

# Global Analysis of Fragmentation Functions

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Global Analysis of Polarized Parton Distributions in the RHIC Era

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# Disclaimer

We (TH) can compute observables in pQCD with high precision

But about errors ..hmmmmmmm

$$1.1 \pm 0.1 \pm 0.2$$

$$1.1 \pm 0.1 \pm 0.2$$

$$1.1 \pm \sqrt{0.1^2 + 0.2^2}$$

# Fragmentation functions

Represent the probability that a parton hadronizes in  $h$

From TH point of view at the same level of pdfs

Relevant any time a hadron is produced in high energy collisions

$e^+e^-$  : primary “source”

SIDIS : complement DIS to allow flavor separation

pp collisions: signal and “background” for a lot of physics

Heavy ions  
polarized pdfs

Global Fit (DSS) that includes all those processes

In the framework of this workshop:  
Fragmentation functions play a very relevant role

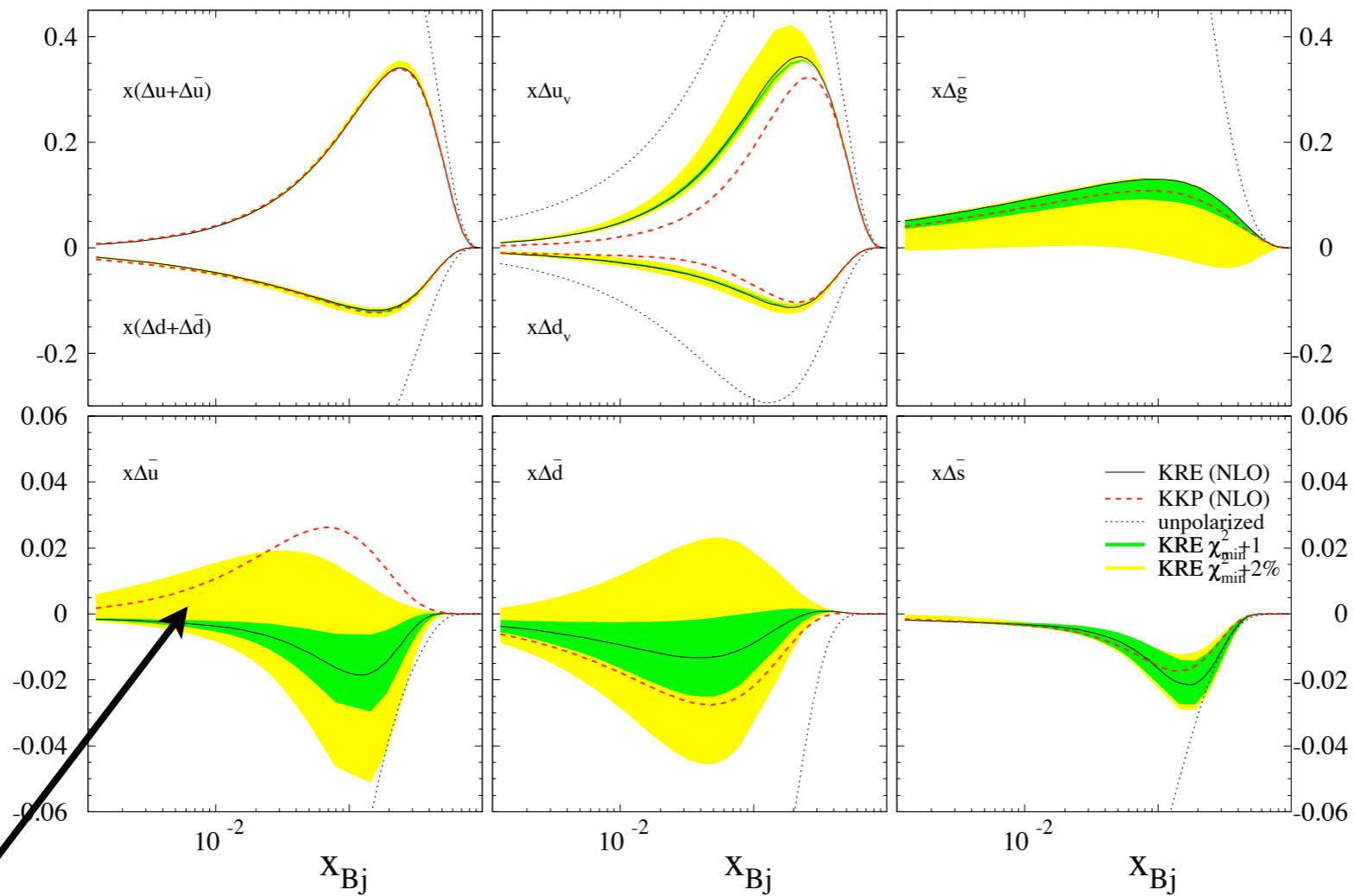
Single hadron production in polarized pp collisions great tool to  
unveil the gluon distribution : gluons enter at LO

Precise FF needed to perform pdf extraction

Otherwise: “error” in FF propagates to pdf

Trivial example: if gluon FF too small, the mistake in analysis of pp  
collisions will result in a gluon pdfs too large to compensate!

# Actual example: analysis of polarized pdfs from SIDIS using different FF (see Rodolfo's talk)

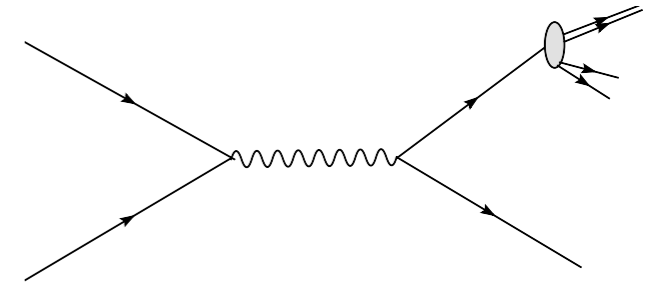


DdeF, G.Navarro, R.Sassot

Using Kretzer or KKP can lead to very different sea distributions

# $e^+e^-$ (SIA) single-inclusive annihilation

$$e^+e^- \rightarrow (\gamma, Z) \rightarrow H$$



Cross section depends on two structure functions

$$\frac{1}{\sigma_{tot}} \frac{d\sigma^H}{dz} = \frac{\sigma_0}{\sum_q \hat{e}_q^2} [2 F_1^H(z, Q^2) + F_L^H(z, Q^2)]$$

$$\sigma_{tot} = \sum_q \hat{e}_q^2 \sigma_0 \left[ 1 + \frac{\alpha_s(Q^2)}{\pi} \right]$$

At NLO they can be written as

$$2F_1^H(z, Q^2) = \sum_q \hat{e}_q^2 \left\{ [D_q^H(z, Q^2) + D_{\bar{q}}^H(z, Q^2)] + \frac{\alpha_s(Q^2)}{2\pi} [C_q^1 \otimes (D_q^H + D_{\bar{q}}^H) + C_g^1 \otimes D_g^H](z, Q^2) \right\}$$

Fragmentation functions depend on both energy fraction ( $z$ ) and energy scale : AP evolution

$$\frac{d}{d \ln Q^2} \vec{D}^H(z, Q^2) = [\hat{P}^{(T)} \otimes \vec{D}^H](z, Q^2)$$

Notice that SIA can only give information on the sum  $D_q^H(z, Q^2) + D_{\bar{q}}^H(z, Q^2)$

# $e^+e^-$ (SIA) single-inclusive annihilation

## Advantages

Very precise data from LEP/SLD

Heavy quark tagged data

Only fragmentation functions enter (clean process)

## Disadvantages

SIA data **dominated** by **precise** LEP/SLD measurements at  $M_Z$

weak scale dependence (bad resolution for g fragmentation)

**mostly determine “singlet” distribution (high precision)**

$$\Sigma = D_u + D_{\bar{u}} + D_d + D_{\bar{d}} + D_s + D_{\bar{s}} + D_c + D_{\bar{c}} + D_b + D_{\bar{b}}$$

not precise at large  $z$  (relevant for pp collisions)

Can not separate  $D_q^h(z, Q^2)$  from  $D_{\bar{q}}^h(z, Q^2)$

$$D_{q+\bar{q}}^{h^+}(z, Q^2) = D_{q+\bar{q}}^{h^-}(z, Q^2) \quad \text{flavor/charge average}$$

$$D_q^{h^++h^-}(z, Q^2) = D_{\bar{q}}^{h^++h^-}(z, Q^2)$$

Some ansatz needed if only SIA data used, like “linear suppression”

$$D_{\bar{q}}^{h^+}(z, Q^2) = (1 - z) D_q^{h^+}(z, Q^2)$$

## Recent NLO analyses

CGGRW(1994), BFGW(2000)

L.Bourhis et al.,  
Eur.Phys.J C19 (2001) 89.

BKK(1995), KKP(2000), AKK(2005)

S. Albino et al.,  
Nuc.Phys.B725 (2005) 206.

KRE(2000)

S. Kretzer, Phys. Rev.D62 (2000) 0540001.

HKNS(2007)

M. Hirai et al., Phys. Rev.D75 (2007) 094009.

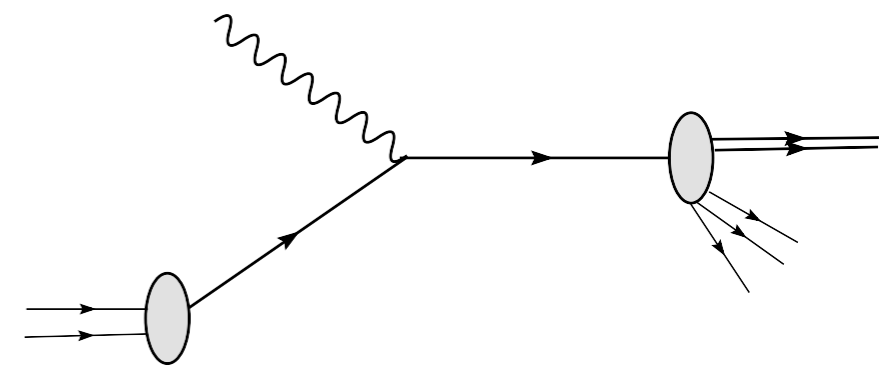
**use only SIA: no separation or ad-hoc assumption**

And some “reasonable” ansatz about charge separation don’t work well



# SIDIS

## Distributions in $x$ and $z$



$$\frac{d\sigma^H}{dx dy dz_H} = \frac{2\pi\alpha^2}{Q^2} \left[ \frac{(1 + (1 - y)^2)}{y} 2F_1^H(x, z_H, Q^2) + \frac{2(1 - y)}{y} F_L^H(x, z_H, Q^2) \right]$$

at LO

$$2F_1^H(x, z_H, Q^2) = \sum_{q, \bar{q}} e_q^2 \cdot q(x, Q^2) D_q^H(z_H, Q^2)$$

“effective charge”

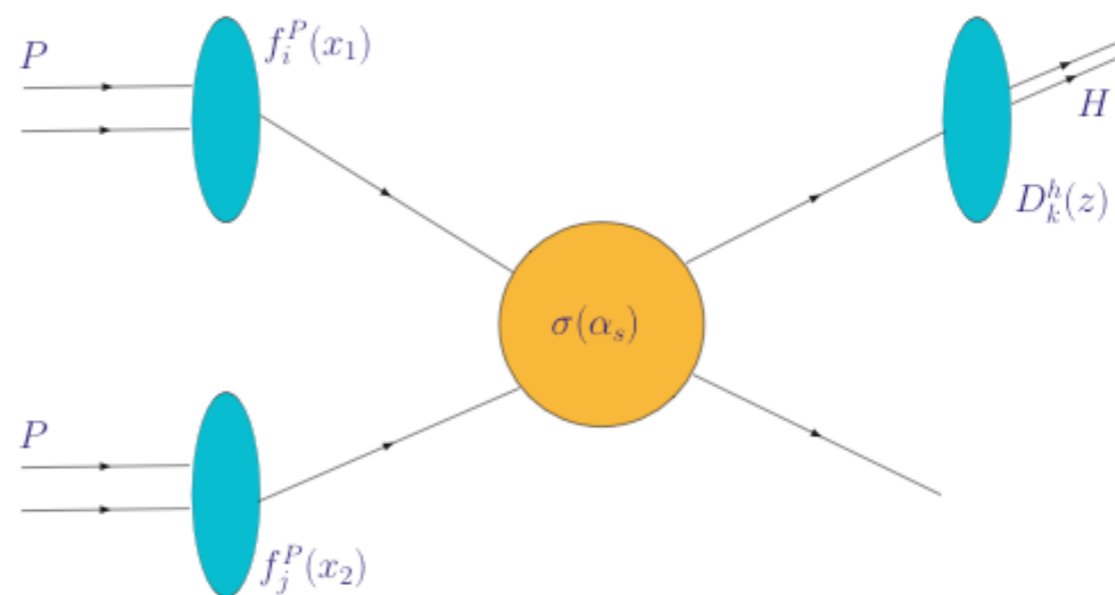
**Advantages :** allows flavor/charge separation  
larger  $z$   
smaller  $Q^2$ , improves scale coverage in evolution :  $D_g$   
Hermes and Compass kinematics

**Disadvantage :** would introduce “dependence” on pdfs (but unpolarized pdfs very well constrained from DIS in the same kinematical range)  
non-perturbative corrections at small  $Q^2$ ?

# Hadron-Hadron collisions

## Transverse momentum distribution

hard scale



$$d\sigma(pp \rightarrow h) = \sum_{i,j,k} \int_0^1 dx_1 f_i^P(x_1, \mu_{FI}^2) \int_0^1 dx_2 f_j^P(x_2, \mu_{FI}^2) \int_0^1 dz D_k^h(z, \mu_{FF}^2) d\hat{\sigma}(ij \rightarrow k)$$

$$q\bar{q} \rightarrow g$$

$$qq \rightarrow q$$

$$qg \rightarrow g$$

$$qg \rightarrow q$$

$$gg \rightarrow q$$

$$gg \rightarrow g$$

**Advantages :** allows flavor/charge separation from hadron measurements

several subprocesses

much larger z

large contribution from  $D_g$

analysis of data allows to have FF that work at RHIC and

can be used in polarized/heavy ion collisions!

**Disadvantage :** Problems for fixed target experiments, use only colliders

Otherwise Threshold resummation needed (**not in first approach**)

larger TH uncertainty (scale dependence)

# Global FIT

**Advantages :** Constrain FF with almost all available data  
Check of pQCD framework  
Precise determination of distributions and estimation of uncertainties

**Disadvantage :** more work required !  
Feasible if Mellin technique used (Marco's talk)  
Tension between different observables requires careful analysis

Complicated observables: without simple NLO interpretation  
involving MC  
“averaged” bins  
large scale dependence  
Weights

# DSS global analysis

fragmentation functions for  $\pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \bar{K}^0, p, \bar{p}, n, \bar{n}, h^+, h^-$   
residual

$$h^+ = \pi^+ + K^+ + p + res^+$$

LO and NLO global fits available

SIA data includes: TPC, TASSO, SLD, ALEPH, DELPHI, OPAL +  
“flavor” tag

SIDIS data from : HERMES, EMC

pp data from : PHENIX, STAR, BRAHMS, CDF, UAI, UA2

Checks with other data sets: STAR,  $p_T$  distributions at HERA (HI)

Estimation of uncertainties using Lagrange multipliers

# Technical details

## Flexible parametrization

$$D_i^H(z, Q_0^2) = N_i z^{\alpha_i} (1 - z)^{\beta_i} [1 + \gamma_i (1 - z)^{\delta_i}]$$

at initial scale

$$Q_0^2 = 1 \text{ GeV}^2 \quad u, d, s, g$$

$$Q_0^2 = m_Q^2 \quad c, b$$

with

$\alpha_s$  and  $\Lambda_{QCD}$  from MRST

Normalizations for different experiments (if not included in syst.)

Try to avoid Isospin symmetry assumptions

Allowing for possible breaking  
of SU(3) of sea and SU(2) in  
favored distributions

$$D_{d+\bar{d}}^{\pi^+} = N D_{u+\bar{u}}^{\pi^+}$$

$$D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} = N' D_{\bar{u}}^{\pi^+}$$

unless data can not discriminate  
for unfavored fragmentations

$$D_{\bar{u}}^{\pi^+} = D_d^{\pi^+}$$

$$D_{\bar{u}}^{K^+} = D_s^{K^+} = D_d^{K^+} = D_{\bar{d}}^{K^+}$$

# Some plots

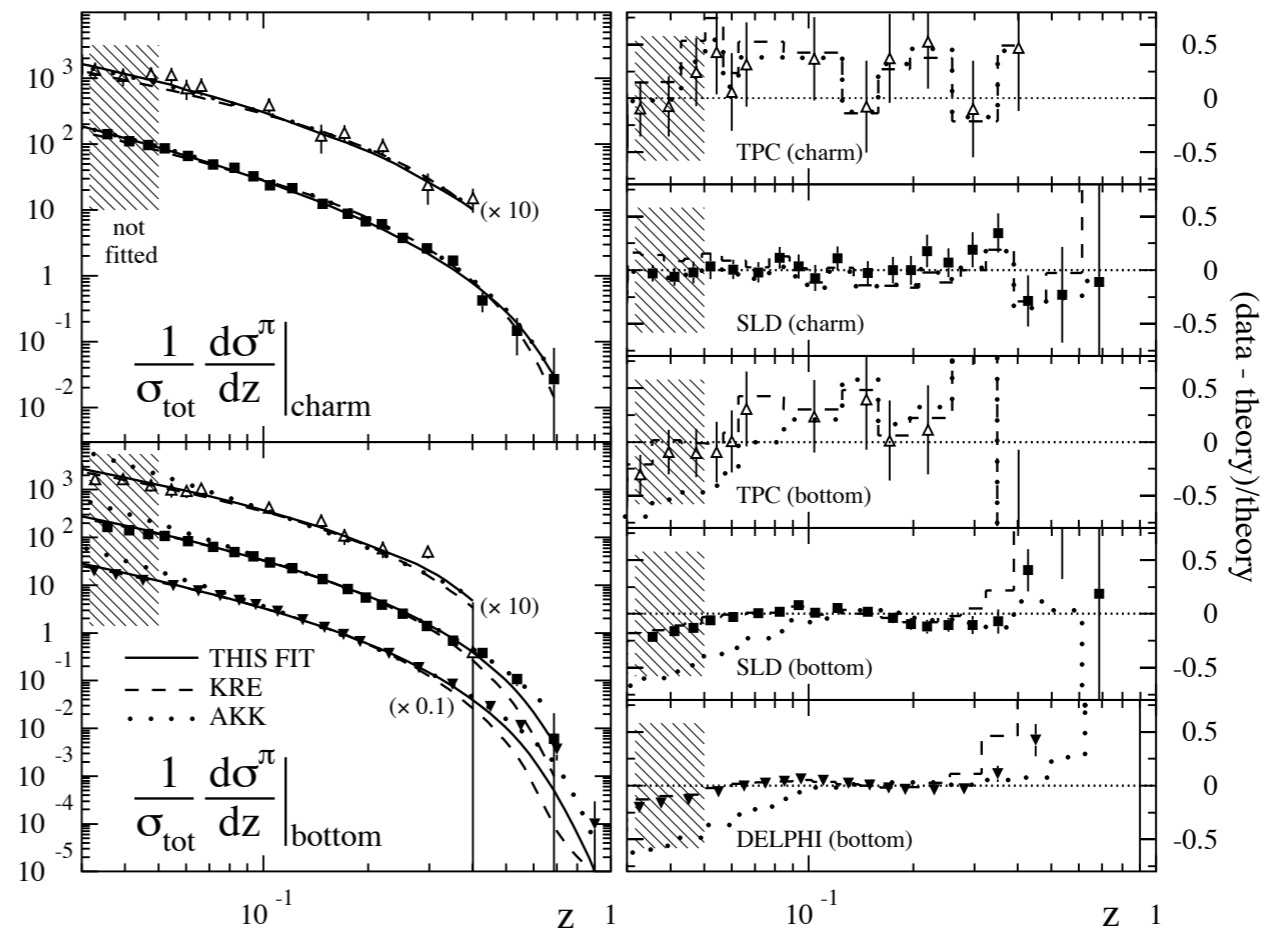
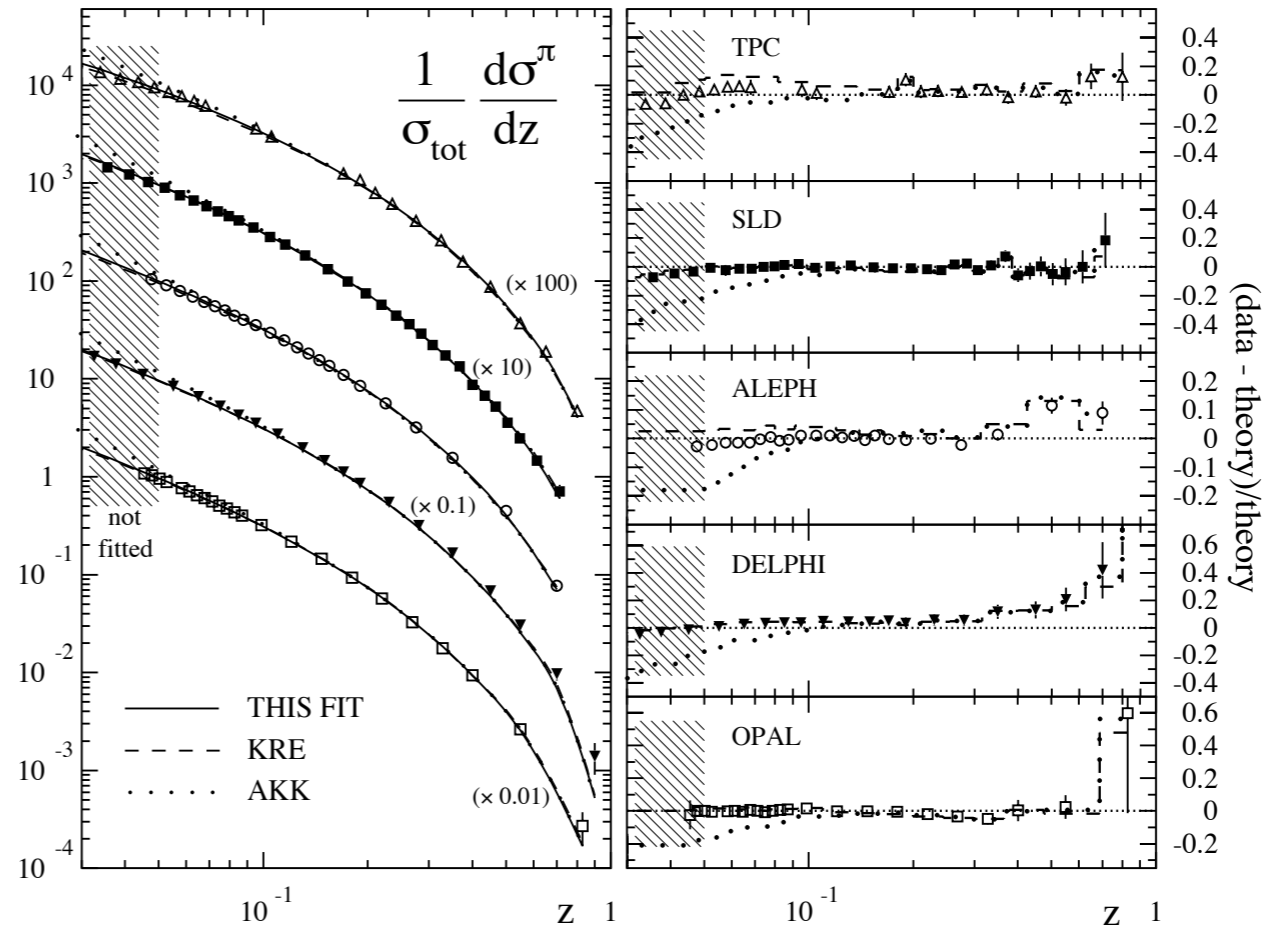
$z > 0.05$  (0.1 for Kaons/Protons)

SIA: still works very well within the global fit

“inclusive”

Large errors at  $z > 0.5$

heavy quark tagged

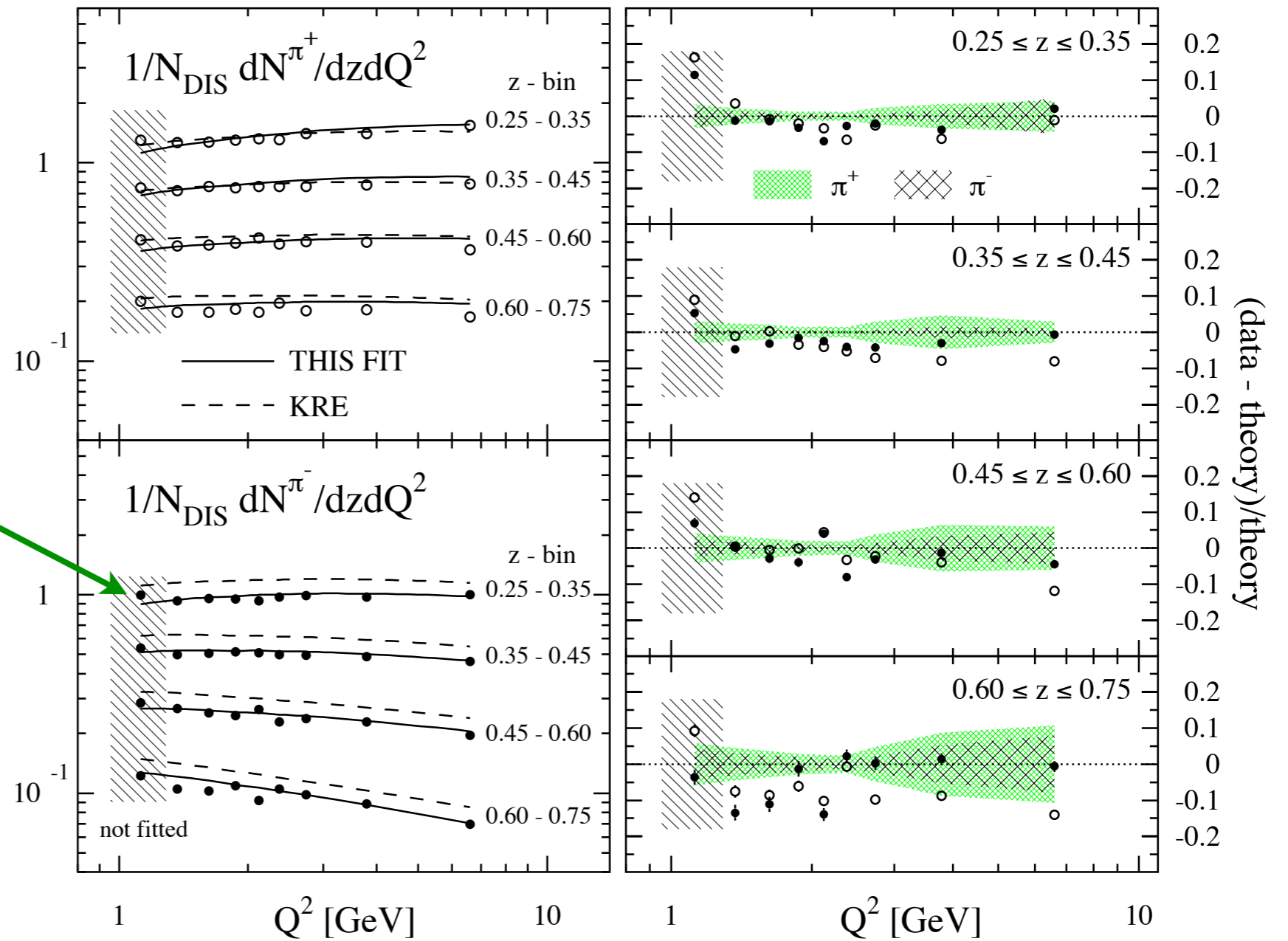


✓ extrapolation to smaller  $z$

# SIDIS

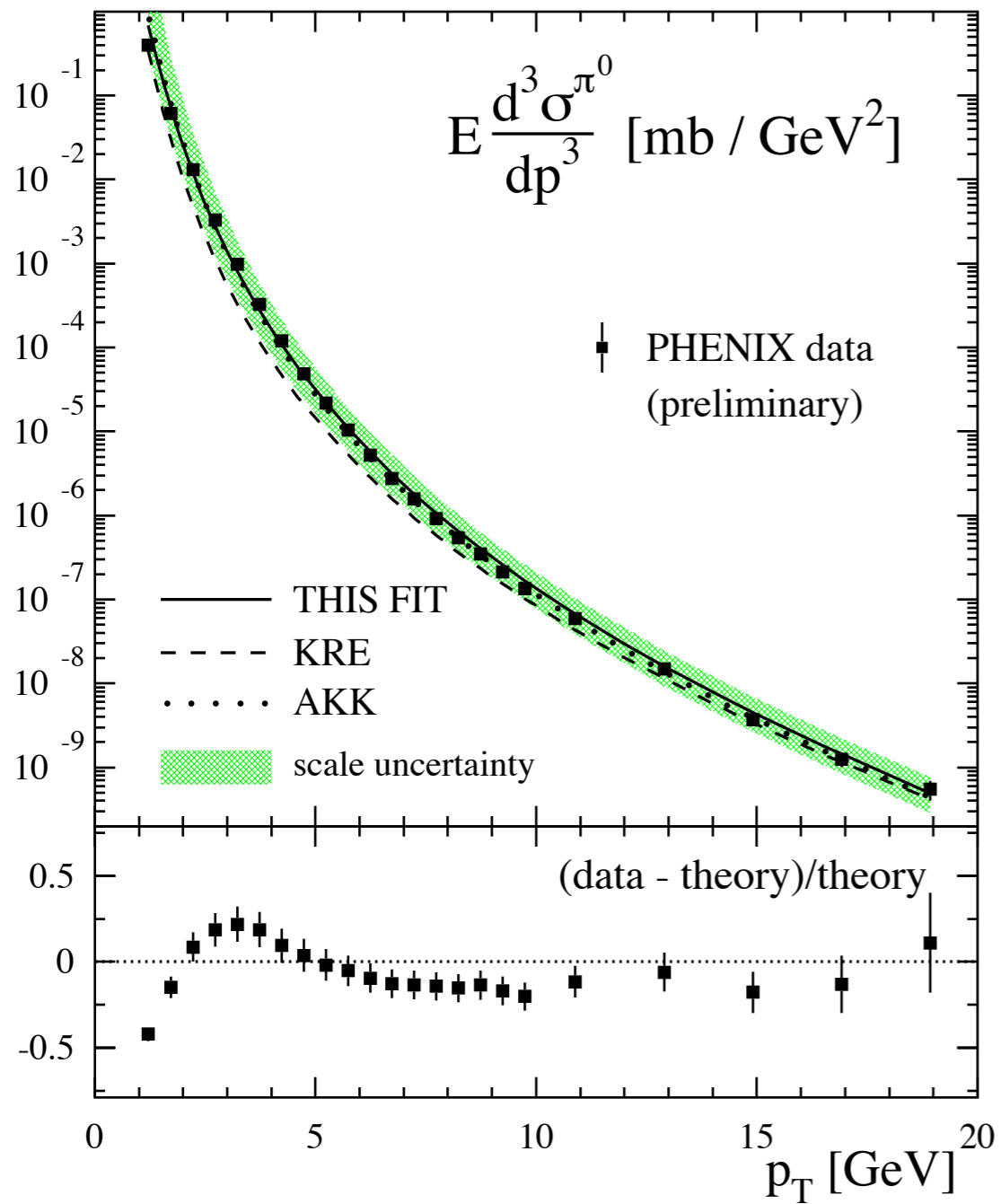
## Hermes

ad-hoc charge separation  
from Kretzer fails



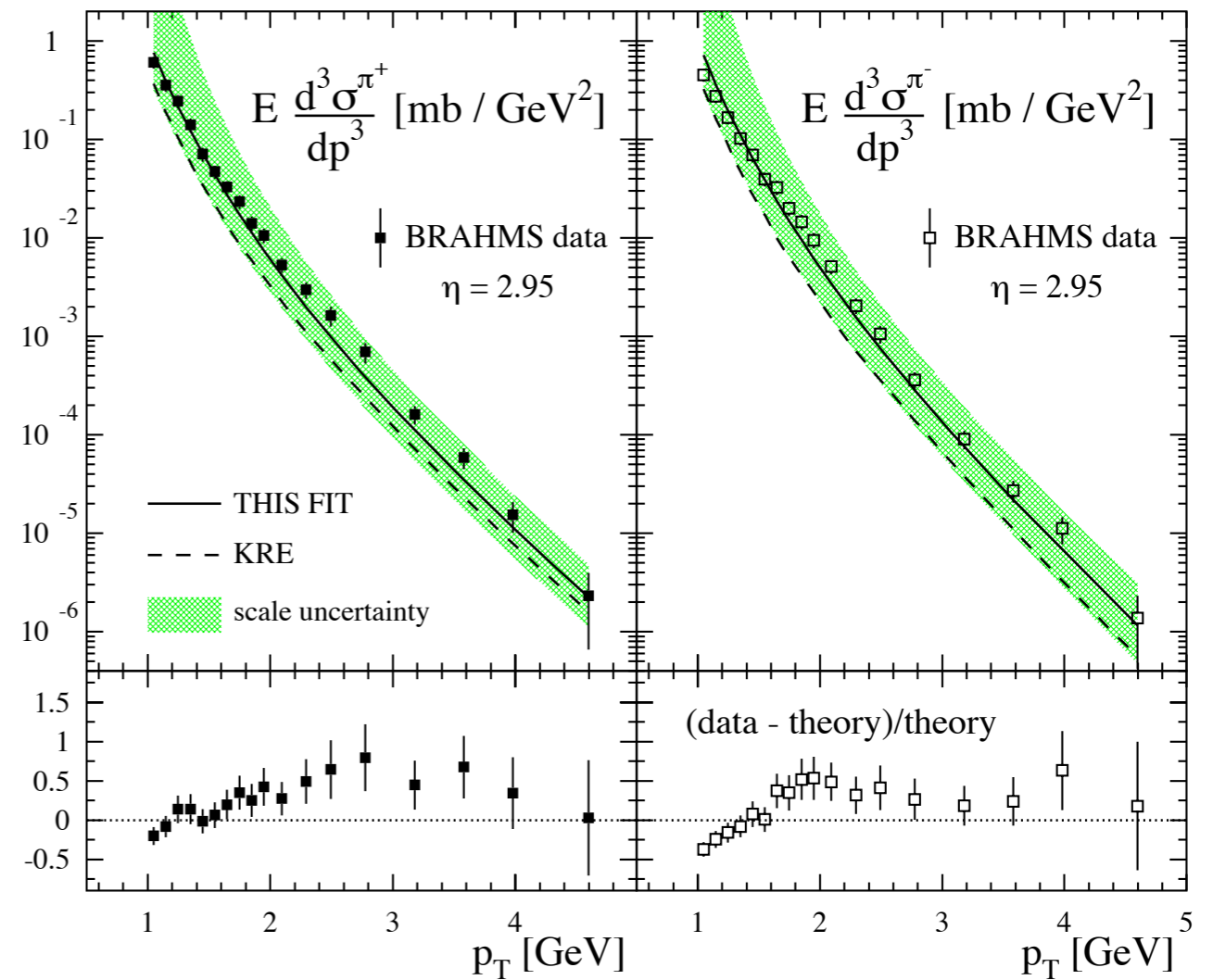
large  $z$  covered

# pp data



Neutral pions at Phenix

$$\mu_F = \mu_R = p_T$$

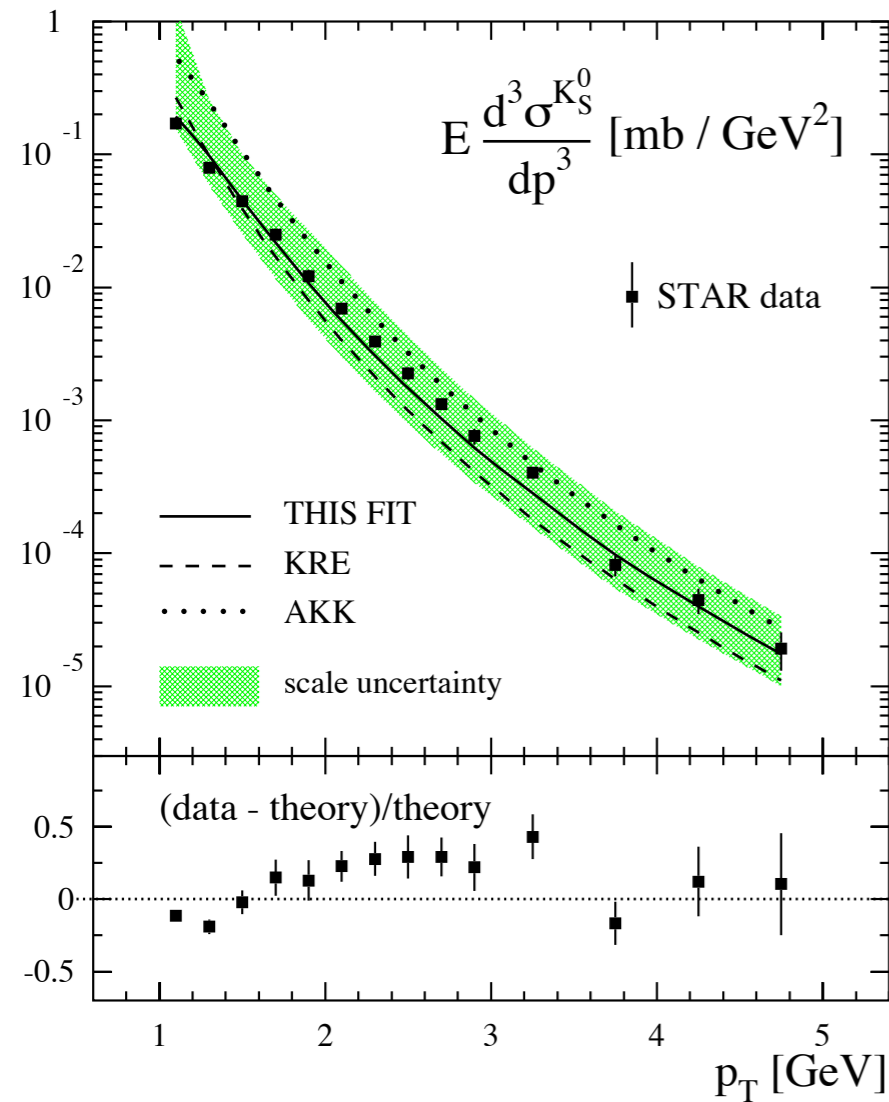


Charged pions (at large z)

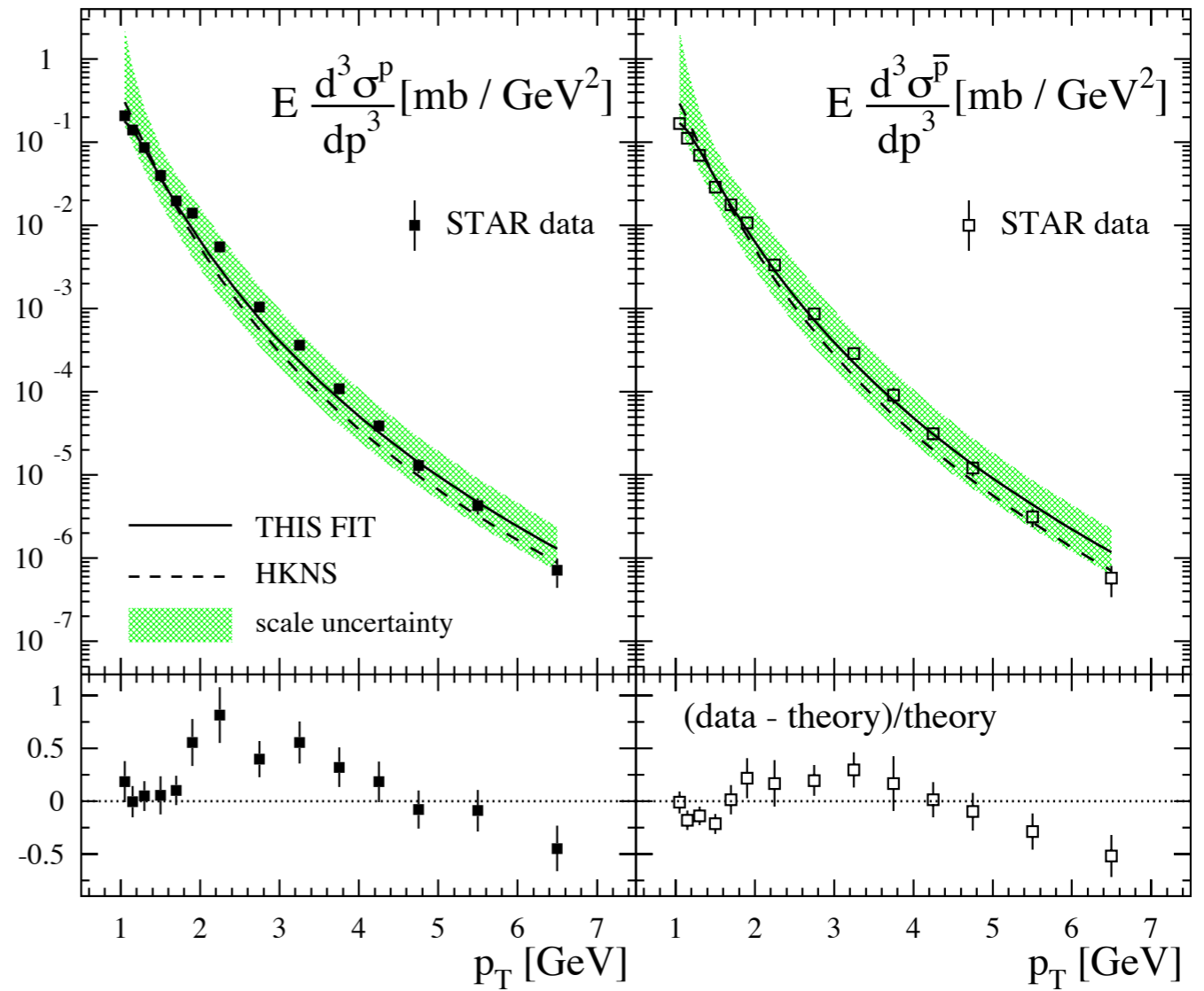


# pp data

## Kaons



## Protons



Good agreement with all RHIC data and visible differences with other sets

Typically  $\chi^2/dof \sim 2$

Large  $\chi^2$  from a few isolated data points (small  $z$  in SIA, and some SIDIS and pp)

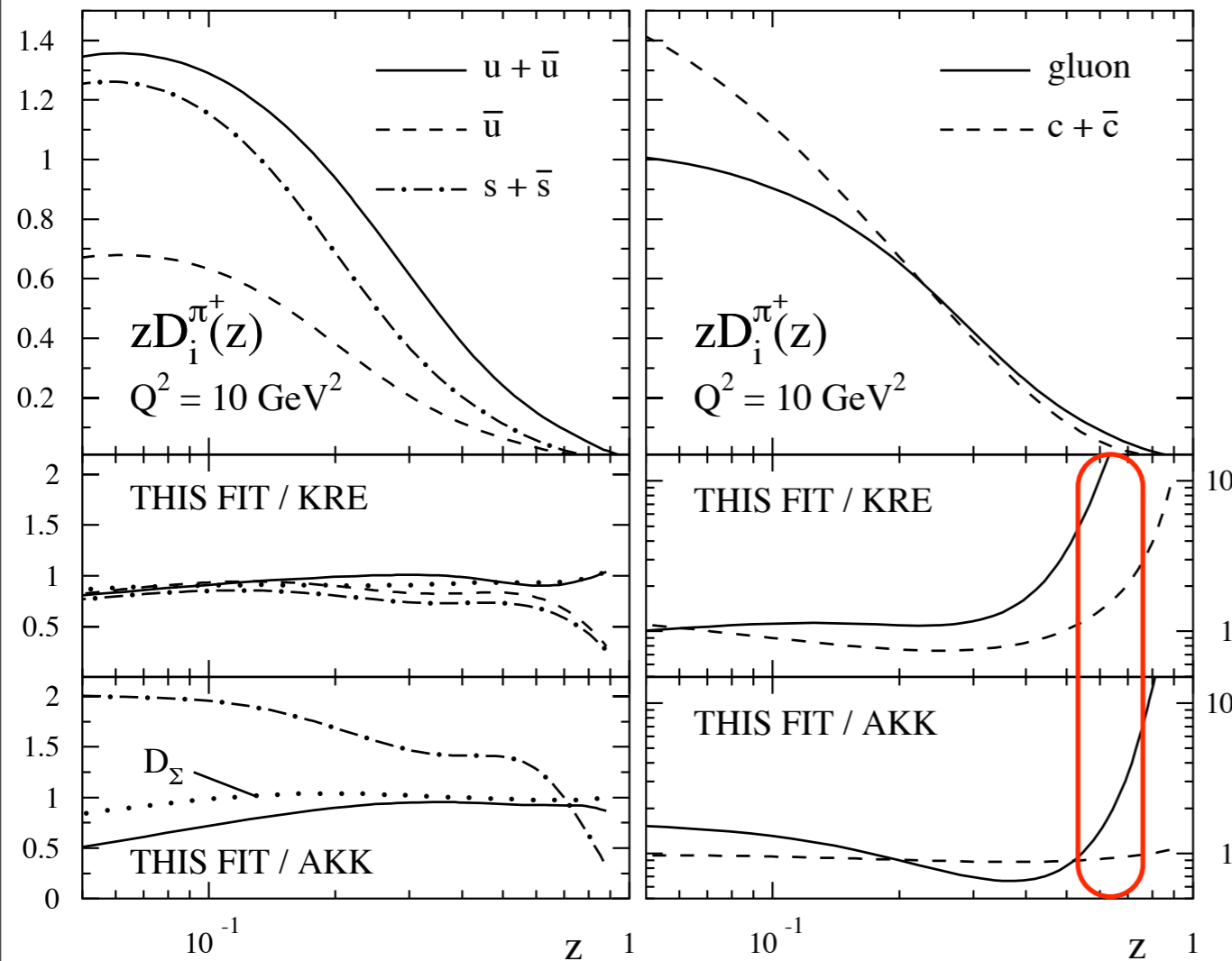
very precise SIA

Also tension between experiments (like Delphi at large  $z$ )

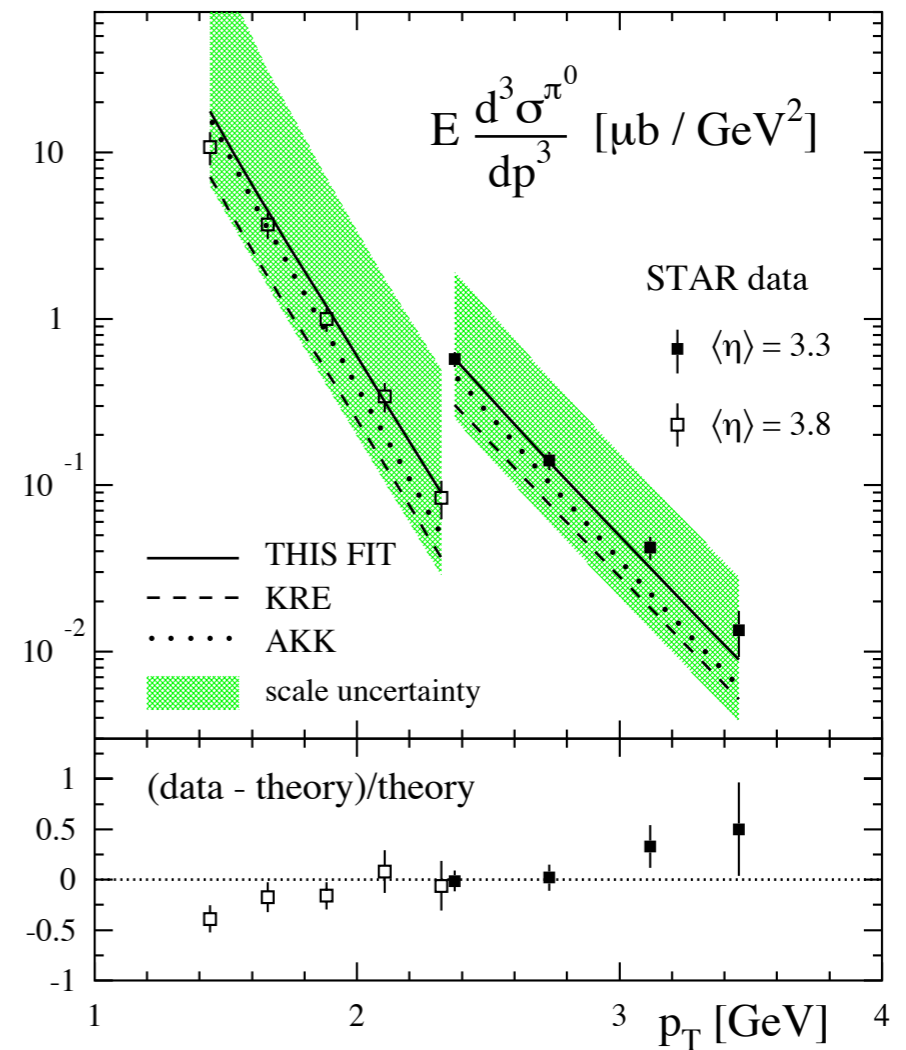
$\chi^2$  grows ( $\sim 25\%$ ) for LO fits : mostly from pp data  
where NLO corrections are very large

Pions + kaons + protons almost saturate charged hadrons:  
residual only sizable for HQ

# Distributions (pions)



# large rapidity at STAR



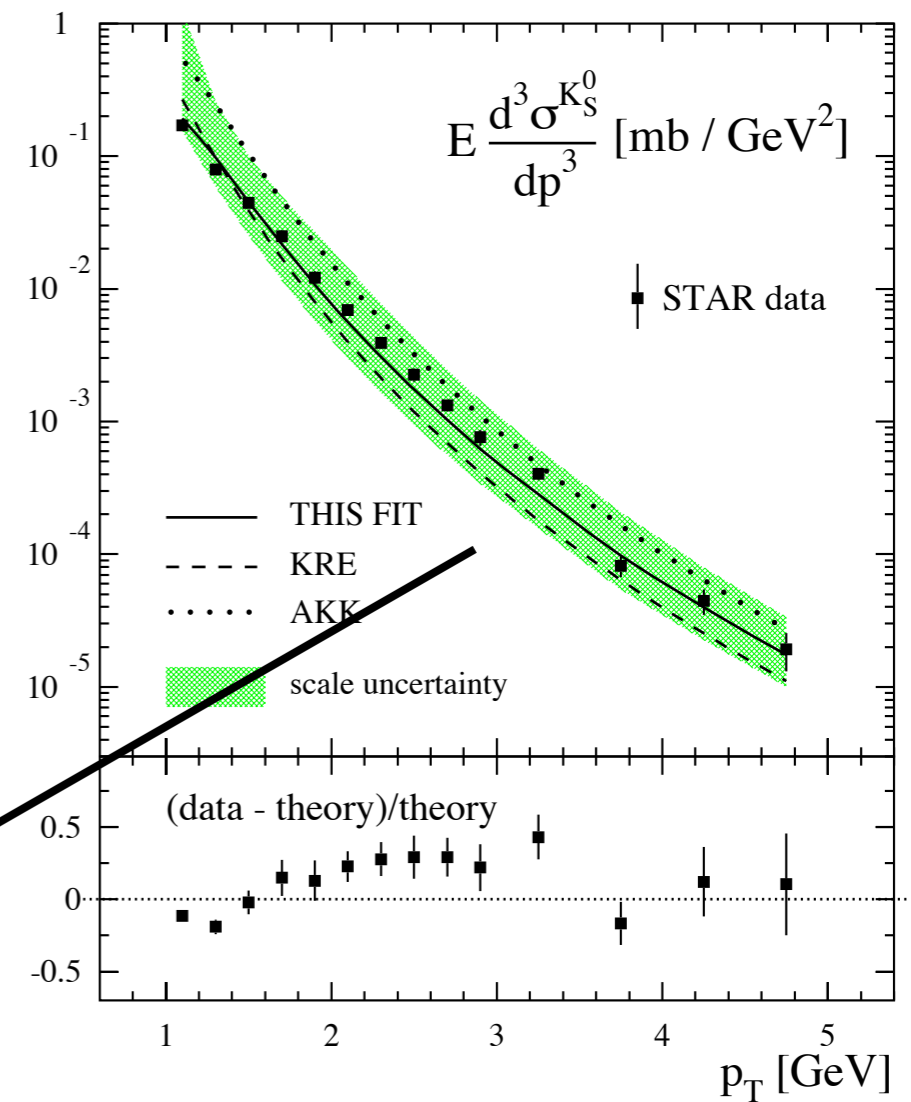
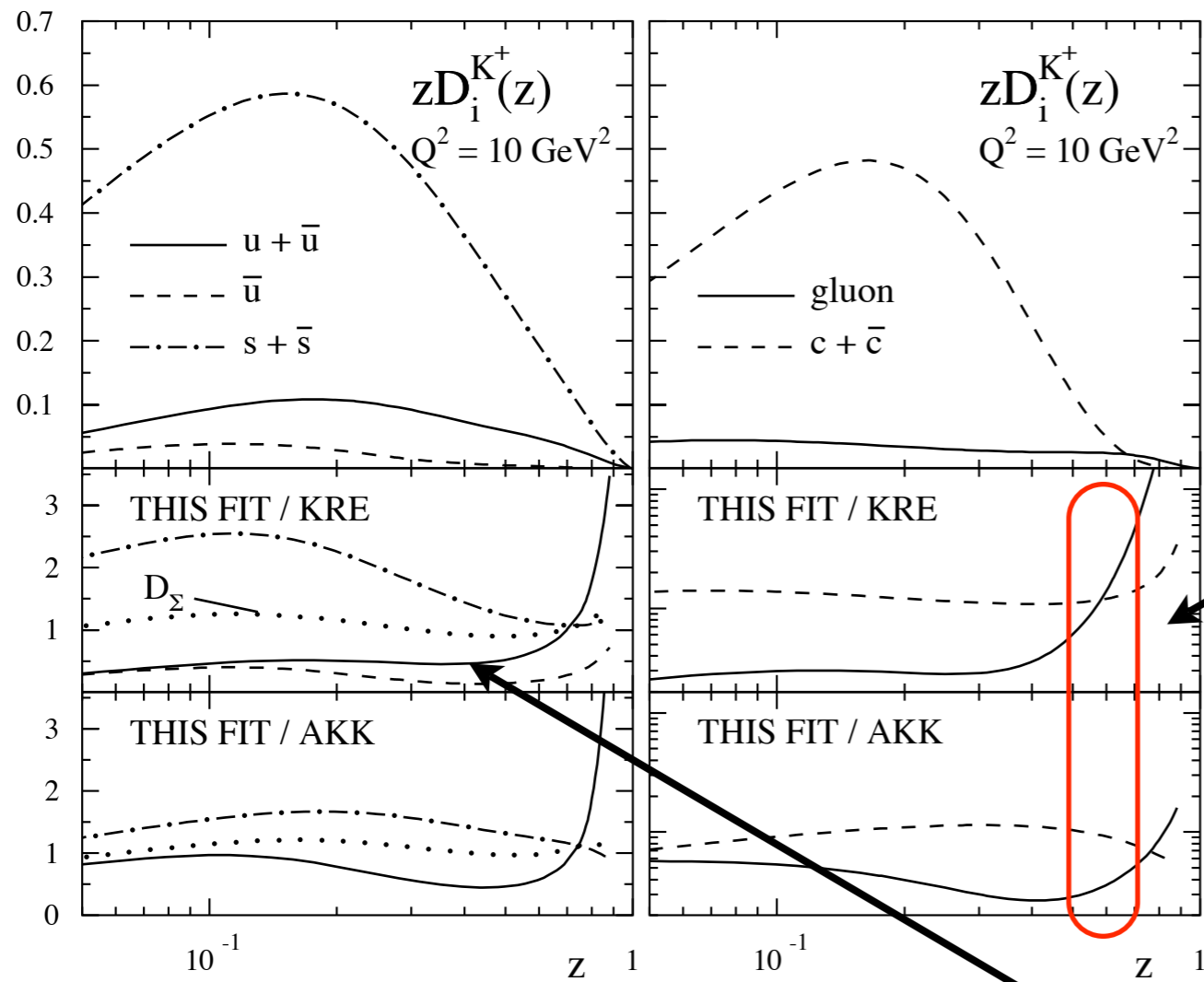
Large differences visible in the gluon at large  $z$  : explains pp

Large differences in unfavored distributions : explains SIDIS and pp

For pions  $u$  fragmentation smaller than AKK : required by SIDIS and compensated in SIA by larger  $s$

Similar singlet

# Distributions (Kaons)

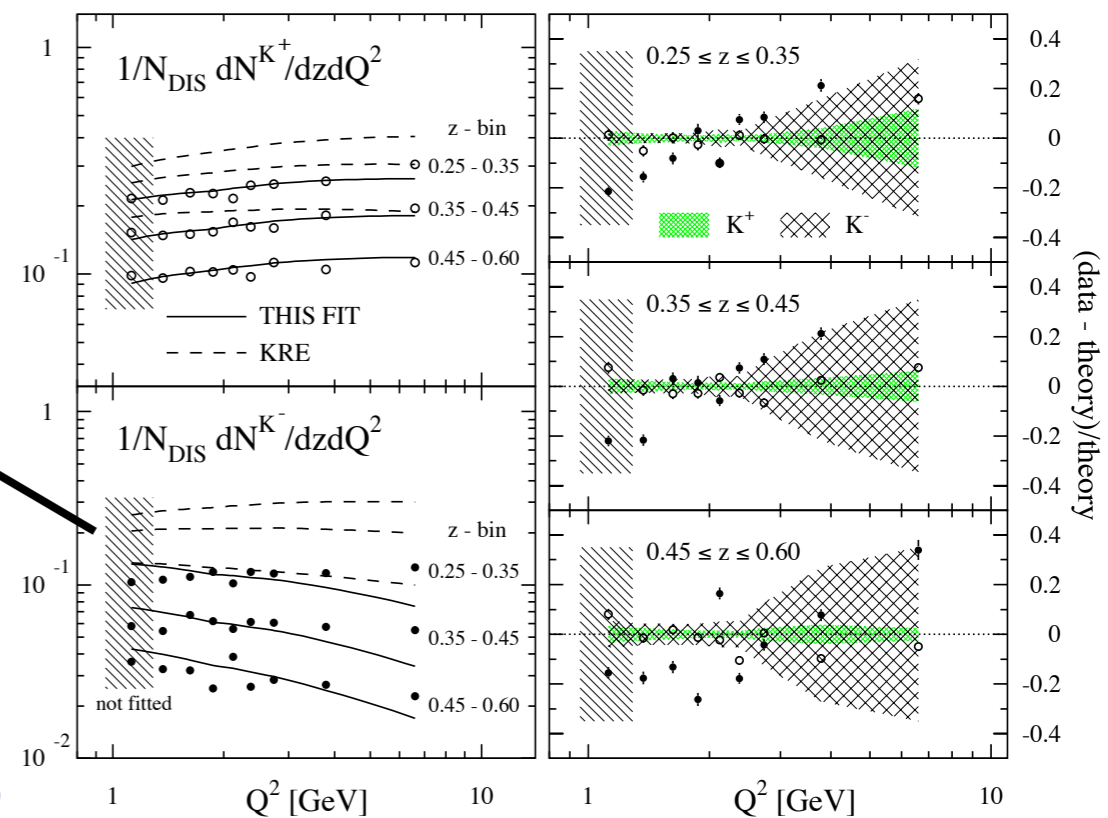


Smaller  $u$  required by SIDIS

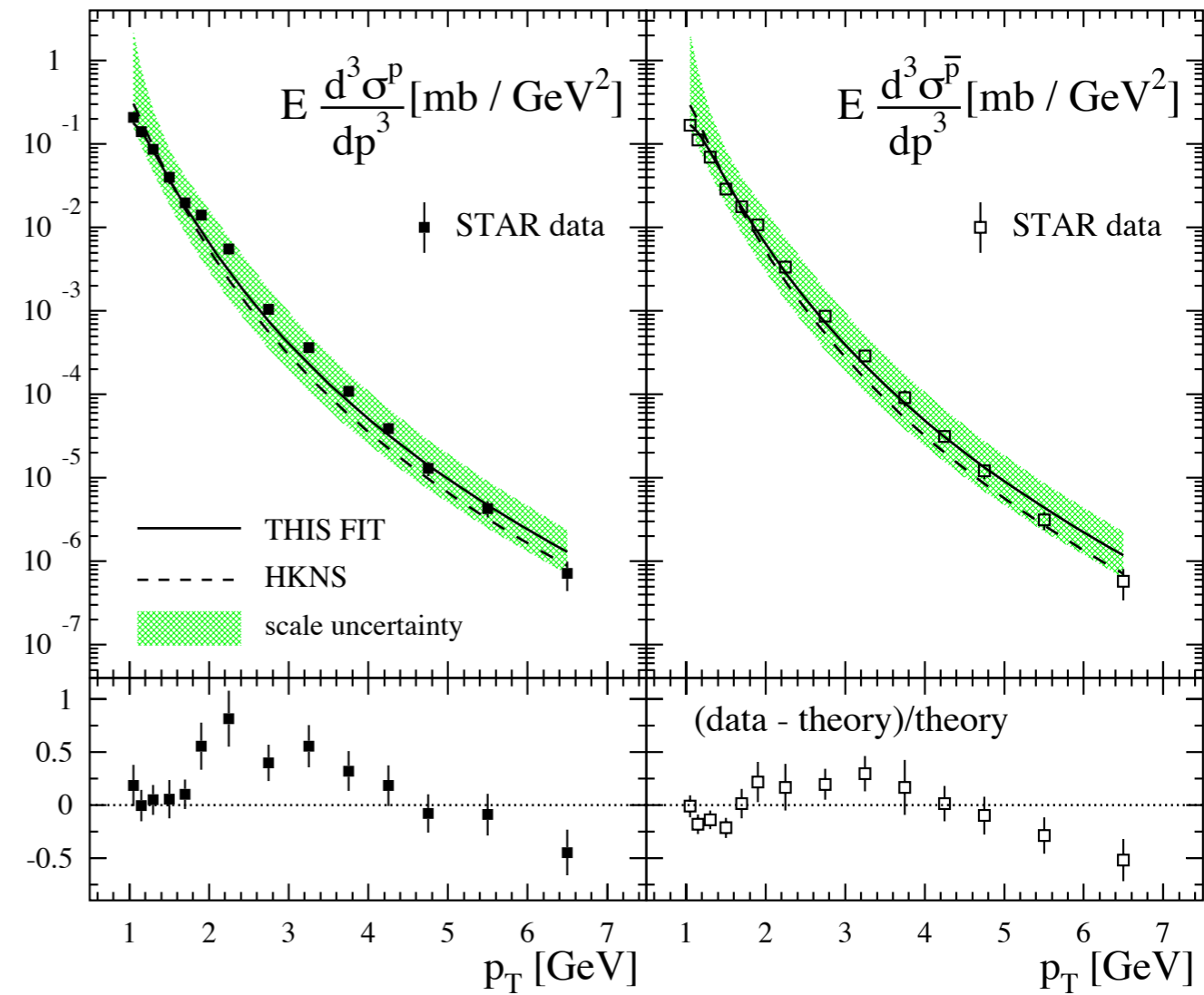
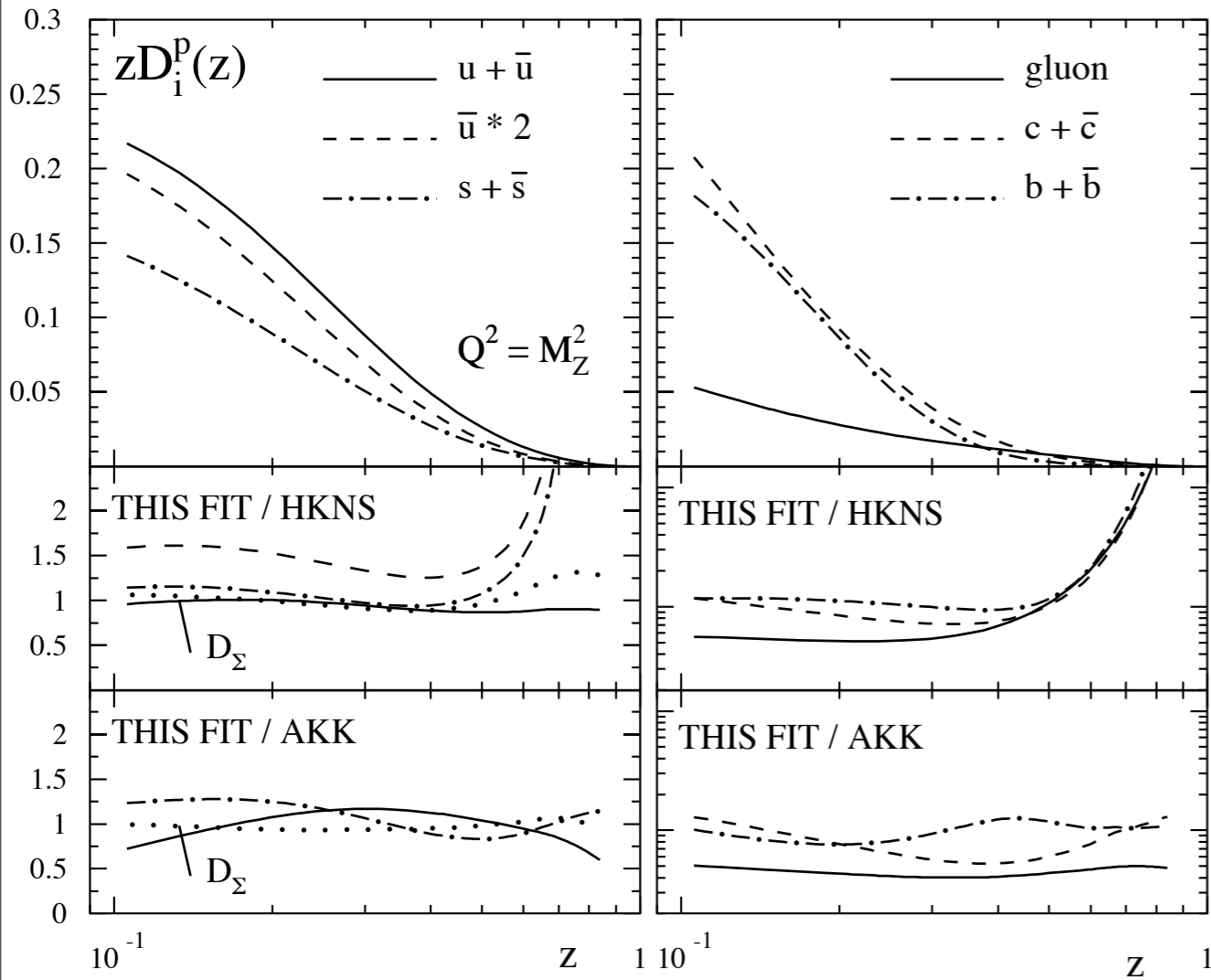
VERY different gluon by pp

Similar singlet (larger dominant  $s$ )

Issues for  $K^-$  ( $s$  and  $\bar{u}$  in proton at large  $x$ )



# Distributions (protons)



Differences with HKNS also sizeable : gluons, unfavored and large  $z$  (pp)

Similar singlet, but large diff. between HKNS and AKK (using same data)

In general differences look much larger than for pdf fits :  
comparison between GLOBAL fits

To “estimate” uncertainties using different sets (like MRST vs CTEQ),  
more global fits needed

# Uncertainties

Use Lagrange multipliers technique to estimate uncertainties (from exp. errors) on some observables

$$\Phi(\lambda_i, \{a_j\}) = \chi^2(\{a_j\}) + \sum_i \lambda_i \mathcal{O}_i(\{a_j\})$$

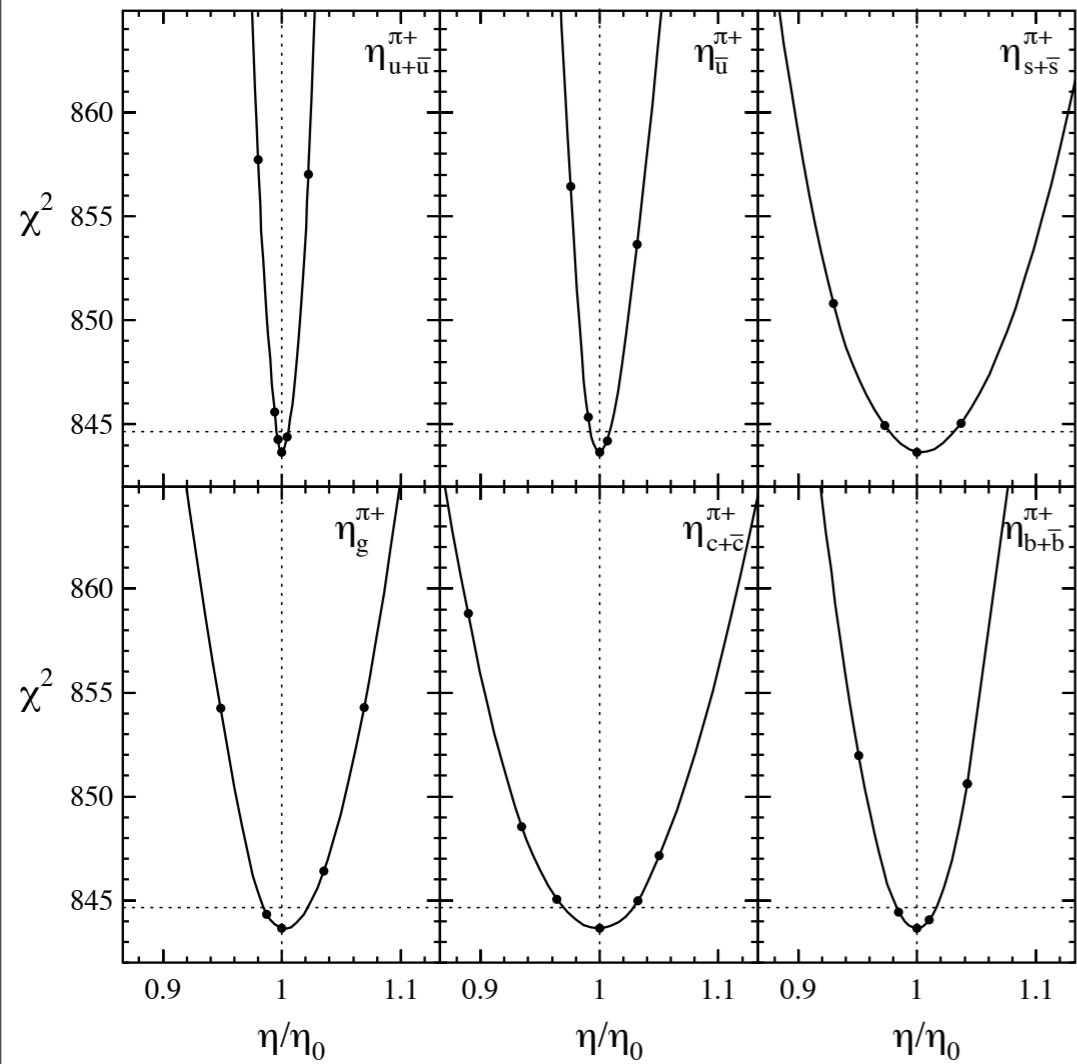
See how fit deteriorates when FFs forced to give different prediction for  $\mathcal{O}_i$

$\Delta\chi_n^2$  should be parabolic if data set can determine the observable  
(otherwise monotonic or flat)

We study truncated moments:  $\int_{0.2}^1 z D_i^H(z, Q = 5 \text{ GeV}) dz$

# Uncertainties

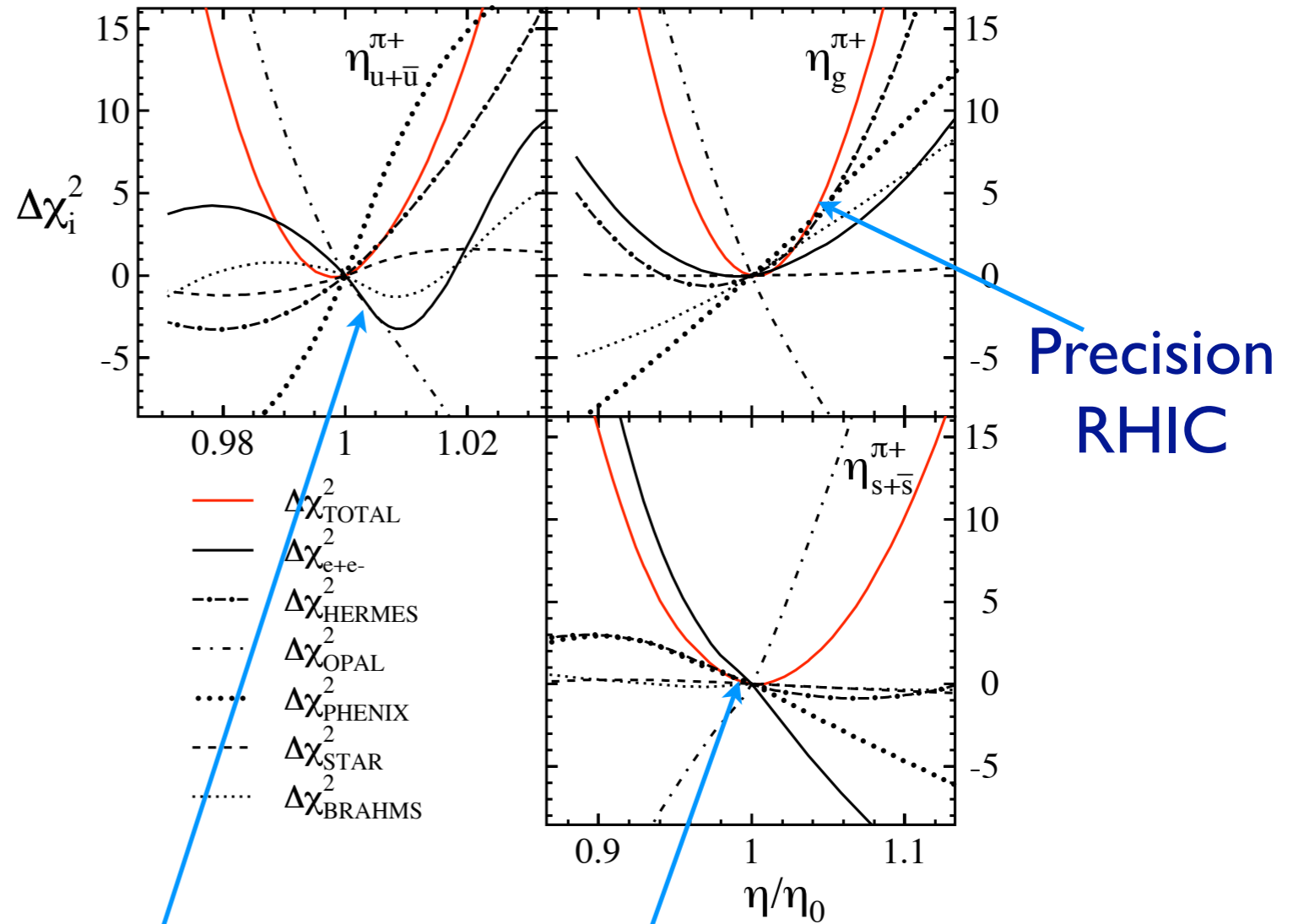
## Individual profiles



$$\Delta\chi^2 = 15 (\sim 2\%)$$

$u < 5\%$   
 $s \sim 10\%$

dominated by  $z \sim 0.2$



Tension

Complementarity

Constrained parabola as a result of global fit



# Conclusions

FFs are a fundamental tool to describe HEP observables within pQCD

NLO (and LO) fragmentation functions from a global fit: electron-positron, lepton-nucleon and hadron-hadron scattering

Charge and flavor separation from data (no ad-hoc assumptions)

For this workshop: ffs work in the kinematic range relevant for polarized pdfs extraction: RHIC, Hermes, Compass

First “global” fit fully developed using Mellin techniques : it can be done!