Generalized Parton Distributions: Recent Results

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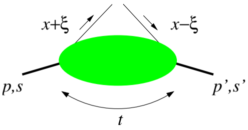
PANIC 2005, 28 October 2005

Generalized parton distributions in a nutshell

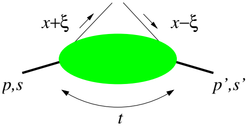


- GPDs $\leftrightarrow$ matrix elements $\langle p'|\mathcal{O}|p\rangle$
  $\mathcal{O}$ = non-local operator with quark/gluon fields
  e.g. $\bar{q}(-z)\, z_\mu \gamma^\mu\, q(z)|_{z^2=0}$
- $p \neq p' \rightsquigarrow$ depend on two momentum fractions* x, ξ and on $t = (p-p')^2$
- for unpolarized quarks two dist's:
  - H^q conserves proton helicity
  - E^q responsible for proton helicity flip

  analogs for polarized quarks and gluons

* fractions of light-cone momentum $k^+ = \frac{1}{\sqrt{2}}(k^0 + k^3)$

Generalized parton distributions in a nutshell



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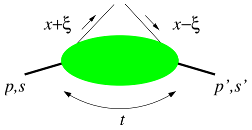
  - for unpolarized quarks two dist's:
    - H^q conserves proton helicity
    - E^q responsible for proton helicity flip
  - if $p = p' \rightsquigarrow$ ordinary parton densities

$$H^q(x, \xi = 0, t = 0) = \begin{cases} q(x) & \text{for } x > 0 \\ -\bar{q}(x) & \text{for } x < 0 \end{cases}$$

Generalized parton distributions in a nutshell



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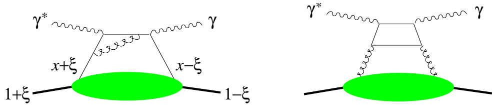
- for unpolarized quarks two dist's:
  - H^q conserves proton helicity
  - E^q responsible for proton helicity flip
- $\int dx\, x^n\, \mathrm{GPD}(x, \xi, t)$ → local operators → form factors

$$\sum_q e_q \int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1(t) \qquad \text{Dirac}$$

$$\sum_q e_q \int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2(t) \qquad \text{Pauli}$$

Processes

factorization theorems: GPDs appear in hard exclusive processes



full results at NLO in α_s :

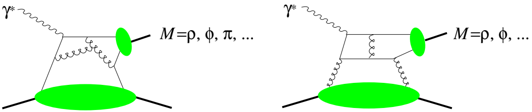
- deeply virtual Compton scattering (DVCS) $\gamma^* p \to \gamma p$
  first results at NNLO D. Müller and A. Schäfer '05

$$\mathcal{A}(\xi,t) = \sum_i \int dx\, C_i(x,\xi)\, \mathrm{GPD}_i(x,\xi,t) \qquad \xi = \frac{x_B}{2-x_B}$$

up to $\log Q^2$ dependence from evolution and running α_s
(calculable in perturbation theory)

Processes

factorization theorems: GPDs appear in hard exclusive processes



full results at NLO in α_s :

- deeply virtual Compton scattering (DVCS) $\gamma^* p \to \gamma p$
  first results at NNLO D. Müller and A. Schäfer '05
- light meson production $\gamma^* p \to \rho\, p,\ \pi\, n,\ \ldots$
  A. Belitsky and D. Müller '01, D. Ivanov et al. '04
- $\gamma p \to J/\Psi\, p$ D. Ivanov et al. '04
- in meson production NLO corrections can be large
  $\rightsquigarrow$ more detailed studies needed

Evolution

- GPDs depend on resolution scale μ
  $\sim$ large momentum in hard process
- evolution interpolates between DGLAP eqs. (parton densities) and ERBL eqs. (meson distribution amplitudes)
- kernels known to NLO
- new: explicit solution of LO evolution A. Manashov et al. '05
  - usual parton densities: Mellin transform

    $M^j(\mu) = \int dx\, x^{j-1}\, q(x,\mu)$ evolves multiplicatively

    $q(x,\mu) = -\frac{1}{2\pi i} \int_C dj\, x^{-j}\, M^j(\mu)$
  - generalization for $\xi \neq 0$ involves Legendre functions
  - $\to$ fast numeric implementation
    analytic approximations, e.g. in small-x limit

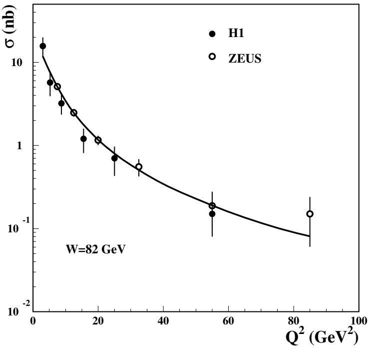
typical strategy for modeling GPDs:

- use conventional $g(x), q(x), \bar{q}(x)$ as input
- generate ξ dependence consistent with Lorentz invariance (polynomiality) relations

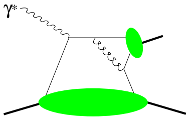
ansätze based on

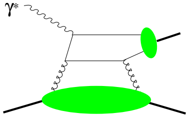
- "double distributions"
  I. Musatov and A. Radyushkin '99
- evolution for small x, ξ
  A. Shuvaev et al. '99
- moments
  D. Müller and A. Schäfer '05
  V. Guzey and M. Polyakov '05 $\longrightarrow$
  $\rightsquigarrow$ with LO calculation good description of DVCS at low x_B

$\gamma^* p \longrightarrow \gamma p$



hep-ph/0507183



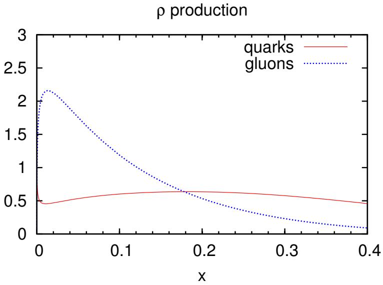


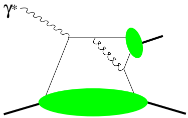
- vector meson production: quark and gluon GPDs at same $O(\alpha_s)$
- schematically:

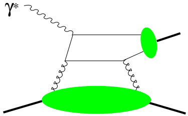
$$\mathcal{A}_{\rho^0} \propto \frac{1}{\sqrt{2}}\left[\tfrac{2}{3}(u+\bar{u})+\tfrac{1}{3}(d+\bar{d})+\tfrac{3}{4}g\right]$$

$$\mathcal{A}_{\phi} \propto \tfrac{1}{3}(s+\bar{s})+\tfrac{1}{4}g$$

- ordinary parton densities ⟶ CTEQ6L at $\mu = 2$ GeV







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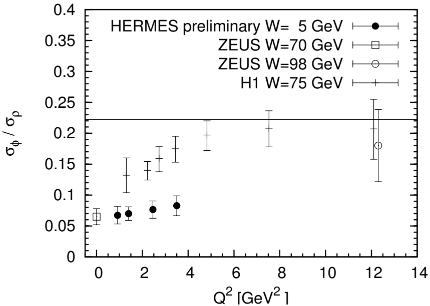
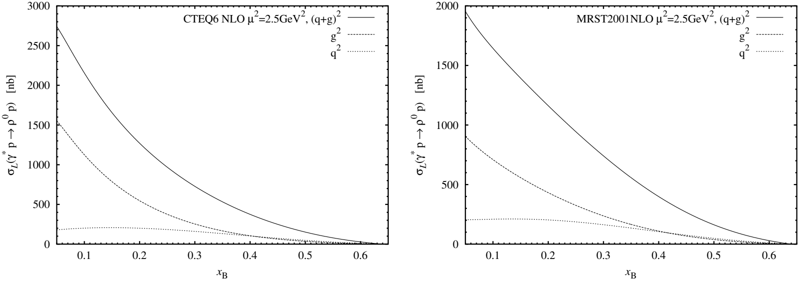
- prelim. HERMES data on σ_ϕ/σ_ρ $\Rightarrow$ substantial gluon contrib'n in ρ^0 production at $x_B \sim 0.1$

M.D. and A. Vinnikov '04



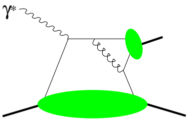
- calculation at leading twist $(Q^2 \to \infty)$ and LO in α_s
  using double distribution model for GPDs M.D. et al. '05

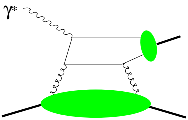


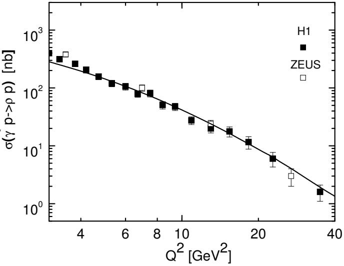
CTEQ6NLO $Q^2 = 2.5$ GeV^2 MRST2001NLO

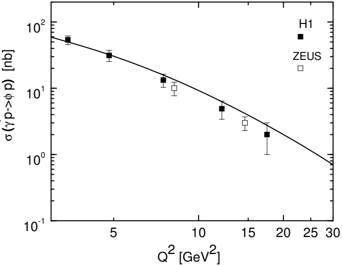
- gluons may be non-negligible even in "valence quark region"
- substantial uncertainties on conventional gluon densities

- leading-twist calculations for vector meson production overshoot data
  factors of several at $Q^2 \lesssim 5$ GeV^2
- strong suppression from meson k_T in hard scattering
  L. Frankfurt et al. '95; M. Vanderhaeghen et al. '99
- new analysis for small x_B (gluons only)
  P. Kroll, S. Goloskokov '05



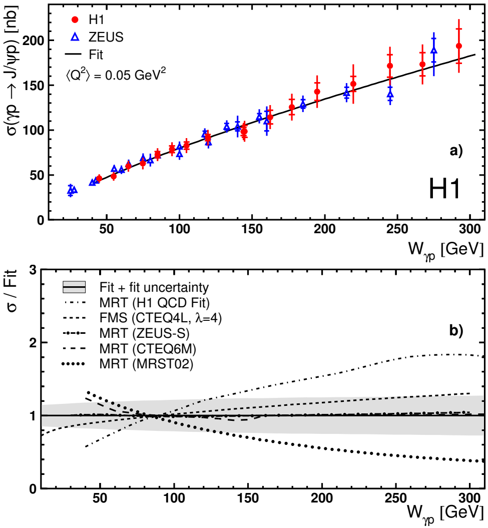






hep-ph/0501242, CTEQ5M gluon, double distribution model

J/Ψ production at small x_B



data clearly disfavor some gluon distrib's in conjunction with model for $g(x) \rightsquigarrow$ GPD

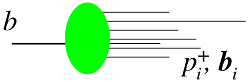
H1 Collab., hep-ex/0510016

Localizing partons: impact parameter

- states with definite light-cone momentum p^+
  can stay in frame where proton moves fast
  and transverse position (impact parameter)
  eigenstates of 2 dim. position operator

$$|p^+, \boldsymbol{b}\rangle = \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{b}\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle$$

- $\boldsymbol{b}$ is center of momentum of the partons in proton



$$\boldsymbol{b} = \frac{\sum_i p_i^+ \boldsymbol{b}_i}{\sum_i p_i^+} \qquad (i = q, \bar{q}, g)$$

consequence of Lorentz invariance
nonrelativistic analog: Galilei invariance $\Rightarrow$ center of mass

Impact parameter GPDs

- impact parameter distribution

$$q(x, b^2) = (2\pi)^{-2} \int d^2\boldsymbol{\Delta}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}}\, H^q(x, \xi = 0, t = -\boldsymbol{\Delta}^2)$$

gives distribution of quarks with
• longitudinal momentum fraction x
• transverse distance b from proton center M. Burkardt '00
can generalize to $\xi \neq 0$

- average impact parameter

$$\langle b^2 \rangle_x = \frac{\int d^2b\, b^2\, q(x, b^2)}{\int d^2b\, q(x, b^2)} = 4\, \frac{\partial}{\partial t} \log H(x, \xi = 0, t)\Big|_{t=0}$$

Impact parameter GPDs

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  gives distribution of quarks with
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- measured t dependence $\rightsquigarrow$ spatial distribution of partons
  data at small x for
  - J/Ψ production ZEUS '04, H1 '05 $\leftrightarrow$ gluons
  - DVCS H1 '05 $\leftrightarrow$ sea quarks and gluons

impact parameter distributions depend on resolution scale μ

- $q(x, b^2)$ fulfills usual DGLAP evolution equation
- for non-singlet distribution (e.g. valence $q_v = q - \bar{q}$) average impact parameter

$$\langle b^2 \rangle_x = \frac{\int d^2 b \, b^2 \, q_v(x, b^2)}{\int d^2 b \, q_v(x, b^2)}$$

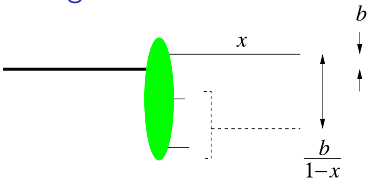
evolves as

$$\mu^2 \frac{d}{d\mu^2} \langle b^2 \rangle_x = -\frac{1}{q_v(x)} \int_x^1 \frac{dz}{z} P_{qq}\Big(\frac{x}{z}\Big) \, q_v(z) \Big[\langle b^2 \rangle_x - \langle b^2 \rangle_z \Big]$$

M.D. et al. '04

analogous for quark singlet and gluons

Large x:



- for $x \to 1$ get $b \to 0$
  nonrel. analog:
  center of mass of atom
- $\Leftrightarrow$ t dependence becomes flat

- $d = b/(1-x)$
  $=$ distance of selected parton from spectator system
  gives lower bound on overall size of proton
- finite size of configurations with $x \to 1$ implies

$$\langle b^2 \rangle_x \sim (1-x)^2$$

M. Burkardt '02, '04

Small x: simplest Regge behavior gives

$$\text{GPD} \sim x^{-(\alpha+\alpha' t)}\, e^{tB} \quad \rightsquigarrow \quad \langle b^2 \rangle_x \sim B + \alpha' \log(1/x)$$

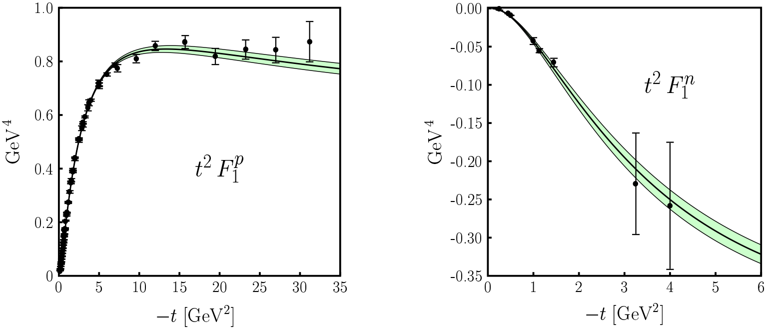
Constraints from Dirac form factors

sum rule $\sum_q e_q \int dx\, H^q(x,\xi,t) = F_1(t)$

$\oplus$ functional ansatz for valence $(q - \bar{q})$ part of $H^q(x,\xi=0,t)$

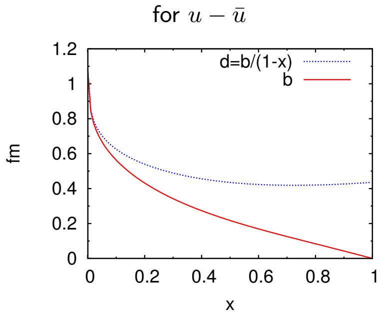
$\rightsquigarrow$ good description of proton and neutron Dirac form factors

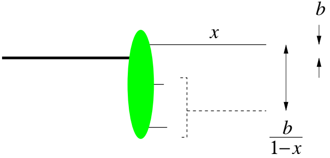
M.D. et al. '04, M. Guidal et al. '04



M.D. et al, hep-ph/0408173

Lesson from the fit

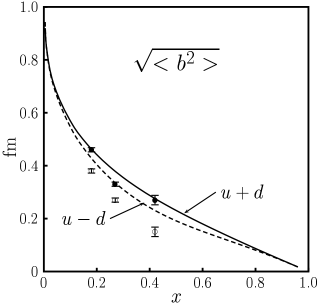




- clear drop with x of average distance $d = b/(1-x)$
  $\leftrightarrow$ strong correlation of x and t dependence
- trend also clearly seen in lattice calculations

Compare with lattice results

matrix elements of local operators $\leftrightarrow$ form factors
calculate in lattice QCD

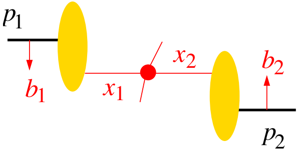


J. Negele et al., hep-lat/0404005

- Wilson fermions
- $m_\pi = 870$ MeV
- typical x in $\int dx\, x^n q(x,b)$ estimated as

$$\langle x \rangle = \frac{\int dx\, x^{n+1} q(x)}{\int dx\, x^n q(x)}$$

Consequences for hadron-hadron collisions



- hard inclusive process, e.g. $pp \to \text{jet jet} + X$
  $\to$ no impact parameter dependence
  integrate over b_1 and b_2 independently

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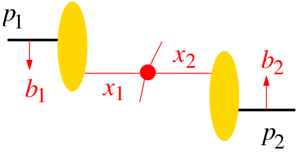
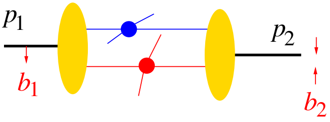




- hard inclusive process, e.g. $pp \to \text{jet jet} + X$
  $\to$ no impact parameter dependence
  integrate over b_1 and b_2 independently
- secondary soft or hard interactions
  do not affect inclusive cross section
  but change event structure
- larger mom. fractions x_1, x_2 in hard subprocess
  $\rightsquigarrow$ more central collision
  $\rightsquigarrow$ more secondary interactions

  M. Strikman, C. Weiss, session II.1

Transverse spin and helicity flip

- $E \leftrightarrow$ nucleon helicity flip $\langle\downarrow|\mathcal{O}|\uparrow\rangle$

  $\leftrightarrow$ transverse pol. difference $|X\pm\rangle = \frac{1}{\sqrt{2}}(\,|\uparrow\rangle \pm |\downarrow\rangle\,)$

  $\langle X{+}|\mathcal{O}|X{+}\rangle - \langle X{-}|\mathcal{O}|X{-}\rangle = \langle\uparrow|\mathcal{O}|\downarrow\rangle + \langle\downarrow|\mathcal{O}|\uparrow\rangle$

- quark density in proton state $|X{+}\rangle$

$$q_v^X(x,\boldsymbol{b}) = q_v(x,\boldsymbol{b}^2) - \frac{b^y}{m}\frac{\partial}{\partial \boldsymbol{b}^2}\, e_v^q(x,\boldsymbol{b}^2)$$

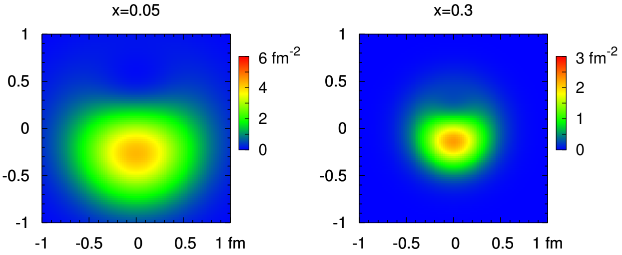
  shifted in y direction M. Burkardt '02

  $e_v^q(x,b)$ is Fourier transform of $E_v^q(x,\xi=0,t)$

quark density in proton state $|X+\rangle$

$$q_v^X(x,\boldsymbol{b}) = q_v(x,\boldsymbol{b}^2) - \frac{b^y}{m}\frac{\partial}{\partial \boldsymbol{b}^2}\, e_v^q(x,\boldsymbol{b}^2)$$

is shifted



$(d-\bar{d})$ density in transverse plane M.D. et al. '04

- $\int dx\, E^u(x,0,0) = \kappa^u \approx 1.67$
  $\int dx\, E^d(x,0,0) = \kappa^d \approx -2.03$
  $\rightarrow$ large spin-orbit correlations
- relation with transverse momentum dependent densities
  $\rightarrow$ Sivers effect M. Burkardt et al. '04
  also: F. Yuan, session III.6

- density representation

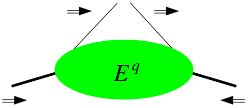
$$q_v^X(x,\boldsymbol{b}) = q_v(x,\boldsymbol{b}^2) - \frac{b^y}{m}\frac{\partial}{\partial \boldsymbol{b}^2}\, e_v^q(x,\boldsymbol{b}^2)$$

gives positivity bound M. Burkardt '03

$$\Big|E^q(x,\xi=0,t=0)\Big| \quad\leq\quad q(x)\, m\sqrt{\langle \boldsymbol{b}^2\rangle_x}$$

have more restrictive bounds involving polarized distributions

$\Rightarrow$ E^q must fall faster than H^q at large x



- $E \leftrightarrow$ orbital angular momentum
  $\Rightarrow$ carried by partons with x not large

Constraints from Pauli form factors

- sum rule $\sum_q e_q \int dx\, E^q(x,\xi,t) = F_2(t)$
  $\oplus$ functional ansatz for valence part of $E^q(x,\xi=0,t)$
  do not know forward limit $E^q(x,\xi=0,t=0)$
  M.D. et al. '04, M. Guidal et al. '04
- obtain good fits of $F_2^p(t)$ and $F_2^n(t)$ data
  large allowed regions of fit parameters
  but positivity constraints seriously limit parameter space
- can estimate orbital angular momentum
  carried by valence quarks $(q - \bar{q})$ Ji's sum rule

$$\langle L_v^q \rangle = \frac{1}{2}\int dx\, x\Big[E_v^q(x,0,0) + q_v(x)\Big] - \frac{1}{2}\int dx\, \Delta q_v(x)$$

$2\langle L_v^u \rangle = -(0.47 \div 0.54)$ and $2\langle L_v^d \rangle = 0.30 \div 0.38$ M.D. et al

$2\langle J_v^u \rangle = 2\langle L_v^u \rangle + 0.93$ and $2\langle J_v^d \rangle = 2\langle L_v^d \rangle - 0.34$

Comparison with lattice calculations

- form factor analysis — M.D. et al '04

  $2\langle L_v^u - L_v^d \rangle = -(0.77 \div 0.92)$

  $2\langle L_v^u + L_v^d \rangle = -(0.11 \div 0.22)$

- lattice results — QCDSF, G. Schierholz at LC 2005

  $2\langle L_v^u - L_v^d \rangle = -0.9 \pm 0.12$

  $2\langle L_v^u + L_v^d \rangle = 0.06 \pm 0.14$

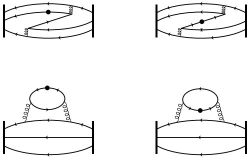
- lattice for $m_\pi = 897$ MeV — LHPC, from hep-ph/0410017

  $2\langle L_v^u - L_v^d \rangle = -0.25 \pm 0.05$

  $2\langle L_v^u + L_v^d \rangle = -0.10 \pm 0.05$

all results for $\mu = 2$ GeV

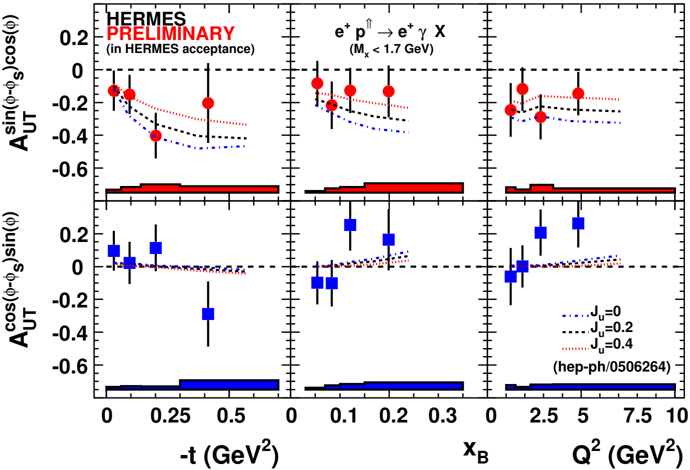
lattice "valence" contributions in sense of "connected quark diagrams" $\nearrow$



- e.m. form factors $\rightsquigarrow$ only $q - \bar{q}$ and as $\int dx$
- DVCS $\rightsquigarrow$ sea quarks and correlated with x
- ongoing experimental effort
  - HERMES transverse p target
    $\rightsquigarrow$ sensitive to $\frac{4}{9}E^u + \frac{1}{9}E^d$ E. Aschenauer, session III.5
  - JLAB Hall A Hall A E03-106 n target
    $\rightsquigarrow$ sensitive to $\frac{1}{9}E^u + \frac{4}{9}E^d$
  - JLAB Halls A and B, HERMES, H1 and ZEUS
    unpolarized proton target $\rightsquigarrow$ mainly $\frac{4}{9}H^u + \frac{1}{9}H^d$ and H^g
  - in addition: vector meson production J. Dreschler, session III.5
- future prospects: Compass, JLAB upgrade, eRHIC/ELIC

DVCS transverse target spin asymmetries

proton polarization ⊥ or ∥ to plane spanned by final $\gamma\, p$



predictions from model with $\langle J^u \rangle$ as parameter in $E^u(x, \xi, t)$

Many other things not covered in this talk, e.g.

- dynamical models Kvinihkidze, Blankleider '04; Pasquini et al. '04; Tiburzi et al. '04; Noguera et al. '04, Scepetta, Vento '04; Mineo et al '05; Ossmann et al. '05
- transverse lattice Dalley '04
- nuclear GPDs Scopetta '04; Liuti, Taneja '04, '05; Guzey, Siddikov '05
- exclusive dijet production Braun, Ivanov '05; D. Ashery session II.4
- exotic meson production Anikin et al. '04, '05
- hadron-photon and baryon-meson transitions Pire, Szymanowski '04, '05; Tiburzi '05
- large-angle processes Miller '04; M.D. et al. '04; Kroll, Schäfer '05
- νp scattering Amore et al. '04; Psaker '04

Conclusions

- technical progress: evolution, full NLO, first NNLO results
- vector meson production:
  very sensitive to gluon distrib'n even in fixed-target kinematics
  theory description more involved than for DVCS
- impact parameter representation:
  $F_1(t)$ data and lattice $\rightsquigarrow$ strong decrease of $\langle \boldsymbol{b}^2 \rangle$ with x
- $E \quad \longrightarrow$ physics of transverse spin
  and of orbital angular momentum
- towards quantitative understanding of L^q
  - progress in lattice calculations
  - first model dependent analyses of form factor and DVCS data

  $\rightsquigarrow$ "valence" part of L^{u-d} big and of L^{u+d} small