RESILIENT QUANTUM COMPUTING

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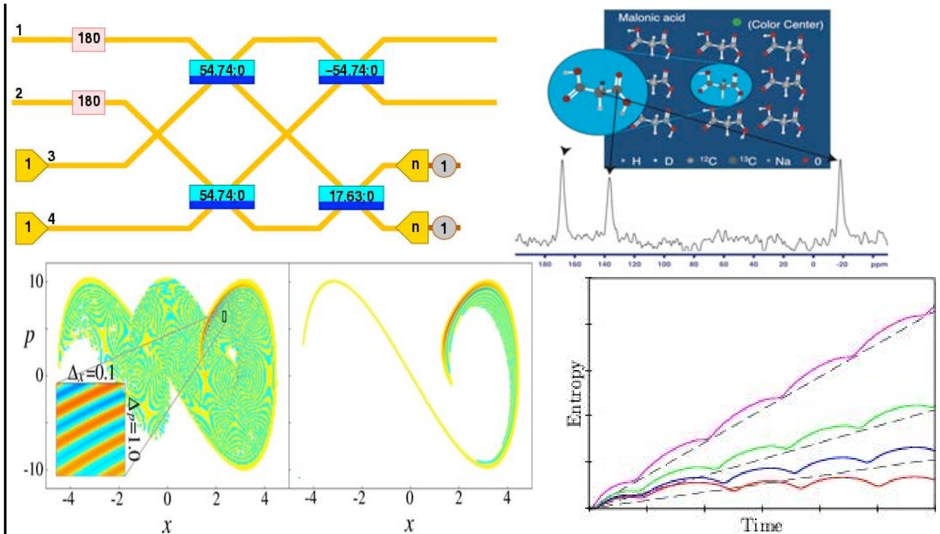
Objective

* Understand origins of the fragility of quantum computers in theoretical and laboratory settings.
* Find ways to make quantum information processing robust against corruption both at the theoretical and experimental levels.



Objective Approach

* Theoretical study:
  - decoherence / benchmarks
  - noise control / error correction
  - simulations
* Experimental work:
  - characterize decoherence / noise in physical settings
  - implement control in the laboratory

Status

* Loschmidt echo as a benchmark / related it to decoherence.
* Investigated role of instability in the environment for the decoherence rates.
* Determined limitations on postselected quantum gates in the KLM QC.
* Made progress in both liquid state and solid state NMR QC's.

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- Progress on last year's objectives

- Established a connection between Loschmidt echo (fidelity benchmark) & decoherence.
- Developed a solvable model of an unstable environment / investigated consequences.
- Synthesized a 10 qubit liquid state NMR QC molecule (checking spectra).
- Developed a new quantum algorithm to characterize spectra of quantum systems.
- Determined limitations on postselected quantum gates in KLM (linear optics) QC.
- Thermodynamic interpretation of the measure of quantumness -- the quantum discord.
- Set up a solid state NMR laboratory / obtained spectra of malonic acid (3 qubits).

- Research plan for the next 12 months

- Characterize the information lost to environment: where is it, can it be recovered / used to control the system & counteract decoherence?
- Investigate decoherence due to non-standard environments.
- Implement 10 qubit NMR QC. Continue progress towards solid state NMR QC (cool the sample / polarise ~1 / implement phase error correction in this setting).
- Characterize linear optics gates / investigate photon loss errors.

-Long term objectives (demonstrations)

- Characterize decoherence in general models as well as in specific implementations.
- Devise theoretical means to benchmark, control and protect quantum information.
- Implement quantum information processing in experimental settings.

Resilient Quantum Computing Road Map

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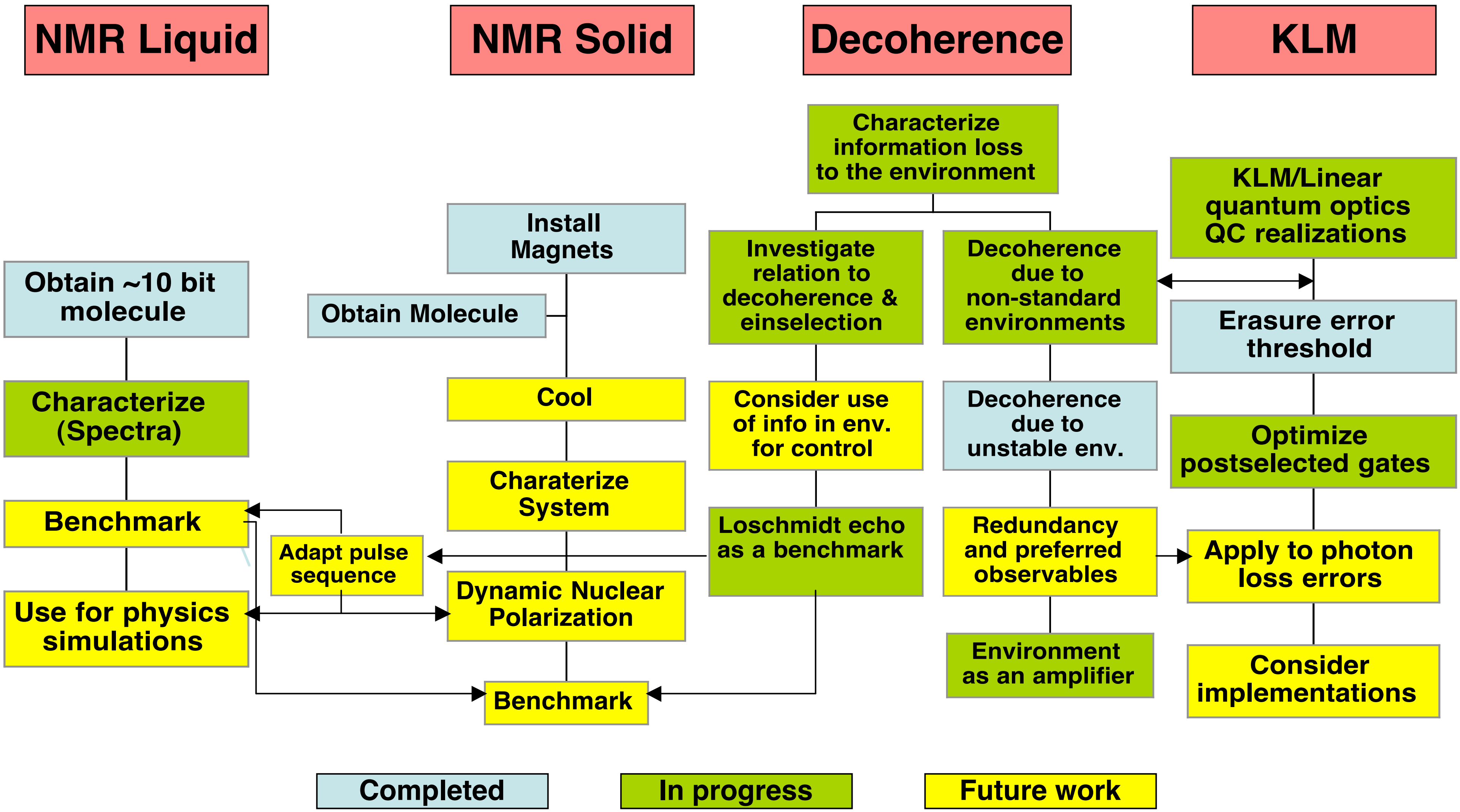

Resilient Quantum Computing

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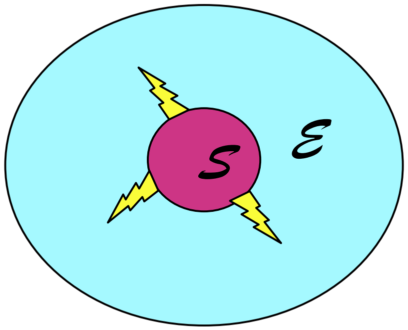
SELECTED PUBLICATIONS: OCTOBER 2002-AUGUST 2003

1. *Decoherence, einselection, and the quantum origins of the classical*
   W. H. Zurek, **REVIEWS OF MODERN PHYSICS, 75, 715-765 (2003**)
2. *Robust dynamical decoupling of quantum systems with bounded controls*
   Viola, L; Knill, E, **PHYSICAL REVIEW LETTERS; JAN 24 2003; v.90, no.3, p.037901-7901**
3. *Quantum gates using linear optics and postselection*
   Knill, E, **PHYSICAL REVIEW A; NOV 2002; v.66, no.5, p.052306-2306**
4. *Environment-assisted invariance, entanglement, and probabilities in quantum physics*
   Zurek, WH, **PHYSICAL REVIEW LETTERS; MAR 28 2003; v.90, no.12, p.120404-404**
5. *Quantum discord and Maxwell's demons*
   Zurek, WH, **PHYSICAL REVIEW A; JAN 2003; v.67, no.1, p.012320-2320**
6. *Testing integrability with a single bit of quantum information*
   David Poulin, Raymond Laflamme, G.J. Milburn, Juan Pablo Paz, **quant-ph/0303042. PHYSICAL REVIEW A, (2003), in press**
7. *Generalizations of entanglement based on coherent states and convex sets*
   H. Barnum, E. Knill, G. Ortiz, L. Viola, **quant-ph/0207149, PHYSICAL REVIEW A (2003), in press**
8. *Quantum chaotic environments, the butterfly effect, and decoherence*
   Karkuszewski, ZP; Jarzynski, C; Zurek, WH, **PHYSICAL REVIEW LETTERS; OCT 21 2002; v.89, no.17, p.170405-405**
9. *Decoherence from a Chaotic Environment: An Upside Down "Oscillator" as a Model*
   Robin Blume-Kohout, Wojciech H. Zurek, **quant-ph/0212153, PHYSICAL REVIEW A (2003), in press**
10. *Proposal for realization of a Toffoli gate via cavity-assisted collision*
    H. Ollivier, P. Milman, **quant-ph/0306064, PHYSICAL REVIEW A (2003), submitted**
11. *Decoherence and the Loschmidt echo*
    F. Cucchietti, D.A. Dalvit, J.P. Paz and W. Zurek, **quant-ph/0306154, PHYSICAL REVIEW LETTERS. (2003), submitted**
12. *Robust polarization-based quantum key distribution over collective-noise channel*
    L. Boileau, D. Gottesman, R. Laflamme, D. Poulin, D. Specken, **quant-ph/0306199, PHYSICAL REVIEW LETTERS. (2003), submitted**

R. Blume-Kohout, H. Barnum, F. Cucchietti, D. Dalvit, R. Martinez,
H. Ollivier, R. Onofrio, G. Ortiz, D. Poulin, R. Somma, L.Viola…

Decoherence: Consequences for quantum computation (intro)



$$\rho_{\mathcal{S}}(t) = Tr_{\mathcal{E}}\left|\Phi_{\mathcal{SE}}(t)\right\rangle\left\langle\Phi_{\mathcal{SE}}(t)\right| = \sum_i \left|\alpha_i\right|^2 \left|\sigma_i\right\rangle\left\langle\sigma_i\right|$$

•Loss of phase coherence between $|\sigma_k\rangle$.

•Emergence of the preferred set of states: *pointer states* (when degenerate known as *decoherence free subspaces*).

•The ability to model measurements (*quantum correlations* converted into *classical correlations*).

•The loss of quantum coherence: source of errors -- at best, a quantum computer will become a classical computer.

W. H. Zurek, Rev. Mod. Phys. **75**, 715-765 (2003).

(i) Models: Not just "Quantum Brownian motion" (required to describe new experimental data?)

(ii) Benchmarking tools: Loschmidt echo and decoherence.

(iii) Quantum discord: the measure of a "quantumness" of a correlation. (WHZ, Ann. der Phys. (Leipzig) 2000; Ollivier & WHZ, PRL '02; WHZ, PRA '03)

Why trace out the environment?

(iv) What happens if you do not trace over the environment? (…WHZ RMP 2003; Ollivier, Poulin, & WHZ, quant-ph0307229)

(v) Why does inaccessibility of the environment imply loss of information? (WHZ, PRL 2003)

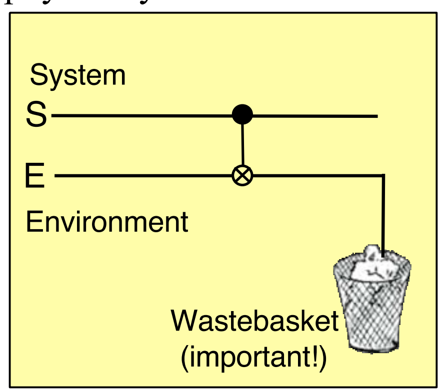
Models of a Decohering Environment

Decoherence from a Chaotic Environment: An “upside-down” Oscillator as a Model.

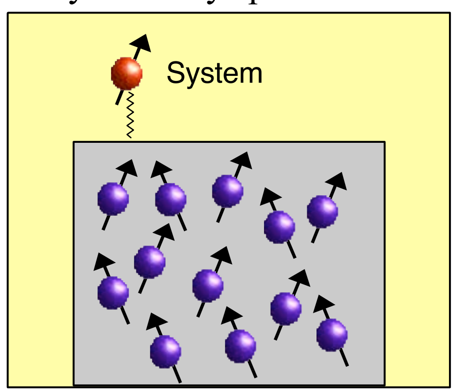
Robin Blume-Kohout, Wojciech H. Zurek, quant-ph/0212153. PHYSICAL REVIEW A, (2003) in press

Problem: Model decoherence -- the destruction of quantum coherences by an environment.
Previous solutions:

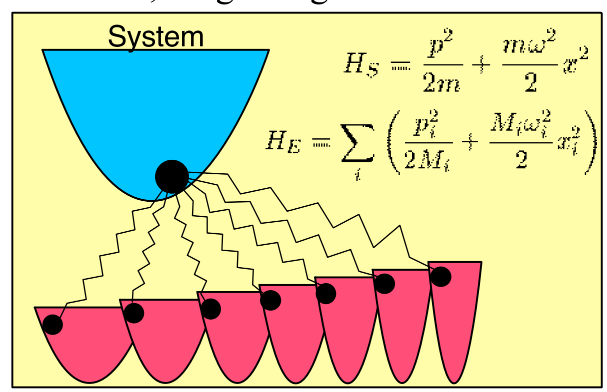
QC Theorists' favorite model. Simple, intuitive, but not physically motivated.



Spin interacting with other spins (... Zurek, 1982...). Depends on a very arbitrary spectrum.

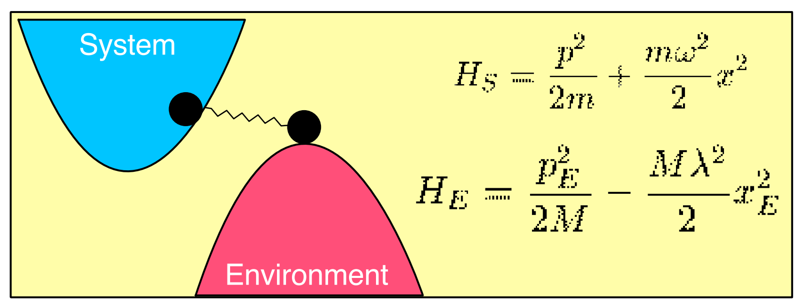


Quantum Brownian Motion (... Feynman & Vernon, 1963...). Uses a bath of oscillators; long thought to be universal.



New Model: an Unstable Linear Environment

- As a linear system, it is easy to solve exactly.
- The instability of the environment makes it highly sensitive to the state of the system.
- Such sensitivity may be generic in the real world.



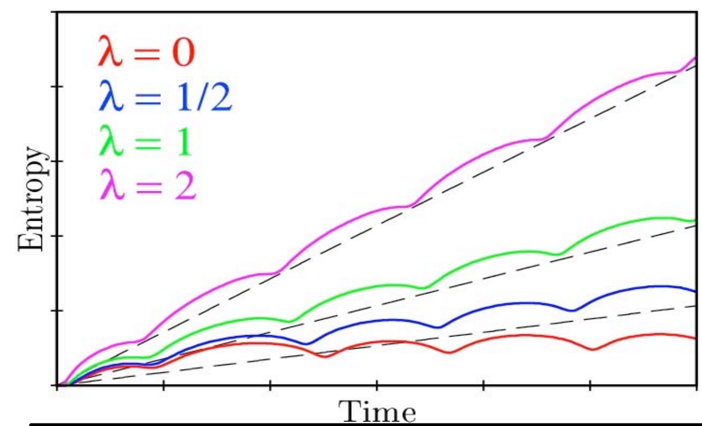
Analysis of an Unstable Decohering Environment

1. A new method for obtaining a master equation.

- **In particular, we can identify when and why a given master equation behaves pathologically.**

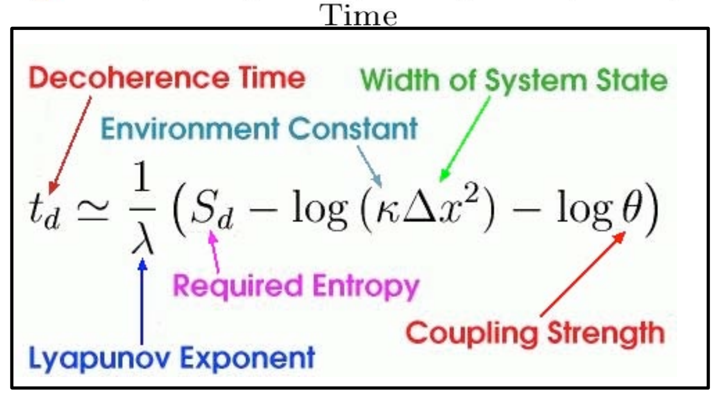
2. Rate of decoherence depends on the “spring constant” of the unstable oscillator.

- The instability leads to a constant rate of entropy growth in the system – “saturation” never occurs.
- The rate at which entropy grows is proportional to the strength of the instability.
- A “free” environment, on the brink of instability, makes entropy grow logarithmically with time.



3. The unstable environment is much more effective than QBM at decohering a system.

- Typical decoherence times are **logarithmic** in the coupling strength.
- Increasing the coherence time of the system is exponentially difficult compared with QBM!



4. Implications for error correction: Perform error correction *frequently*.

- Unstable environments do not destroy coherences arbitrarily quickly, but coherence times are fixed.
- Thus, decoherence errors must be corrected frequently, before they become uncorrectable.

Decoherence and the Loschmidt Echo

Loschmidt echo: A way to characterize sensitivity to perturbations. Can be used as a benchmark: fidelity decay.

$$M(t) = Tr(\rho_0(t)\rho_\Delta(t)) \qquad \rho_\Delta(t) = U_\Delta(t)\rho(0)U_\Delta^+(t)$$

Connection with decoherence: evident when average echo is considered

$$\overline{M}(t) = \int d\Delta\, P(\Delta)\, Tr(\rho_0(t)\rho_\Delta(t)) = Tr(\rho_0(t)\overline{\rho}(t))$$

$$\overline{\rho}(t) = \int d\Delta\, P(\Delta)\, \rho_\Delta(t)$$ **analogous to a decohered density matrix! (it obeys a master equation, etc)**

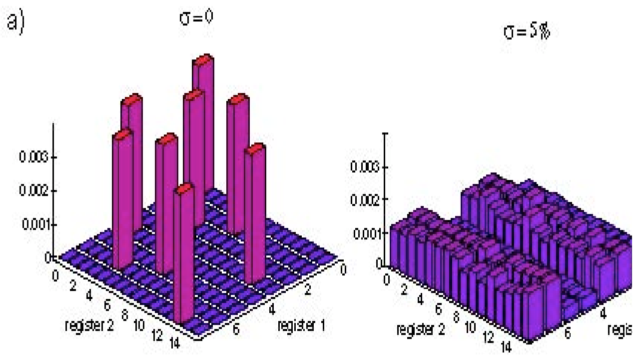
Analogy between measure of decoherence (purity decay) and Loschmidt echo (fidelity decay)

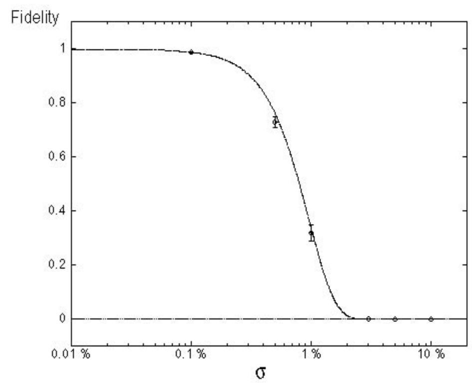
$$\varsigma(t) = Tr(\overline{\rho}^2(t)) \qquad \overline{M}(t) = Tr(\rho_0(t)\overline{\rho}(t))$$

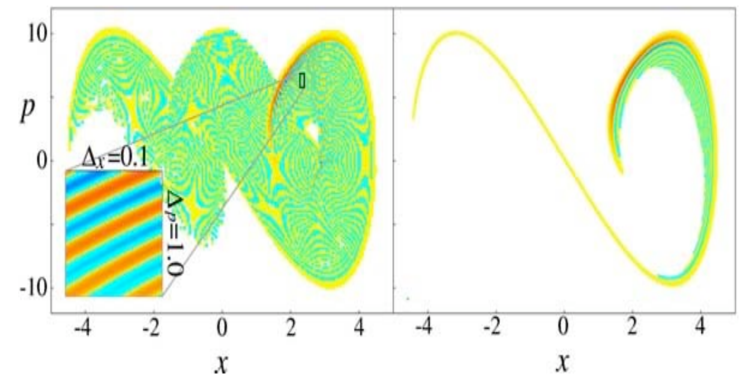
Cucchieti, Dalvit, Paz, and Zurek, quant-ph/0306154

Consequences of Decoherence-Echo Connection

"*Deja vu* all over again": We have used similar benchmarks before: Miquel, Paz, & Zurek 1997; "Schroedinger cat" in NMR…







Use master equation for the ensemble averaged density matrix.
Show that for unstable systems

quant-ph/0306154

$$\overline{M}^2(t) = a\exp(-\lambda t) + b\exp(-\Gamma t)$$

System dependent decay
(environment independent, Lyapunov regime)

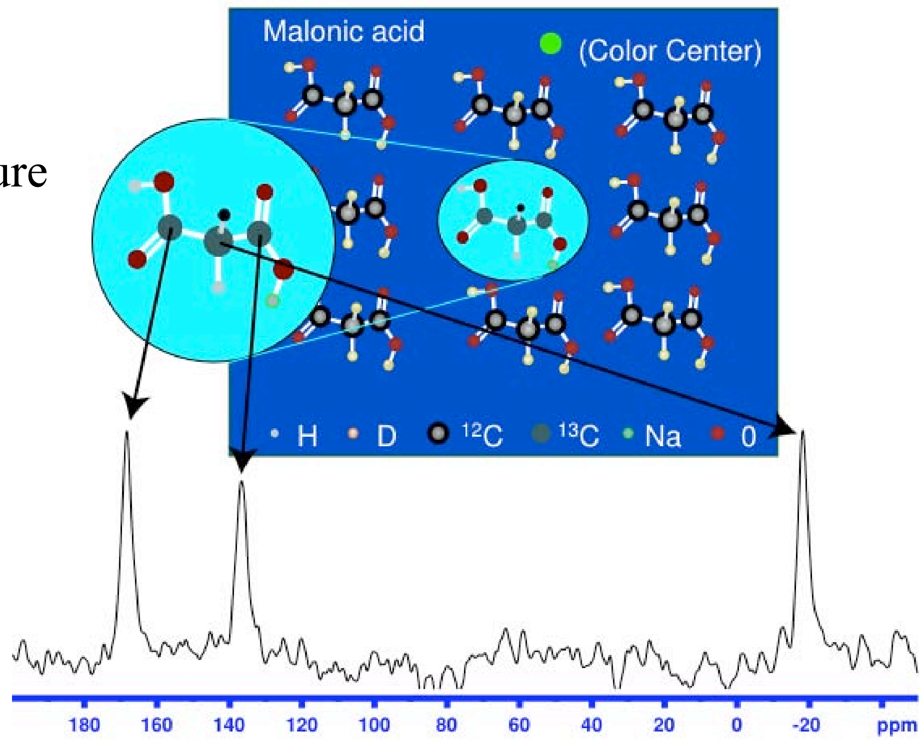
Environment dependent decay
(system independent, Fermi GR regime)

Analogies with decoherence, can be used to establish connections between dynamics (algorithm) and the loss of echo signal.

Progress in Solid State NMR

Achievements:
-establish solid state NMR laboratory
-found a 3 qubit suitable crystal
-characterize the qubits at room temperature
(chemical shift, …

Next year goals
-polarize nuclei from electrons
using DNP and Schulman-Vazirani
-benchmark quantum control
1 and 2bit gates and noise model
-implement 3 qubit QEC

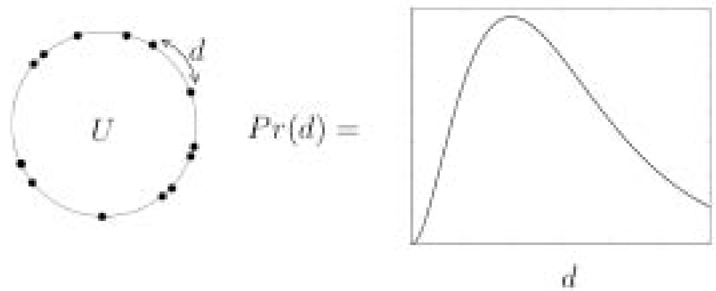


Long term goals:
-investigate scalability of control
-derive methods to characterize error model in physical systems
-optimize error control methods to improve precision of quantum manipulations
-reach control at the level of threshold accuracy

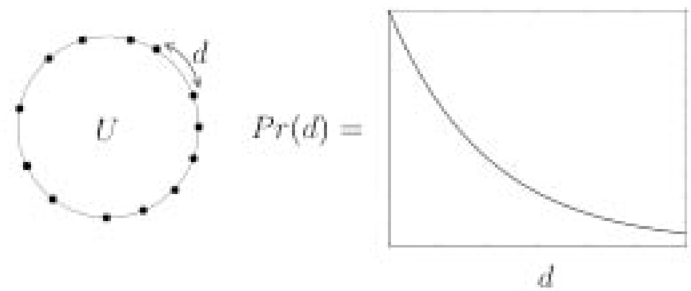
A new algorithm to characterize spectra of quantum systems

Task: Determine if a unitary operator belongs to one of two families

Chaotic (non-integrable)

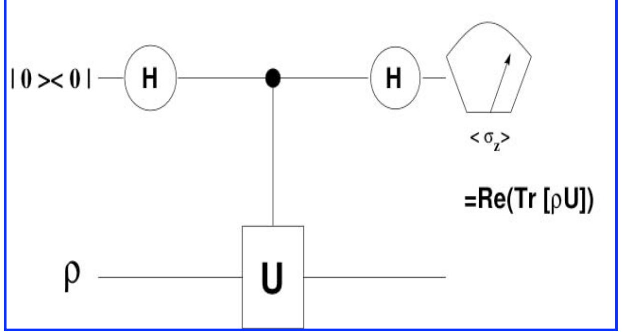


Regular (integrable)



Method: Use scattering circuit to measure traces of unitary operator

Resources: One qubit in a pure state, log(N) qubits in a maximally mixed state



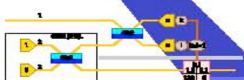
$$Tr\left(\frac{U}{N}\right) \approx \begin{cases} 1/N & \textit{Chaotic} \\ 1/\sqrt{N} & \textit{Regular} \end{cases}$$

The quantum algorithm requires O(N) repetitions while classical methods seem to require O(N^2)
Quadratic speedup!

David Poulin, Raymond Laflamme, G.J. Milburn, Juan Pablo Paz,

quant-ph/0303042. PHYSICAL REVIEW A, (2003), in press

Linear Optics Quantum Computation

Optical methods	Capabilities realized
 Optical systems & ops:	Qubits and one-qubit ops
 Postselection:	QC, if lucky
 Teleportation:	Standard QC
 Z-encoding:	Efficient QC
 Loss detection, Erasure codes:	Robuster QC
 Accuracy threshold:	Scalable QC

Implementation issues:

- Resource requirements
- Technological challenges

Knill&Laflamme&Milburn [1] *Nature* 2001

LOQC Progress and Problems

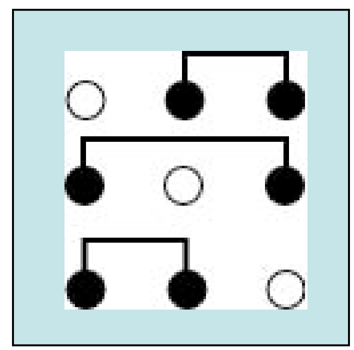
- Progress:
  - Proved that without feedback, the probability of success of the postelected nonlinear gates NS and CS are at most $1/2$ and $3/4$ respectively.
  - Preliminary studies of error-correcting codes tuned to the elimination of errors due to detected photon loss.
- Problems:
  - There is still a big gap between the best constructions and the theoretical upper bound for the probability of success of NS and CS gates.
  - Firm up the requirements on the efficiencies of photodetectors and single photon sources?
  - If the only problem is detected photon loss, what rate of photon loss is tolerable?

Robust polarization based QKD over collective noise channel

(Boileau, Gottesman, Laflamme, Poulin, Specken, quant-ph/0306199)

Use three encoded states to implement BB92-type protocol using output of down converted photon in the singlet state and a photon in a random state:

$$\rho_1 = \mathbb{1}_1 \otimes (\mathbb{1}_{23} - \vec{\sigma}_2 \cdot \vec{\sigma}_3)/2$$
$$\rho_2 = \mathbb{1}_2 \otimes (\mathbb{1}_{13} - \vec{\sigma}_1 \cdot \vec{\sigma}_3)/2$$
$$\rho_3 = \mathbb{1}_3 \otimes (\mathbb{1}_{12} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)/2$$

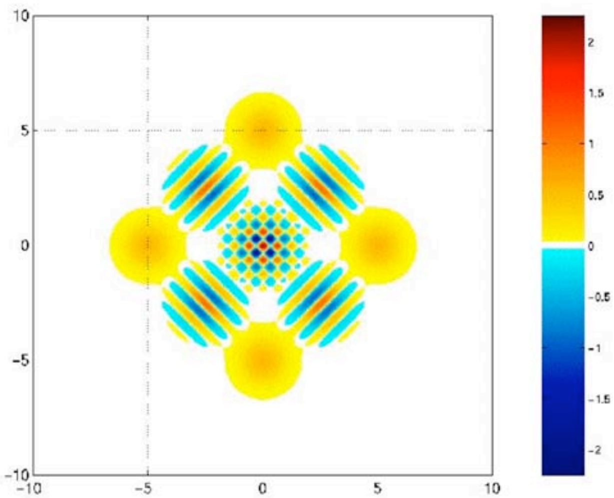


These states are part of a noiseless subsystem (Knill et al. PRL85,2525, 2001) and the protocol can be implemented with today's technology

Ultrasensitive position monitoring

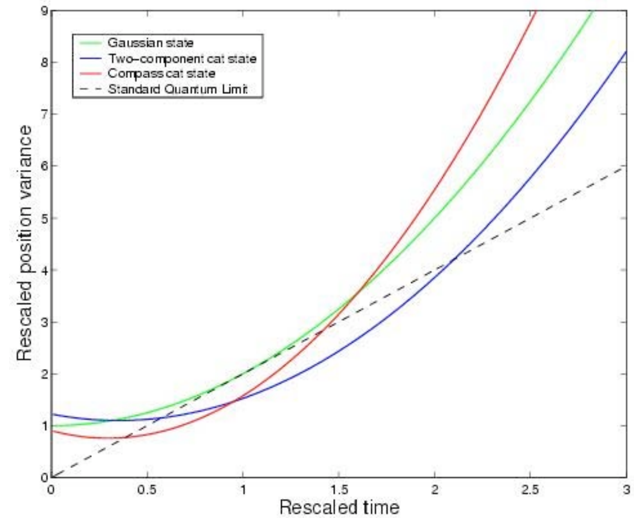
Non-classical states can be exploited to improve the sensitivity of position measurements, beating the limits holding for conventional, classical states.

Schroedinger cat states: quantum interference may make them more sensitive to the interplay between position and momentum in the phase space structure.



W.H. Zurek, **NATURE 412, 712-717 (2001)**

Beating the standard quantum limit for harmonic oscillator systems



L. Viola & R. Onofrio, **NEW JOURNAL OF PHYSICS 5, 5.1-5.21(2003)**

Contractivity features in the position variance for free particle systems

Compass states can have a better position resolution for *both* free particles and harmonic systems. Compass states may be generated as *conditional states* for number operator measurements.

Goals to be achieved by January 2004 (from proposal)

January 2004

- Develop exact master equation for a simple model of a system/environment in the case when either/both exhibit exponential sensitivity to initial conditions.
- Develop conditional dynamics of open quantum systems in the case of several observers.
- To find exact solutions for quantum walks on graphs with decoherence. To use phase space tools to understand the differences and similarities between quantum and classical regimes for these systems.
- To generalize tomographic schemes available for efficiently measuring the discrete Wigner function in order to efficiently evaluate other distribution functions (Husimi, Kirkwood, etc).
- Learn bounds on the ability to realize the postselected gates at the foundation of eLOQC.
- Finally obtain an accessible 10 qubit molecule for liquid state NMR.
- NMR control strategies for a 10 qubit molecule.
- Fabricate enclosed probes suitable for low temperature operation.
- Investigate of efficiency of decoupling schemes for decoupling protons and ^{13}C or D in the solid state proposal (with MIT).
- Search for suitable molecules (such as pyruvic and malonic acid) for QIP with solid state NMR, grow a crystal with molecules containing a small numer of qubits, and characterize the strength of their dipolar couplings, chemical shifts, T1 and T2 (with MIT).