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Large angle in-plane light scattering from rough surfaces: comment

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A recent paper by Karabacak *et al*., which discussed the scattering from rough surfaces in directions out of the plane of incidence, exhibited an error in the derivation of a polarization factor. An asymmetry in the scattering function for directions out of the plane of incidence and for circularly polarized incident light is predicted by the correct derivation of this factor and can be observed in their data.

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It is sometimes assumed that circularly polarized light can be used interchangeably with unpolarized light in an optical experiment. The former is a wave whose projections along two orthogonal directions have equal amplitude and a fixed relative phase π/2, while the latter has a random phase between the two projections, varying either in time or in space. A specific example of how these two optical states are not equivalent can be found in the scattering of light from a single rough metallic interface. When light is incident on such a surface at an oblique angle with circular polarization, the intensity of light diffusely scattered towards one side of the plane of incidence differs from that scattered towards the other side. This effect can be observed in data recently published by Karabacak *et al.*[1] for scattering from an electrodeposited copper surface. However, due to an error in their derivation of a polarization factor, they overlooked this effect. This Comment seeks to point out this interesting phenomenon and the error in their derivation.

The differential intensity dP_s of light scattered by a single rough interface in the small amplitude perturbation limit is related to the power spectral density (PSD) function $S(\mathbf{k}_\parallel)$ of the surface height function by the expression[2]

$$dP_\mathrm{s} = (16\pi^2/\lambda^4)\cos\theta_\mathrm{i}\cos^2\theta_\mathrm{s}\;Q\,S(\mathbf{k}_\parallel)\,P_\mathrm{i}\,d\Omega\,, \tag{1}$$

where λ is the wavelength of the light, θ_i is the incident angle, θ_s is the scattering angle, P_i is the incident power, and dΩ is the differential solid angle. The surface wavevector $\mathbf{k}_\parallel$ probed by a specific scattering geometry is given by the Bragg relationship and has components

$$k_x = 2\pi(\sin\theta_\mathrm{s}\cos\phi_\mathrm{s} - \sin\theta_\mathrm{i})/\lambda\,, \tag{2a}$$

$$k_y = 2\pi\sin\theta_\mathrm{s}\sin\phi_\mathrm{s}/\lambda\,. \tag{2b}$$

The factor Q is a factor derived from the scattering matrix elements,

$$q_{ss} = \frac{(1-\varepsilon)\cos\phi_\mathrm{s}}{(\cos\theta_\mathrm{i}+\sqrt{\varepsilon-\sin^2\theta_\mathrm{i}})(\cos\theta_\mathrm{s}+\sqrt{\varepsilon-\sin^2\theta_\mathrm{s}})}\,, \tag{3a}$$

$$q_{sp} = \frac{(\varepsilon-1)\sqrt{\varepsilon-\sin^2\theta_\mathrm{i}}\,\sin\phi_\mathrm{s}}{(\varepsilon\cos\theta_\mathrm{i}+\sqrt{\varepsilon-\sin^2\theta_\mathrm{i}})(\cos\theta_\mathrm{s}+\sqrt{\varepsilon-\sin^2\theta_\mathrm{s}})}\,, \tag{3b}$$

$$q_{ps} = \frac{(1-\varepsilon)\sqrt{\varepsilon - \sin^2\theta_\mathrm{s}}\,\sin\phi_\mathrm{s}}{(\cos\theta_\mathrm{i} + \sqrt{\varepsilon-\sin^2\theta_\mathrm{i}})(\varepsilon\cos\theta_\mathrm{s} + \sqrt{\varepsilon - \sin^2\theta_\mathrm{s}})}, \tag{3c}$$

$$q_{pp} = \frac{(\varepsilon-1)(\varepsilon\sin\theta_\mathrm{i}\sin\theta_\mathrm{s} - \sqrt{\varepsilon-\sin^2\theta_\mathrm{i}}\sqrt{\varepsilon-\sin^2\theta_\mathrm{s}}\cos\phi_\mathrm{s})}{(\varepsilon\cos\theta_\mathrm{i} + \sqrt{\varepsilon-\sin^2\theta_\mathrm{i}})(\varepsilon\cos\theta_\mathrm{s} + \sqrt{\varepsilon - \sin^2\theta_\mathrm{s}})}, \tag{3d}$$

where ε is the complex dielectric function of the material evaluated at the wavelength λ, ϕ_s is the azimuthal angle of scattering, the first index represents the polarization of the scattered light, while the second index represents the polarization of the incident light.[3] When p-polarized light is incident upon a sample, for example, the factor Q is

$$Q_p = |q_{pp}|^2 + |q_{sp}|^2. \tag{4}$$

The article by Karabacak *et al.*[1] describes a measurement whereby circularly polarized light was incident upon the sample, and the intensities scattered by rough silicon and copper surfaces were measured with a polarization-insensitive detector. The measurements were performed by varying ϕ_s with fixed $\theta_\mathrm{i} = \theta_\mathrm{s}$, in what they refer to as an "in-plane" geometry.[3] For right-hand and left-hand circularly polarized light incident upon the sample, the respective factors Q are given by

$$Q_R = |q_{RR}|^2 + |q_{LR}|^2, \tag{5a}$$

$$Q_L = |q_{RL}|^2 + |q_{LL}|^2, \tag{5b}$$

where the matrix elements in the circular basis are related to those in the plane-polarization basis by

$$q_{RR} = [-q_{pp} + q_{ss} + i(q_{ps} + q_{sp})]/2, \tag{6a}$$

$$q_{RL} = [q_{pp} + q_{ss} - i(q_{sp} - q_{ps})]/2, \tag{6b}$$

$$q_{LR} = [q_{pp} + q_{ss} + i(q_{sp} - q_{ps})]/2, \tag{6c}$$

$$q_{LL} = [-q_{pp} + q_{ss} - i(q_{ps} + q_{sp})]/2. \tag{6d}$$

Substituting Eqs. (6) into Eqs. (5) and performing some algebra, we arrive at

$$\begin{aligned} Q_R = \Big(&|q_{ss}|^2 + |q_{pp}|^2 + |q_{sp}|^2 + |q_{ps}|^2 \\ &+2\,\mathrm{Im}\,q_{ps}\,\mathrm{Re}\,q_{pp} - 2\,\mathrm{Im}\,q_{pp}\,\mathrm{Re}\,q_{ps} \\ &+2\,\mathrm{Im}\,q_{ss}\,\mathrm{Re}\,q_{sp} - 2\,\mathrm{Im}\,q_{sp}\,\mathrm{Re}\,q_{ss}\Big)/2, \end{aligned} \tag{7a}$$

$$\begin{aligned} Q_L = \Big(&|q_{ss}|^2 + |q_{pp}|^2 + |q_{sp}|^2 + |q_{ps}|^2 \\ &-2\,\mathrm{Im}\,q_{ps}\,\mathrm{Re}\,q_{pp} + 2\,\mathrm{Im}\,q_{pp}\,\mathrm{Re}\,q_{ps} \\ &-2\,\mathrm{Im}\,q_{ss}\,\mathrm{Re}\,q_{sp} + 2\,\mathrm{Im}\,q_{sp}\,\mathrm{Re}\,q_{ss}\Big)/2. \end{aligned} \tag{7b}$$

These expressions for Q_R and Q_L differ from those given after Eqs. (A14) and (A15) of Karabacak *et al.*[1] They determined, incorrectly, that these factors should be identical to the factor appropriate for unpolarized incident light,

$$Q_U = \left(|q_{ss}|^2 + |q_{pp}|^2 + |q_{sp}|^2 + |q_{ps}|^2\right)/2. \tag{8}$$

Such an error can be made if one ignores the relative phases between the different matrix elements q_{ss}, q_{pp}, q_{sp}, and q_{ps}.

The factors Q_R and Q_L are only equal to Q_U in certain cases, such as when $\varepsilon - 1$ is real and positive, or in the plane of incidence, where q_{sp} and q_{ps} are zero. For silicon at the wavelength used in the study, λ = 633 nm, the dielectric function is[4] ε = 15.1 + 0.1i, which approximately satisfies the first of these conditions. For copper, the dielectric function is[4] $\varepsilon = -11.6 + 1.7\mathrm{i}$, and the cross terms in Eq. (7) do not vanish. Copper at visible wavelengths is not a perfect conductor, so that using the perfectly conducting form of the scattering matrix would be a poor approximation. If the power spectrum of the roughness is isotropic [$S(\mathbf{k}_{\parallel}) = S(k_{\parallel})$], the cross terms in Eqs. (7) lead to a difference between the scattering intensity for positive and negative ϕ_s for either left or right circularly polarized light incident at an oblique angle. This effect is only evident for non-zero ϕ_s, that is, in directions out of the plane of incidence, a geometry that exhibits handedness.

The predicted asymmetry can be observed in the data of Karabacak *et al*. Figure 1 shows their data, for scattering from silicon and copper, as originally plotted and for $\phi_s \to -\phi_s$. The incident light was right-circularly polarized (clockwise when viewed in the direction of propagation[3]). The data for the rough backside of a silicon wafer shows a symmetric scattering distribution. In comparison, a distinct asymmetry to the scattering function for the electroplated copper sample can be observed. Figure 2 shows the $Q_R(\phi_s)/Q_R(-\phi_s)$ predicted by Eqs. (3) and (7) compared to that extracted from the data. The sign of the effect is reproduced, while the absolute magnitude of the effect is underestimated. To account for an oxide film which may have been present, we have calculated the predicted behavior of a 12 nm conformal Cu_2O film using small amplitude perturbation theory.[5] The existence of a film, however, is difficult to account for without knowing its thickness, its composition, and the statistics of the roughness functions of the two interfaces.

It is relatively easy to understand why incident light polarized linearly at 45° from either *p*- or *s*-polarization would generate an asymmetric scattering pattern.[6] Induced dipoles in the material will be preferentially aligned in the direction of the local electric field, and will radiate perpendicular to those dipoles. The intensity of the light will therefore be lower on that side of the plane of incidence pointed to by the electric field. This effect can be observed even for insulating rough surfaces. For circularly polarized incident light, an understanding of the effect is more subtle. Circularly polarized light incident upon a metallic surface or a surface with films generally becomes elliptically polarized upon reflection with its major axis tilted from the plane of incidence. The side towards which the major axis becomes tilted depends upon the handedness of the incident circular polarization. As for linearly polarized incident light, the material would be expected to radiate less in the direction of the major axis of the reflected polarization ellipse.

Finally, a typographical error exists in the q_{sp} in Eq. (A12) of Karabacak, *et al*. The expression for q_{sp} should read as shown above in Eq. (3b) or in Ref. 2.

I would like to thank Tansel Karabacak for sharing their original data and John C. Stephenson for some insightful comments.

REFERENCES AND NOTES

1. T. Karabacak, Y. Zhao, M. Stowe, B. Quayle, G.-C. Wang, and T.-M. Lu, "Large-angle in-plane light scattering from rough surfaces," Appl. Opt. **39,** 4658-4668 (2000).

2. D. E. Barrick, "Rough Surfaces," in *Radar Cross Section Handbook*, George T. Ruck, Ed. (Plenum, New York, 1970) Chap. 9.

3. Some conventions used by Ref. 1 differ from those often used in other works, including those used for the order of the indices of matrix elements, the definition of handedness of circularly polarized light, and the use of the terms "in-plane" and "out-of-plane." In this Comment, I have attempted to follow the conventions used by Ref. 1.

4. E. D. Palik, *Handbook of Optical Constants of Solids*, (Academic, San Diego, 1985).

5. J. M. Elson, "Light scattering from surfaces with a single dielectric overlayer," J. Opt. Soc. Am. **66,** 682-694 (1976).

6. T. Kawanishi, H. Ogura, and Z. L. Wang, "Polarization characteristics of an electromagnetic wave scattered from a slightly random surface: +45° polarized incidence," Waves in Random Media **7,** 593-605 (1997).

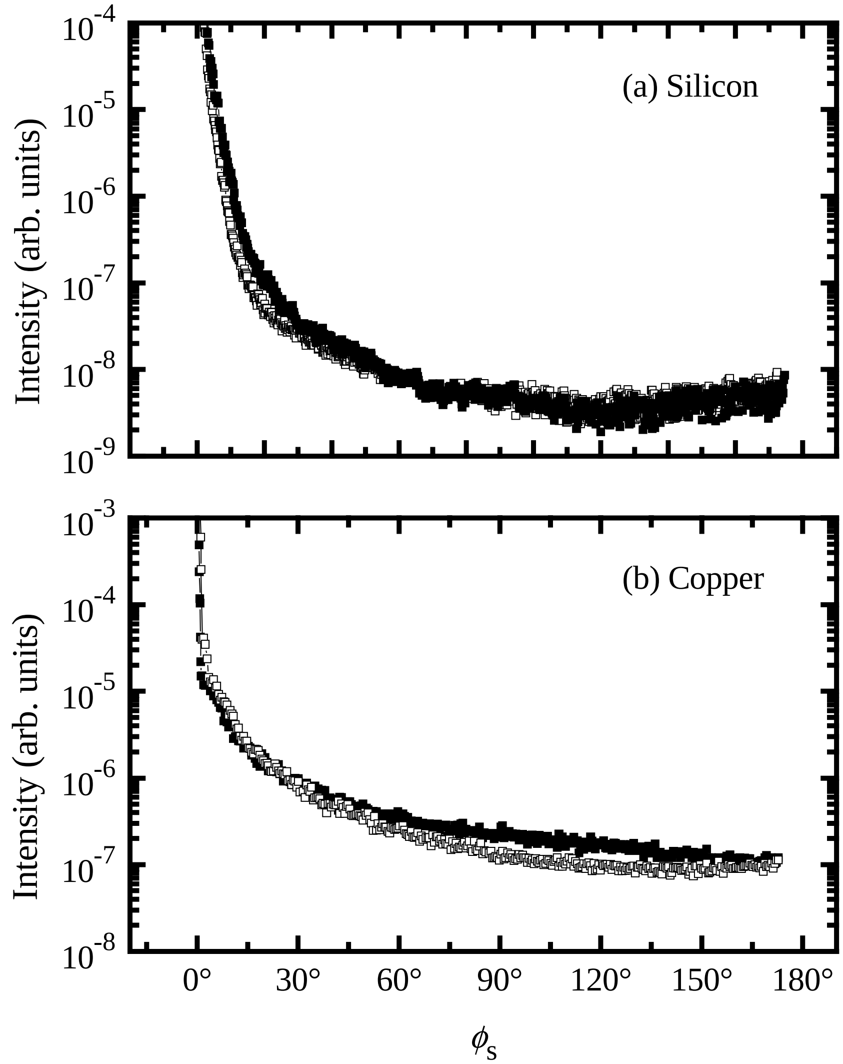

Fig. 1. (a) Normalized scattering function at an incident angle of 83° as a function of azimuthal scattering angle ϕ_s from the rough backside of a silicon wafer, shown with the original data (open symbols) and after the transformation $\phi_s \rightarrow -\phi_s$ (closed symbols). (b) Normalized scattering function at an incident angle of 52° as a function of azimuthal scattering angle ϕ_s from an electroplated copper film, shown with the original data (open symbols) and after the transformation $\phi_s \rightarrow -\phi_s$ (closed symbols). Data are from Ref. 1.

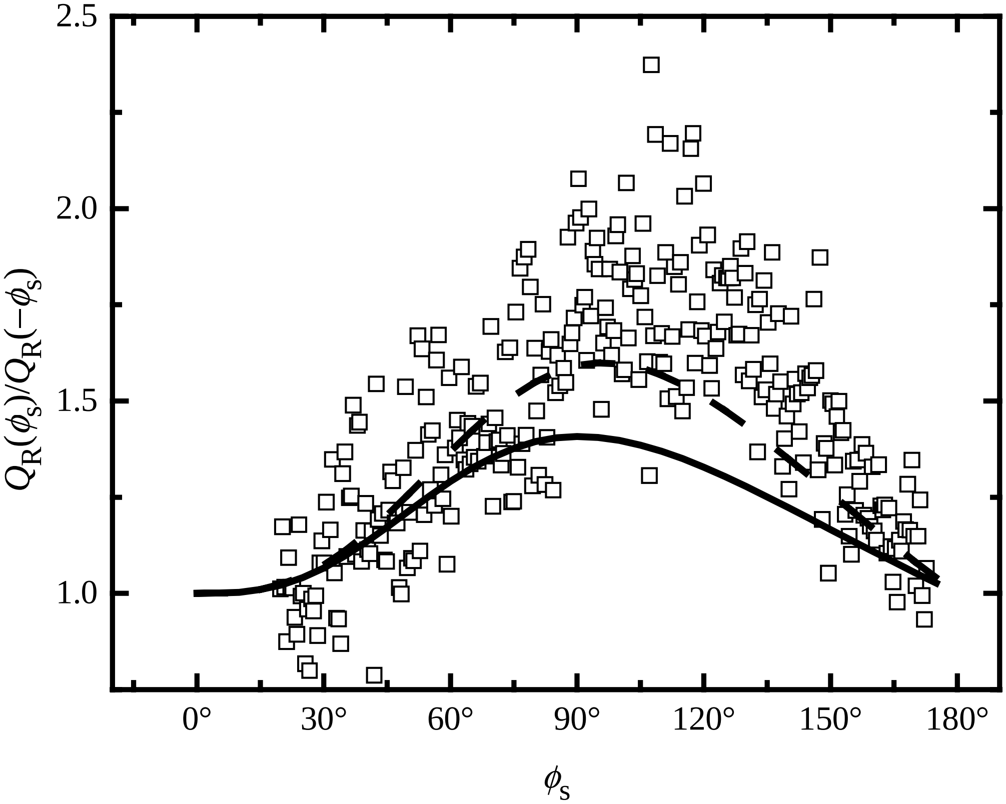

Fig. 2. The ratio $Q_R(\phi_s)/Q_R(-\phi_s)$ measured by Karabacak, *et al*. (points), calculated from Eqs. (3) and (7) (solid curve), and calculated for a 12 nm conformal Cu_2O film.