# Reference Data for the Density and Viscosity of Liquid Aluminum and Liquid Iron

**Marc J. Assael**[a] **and Konstantinos Kakosimos**
*Chemical Engineering Department, Aristotle University, 54124 Thessaloniki, Greece*

**R. Michael Banish**
*Center for Materials Research, University of Alabama in Huntsville, Huntsville, Alabama 35899*

**Jürgen Brillo and Ivan Egry**
*Deutsches Zentrum für Luft- und Raumfahrt, Institut für Raumsimulation, Köln, D-51170, Germany*

**Robert Brooks, Peter N. Quested, and Kenneth C. Mills**
*Division of Engineering and Process Control, National Physical Laboratory, Teddington TW11 OLW, United Kingdom*

**Akira Nagashima**
*Department of System Design, Keio University, Hiyoshi, Yokohama 223-8522, Japan*

**Yuzuru Sato**
*Department of Metallurgy, Graduate School of Engineering, Tohoku University, Sendai 980-8579, Japan*

**William A. Wakeham**
*University of Southampton, Highfield, Southampton SO17 1BJ, United Kingdom*

(Received 4 May 2005; revised 13 July 2005; accepted 16 September 2005; published online 10 February 2006)

The available experimental data for the density and viscosity of liquid aluminum and iron have been critically examined with the intention of establishing a density and a viscosity standard. All experimental data have been categorized into primary and secondary data according to the quality of measurement specified by a series of criteria. The proposed standard reference correlations for the density of the aluminum and iron are characterized by standard deviations of 0.65% and 0.77% at the 95% confidence level, respectively. The overall uncertainty in the absolute values of the density is estimated to be one of ±0.7% for aluminum and 0.8% for iron, which is worse than that of the most optimistic claims but recognizes the unexplained discrepancies between different methods. The standard reference correlations for the viscosity of aluminum and iron are characterized by standard deviations of 13.7% and 5.7% at the 95% confidence level, respectively. The uncertainty in the absolute values of the viscosity of the two metals is thought to be no larger than the scatter between measurements made with different techniques and so can be said to be ±14% in the case of aluminum and ±6% in the case of iron. © *2006 American Institute of Physics.* [DOI: 10.1063/1.2149380]

Key words: aluminum; density; iron; melt; reference data; viscosity.

## Contents

---

[a] Author to whom correspondence should be addressed; E-mail: assael@auth.gr

## List of Tables

## List of Figures

## 1. Introduction

There is an increasing use of mathematical models to simulate a variety of processes involving liquid metals such as ''cast to shape,'' primary and secondary metal production, powder production by spray forming, and welding. Depending on what aspect of the process is modeled, a need for viscosity data of relevant alloys exists. Historically there are wide discrepancies in the viscosity data reported for the metallic elements and alloys [Iida and Guthrie (1988)]. For example there is a spread of about 400% in the reported values for molten aluminum and about 100% for molten iron.

These discrepancies prompted this work to review the values in the literature and attempt to derive recommended values for the viscosity of aluminum and iron. These elements form the basis of a wide range of aluminum-based alloys and steels which still contribute a crucial part in manufacturing industry. It was also recognized that values of the density of these elements are important information in many of the experiments to measure the viscosity or are needed in the conversion of kinematic viscosity to dynamic viscosity.

## 2. Primary and Secondary Data

According to the recommendation adopted by the Subcommittee of Transport Properties (now known as the International Association for Transport Properties) of the International Union of Pure and Applied Chemistry, experimental data can be categorized into two categories according to the quality of the data: primary and secondary data. The primary data were identified by the following criteria [Assael *et al.* (1990)]:

(i) Measurements must have been made with a primary experimental apparatus, i.e., a complete working equation must be available.
(ii) The form of the working equation should be such that sensitivity of the property measured to the principal variables does not magnify the random errors of measurement.
(iii) All principal variables should be measurable to a high degree of precision.
(iv) The published work should include some description of purification methods and a guarantee of purity.
(v) The data reported must be unsmoothed data. While graphs and fitted equations are useful summaries for the reader, they are not sufficient for standardization purposes.
(vi) The lack of accepted values of the viscosity of standard reference materials implies that only absolute and not relative measurement results can be considered.

(vii) Explicit quantitative estimates of the uncertainty of reported values should be given, taking into account the precision of experimental measurements and possible systematic errors.

(viii) Owing to the desire to produce low-uncertainty reference values, limits must be imposed on the uncertainty of the primary data sets. These limits are determined after critical evaluation of the existing data sets.

These criteria have been successfully employed to the proposition of standard reference values for the viscosity and thermal conductivity of fluids over a wide range of conditions, with uncertainties in the region of 1%.

In the case, however, of the liquid metals, it was felt and agreed that these criteria needed to be relaxed slightly, especially since the uncertainty of the measurements is much higher, primarily due to: (i) the difficulties associated with the techniques employed in such high temperatures and (ii) the purity of the liquid metal sample which can be strongly affected by the surrounding atmosphere and the container for the melt. It should also be mentioned that, where data were given only in diagram form, values were obtained by high-resolution digital scanning.

## 3. Density

### 3.1. Experimental Techniques

One of the oldest techniques employed for the measurement of the density is the Archimedean technique. According to this method, a solid sinker of known weight in air is suspended by a wire attached to the arm of a balance. When the sinker is immersed in the liquid metal specimen an apparent loss of weight is observed, arising mainly from the buoyant force exerted by the liquid metal sample. To obtain very accurate data, the volumes of the sinker and the immersed suspension wire or rod must always be corrected to allow for thermal expansion at the operating temperature, since their volumes are, in general, determined experimentally at room temperature. The weight loss of the sinker must also be corrected for the effect of surface tension on the weight of the wire. Both effects can be minimized by using a fine suspension wire. A modification of the Archimedean technique is to drive the sinker into the liquid and continuously monitor the weight of the sinker as function of depth. This method negates the need for a correction for the surface tension forces.

The Pycnometric technique refers to the filling of a vessel or pot of known volume by the liquid metal. Upon freezing, the solid metal specimen contained within the pot is weighed at room temperature. The method is absolute and allows very accurate measurements provided the material of the vessel is characterized by excellent machinability, does not react with the liquid metal and has a low and well-known coefficient of thermal expansion.

The maximum-bubble-pressure technique is based upon the formation of a hemispherical bubble of an inert gas at the tip of a capillary tube immersed to a certain depth in a liquid. In such a case bubbles of gas will detach from the tip of the capillary. The density of the liquid specimen can be determined by measuring the difference in the overpressure required to form a hemispherical bubble of the inert gas at the tip of the capillary at different depths in the liquid. The technique is not as accurate as the pycnometric method but allows density measurements at high temperatures. Its main disadvantages are the corrections to the spherical shape, the high precision of the maximum bubble pressure measurement and the presence of nonwetting effects in the gas/liquid metal systems that can lead to spreading away from an inside nozzle perimeter and across the surfaces of the capillary tube. When measurements are carried out on aluminum, alumina can be formed by reaction with oxygen in the atmosphere; this can accumulate in the capillary tube and reduces the effective radius of the tube.

The Sessile-Drop technique employs a liquid drop of known mass resting on a plate or substrate. Provided the shape of the drop is fully symmetrical, the volume of the drop and hence its density, can accurately be calculated. To prevent a sessile drop to be formed asymmetrically, the ‘‘large-drop method’’ is proposed. According to this technique, a sessile drop is formed from an accurately ground cylindrical tube with sharp edges which is most likely to yield a drop that is perfectly symmetrical. Very accurate measurements have thus been performed. The best fit of the shape of the drop (when it departs from the pure sphere) can be calculated using various algorithms and can improve the accuracy of the computed densities.

In the levitation technique, a small drop of the liquid can be supported by three techniques: (a) aerodynamically by gas flow in a convergent/divergent nozzle; (b) electrostatically by electrically charging the drop and holding it steady by an electrical potential; or (c) by electromagnetic forces using a high frequency coil.

In the case of aerodynamic and electrostatic levitation the drop is heated by a high power laser but frequently the electromagnetic field is used to both levitate and heat the drop. The volume of the drop is obtained from sectional images which are frequently taken from three orthogonal directions. The density is obtained from the sectional images and the mass of the drop. Generally the method is inferior to the pycnometric or the Archimedean methods. It has however the advantage that it is able to be employed with highly reactive metallic elements since it is effectively containerless.

The gamma radiation attenuation technique is based on the attenuation of a $\gamma$-ray beam passing through the liquid metal. The incident beam is attenuated according to the mass of the liquid metal, and is recorded by a radiation counter. In determining the liquid metal’s density the mass attenuation coefficient must be known. An advantage of this technique is that surface tension effects and chemical contamination of the liquid metal’s surface are not involved, since the $\gamma$-ray beam penetrates the bulk of the specimen.

TABLE 1. Data sets considered for the density of liquid aluminum

| Reference | Purity (mass %) | Technique employed[a] | Uncertainty quoted/% | Form of data[b] |
|---|---|---|---|---|
| Primary data | | | | |
| Smith *et al.* (1999) | 99.99 | X-Ray (Rel) | — | D |
| Yatsenko *et al.* (1972) | 99.996 | Sessile Drop (Abs) | 1.5 | D |
| Levin *et al.* (1968) | >99.996 | Sessile Drop (Abs) | 1.0 | D |
| Coy and Mateer (1955) | 99.997 | Bubble Pressure (Abs) | — | P |
| Gebhardt *et al.* (1955) | 99.996 | Archimedean (Abs) | — | P |
| Secondary data | | | | |
| Sarou-Kanian *et al.* (2003) | 99.99 | Levitation (Rel) | 1.5 | D |
| Nasch and Steinemann (1995) | — | $\gamma$-ray (Abs) | 0.75 | E |

[a] Abs = absolute; Rel = relative.
[b] D = diagram; E = equation; P = points.

### 3.2. Data Compilation

Tables 1 and 2 present the data sets found for the measurement of the density of liquid aluminum and liquid iron, respectively. Papers prior to 1930 were not considered, as sample purity was disputed before that time. In these tables, the purity of the sample, the technique employed and the uncertainty quoted are also presented. Furthermore the form of the data is noted. The data sets have been classified into primary and secondary sets according to the criteria presented in Sec. 2 and in conjunction with the techniques described in Sec. 3.1. More specifically, following the brief presentation of the various techniques employed for the measurement of the density of the two liquid metals, the following can be noted.

The Archimedean technique was successfully employed by Lucas (1972), Kirshenbaum and Cahill (1962), Lucas (1960), and Gebhardt *et al.* (1955). In these papers the technique is well described and these data sets are considered as Primary data sets. In the case of Adachi *et al.* (1971), the description of the instrument is insufficiently detailed. They seemed to employ an alumina sinker, which is less dense than the iron melt, attached to a tungsten wire. In this case, surface tension effects are identified to be the main reason for their high values. This data set was not included in the primary data set.

A pycnometer was employed very successfully by Sato (2003) and this set was included in the primary data set.

The maximum-bubble-pressure technique was successfully employed by Coy and Mateer (1955), and Frohberg and Weber (1964). These data sets are thus considered as primary data sets. In the case of the measurements of Watanabe (1971), the scatter of data is much higher than the uncertainty quoted. Hence this data set was not considered as a primary data set.

The Sessile-Drop technique was successfully employed by Yatsenko *et al.* (1972), while Levin *et al.* (1968) employed the large-drop method. Both sets of measurements give full description of methods and procedures employed and are thus considered as primary data sets.

The levitation technique was successfully employed by Saito *et al.* (1969) and Sarou-Kanian *et al.* (2003). The measurements of Saito *et al.* (1969) were included in the primary data set. The measurements of Sarou-Kanian *et al.* (2003), although of high quality, were not included as they were in a

TABLE 2. Data sets considered for the density of liquid iron

| Reference | Purity (mass %) | Technique employed[a] | Uncertainty quoted/% | Form of data[b] |
|---|---|---|---|---|
| Primary data | | | | |
| Brillo and Egry (2004) | 99.995 | Elect. Levitation (Abs) | 1.0 | P |
| Sato (2003) | 99.99 | Pycnometer (Abs) | 0.5 | E |
| Lucas (1972) | >99.96 | Archimedean (Abs) | 0.3 | D |
| Saito *et al.* (1969) | >99.9 | Levitation (Abs) | 1.5 | D |
| Frohberg and Weber (1964a) | 99.98 | Bubble Pressure (Abs) | — | D |
| Kirshenbaum and Cahill (1962) | >99.9 | Archimedean (Abs) | 0.05 | P |
| Lucas (1960) | 99.9 | Archimedean (Abs) | — | D |
| Secondary data | | | | |
| Wille *et al.* (2002) | 99.99 | Levitation (Rel) | — | D |
| Nasch and Steinemann (1995) | — | $\gamma$-Ray (Abs) | 0.75 | E |
| Adachi *et al.* (1971) | 99.97 | Archimedean (Abs) | — | D |
| Watanabe (1971) | >99.9 | Bubble Pressure (Abs) | 0.7 | D |

[a] Abs = absolute; Rel = relative.
[b] D = diagram; E = equation; P = points.

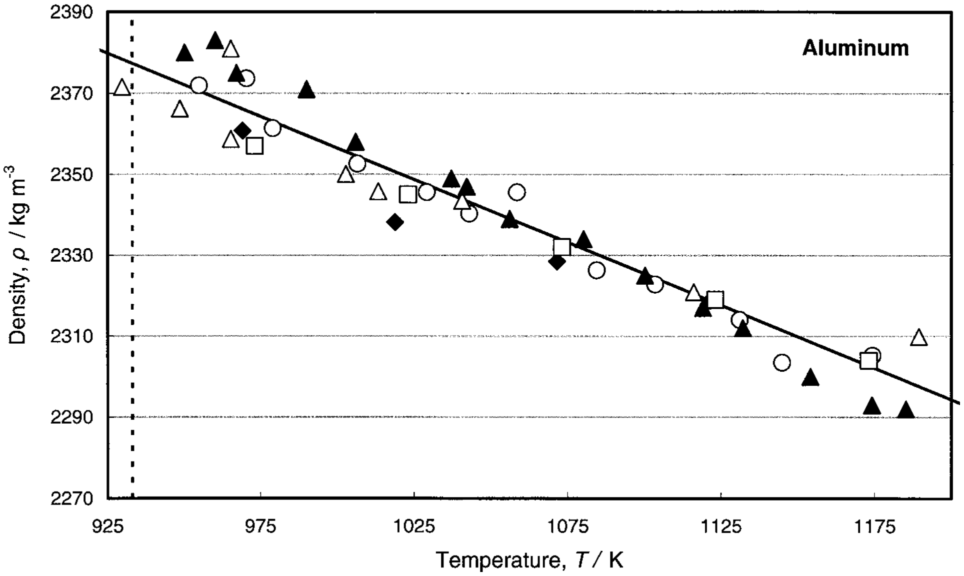

FIG. 1. Density primary data for liquid aluminum as a function of the temperature: (◆) Smith *et al.* (1999); (○) Yatshenko *et al.* (1972); (△) Levin *et al.* (1968); (▲) Coy and Mateer (1955); (□) Gebhardt *et al.* (1955); (—) fitted equation; (- - -) melting point.

much higher temperature region than all other sets of measurements Also the measurements of Wille *et al.* (2002) were not included in the primary data set as their scatter was considered very high (about 5%), in relation to all other sets considered. Finally the measurements of Brillo and Egry (2004) performed by the electromagnetic levitation technique fulfilled all the requirements for the primary data, and were thus included in that set.

The gamma radiation attenuation technique was employed by Nasch and Steinemann (1995). This data set was not included in the primary data sets as it was felt that the mass attenuation coefficient should have not been considered as temperature independent. This is probably the reason why as temperature increases, these data are systematically higher than all other sets. Smith *et al.* (1999) employed the x-ray attenuation technique in a relative manner and these data were included in the primary data set.

### 3.3. Density Reference Correlation

The primary density data for liquid aluminum and iron, shown in Tables 1 and 2, respectively, were employed in a linear regression analysis as a function of the temperature. The following equations were obtained for the density, $\rho$, as a function of the absolute temperature, $T$:

For aluminum (temperature range 933–1190 K)

$$\rho = c_1 - c_2(T - T_{\mathrm{ref}}), \qquad (1)$$

where $c_1 = 2377.23\ \mathrm{kg\,m^{-3}}$, $c_2 = 0.311\ \mathrm{kg\,m^{-3}\,K^{-1}}$, and $T_{\mathrm{ref}} = 933.47$ K is the melting point of aluminum [Preston-Thomas (1990)]. The standard deviation of the above equation at the 95% confidence level is 0.65%.

For iron (temperature range 1809–2480 K)

$$\rho = c_3 - c_4(T - T_{\mathrm{ref}}), \qquad (2)$$

where $c_3 = 7034.96\ \mathrm{kg\,m^{-3}}$, $c_4 = 0.926\ \mathrm{kg\,m^{-3}\,K^{-1}}$, and $T_{\mathrm{ref}} = 1811.0$ K is the melting point of iron [Dinsdale (1991)]. The standard deviation of the above equation at the 95% confidence level is 0.77%.

In Figs. 1–4 the primary data and their percentage deviations from the above two equations for aluminum and iron are shown, respectively. The deviations are consistent with the quoted uncertainties.

The overall uncertainty in the absolute values of the density is estimated to be one of ±0.7% for aluminum and 0.8% for iron, which is worse than that of the most optimistic claims but recognizes the unexplained discrepancies between different methods.

Density values calculated from the above two equations are shown in Table 3.

## 4. Viscosity

### 4.1. Experimental Techniques

There is a large number of methods to measure the viscosity of liquids, but those suitable for liquid metals are limited by the low viscosities of metals (of the order of 1 mPa s); their chemical reactivity and generally high melting points. Proposed methods include: capillary, oscillating vessel, rotational bob or crucible, oscillating plate, draining vessel, and levitation using the damping of surface oscillations in levitated and acoustic methods.

#### 4.1.1. The Capillary Rheometer

The capillary rheometer is generally thought to be the best method for the measurement of the viscosity of liquids [Iida and Guthrie (1988)] and is based upon the time for a finite volume of liquid to flow through a narrow bore tube under a given pressure (Fig. 5). The relation between viscosity and

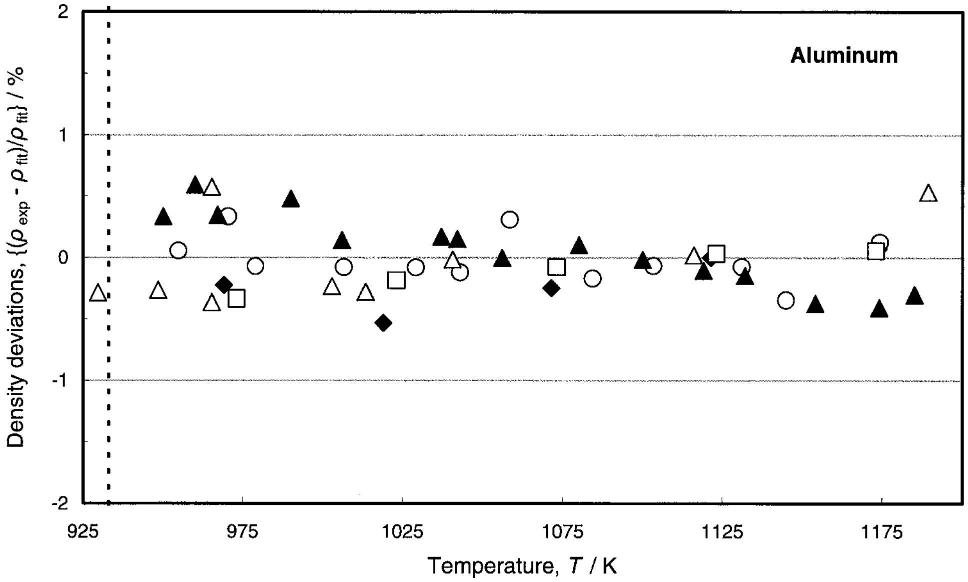

FIG. 2. Percentage deviations of the density primary data for liquid aluminum from the values calculated by Eq. (19), as a function of the temperature: (◆) Smith *et al.* (1999); (○) Yatshenko *et al.* (1972); (△) Levin *et al.* (1968); (▲) Coy and Mateer (1955); (□) Gebhardt *et al.* (1955); (- - -) melting point.

efflux time is given by the modified Poiseuille equation or Hagen-Poiseuille equation [Nagashima *et al.* (1991)] as follows:

$$\eta = \frac{\pi r^4 \rho g h t}{8V(l+nr)} - \frac{m\rho V}{8\pi(l+nr)t}, \tag{3}$$

where $r$ and $l$ are the radius and length of the capillary, respectively, $h$ is the effective height of the column of liquid, $\rho$ is the liquid density, $V$ is the volume discharged in time $t$, and $m$ and $n$ are constants which can be determined experimentally. The product $\rho g h$ may be replaced by $\Delta P$, the pressure drop along the capillary, and $nr$ is called the end-correction, and corrects for surface tension effects as the liquid is expelled from the capillary. For liquid metals, with their relatively high densities, the second term, which corrects for kinetic energy, is particularly important.

This technique is often used as a relative, rather than absolute, method, as the experimental procedures are simple, and any errors incidental to the measurement of dimensions are thereby avoided. For a viscometer in which $r$, $l$, $h$, and $V$ are fixed, Eq. (3) reduces to

$$\frac{\eta}{\rho} = C_1 t - \frac{C_2}{t}, \tag{4}$$

where the values of $C_1$ and $C_2$ are easily evaluated using viscosity standard reference samples, but are constants equal to:

$$C_1 = \frac{\pi r^4 g h}{8V(l+nr)}, \quad C_2 = \frac{mV}{8\pi(l+nr)}. \tag{5}$$

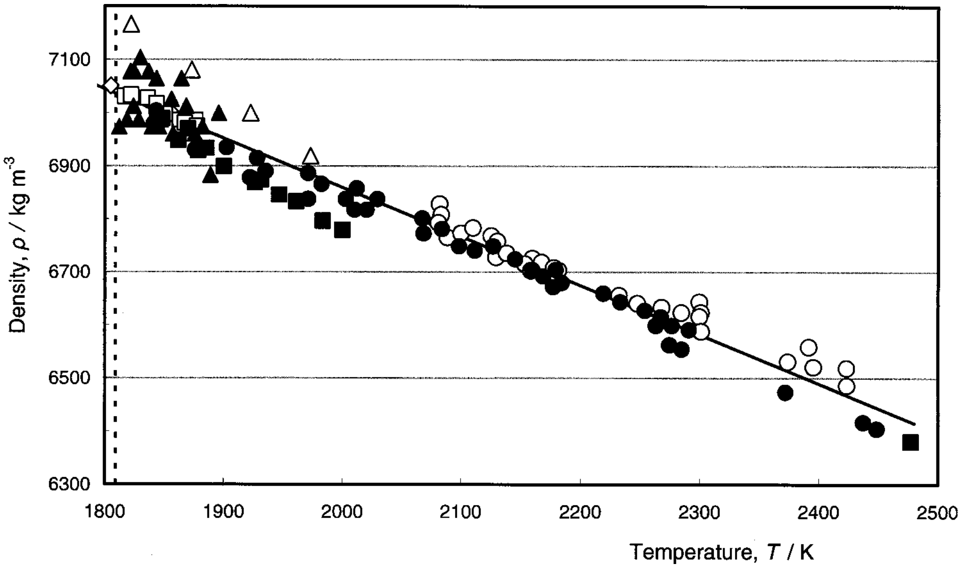

FIG. 3. Density primary data for liquid iron as a function of the temperature: (◇) Brillo and Egry (2004); (□) Sato (2003); (△) Lucas (1972); (○) Saito *et al.* (1969); (▲) Frohberg and Weber (1964a); (●) Kirshenbaum and Cahill (1962); (■) Lucas (1960); (—), fitted equation; (- - -) melting point.

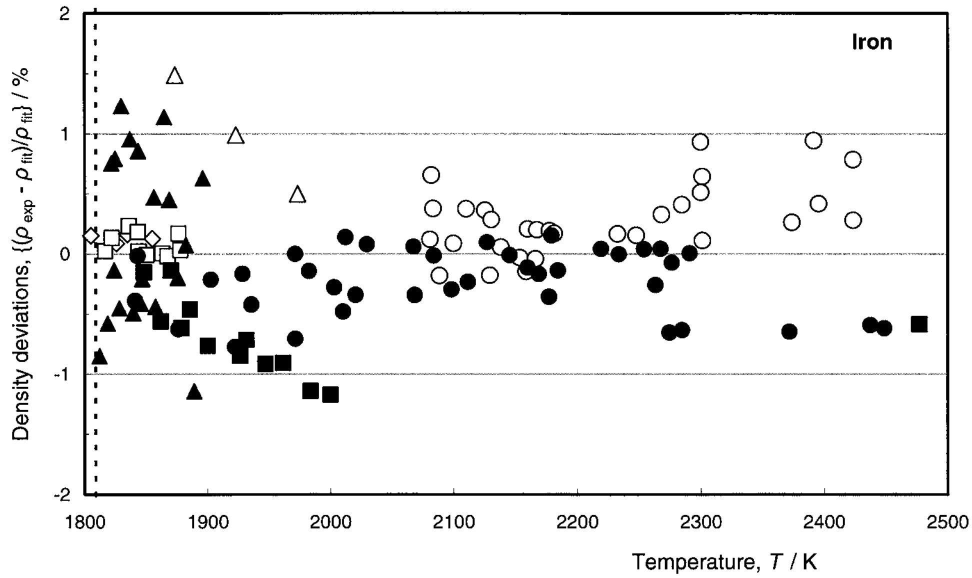

FIG. 4. Percentage deviations of the density primary data for liquid iron from the values calculated by Eq. (2), as a function of the temperature: (◇) Brillo and Egry (2004); (□) Sato (2003); (△) Lucas (1972); (○) Saito *et al.* (1969); (▲) Frohberg and Weber (1964a); (●) Kirshenbaum and Cahill (1962); (■) Lucas (1960); (- - -) melting point.

In determining the viscosities of metallic liquids by the capillary method, an especially fine and long-bore tube (in general, $r<0.15-0.2$ mm, $l>70-80$ mm) is needed so as to satisfy the condition of a low Reynolds number for ensuring laminar flow. This in turn requires a furnace with a similarly long and uniform hot zone. Blockage of the capillary by bubbles or oxide inclusions is a common problem, particularly with aluminum alloys, and due to materials problems a temperature limit of 1200 °C is often imposed, but metals such as bismuth have been successfully measured [Iida and Guthrie (1988)].

### 4.1.2. The Oscillating Vessel Viscometer

Most measurements of the viscosity of metals use some form of oscillating vessel viscometer. A vessel, normally a cylinder, containing the test liquid is set in motion about a vertical axis and the motion is damped by frictional energy absorption and dissipation within the liquid. The viscosity is determined from the decrement and time period of the motion. The main advantages of the method are that the time period and decrement are easily measured and the amount of liquid is relatively small which allows stable temperature profiles to be attained. One of the major difficulties is relating the measured parameters to the viscosity through the second order differential equation for the motion of an oscillating system and there are a number of mathematical treatments appearing to yield different results [Knappworst (1952); Shvidkovskiy (1955); Roscoe (1958); Kestin and Newell (1957); Brockner *et al.* (1979)] with the same experimental data. Since this is the most commonly used method it will be discussed in more detail below.

TABLE 3. Recommended values for the density of liquid aluminum and liquid iron

| Liquid aluminum | | Liquid iron | |
|---|---|---|---|
| Temperature ($T$/K) | Density ($\rho$/kg m$^{-3}$) | Temperature ($T$/K) | Density ($\rho$/kg m$^{-3}$) |
| 950 | 2372 | 1850 | 6999 |
| 975 | 2364 | 1900 | 6953 |
| 1000 | 2357 | 1950 | 6906 |
| 1025 | 2349 | 2000 | 6860 |
| 1050 | 2341 | 2050 | 6814 |
| 1075 | 2333 | 2100 | 6767 |
| 1100 | 2325 | 2150 | 6721 |
| 1125 | 2318 | 2200 | 6675 |
| 1150 | 2310 | 2250 | 6628 |
| 1175 | 2302 | 2300 | 6582 |
| 1200 | 2294 | 2350 | 6536 |
| | | 2400 | 6490 |
| | | 2450 | 6443 |
| | | 2500 | 6397 |

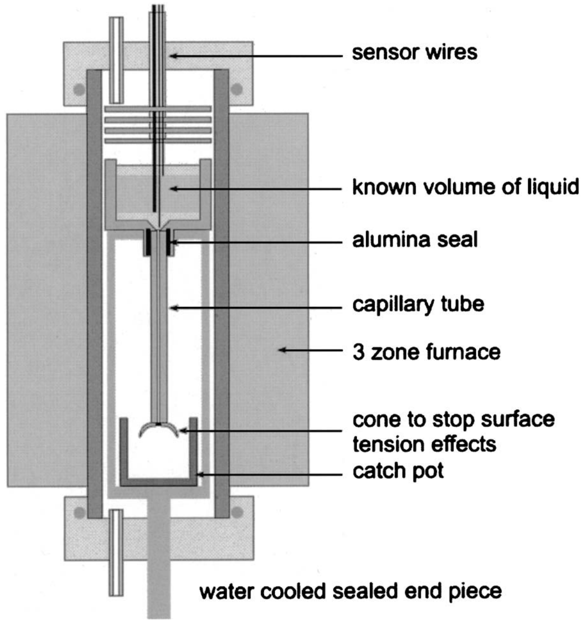

FIG. 5. Diagram of a capillary viscometer.

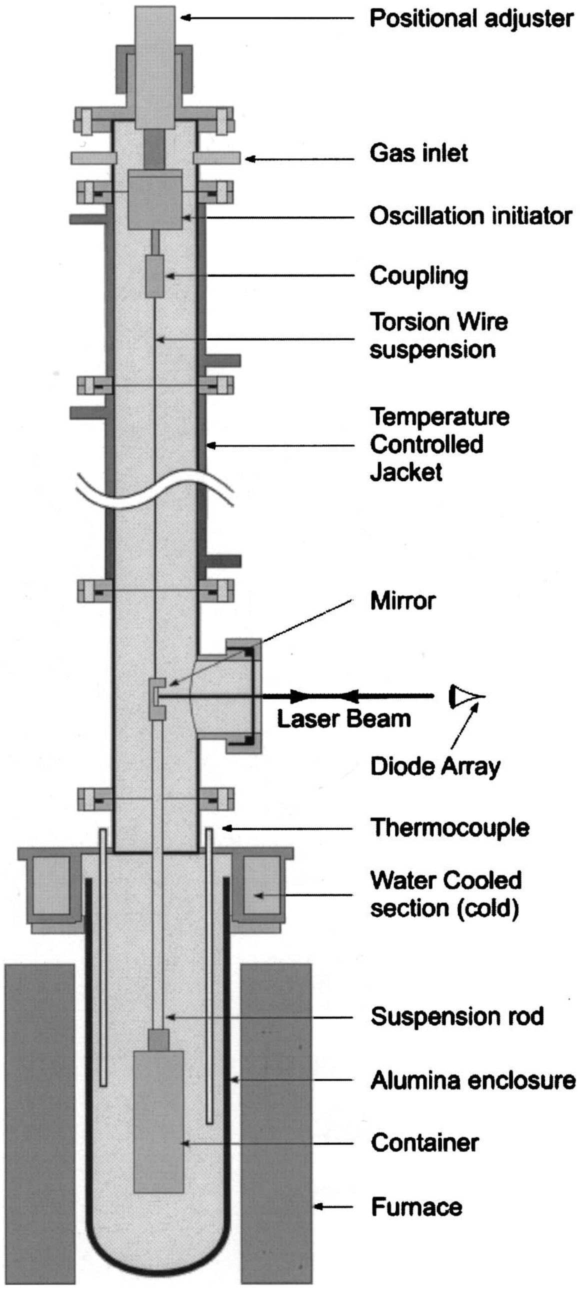

FIG. 6. Diagram of the NPL oscillating viscometer.

Figure 6 shows the experimental arrangement for an oscillating vessel viscometer and details of this particular design are given elsewhere [Brooks *et al.* (2001)]. The sample is contained within an alumina crucible (105 mm long by 14 mm internal diameter), which is screwed into a molybdenum lid and suspension rod and suspended on a torsion wire. A rotary solenoid is used to impart oscillatory motion to the crucible and an optical pointer with a diode array is used to measure the time constant and decrement of the system. The sample is heated by a two zone furnace. These authors claim an uncertainty of measurement of ±9% within a 95% confidence limit. A major contribution to the uncertainty is the extrapolation of the dimensions of the crucible and the height of liquid at high temperatures.

For a right circular cylinder that is infinitely long containing a fluid the equation of motion of the damped cylinder is:

$$I_0(d^2\theta/dt^2)+L(d\theta/dt)+f\theta=0, \tag{6}$$

where $I_0$ is moment of inertia of empty cup and suspension; $t$ is the time; $f$ the force constant of the torsion wire; and $\theta$ is the angle of displacement of any small segment of the fluid from its equilibrium position. $L$ is a function of the density and viscosity of the fluid; the internal radius of crucible and height of liquid expressions for $L$ are determined by solving the Navier–Stokes equations for the motion of the liquid within the vessel (neglecting nonlinear terms). The working formulas for the oscillating cup viscometer reviewed by Iida and Guthrie (1988) are Knappworst (1952), Shvidkovskkii (1955), and Roscoe (1958). The majority of measurements made by this method have used the analysis by Roscoe, mainly for reasons of simplicity of presentation of the working formulas of that reference and following other workers in the liquid metals field. A more complete model of oscillating cup viscometers was described by Kestin and Newell (1957), and Beckwith and Newell (1957). The resulting working equations were not stated explicitly in the early references and required some further analysis for their derivation. These have been adopted by chemical engineers and it has been suggested [Ferris *et al.* (2002) and Wang and Overfelt (2002)] that the Beckwith–Newell model is more accurate and comprehensive than that of Roscoe and should therefore be adopted for future analysis, a conclusion with which the present authors agree. We have verified that in all cases of primary data included in this work the consequences of employing less than complete working equations are negligible relative to the other sources of discrepancy between the results of different workers.

*4.1.2.1. The Roscoe Analysis.* Roscoe (1958) describes the derivation of the necessary formulas for determining viscosity from oscillatory spherical and cylindrical viscometers. The logarithmic decrement of the oscillations is related to the properties of the fluid by the equations

$$I\Delta=\frac{R^3H}{2}(\pi\eta\rho T)^{1/2}\left[\left(1+\frac{1}{4}\frac{R}{H}\right)a_0-\left(\frac{3}{2}+\frac{4}{\pi}\frac{R}{H}\right)\frac{1}{p}\right.$$
$$\left.+\left(\frac{3}{8}+\frac{9}{4}\frac{R}{H}\right)\frac{a_2}{2p^2}-\left(\frac{63}{128}-\frac{45}{64}\frac{R}{H}\right)\frac{a_4}{4p^4}\cdots\right], \tag{7}$$

where

$$a_0=(1-\Delta)\left\{\frac{\sqrt{(1+\Delta^2)}+1}{2}\right\}^{1/2}-(1+\Delta)\left\{\frac{\sqrt{1+\Delta^2}-1}{2}\right\}^{1/2}$$
$$=1-\frac{3}{2}\Delta-\frac{3}{8}\Delta^2-\frac{1}{16}\Delta^3\ldots, \tag{8}$$

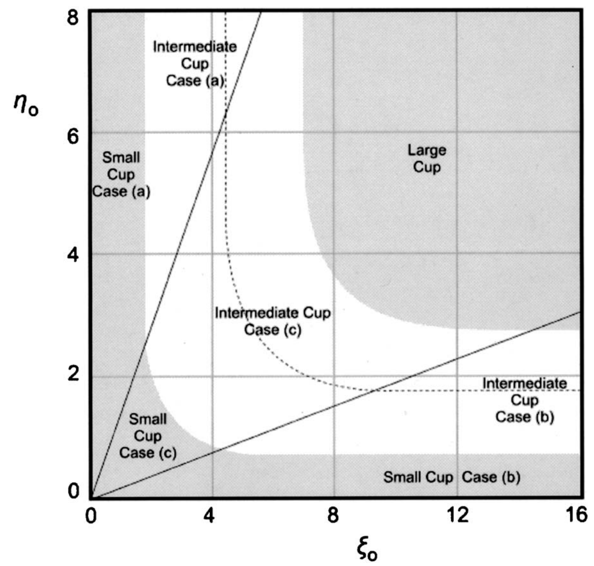

FIG. 7. Diagram [Beckwith (1957)] showing region of interest $8<\xi_0<23$, $68<\eta_0<200$ to be within the large cup regime [$\eta_0=H/(\eta T_0/2\pi\rho)^{1/2}$, and $\xi_0=R/(\eta T_0/2\pi\rho)^{1/2}$].

$$a_2=\left\{\frac{\sqrt{1+\Delta^2}+1}{2}\right\}^{1/2}+\left\{\frac{\sqrt{1+\Delta^2}-1}{2}\right\}^{1/2}$$

$$=1+\frac{\Delta}{2}+\frac{1}{8}\Delta^2-\frac{1}{16}\Delta^3\ldots, \tag{9}$$

$$a_4=a_2/(1+\Delta^2)^{1/2}=1+\frac{\Delta}{2}-\frac{3}{8}\Delta^2-\frac{5}{16}\Delta^3\ldots, \tag{10}$$

and

$$p=(\rho\pi/\eta T)^{1/2}R. \tag{11}$$

In Eq. (7), $R$ is the cylinder radius, $H$ the height of the liquid within it, $I$ is the moment of inertia of the suspension, $2\pi\Delta$ the logarithmic decrement between consecutive swings, $\rho$ is the fluid density, $\eta$ the viscosity, and $T$ is the period of oscillation.

In his review of the equations Ferriss *et al.* (2002), found that Eq. (8) was printed incorrectly in the original paper by Roscoe (1958), and should read $(-3\Delta/2)$, as given above.

*4.1.2.2. The Beckwith, Kestin, and Newell Analyses.* The analyses due to Kestin and Newell (1957), and Beckwith and Newell (1957) identify the "small," "intermediate," and "large" cup regimes according to the magnitude of certain dimensionless parameters (Fig. 7). Ferriss *et al.* (2002) has shown that these values for the NPL design correspond to the "large cup" regime.

The corresponding operating equation due to Beckwith and Newell, in the form of Brockner *et al.* (1979), is

$$\frac{\pi\rho H R^4}{2I}\left[A(p-\Delta q)\frac{1}{x}-\frac{B}{x^2}+\frac{Cq}{x^3}\right]=2(\Delta-\Delta_0/\omega), \tag{12}$$

where

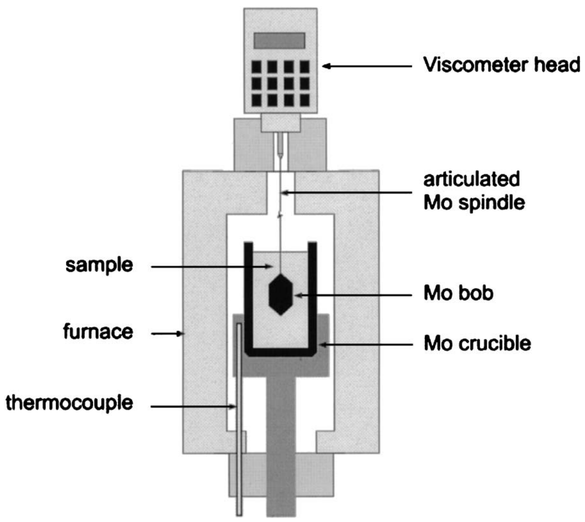

FIG. 8. Diagram of a rotating bob viscometer.

$$A=4+\frac{R}{H};\quad B=6+\frac{16}{\pi}\cdot\frac{R}{H};\quad C=\frac{3}{2}+\frac{9R}{H}, \tag{13}$$

$$\omega=T_0/T;\quad p=1/(2(\Delta+(1+\Delta^2)^{1/2}))^{1/2};\quad q=1/(2p), \tag{14}$$

and

$$x=R(2\pi\rho/(\eta T))^{1/2}, \tag{15}$$

where $T_0$ is the oscillation period and $2\pi\Delta_0$ is the logarithmic decrement for the empty cup. The other quantities are as defined in the previous section.

Comparison of the oscillating viscometer with the capillary viscometer reveals some differences. Iida and Guthrie (1988) suggest that the finite length of the oscillating cylinder is inadequately weighted in the Roscoe treatment and should also include the effect of the liquid meniscus on the height of liquid, suggesting that a correction factor, $\xi$, is introduced into the (uncorrected) formula of $1\pm0.04$. Brockner *et al.* (1979) actually demonstrated and corrected for the meniscus effect. Various studies [Iida and Guthrie (1988); Kimura and Terashima (1994); Kimura and Terashima (1995); Sasaki *et al.* (1995); and Sato and Yamamura (1999)], have suggested that wetting of the crucible may also be important, and that if the metal does not wet the crucible it may slip during the oscillation and thus provide greater damping. Aspect ratio of the sample may be important in both these assessments, and further work is required to justify modification of the Roscoe equation.

#### 4.1.3. The Rotating Cylinder Viscometer

For the rotating cylinder technique the torque on a cylinder rotated in a liquid is related to the viscosity of the fluid. Viscometers of this type consist of two concentric cylinders (see Fig. 8) named a bob (internal) and a crucible (external). The viscosity is determined from measurements of the torque generated on the rotor arm of the rotating cylinder. When rotating the cylinder at a constant speed the viscosity can be obtained from the following equation:

$$\eta=\left(\frac{1}{r_1^2}-\frac{1}{r_0^2}\right)\frac{M}{8\pi^4 nh}, \tag{16}$$

where $M$ is the torque; $n$ is the number of revolutions per second; $r_1$ is the radius of the bob; $r_0$ is the radius of the crucible; and $h$ the height of the bob. The theory is applicable to infinitely long cylinders and it is normal to calibrate the system with reference materials using the equation:

$$\eta=G=S/n, \tag{17}$$

where $S$ is the scale deflection and $G$ is the apparatus constant.

This is the most common method for the measurement of slag viscosity and for several practical reasons it is more customary to rotate the bob rather than the crucible. Probably the most important reason is that the rotating bob viscometer is based upon readily available and cheap commercial instruments. It is also easier to center the bob but the viscosity range is less than for a rotating crucible instrument. Nakashima *et al.* (1997) showed a modern example of a rotating crucible method.

In order to obtain the necessary sensitivity to measure the low viscosity of liquid metals the clearance between the stationary and rotating parts has to be made very small, and it is difficult to maintain the system coaxial. In spite of the experimental difficulties the rotating bob technique has been used to measure the viscosity of aluminum and its alloys by Jones and Bartlett (1952–1953).

### 4.1.4. The Draining Vessel Method

It is common for comparative measurements of viscosity to be made for oils and slurries in industrial applications by use of a flow cup (see Fig. 9), where the time taken for a volume of sample to flow through a small orifice in the bottom of a cup is measured and viscosity is derived from look-up tables. Roach *et al.* (2001) have derived equations to adapt this method for liquid metals, and to provide values of viscosity, surface tension, and density. Values for aluminum are lower than usually quoted, but the method is robust and experiments simple to perform.

### 4.1.5. The Oscillating (Levitated) Drop Method

The oscillating drop method is widely used for surface tension measurements of liquid, levitated samples. It is based on the fact that the frequencies of the surface oscillations of a liquid drop free from other forces are related to the surface tension by Rayleigh's formula [Lord Rayleigh (1879)]. For earthbound levitation, the Rayleigh formula has been corrected by Cummings and Blackburn (1991). For a viscous drop, these oscillations are damped owing to the viscosity of the liquid. Therefore, it is, in principle, also possible to determine the viscosity from the damping. The damping constant $\Gamma$ is given by:

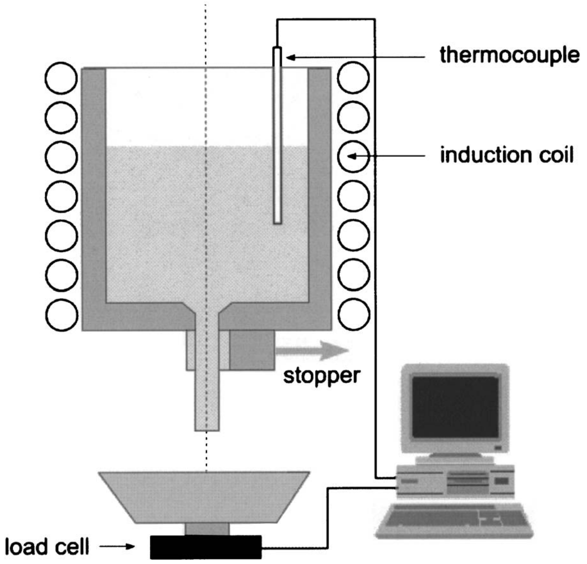

FIG. 9. Diagram of a draining vessel viscometer.

$$\Gamma=\frac{20\pi}{3}\frac{R\eta}{m}, \tag{18}$$

where $R$ and $m$ are the radius and the mass of the droplet, respectively, and $\eta$ is the viscosity. In order to apply the above formula, the following conditions must be satisfied:

(a) the liquid drop must be spherical;
(b) the oscillations must persist undisturbed for a time, $t=1/\Gamma$; and
(c) there must be no additional damping mechanism present.

It is often thought that none of these conditions is met in terrestrial experiments; the sample is deformed, and there is an overlap of self-excited oscillations, making the decay of one single excitation hard to detect. Also, the electromagnetic fields necessary for earthbound levitation inevitably induce turbulent fluid flows inside the sample which lead to additional damping, although Rhim *et al.* (1999) have claimed success terrestrially using this method with electrostatic levitation. Under microgravity conditions, the external fields can be much weaker, and a laminar fluid flow can be expected, at least for sufficiently viscous materials. In microgravity an electromagnetic positioning field is used, with a separate heating coil. A pulse of power initiates the oscillations which are monitored with a video camera. The temperature is recorded with a pyrometer. Image analysis is used to obtain the frequency and decay of the oscillations as shown in Fig. 10. Microgravity experiments utilizing the oscillating drop technique were carried out on liquid gold in 1994. This material has a low viscosity and unphysically high results for the viscosity were obtained indicating turbulent flows in the sample. During the MSL-1 Spacelab mission STS-83, the same experiment was carried out on the highly viscous eutectic system $Cu_6Pd_{78}Si_{16}$. Values agree

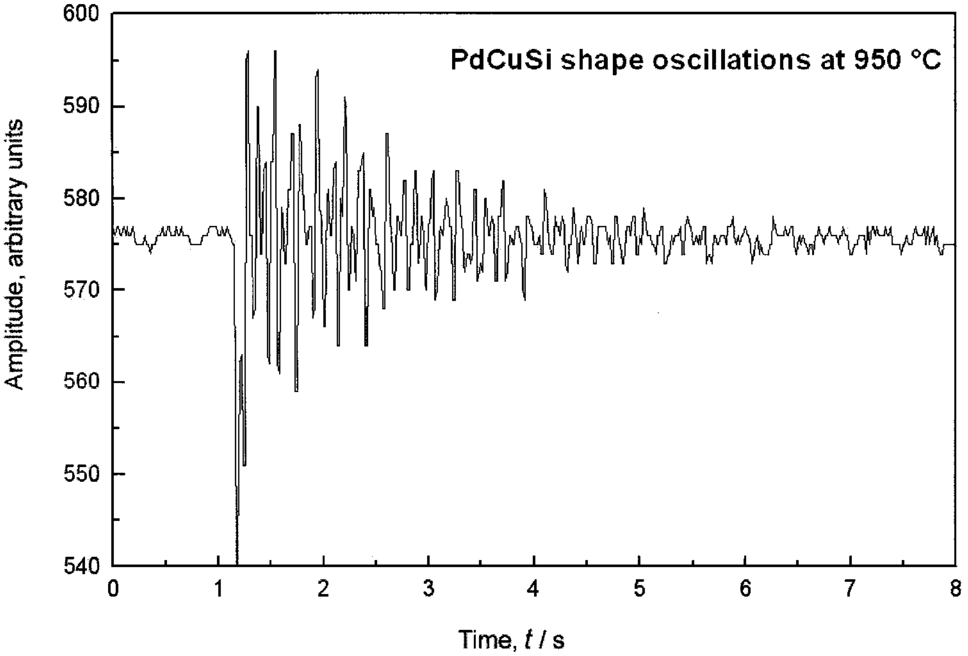

FIG. 10. Damping of shape oscillations at 950 °C.

well with one set of previously published data for the viscosity of this alloy near the melting point, although there is debate as to the true viscosity of this system. More recently Wunderlich *et al.* (2004) has carried out measurements on parabolic and sounding rocket flights, on the nickel based superalloy CMSX-4 which show reasonable agreement with terrestrial measurements of the same alloy using an oscillating cylinder method.

#### 4.1.6. Acoustic Wave Damping

The viscosity of a liquid can be measured by measuring the damping of an acoustic wave. There is one example of this technique [Nozdrev *et al.* (1979)], for the measurement of aluminum and some alloys using hole theory to interpret the results.

### 4.2. Data Compilation

Tables 4 and 5 present the data sets found for the measurement of the viscosity of liquid aluminum and liquid iron, respectively. As in the case of the density measurements, papers prior to 1930 were not considered, as sample purity was disputed before that time. In these tables, for every data set, the sample purity, the technique employed, the uncertainty quoted, the form of data, the material in contact with the sample, the density source, and the equation employed are also shown. The data sets have been classified into primary and secondary sets according to the criteria presented in Sec. 2 and in conjunction with the techniques described in Sec. 4.1.

In the case of the viscosity data sets and in relation to the discussion in Sec. 4.1, the following points can be noted: Data sets that employ the equation of Knappwost (1952) have not been considered as primary data sets. Furthermore, data sets that did not give details for most of the columns in the table were also not considered as primary data.

### 4.3. Viscosity Reference Correlation

The primary viscosity data for liquid aluminum and iron, shown in Tables 4 and 5, respectively, were employed in a regression analysis as a function of the temperature. The following equations were obtained for the density, $\eta$ [mPa s], as a function of the absolute temperature, $T$ [K]:
for aluminum (temperature range 933–1270 K)

$$\log_{10}(\eta/\eta^0) = -a_1 + \frac{a_2}{T}, \tag{19}$$

where $\eta^0 = 1$ mPa s, $a_1 = 0.7324$, and $a_2 = 803.49$ K. The standard deviation of the above equation at the 95% confidence level is 13.7%:

For iron (temperature range 1809–2480 K)

$$\log_{10}(\eta/\eta^0) = -a_3 + \frac{a_4}{T}, \tag{20}$$

where $\eta^0 = 1$ mPa s, $a_3 = 0.7209$, and $a_4 = 2694.95$ K. The standard deviation of the above equation at the 95% confidence level is 5.7%.

In Figs. 11–14 the primary data and their percentage deviations from the above two equations for aluminum and iron, are shown, respectively.

Viscosity values calculated from the above two equations are shown in Table 6. The uncertainty in the absolute values of the viscosity of the two metals is thought to be no larger

TABLE 4. Data sets considered for the viscosity of liquid aluminum

| Reference | Purity (mass %) | Technique employed[a] | Uncertainty quoted/% | Form of data[b] | Material in contact with sample | Density source[c] | Equation employed |
|---|---|---|---|---|---|---|---|
| Primary data | | | | | | | |
| Sato (2004) | 99.99 | OV | 1 | P | Graphite | L | Roscoe |
| Mills (2002) | 99.995 | OV | 2.5 | D | Alumina | L | Roscoe |
| Wang and Overfelt (2002) | 99.995 | OV | 4 | P | Graphite | M | Wang and Overfelt |
| Yamasaki *et al.* (1993) | 99.99 | OV | — | D | — | — | Roscoe |
| Arsent'ev and Polyakova (1977a) | Various | OV | 5 | D | Alumina | — | Shvidkovskiy |
| Pakiewicz (1970) | 99.999 | OV | 1 | P | Graphite | L | Roscoe |
| Rothwell (1961–1962) | 99.9 | OS | <1 | P | Graphite | L | Brewer modified Andrade |
| Gebhardt and Becker (1955) | 99.996 | OV | — | P | Carbon | M | Calibration |
| Secondary data | | | | | | | |
| Efimenko *et al.* (1988) | — | — | — | D | — | — | — |
| Arsent'ev *et al.* (1987) | Various | OV | — | D | — | — | — |
| Kisun'ko *et al.* (1983)[d] | — | OV | 4.5 | D | | L | — |
| Popel (1983) | — | OV | — | D | BeO | M | — |
| Kisun'ko *et al.* (1980)[d] | — | OV | — | D | — | L | — |
| Nozdrev *et al.* (1979) | — | AC | — | D | — | — | — |
| Arsent'ev and Polyakova (1978) | Various | OV | — | D | Alumina | — | — |
| Novokhatskii *et al.* (1978) | — | — | — | D | — | — | — |
| Arsent'ev and Polyakova (1977b) | Various | OV | — | D | Alumina | — | — |
| Levin (1971)[d] | 99.99 | OV | — | D | — | L | — |
| Petrushevskii *et al.* (1971) | 99.99 | OV | 5 | D | — | — | Shvidkovskiy |
| Kononenko *et al.* (1969)[d] | 99.7 | OV | — | D | Graphite | L | Empirical |
| Lihl *et al.* (1968) | 99.99 | OV | — | D | Alumina | — | Empirical |
| Vignau *et al.* (1967) | 99.996 | OV | — | D | — | — | Roscoe |
| Lihl and Schwaiger (1967) | 99.99 | OV | — | D | — | — | — |
| Gebhardt and Detering (1959) | 99.996 | OV | — | P | — | L | Empirical |
| Secondary data | | | | | | | |
| Navarro and Kondic (1956) | 99.996 | OV | — | P | Alumina or Graphite | — | Empirical |
| Shvidkovskiy (1955)[d] | 99.7 | OV | — | D | Graphite | L | Shvidkovskiy |
| Gebhardt *et al.* (1953) | 99.996 | OV | — | P | — | M | — |
| Jones and Bartlett (1952–1953) | — | OV | 0.5 | D | Graphite | — | Empirical |
| Yao and Kondic (1952–1953) | 99.9935 | OV | — | P | Graphite | — | Empirical |
| Salceanu (1948) | — | OV | — | P | — | — | — |
| Sergeev and Polyak (1947) | — | OS | — | D | — | — | — |

[a] AC = acoustic; OV = oscillating vessel; OS = oscillating sphere.

[b] D = diagram; P = points.

[c] M = measured; L = literature.

[d] Kinematic viscosity was measured.

than the scatter between measurements made with different techniques and so can be said to be ±14% in the case of aluminum and ±6% in the case of iron.

In the particular case of aluminum it is necessary to add one caveat to this statement of uncertainty. Some of the experiments that we have characterized as primary have used an inert gas atmosphere when studying aluminum to prevent the formation of the oxide of aluminum on the surface of the melt and its subsequent entrainment in the body of the liquid. It is exceedingly unlikely that any experiments have succeeded in achieving a partial pressure of oxygen in the atmosphere that approaches the $10^{-23}$ that is necessary to avoid oxide formation. Accordingly, it is likely that alumina was present to some extent in all samples. It can also be expected

TABLE 5. Data sets considered for the viscosity of liquid iron

| Reference | Purity (mass %) | Technique employed[a] | Uncertainty quoted/% | Form of data[b] | Material in contact with sample | Density source[c] | Equation employed |
|---|---|---|---|---|---|---|---|
| Primary data | | | | | | | |
| Sato *et al.* (2002) | 99.8 | OV | 1 | D | Alumina | M | Roscoe |
| Brooks *et al.* (2001) | 99.8 | OV | 4.5 | D | Alumina | L | Roscoe |
| Kaplun and Avaliani (1977) | 99.92 | OV | 5 | D | Zircon or Alumina | — | Shvidkovskiy |
| Cavalier (1959)[d] | 99.996 | OV | — | D | Alumina | L | Calibration |
| Secondary data | | | | | | | |
| Arsent'ev *et al.* (1987) | — | OV | — | D | — | — | — |
| Sroka and Skala (1979) | — | OV | — | D | Alumina | — | Knappwost |
| Arsent'ev (1978)[d] | — | OV | — | D | — | — | — |
| Arkharov *et al.* (1977) | 99.997 | — | — | P | — | — | — |
| Frohberg and Cakici (1977) | 99.91 | OV | — | D | — | — | Modified Roscoe |
| Ershov (1976) | — | OV | — | P | Alumina | — | — |
| Borgmann and Frohberg (1974) | — | OV | — | D | Alumina | L | Modified Roscoe |
| Ogino *et al.* (1973) | — | OV | — | D | — | — | — |
| Kisun'ko *et al.* (1973)[d] | 99.98 | OV | 1 | D | — | — | — |
| Arsent'ev and Filippov (1971) | 99.97 | OV | — | E | — | — | — |
| Adachi *et al.* (1971) | 99.97 | OV | — | D | Alumina | M | Knappwost |
| Ogino *et al.* (1971) | 99.97 | OV | — | D | Alumina | M | Knappwost |
| Orecki *et al.* (1971) | — | OV | — | P | — | — | Shvidkovskiy |
| Adachi *et al.* (1970) | 99.97 | OV | — | D | — | M | — |
| Arsent'ev *et al.* (1974) | 99.91 | OV | — | D | — | — | — |
| Ogino *et al.* (1970) | 99.97 | OV | — | D | — | M | — |
| Saito *et al.* (1967) | 99.987 | OV | — | D | Alumina | — | — |
| Frohberg and Weber (1964b) | — | OV | — | P | — | — | — |
| Lucas (1964) | — | OV | — | P | — | — | — |
| Romanov and Kochegarowv (1964)[d] | 99.95 | OV | — | D | — | L | — |
| Cavalier (1963) | 99.992 | OV | — | P | — | L | — |
| Primary data | | | | | | | |
| Vatolin *et al.* (1963) | 99.95 | OV | — | P | Alumina | L | Shvidkovskiy |
| Schenck *et al.* (1963) | 99.995 | OV | — | P | — | L | Knappwost |
| Wen Li-Shih and Arsent'ev (1961) | 99.91 | OV | 5 | — | Alundum | — | Shvidkovskiy |
| Romanov and Kochegarov (1960)[d] | — | — | — | D | — | L | — |
| Barfield and Kitchener (1955) | 99.94 | OV | 4 | D | Alumina | L | Toye and Hopkins |
| Salceanu (1948) | — | OV | — | P | — | — | — |

[a] OV = oscillating vessel.

[b] D = diagram; P = points.

[c] M = measured; L = literature.

[d] Kinematic viscosity was measured.

therefore that there would be some differences between the use of alumina vessels for containment and that with other materials such as graphite. These observations are supported by the fact that experiments have shown that the apparent viscosity of aluminum increases as the partial pressure of oxygen in the atmosphere increases and with increasing time of exposure [Mills (2002), Vigneau *et al.* (1967), Shvidkovskiy (1955)]. For this reason we suggest that our stated values for the viscosity of aluminum be taken as those appropriate to circumstances created in a well-controlled gaseous atmosphere. These are likely to be the circumstances encountered in usual industrial practice.

## 5. Conclusions

The available experimental data for the density and viscosity of liquid aluminum and iron have been critically examined with the intention of establishing a density and a

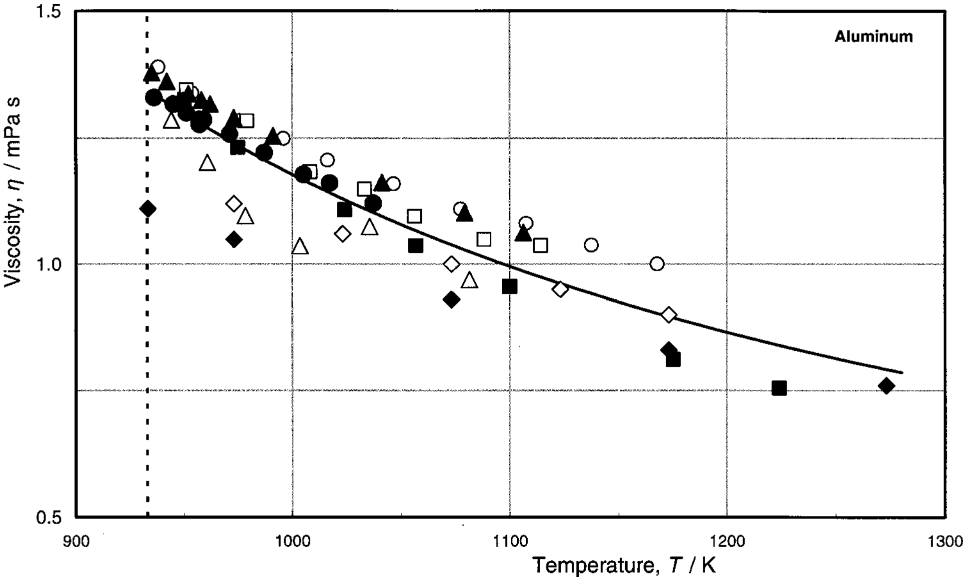

FIG. 11. Viscosity primary data for liquid aluminum as a function of the temperature: (○) Sato (2004); (□) Wang and Overfelt (2002); (◆) Mills (2001); (△) Yamasaki *et al.* (1993); (■) Arsent'ev and Polyakova (1977); (●) Pakiewicz (1970); (▲) Rothwell (1961–1962); (◇) Gebhardt and Becker (1955); (—) fitted equation; (- - -) melting point.

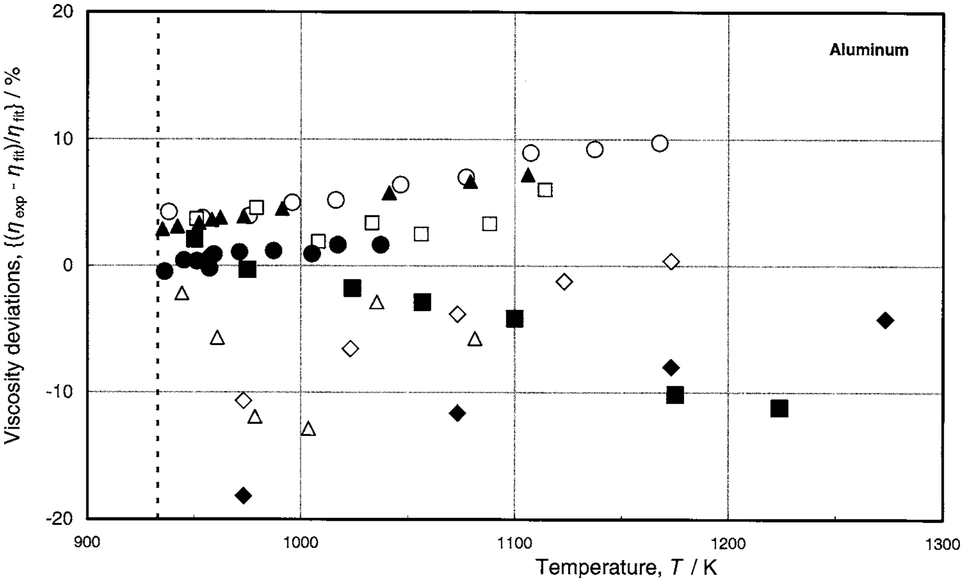

FIG. 12. Percentage deviations of the viscosity primary data for liquid aluminum from the values calculated by Eq. (19), as a function of the temperature: (○) Sato (2004); (□) Wang and Overfelt (2002); (◆) Mills (2001); (△) Yamasaki *et al.* (1993); (■) Arsent'ev and Polyakova (1977); (●) Pakiewicz (1970); (▲) Rothwell (1961–1962); (◇) Gebhardt and Becker (1955); (- - -) melting point.

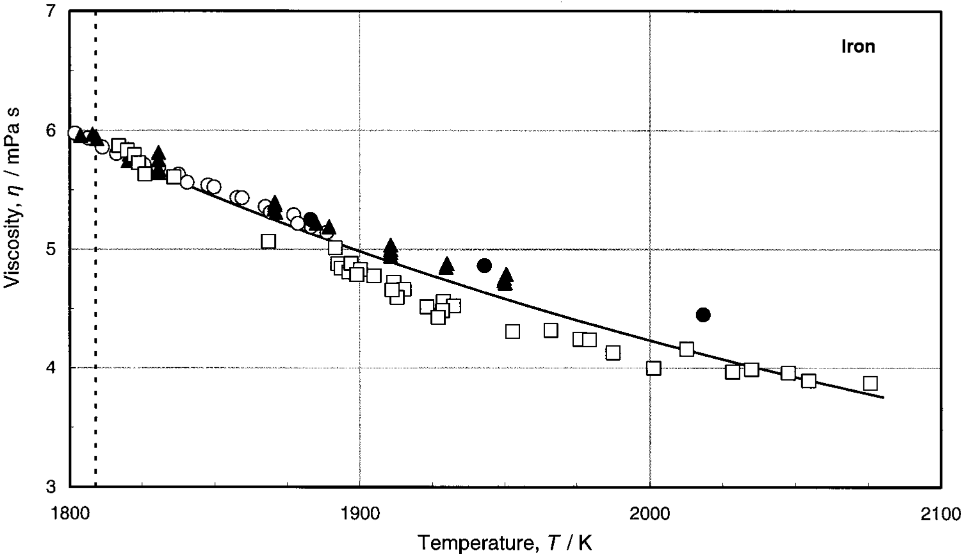

FIG. 13. Viscosity primary data for liquid iron as a function of the temperature: (○) Sato (2003); (▲) Brooks (2001); (●) Cavalier (1959); (□) Kaplun (1977); (—) fitted equation; (- - -) melting point.

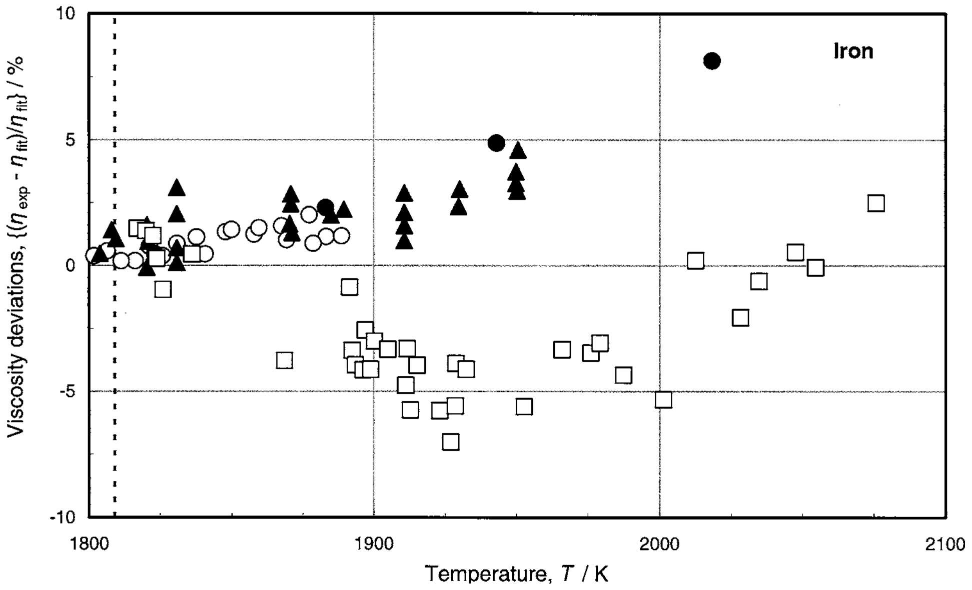

FIG. 14. Percentage deviations of the viscosity primary data for liquid iron from the values calculated by Eq. (20), as a function of the temperature: (○) Sato (2003); (▲) Brooks (2001); (●) Cavalier (1959); (□) Kaplun (1977); (- - -) melting point.

viscosity standard. All experimental data have been categorized into primary and secondary data according to the quality of measurement specified by a series of criteria. The proposed standard reference correlations for the density of the aluminum and iron are characterized by 0.65% and 0.77% percentage standard deviation, at the 95% confidence level, respectively. The standard reference correlations for the viscosity of aluminum and iron are characterized by 13.7% and 5.7% percentage standard deviation, at the 95% confidence level, respectively. To lower this viscosity uncertainty, more highly accurate measurements are necessary.

The proposed correlations are for the saturation conditions. Although in some applications, as the flow in a tube or a nozzle, the pressure is higher than the saturation pressure, the pressure dependences of the density and the viscosity of liquid metals are considered as not high.

TABLE 6. Recommended values for the viscosity of liquid aluminum and liquid iron

| Liquid aluminum | | Liquid iron | |
|---|---|---|---|
| Temperature ($T$/K) | Viscosity ($\eta$/mPa s) | Temperature ($T$/K) | Viscosity ($\eta$/mPa s) |
| 950 | 1.298 | 1850 | 5.443 |
| 975 | 1.235 | 1900 | 4.983 |
| 1000 | 1.178 | 1950 | 4.583 |
| 1025 | 1.126 | 2000 | 4.232 |
| 1050 | 1.079 | 2050 | 3.924 |
| 1075 | 1.035 | 2100 | 3.651 |
| 1100 | 0.996 | 2150 | 3.408 |
| 1125 | 0.959 | 2200 | 3.192 |
| 1150 | 0.925 | 2250 | 2.998 |
| 1175 | 0.894 | 2300 | 2.824 |
| 1200 | 0.865 | 2350 | 2.666 |
| | | 2400 | 2.523 |
| | | 2450 | 2.394 |
| | | 2500 | 2.276 |

## 6. Acknowledgments

The work described in this paper was carried out under the auspices of the International Association for Transport Properties (formerly known as the Subcommittee of Transport Properties of the International Union of Pure and Applied Chemistry). The authors gratefully acknowledge the partial financial support of the International Union of Pure and Applied Chemistry (Contract No. 2003-005-1-100, titled: Viscosity of molten iron and aluminum).

## 7. References

Adachi, A., Maehana, T., Morita, Z., and Ogina, Y., Tetsu to Hagane **56**, 1633 (1970).

Adachi, A., Morita, Z., Ogino, Y., Kaito, H., Maehana, T., and Yokotani, K., Structural change of liquid iron and its alloys observed on the density and viscosity measurements, Proc. ICSTIS, Suppl. Trans. ISIJ **11**, 395–399 (1971).

Adachi, A., Morita, Z., Zenichiro, Y., and Ogina, Y., Sci. Technol. Iron Steel **1**, 395 (1971).

Arkharov, V.I., Kisun'ko, V.Z., and Novokhaskii, I., Teplofiz. Vy. Temp. **15**, 1208 (1971).

Arsent'ev, P.P., Vinogradov, B.G., and Lisitskii, B.S., Izv. Vyssh. Uchebn. Zaved., Cher. Metall. **7**, 181 (1974).

Arsent'ev, P.P., Probl. Stal'n. Slitka **7**, 28 (1978).

Arsent'ev, P.P., and Filippov, S.I., Steel in the USSR **1**, 64 (1971).

Arsent'ev, P.P., and Filippov, S.I., Probl. Stal'vn. Slitka, 249 (1974).

Arsent'ev, P.P., and Polyakova, K.I., Izv. Vyssh. Uchebn. Zaved., Tsvetn. Metall. **51**, 5 (1977a).

Arsent'ev, P.P., and Polyakova, K.I., Izv. Akad. Nauk. SSSR. Met. **2**, 65 (1977b).

Arsent'ev, P.P., and Polyakova, K.I., Akad. Nauk. SSSR. Ural, Nauchn. Tsentr. **2**, 16 (1978).

Arsent'ev, P.P., Ryzhonkov, D.I., and Polyakova, K.I., Liteinoe Proizvodsyve **1**, 9 (1987).

Assael, M.J., Ramires, M.L.V., Nieto de Castro, C.A., and Wakeham, W.A., J. Phys. Chem. Ref. Data **19**, 113 (1990).

Barfield, R.N. and Kitchener, J.A., J. Iron Steel Inst. **180**, 324 (1955).

Beckwith, D.A., and Newell, G.F., ZAMP **VIII**, 450 (1957).

Borgmann, F.O., and Frohberg, M.G., Trans. Iron Steel Inst. Japan **14**, 82 (1974).

Brillo, J., and Egry, I, Z. Metallkd **95**, 691 (2004).

Brockner, W., Torklep, K., and Oye, H.A., Ber. Bursenges Phys. Chem. **83**, 1 (1979).

Brooks, R.F, Day, A.P., Andon, R.J.L., Chapman, L.A., Mills, K.C., and Quested, P.N., High Temperatures-High Pressures **33**, 72 (2001).

Cavalier, G., The NPL Symposium on *The Physical Chemistry of Metallic Solutions and Intermetallic Compounds* (HMSO, London, 1959), Paper 4D.

Cavalier, G., Compt. Rend. **256**, 1308 (1963).

Coy, W.J., and Mateer, R.S., Trans. Amer. Soc. Metals **58**, 99–102 (1955).

Cummings, D.L., and Blackburn, D.A., J. Fluid Mech. **224**, 395 (1991).

Dinsdale, A.T., CALPHAD **15**, 317 (1991).

Efimenko, V.P., Kisunk'ko, V.S., Bychkov, Yu.B., and Beloborodov, A.Z., Soviet Cast Technology **9**, 37 (1988).

Ershov, G.S., and Kasatkin, A.A., Izv. Vyssh. Uchebn. Zaved., Chern. Metall., 141 (1976).

Ferriss, D.H., Quested, P.N., Chapman, L.A., and Day, A.P., "The Choice of Equations for the Measurement of Viscosity by the Oscillating Cylinder Method," Presented at ECTP, London, 2002.

Frohberg, M.G., and Cakici, T., Arch. Eisenhüttenw. **48**, 145 (1977).

Frohberg, M.G., and Weber, R., Arch. Eisenhuttenw. **35**, 877 (1964a).

Frohberg, M.G., and Weber, R., Arch. Eisenhüttenw. **35**, 885 (1964b).

Gebhardt, E., Becker, M., and Dorner, S., Z. Metallkde. **44**, 510 (1953).

Gebhardt, E., Becker, M., and Dorner, S., Aluminium **31**, 315 (1955).

Gebhardt, E., and Detering, K., Z. Metallkde. **50**, 379 (1959).

Iida, T., and Guthrie, R.I.L., *The Physical Properties of Liquid Metals* (Clarendon, Oxford, 1988).

Jones, W.R.D., and Bartlett, W.L., J. Inst. Metals **81**, 145 (1952–1953).

Kaplun, A.B., and Avaliani, M.I., High Temp. **15**, 259 (1977).

Kestin, J., and Newell, G.F., ZAMP **VIII**, 433 (1957).

Kimura, S., and Terashima, K. J., Cryst. Growth **139**, 225 (1994).

Kimura, S., and Terashima, K.J., Proc 4th Asian Thermophys. Prof. Conf., Toyko, September, 1995.

Kirshenbaum, A.D., and Cahill, J.A., Trans. Metal. Soc. AIME **224**, 816 (1962).

Kisun'ko, V.Z., Lad'yanov, V.I., Arharov, V.I., and Novokhatskii, I.A., Fiz. Met. Metallkde. (USSR) **36**, 529 (1973).

Kisun'ko, V.Z., Novokhatskii, I.A., Pogorelov, A., Lad'yanov, V.I., and Bychkov, Yu.B., Izv. Akad. Nauk. SSSR Met., 125 (1980).

Kisun'ko, V.Z., Novokhatskii, I.A., Beloborodov, A.Z., Bychkov, Yu.B., and Pogorelov, A.I., Tsvetnye Metally. Non-Ferrous Metals **4**, 74 (1983).

Knappwost, A., Z. Phys. Chem. **200**, 81 (1952).

Kononenko, V.I., Yatsenko, S.P., Rubinshtein, G.M., and Privalov, I.M., Teplofiz. Vysokikh. Temp. (USSR), 243 (1969) [In Russian] High Temp. **7**, 243 (1969) [in English].

Levin, E.S., Izv. Akad. Nauk SSSR. Met. **5**, 72 (1971).

Levin, E.S., Ayushina, G.D., and Gel'd, P.V., High Temperature **6**, 416418 (1968).

Lihl, F., and Schwaiger, A., Z. Metallkde. **58**, 777 (1967).

Lihl, F., Nachitgall, E., and Schwaiger, A., Z. Metallkde. **59**, 213 (1968).

Lord Rayleigh, Proc. Roy. Soc. **29A**, 71 (1879).

Lucas, L.D., Compt. Rend. **250**, 1850–1852 (1960).

Lucas, L.D., Compt. Rend. **259**, 3760 (1964).

Lucas, L.D., Mem. Sci. Rev. Met. **69**, 479–492 (1972).

Mills, K.C., *Recommended Thermophysical Properties for Selected Commercial Alloys* (Woodhead Publishing Limited, Cambridge, England, 2002).

Morita, Z.-I., Ogino, Y., Kaito, H., and Adachi, A., J. Japan Inst. Metals **34**, 248–253 (1970).

Nagashima, A., Sengers, J.V., and Wakeham, W.A., eds., "Experimental thermodynamics" *Measurement of the Transport Properties of Fluids*, Vol. III (Blackwell Scientific Publications, U.K., 1991).

Nakashima, K., Kawagoe, T., Ookado, T., and Mori, K., Molten Slags, Fluxes and Salts'97 Conference, Sydney, Australia, 1997.

Nasch, P.M., and Steinemann, S.G., Phys. Chem. Liq. **29**, 43–58 (1995).

Navarro, J.M., and Kondic, V., Inst. Hierro y Acero **9**, 953 (1956).

Novokhatskii, I.A., Arkharov, V.I., Lad'yanov, V.I., and Kisun'ko, V.Z., Sov. Phys.-Dokl. (USA), **23**, 57 (1978). [Dokl. Akad. Nauk SSSR **243**, 100 (1978)].

Nozdrev, V.F., Stremousov, V.I., and Takuchev, V.V., Russ. J. Phys. Chem. **53**, 677 (1979).

Nozdrev, V.F., Stremousov, V.I., and Tokuchev, V.V., Zh. Fiz. Khim. **53**, 1199 (1979).

Ogino, Y., Borgmann, F.O., and Frohberg, M.G., J. Jap. Inst. Met. **37**, 1230 (1973).

Ogino, Y., Morita, Z., and Adachi, A., Technol. Rep. Osaka Univ. (Japan), **21**, 399 (1971).

Ogino, Y., Morita, Z., Maebana, T., Yokoya, K., and Adachi, A., Tetsu Hagane **56**, 59 (1970).

Orecki, K., Sikora, B., and Zielinski, M., Prace Inst. Hutnicyzh **23**, 353 (1971).

Pakiewicz, M.J., Ph.D Department of Chemistry, Syracuse University, New York, 1970.

Petrushevskii, M.S., Levin, E.S., and Gel'd, P.V., Zhur Fiz Khim. **45**, 3035 (1971).

Popel, P.S., Zam. Russ. Met. **3**, 38 (1983).

Preston-Thomas, H., Metrologia **27**, 3 (1990).

Rhim, W.-K., Ohsaka, K., Paradis, P.-F., and Spjut, R.E., Rev. Sci. Instrum. **70**, 2796 (1999).

Roach, S.J., Henein, H., and Owens, D.C., *Light Metals 2001*, edited by J.L. Aujier (TMS, Warrendale, PA, 2001), pp. 1285–1291.

Romanov, A.A., and Kochegarowv, V.G., Izv. Akad. Nauk, SSSR, Otd, **1**, 30 (1960).

Romanov, A.A., and Kochegarowv, V.G., Fiz. Metal **18**, 869 (1964).

Roscoe, R., Proc. Phys. Soc. **72**, 576 (1958).

Rothwell, E., J. Inst. Metals **90**, 389 (1961–1962).

Saito, T., Nakanishi, K., and Shiraishi, Y., J. Japan Inst. Met. **31**, 881 (1967).

Saito, T., Shiraishi, Y., and Sakuma, Y., Trans. Iron Steel Inst. Japan **9**, 118 (1969).

Salceanu, C., Compt. Red. **226**, 1798 (1948).

Sarou-Kanian, V., Millot, F., and Rifflet, J.C., Int. J. Thermophys. **24**, 277–286 (2003).

Sasaki, H., Tozizaki, E., Terashima, K., and Kimura, S., Jpn. J. Appl. Phys. **34**, 3432 (1995).

Sato, Y., Proc. 16th Europ. Conf. Thermophys. Prop., 2002.

Sato, Y. (private communication 2003).

Sato, Y., "Research and Development of Innovative Casting Simulation supported by New Energy and Industrial Technology Development Organization (NEDO), through the Japan Space Utilization Promotion Center-(JSUP)," Report to the Ministry of Economy, Trade and Industry (METI), Japan, 2004.

Sato, Y., and Yamamura, T. (private communication, Tohoku University, Sendai, 1999).

Schenck, H., Frohberg, M.G., and Hoffmann, K., Arch. Eisenhüt. **34**, 93 (1963).

Sergeev, S.V., and Polyak, E.V., Zavodskaya Lab. **13**, 336 (1947).

Shvidkovskiy, Ye. G., *Certain Problems Related to Viscosity of Fused Metals* (State Publishing House for Technical and Theoretical Literature, Moscow, 1955).

Smith, P.M., Elmer, J.W., and Gallegos, G.F., Scripta Materialia **40**, 937–941 (1999).

Sroka, M., and Skala, J., Vyskup, Kovove Mater. **17**, 369 (1979).

Vatolin, N.V., Vostryakov, A.A., and Yesin, O.A., Fizika Met. **15**, 222 (1963).

Vignau, J.M., Azou, P., and Bastien, P., C.R. Acad. Sci. Paris, 174 (1967).

Wang, D., and Overfelt, R.A., Int. J. Thermophysics **23**, 1063 (2002).

Watanabe, Sh., Trans. JIM **12**, 17–22 (1971).

Wen, L.-Sh., and Arsentyev, P.P., Izc. VUZ Chern. Met. **7**, 5 (1961).

Wille, G., Millot, F., and Rifflet, J.C., Int. J. Thermophys. **23**, 1197–1206 (2002).

Wunderlech, R., Presented at 2nd International Symposium on Physical Sciences in Space, Toronto, Canada, May 23–27, 2004.

Yamasaki, T., Kanatani, S., Ogino, Y., and Inoue, A., J. Non-Cryst. Solids **154–158**, 441 (1993).

Yao, T.P., and Kondic, V., J. Inst. Metals **81**, 17 (1952–1953).

Yatsenko, S.A., Kononenko, V.I., and Sukhman, A.L., High Temperature **10**, 55–59 (1972).