $\bar{\Psi}(r, z)$ allows to calculate the axisymmetric component of magnetic field**

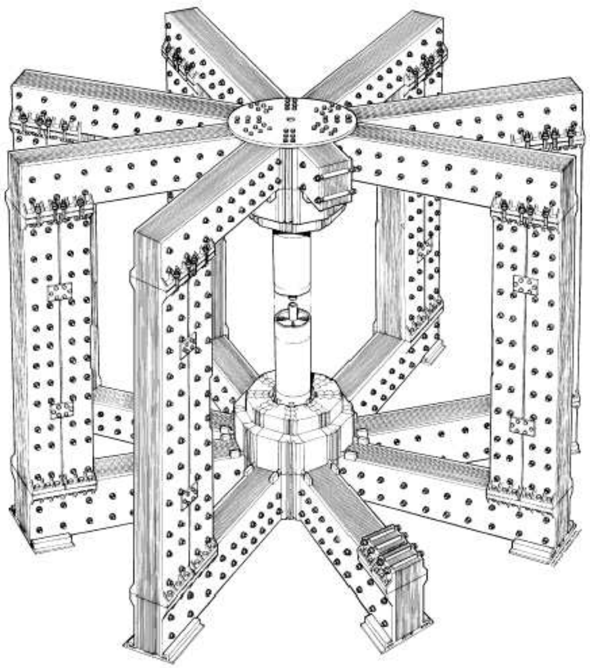
$$\mathrm{B}_0 = \nabla \bar{\Psi}(r, z) \times \nabla \varphi \tag{1.8}$$

## 3-D contribution into magnetic measurements can be found as

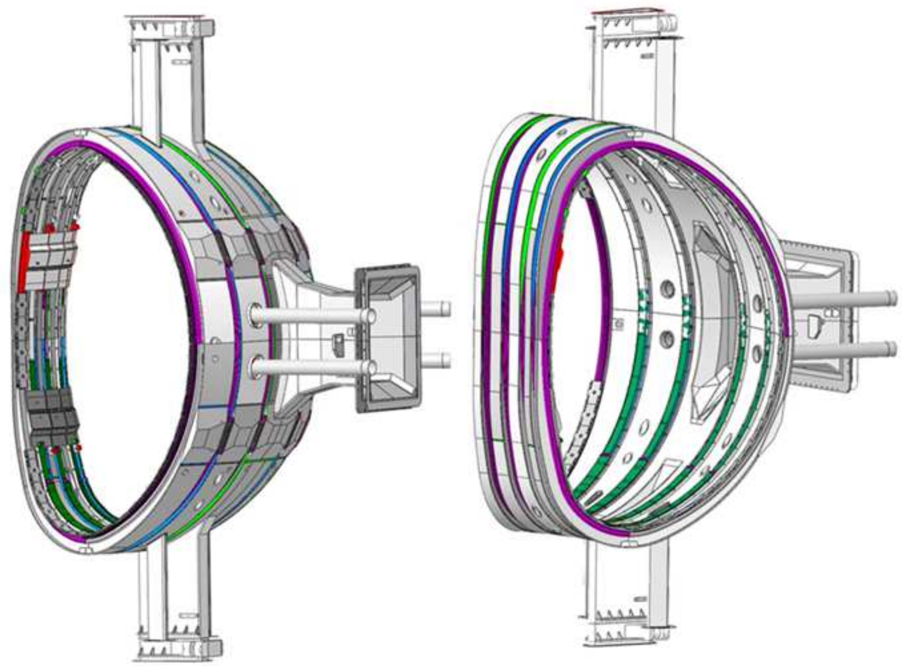
$$B_{\sim} = B_{\sqcup} - B_0 \tag{1.9}$$

**for each sensor.**

**It is assumed the $n \neq 0$ component is generated by the iron core and by eddy currents**



n=8 periodicity of JET iron core

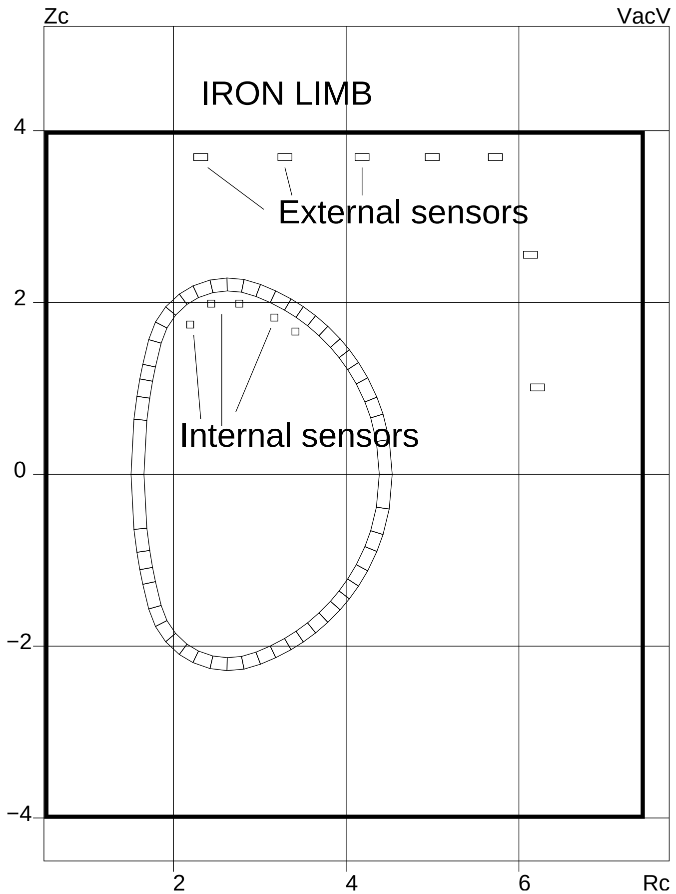


sections of JET vacuum vessel

**3-D effects are more important for sensor readings**

**rather than for plasma equilibrium**

# Sensors between vacuum vessel and iron are necessary




***Field in the opening of iron core is vacuum***

$$\vec{B} = -\nabla\phi \tag{1.10}$$

***There is a linear relationship between signals of external and internal sensors through the "correlation" matrix*** $\mathbf{C}^{iron}$

$$B_i^{int,iron} = \sum_{k=0}^{k<K} C_i^{iron,k} B_k^{ext} \tag{1.11}$$

***The same is valid for oscillating component***

$$B_{i\sim}^{int,iron} = \sum_{k=0}^{k<K} \tilde{C}_i^{iron,k} B_k^{ext} \tag{1.12}$$

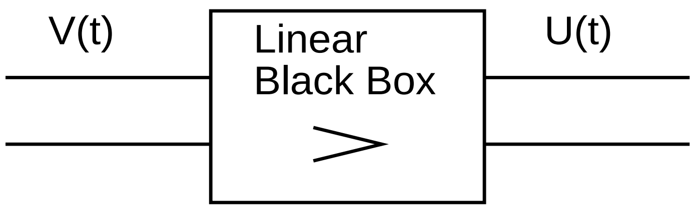
$B_{i\sim}^{int,iron}$ ***can be extracted from*** $B_{i,\sqcup}^{int,iron}$ ***using technique (1.9).***

**Correlation matrix $\tilde{\mathbf{C}}^{iron}$ can be determined using signals of calibration shots during the flat top phase**

## Time history is necessary for eliminating eddy current effects

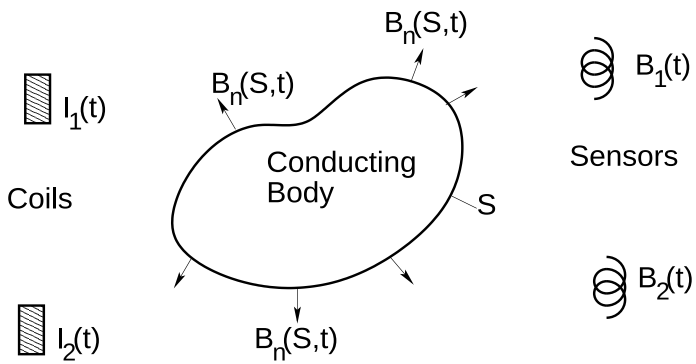
**In general, the linear causality relationships is expressed as**

***Radio-technical device***




$$U(t) = \int_0^t s(t-\tau)\frac{dV(\tau)}{d\tau}d\tau \qquad (1.13)$$

***Electrodynamic example***



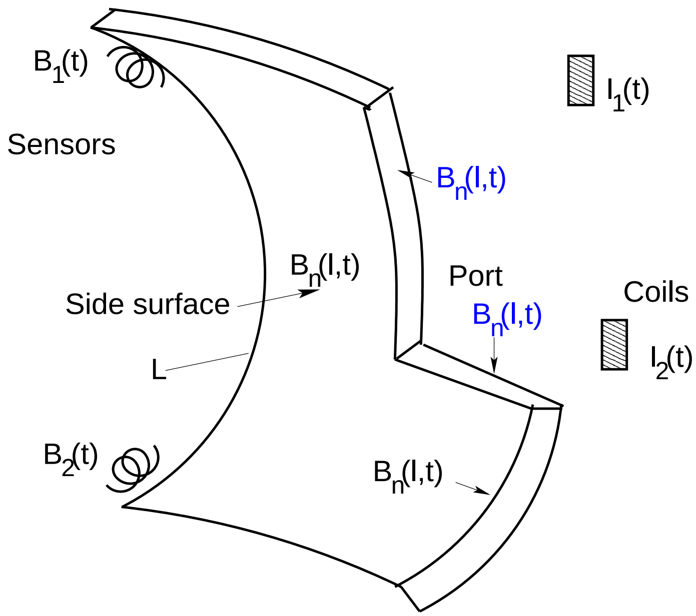

$$B_i(t) = \int_0^t \oint_S s_i(S, t-\tau)\frac{dB_n(S,\tau)}{d\tau}d\tau dS,$$

$$B_i(t) = \sum_k \int_0^t s_i^k(t-\tau)\frac{dI_k(\tau)}{d\tau}d\tau \qquad (1.14)$$

**In presence of an iron core expression (1.14) cannot be used.**

**The kernel of the integrals is the response function $s_i(t)$**

## In tokamak case the thin wall approximation is reasonable




$$B_i(t) = \int_0^t \oint_S s_i(S, t-\tau) \frac{dB_n(S,\tau)}{d\tau} d\tau dS$$

***Only side surface is essential***

$$B_i(t) = \int_0^t \oint_L s_i(l, t-\tau) \frac{d\bar{\Psi}(l,\tau)}{d\tau} d\tau dl$$

*( It is assumed that $n \neq 0$ field of the iron core does not generate eddy currents)*

***3-D contribution of eddy currents can be extracted using technique (1.9)***

$$B_{i,\sim}^{eddy}(t) = B_{i,\sqcup}(t) - B_0(t) - B_{i,\sim}^{iron}(t)$$

$$B_{i,\sim}^{eddy}(t) = \int_0^t \oint_L s_i(l, t-\tau) \frac{d\bar{\Psi}(l,\tau)}{d\tau} d\tau dl. \qquad (1.15)$$

**Calibration shots can be used for extracting response function**

**Due to discrete measurements all integrals should be replaced by summation**

**Axisymmetric solution, necessary for calibration, is obtained from a matrix equation**

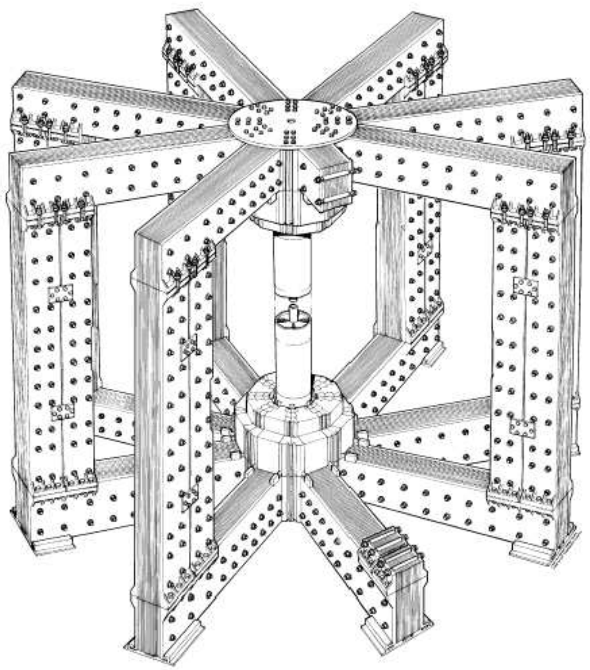
$$\sum_j^{N^v} \mathsf{L}_i^j I_j^{virtual} = \bar{\Psi}(r_i, z_i), \quad \mathsf{L}_i^j \equiv \bar{\psi}(r_i, z_i, r_j, z_j),$$
$$I_j^{virtual} = \sum_i^{N^\psi} \mathsf{L}_j^{-1,i} \bar{\Psi}(r_i, z_i) \tag{2.1}$$

**and**

$$B_{i,0} = \sum_j^{N} b(r_i, z_i; r_j, z_j) I_j^{virtual},$$
$$B_{i,0} = \sum_j^{N} b(r_i, z_i; r_j, z_j) \sum_k^{N^\psi} \mathsf{L}_j^{-1,k} \bar{\Psi}_k \tag{2.2}$$

**The final matrix should be calculated only once**

## Periodicity of iron core geometry may simplify the problem to the level of practicality



***If $n = 8$ as a dominant perturbation by the iron core***

$$\begin{aligned} \mathrm{B}^{iron} &= \nabla\phi, \\ \phi_{\sim}(r, z, \varphi) &= \phi_{\sim}(r, z)\cos 8\varphi, \end{aligned} \qquad (2.3)$$

***the number of external sensors can be reduced to 20-30 in order to get the correlation matrix***

$$B_{i\sim}^{int,iron} = \sum_{k=0}^{k<K} \tilde{C}_i^{iron,k} B_k^{ext} \qquad (2.4)$$

**Compared with air-core inductors,**

**Iron core gives more independent regimes for calibration**

**The matrix formulation is not restricted by assumption of thin wall**

$$B^{eddy}_{i,\sim}(t) = \int_0^t \oint_L s_i(l, t-\tau) \frac{d\bar{\Psi}(l,\tau)}{d\tau} d\tau \quad \rightarrow$$

$$B^{eddy}_{i,\sim}(t) = \sum_j^{N^\psi} \int_0^t s_i^j(t-\tau) \frac{d\bar{\Psi}_j(\tau)}{d\tau} d\tau. \tag{2.5}$$

**Matrix of response functions allows to use the time history**

## Finite number of sensors results in inconsistencies in measurements

**All basic relationships for calibrating the machine**

$$
\begin{aligned}
B_{i,0} &= \sum_{j}^{N} b(r_i, z_i; r_j, z_j) \sum_{k}^{N^\psi} \mathsf{L}_j^{-1,k} \bar{\Psi}_k, \\
B_{i_\sim}^{int,iron} &= \sum_{k=0}^{k<K} \tilde{C}_i^{iron,k} B_k^{ext}, \\
B_{i,\sim}^{eddy}(t) &= \sum_{j}^{N^\psi} \int_0^t s_i^j(t-\tau) \frac{d\bar{\Psi}_j(\tau)}{d\tau} d\tau
\end{aligned}
\tag{2.6}
$$

**should be treated statistically using excessive number of calibration shots.**

**This calibration approach allows to calculate standard deviations using only calibration shots**

**Cbc2e was developed for CDX-U tokamak (no iron core)**

**Because of insufficient number of flux loops on CDX-U, the $n \neq 0$ cannot be eliminated from measurements.**

**Instead, from the signal entire contribution of eddy currents, generated by the PFCoils, was eliminated**

$$B^{eddy}_{i,\sqcup}(t) = \sum_j^{N^\psi} \int_0^t s_i^j(t-\tau) \frac{dI_j^{PFC}(\tau)}{d\tau} d\tau \qquad (3.1)$$

**Cbc2e:**

1. **Solves equation (3.1) for $s_i^j(t-\tau)$**
2. **Provides the necessary service associated with the problem.**
3. **Its outputs is linked with ESC, which was extended to handle the time history.**

**Cbc2e is a good starting point to implement JET calibration**

**A rigorous approach for JET calibration is outlined**

**Using only calibration shots it can generate:**

1. ***the correlation matrix between external and internal (with respect to vacuum vessel) signals in order to eliminate the uncertainties due to the iron core,***
2. ***the matrix of response functions in order to eliminate the uncertainties due to eddy currents.***

**Both correlation matrix and response functions are applicable for plasma discharges.**

**Calibration of JET would allow to pre-process magnetic measurements for further use in reconstruction codes**