Wavelength Dependence of the Range Correction

by
David A. Arnold
94 Pierce Road
Watertown, MA 02472-3035
617-924-6812

Introduction.

One of the ways to determine the atmospheric correction for laser range measurements is by two-wavelength ranging. Since the dispersion can be a factor of 15, very high accuracy is needed in the range correction for the retroreflector array at the two wavelengths. This paper discusses the dependence of the range correction on wavelength for Lageos. The difference in the range correction for the array consists of two parts. The first part is the difference in the optical path length in the retroreflectors at the two wavelengths. The second part is the dependence of the range correction on diffraction.

The analysis in this report was done privately without any outside funding. It is being presented to illustrate the kinds of problems that can be studied and the results that can be obtained to improve the accuracy of two-color ranging.

1. Dependence of the optical path length on wavelength.

The cube corners on Lageos are 1.5 inches in diameter. The length from face to vertex is 1.5 divided by $\sqrt{2}$ or 1.06066 inches (.02694 meters). Multiplying this by the index of refraction gives the one way optical path length. Table 1 below lists the optical path length at 5 different wavelengths. The first column is the wavelength in nanometers, the second is the index of refraction and the third is the optical path length.

λ	n	OPL
355	1.477	.03979
423.5	1.468	.03955
532	1.455	.03920
847	1.453	.03914
1064	1.450	.03906

Table 1. Optical path length for a Lageos cube corner vs wavelength.

The difference in optical path length between 1064 and 532 nm is .14 millimeters. The difference in optical path length between 847 and 423.5 nm is .41 millimeters. If the dispersion is a factor of 15, a difference in optical path length of .41 millimeters is a difference of 6 millimeters in the atmospheric correction.

2. Dependence of the range correction on diffraction.

If all the retroreflectors in an array are at the same orientation and have identical specifications, the diffraction patterns should all be the same except for manufacturing imperfections. If the retroreflectors are at different orientations, the diffraction patterns will be different at each orientation. The range correction will be different at each point in the far field diffraction pattern of the array.

3. Properties of the Lageos cube corners.

The Lageos cube corners have all three dihedral angles offset to 90 deg + 1.25 arc seconds with a tolerance of .5 arc seconds. The geometrical optics solution for this at normal incidence is 6 spots in the form of a hexagon. Because of diffraction effects the pattern is more complicated when the separation of the spots is on the order of λ/d where λ is the wavelength and d is the diameter of the cube corner.

Because the Lageos cube corners are uncoated and rely on total internal reflection, there are polarization effects that cause additional variations in the diffraction pattern. In particular, there is an interaction between linear polarization and the dihedral angle offset that causes a "dumbbell" shaped diffraction pattern with the axis of the dumbbell aligned with the polarization vector. For circular polarization there is no preferred axis of the diffraction pattern.

Because there are a limited number of cube corners active at any one time, the diffraction pattern of the array is different at each orientation of the satellite. The average diffraction pattern over many orientations has a well defined shape that depends on the polarization of the incident laser beam.

The diffraction pattern of a circular aperture has a zero at 1.22 λ/d. The spreading of the diffraction pattern is proportional to the wavelength with the smallest features on the order of λ/d. For this reason, the diffraction pattern at shorter wavelengths has more structure. The diffraction pattern at longer wavelengths is smoother and wider.

4. Cross section and range correction vs wavelength at a single orientation.

Figure 1 shows the cross section and range correction for Lageos vs wavelength and polarization at orientation θ = 20 deg, and ϕ = 150 deg. This orientation was chosen because it is close to the average range correction over many orientations.

The first row is the cross section with linear vertical polarization for each of the 5 wavelengths. At 355 nm the diffraction pattern is closer to being circular which is what one would have for the geometrical optics solution. At longer wavelengths the shape of the pattern is determined more by diffraction and polarization effects. The pattern is closer to the shape of a vertical "dumbbell". The pattern is also smoother at longer wavelengths.

The second row is the range correction with linear vertical polarization. The shape of the range correction matrix has properties similar to those of the cross section. The pattern is more circular at short wavelengths and has more of a "dumbbell" shape at longer wavelengths. Since the shape of the pattern is different at each wavelength, the difference in the range correction between two wavelengths will be different at each part of the diffraction pattern.

The third row is the cross section with circular polarization. There is no preferred orientation of the pattern. The irregularities in the shape are due the limited number of retroreflectors active at any one orientation of the array. The shape is approximately circular. The spreading of the pattern due to the dihedral angle offsets is independent of wavelength. However, there is additional spreading at longer wavelengths due to diffraction.

The fourth row is the centroid range correction with circular polarization. There is no preferred orientation but the shape is irregular due to the limited number of active cube corners.

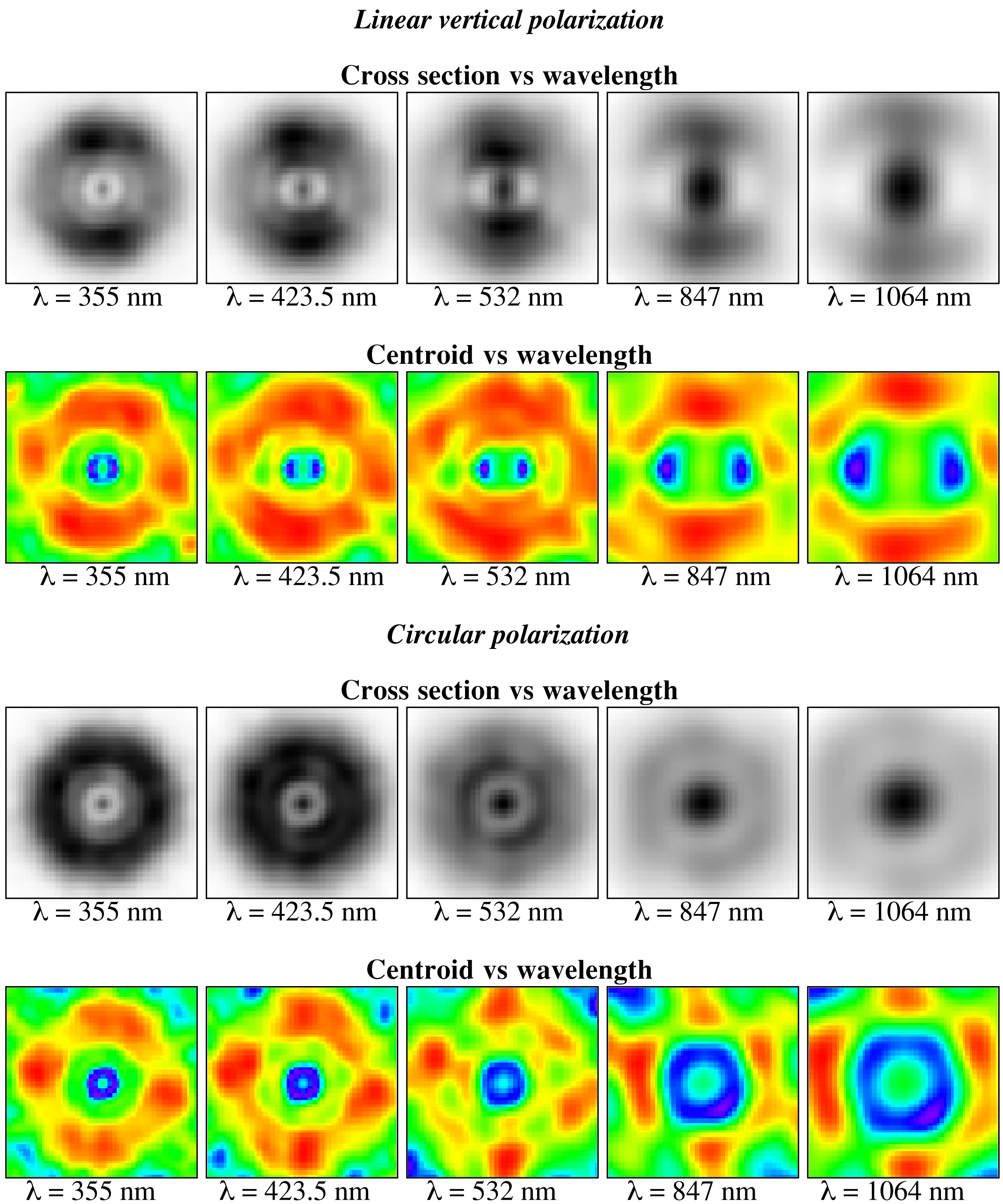


Figure 1. Lageos cross section and centroid range correction at a single orientation.

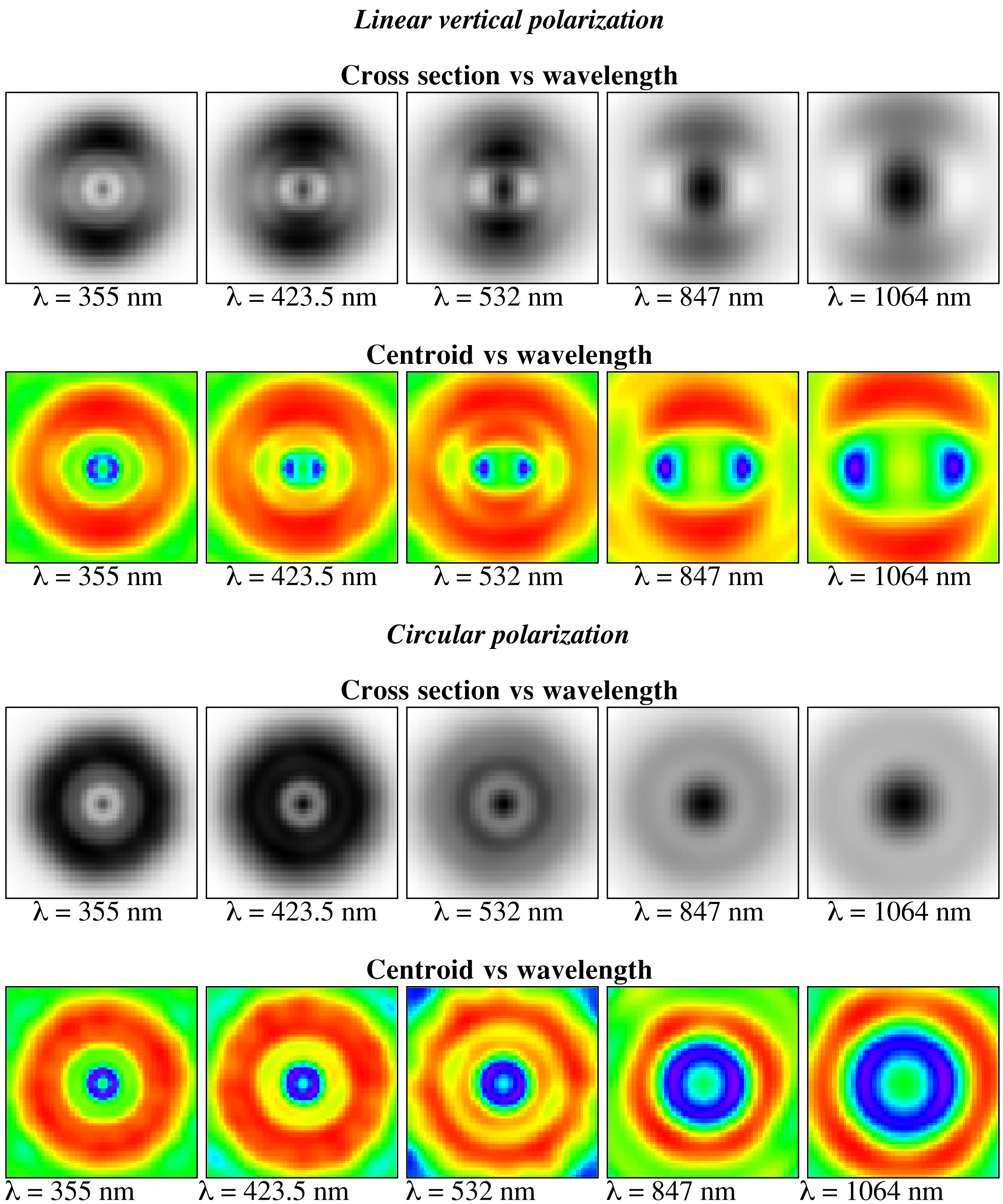


Figure 2. Lageos cross section and centroid range correction averaged over 16 orientations.

5. Cross section and range correction averaged over 16 orientations.

Figure 2 shows the cross section and range correction for Lageos vs wavelength and polarization averaged over 16 orientations. The orientations start at $\theta = 0$ deg, and $\phi = 0$ deg, with each angle incremented by 5 degrees between each case. The final angles are $\theta = 75$ deg, and $\phi = 75$ deg. In general, the shape of the patterns is much more regular than for a single orientation. However, the averaging is not perfect with this limited number of cases. In particular the centroid with circular polarization for $\lambda = 532$ nm in the fourth row shows noticeable irregularities. It is much more circular than for a single orientation and would presumably become more circular by averaging a larger number of cases.

6. Difference in range correction between two wavelengths.

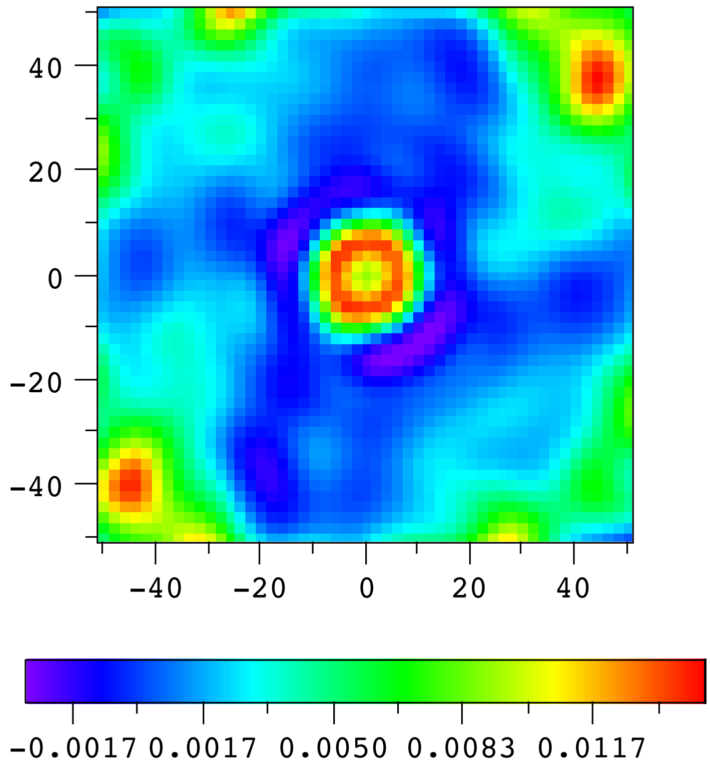
Figure 3 shows the difference in centroid range correction at a single orientation between 847 and 423.5 nm with circular polarization. The table shows the minimum, average, and maximum value of the difference around circles of increasing radius in the far field pattern. In the range between 32 and 38 microradians the average difference is about 1.5 millimeters. However, the difference around the circle can vary by as much as 5 millimeters.

Figure 4 shows the difference in centroid range correction between 847 and 423.5 nm with circular polarization averaged over 16 orientations. The pattern has fairly good circular symmetry. In the range between 32 and 38 microradians the average difference is between 1.5 and 1.9 millimeters. The differences around the circle are less than .7 millimeters.

Figure 5 shows the difference between the centroid range correction at 1064 and 532 nm for circular polarization averaged over 16 orientations. Between 32 and 38 millimeters, the average difference is less than 1 millimeter. The variations around the circle are about one millimeter.

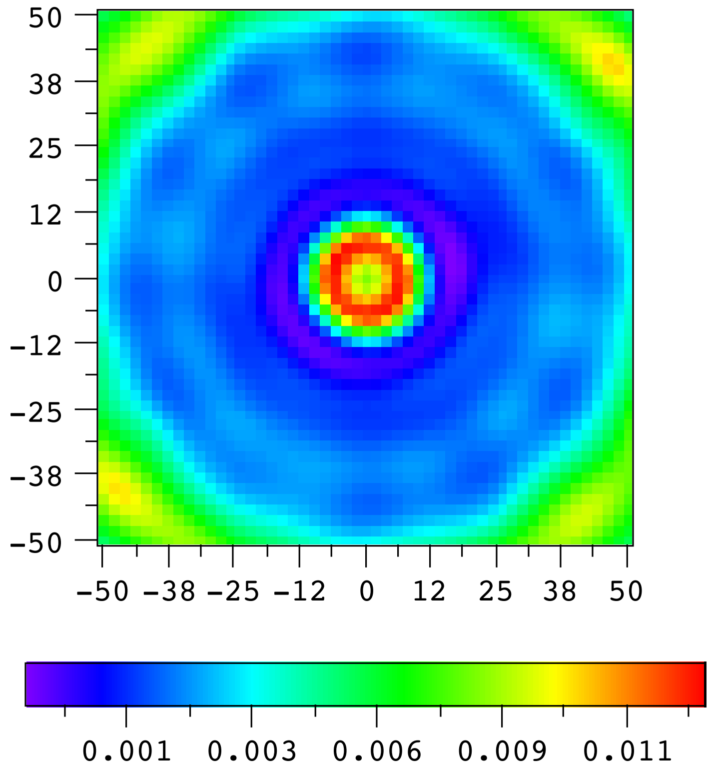
Figure 6 shows the difference in centroid range correction between 847 and 423.5 nm for linear polarization averaged over 16 orientations. The pattern is quite regular but does not have circular symmetry. Between 32 and 38 microradians the average difference is around 1.5 millimeters. The variation around a circle of constant velocity aberration are about 2 millimeters due to the asymmetry of the pattern.

Figure 7 shows the difference in centroid range correction between 1064 and 532 nm for linear polarization averaged over 16 orientations. There is a significant effect from the linear polarization. the average difference in the interval 32 to 38 microradians is less than .5 millimeters but the peak to peak variation around a circle is 3 to 5 millimeters as a function of the angle between the velocity aberration and the polarization vector.



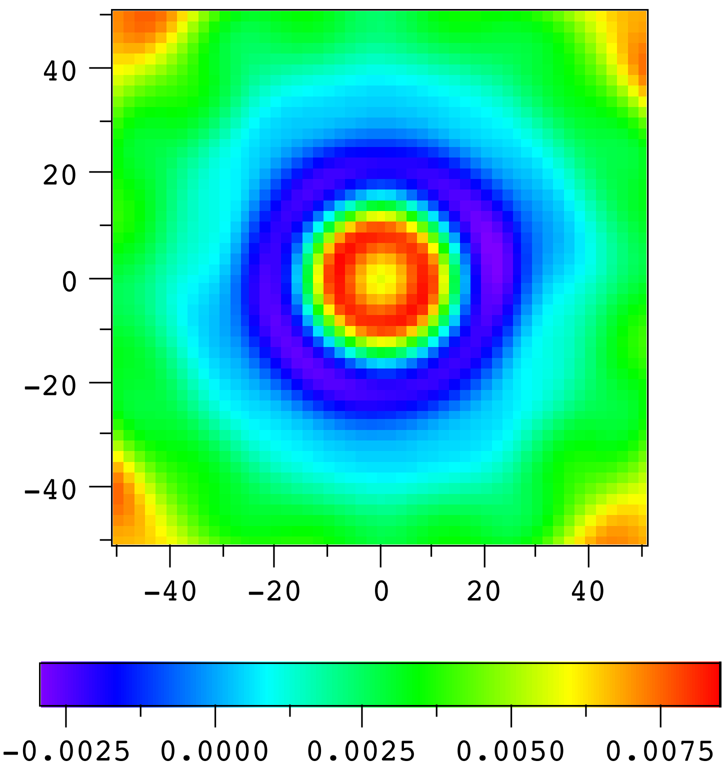
Microrad	Minimum	Average	Maximum	Max - Min
0.0	0.0090280	0.0090280	0.0090280	0.0000000
2.0	0.0096850	0.0098967	0.0100517	0.0003667
4.0	0.0114198	0.0117083	0.0119292	0.0005094
6.0	0.0128080	0.0133172	0.0136457	0.0008377
8.0	0.0111834	0.0120252	0.0130686	0.0018852
10.0	0.0059825	0.0073829	0.0096873	0.0037048
12.0	0.0010526	0.0025919	0.0052541	0.0042015
14.0	-0.0018877	-0.0003592	0.0021126	0.0040003
16.0	-0.0028789	-0.0014734	0.0005714	0.0034503
18.0	-0.0024965	-0.0013465	0.0000231	0.0025195
20.0	-0.0015096	-0.0006318	0.0008024	0.0023120
22.0	-0.0009871	0.0000629	0.0018559	0.0028430
24.0	-0.0010755	0.0004153	0.0024788	0.0035543
26.0	-0.0010526	0.0005334	0.0027009	0.0037535
28.0	-0.0009015	0.0006463	0.0027723	0.0036738
30.0	-0.0006171	0.0008811	0.0028759	0.0034930
32.0	-0.0004476	0.0011837	0.0030507	0.0034983
34.0	-0.0004949	0.0014666	0.0033185	0.0038134
36.0	-0.0008877	0.0016243	0.0036953	0.0045830
38.0	-0.0012723	0.0016542	0.0039258	0.0051981
40.0	-0.0016139	0.0016115	0.0039929	0.0056068
42.0	-0.0016293	0.0015926	0.0038678	0.0054972
44.0	-0.0015634	0.0016931	0.0040515	0.0056150
46.0	-0.0011725	0.0019822	0.0048666	0.0060391
48.0	-0.0007200	0.0024951	0.0063235	0.0070435
50.0	-0.0000875	0.0032949	0.0082541	0.0083415

Figure 3. Difference (meters) between the centroid range correction at 847 and 423.5 nm for circular polarization at a single orientation.



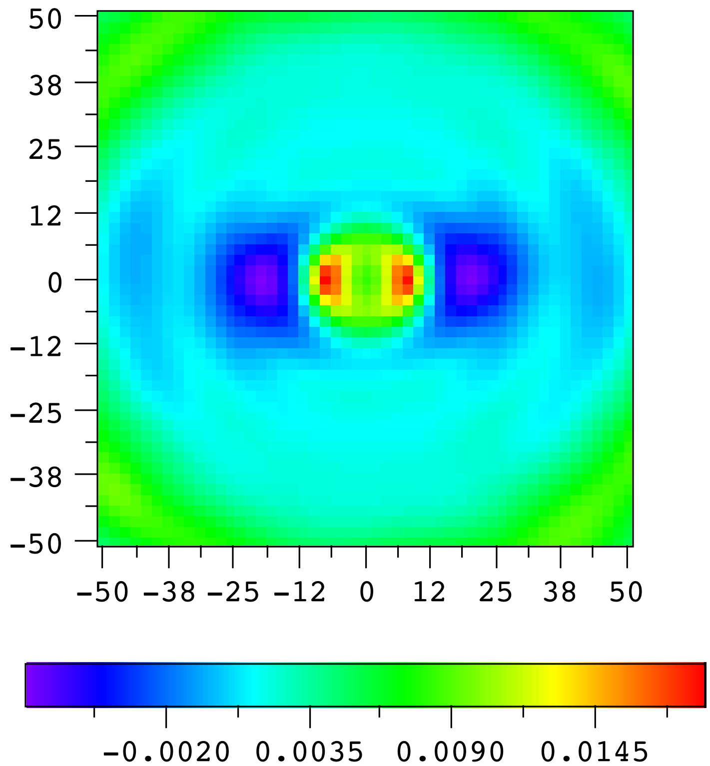
Microrad	Minimum	Average	Maximum	Max - Min
0.0	0.0081900	0.0081900	0.0081900	0.0000000
2.0	0.0088000	0.0089805	0.0090860	0.0002860
4.0	0.0105000	0.0106778	0.0108345	0.0003345
6.0	0.0119810	0.0123710	0.0127220	0.0007410
8.0	0.0110356	0.0115613	0.0119800	0.0009444
10.0	0.0070254	0.0074764	0.0079127	0.0008873
12.0	0.0024372	0.0030304	0.0034551	0.0010179
14.0	-0.0003728	0.0002495	0.0006452	0.0010180
16.0	-0.0014420	-0.0008079	-0.0004564	0.0009856
18.0	-0.0013328	-0.0006971	-0.0003989	0.0009339
20.0	-0.0006807	-0.0000232	0.0002793	0.0009600
22.0	-0.0000379	0.0006269	0.0009603	0.0009982
24.0	0.0003121	0.0009164	0.0013019	0.0009898
26.0	0.0004412	0.0009356	0.0013392	0.0008981
28.0	0.0005613	0.0009555	0.0013410	0.0007797
30.0	0.0007985	0.0011342	0.0014789	0.0006804
32.0	0.0011201	0.0014397	0.0017856	0.0006655
34.0	0.0014235	0.0017527	0.0021139	0.0006904
36.0	0.0015500	0.0019218	0.0023378	0.0007878
38.0	0.0014100	0.0018978	0.0023830	0.0009730
40.0	0.0011500	0.0017434	0.0022895	0.0011395
42.0	0.0009500	0.0016093	0.0021927	0.0012427
44.0	0.0009800	0.0016617	0.0022362	0.0012562
46.0	0.0013300	0.0020065	0.0025431	0.0012131
48.0	0.0020000	0.0026279	0.0031063	0.0011063
50.0	0.0028942	0.0034602	0.0041010	0.0012068

Figure 4. Difference (meters) between the centroid range correction at 847 and 423.5 nm for circular polarization averaged over 16 orientations.



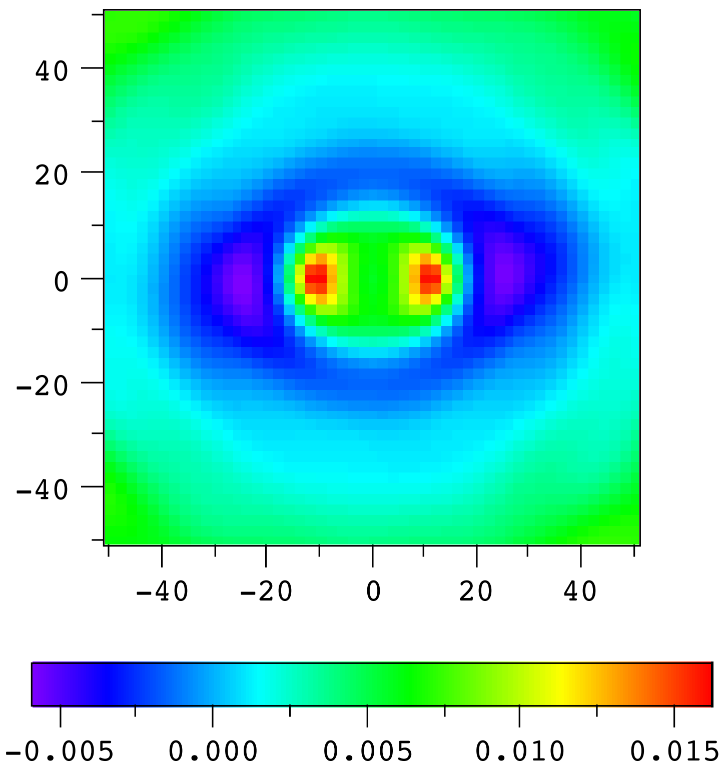
Microrad	Minimum	Average	Maximum	Max - Min
0.0	0.0057300	0.0057300	0.0057300	0.0000000
2.0	0.0059600	0.0060322	0.0060694	0.0001094
4.0	0.0065900	0.0066913	0.0067493	0.0001593
6.0	0.0074527	0.0075902	0.0077148	0.0002621
8.0	0.0079408	0.0082106	0.0084312	0.0004904
10.0	0.0073851	0.0077408	0.0080601	0.0006749
12.0	0.0054771	0.0057711	0.0061118	0.0006347
14.0	0.0026775	0.0029569	0.0032111	0.0005336
16.0	0.0000120	0.0003971	0.0006400	0.0006280
18.0	-0.0018052	-0.0013461	-0.0010500	0.0007552
20.0	-0.0027453	-0.0022328	-0.0018871	0.0008583
22.0	-0.0029604	-0.0024004	-0.0020435	0.0009170
24.0	-0.0026561	-0.0020353	-0.0016951	0.0009609
26.0	-0.0020328	-0.0013562	-0.0010011	0.0010317
28.0	-0.0013200	-0.0006200	-0.0001751	0.0011449
30.0	-0.0007167	-0.0000331	0.0004886	0.0012053
32.0	-0.0003093	0.0003317	0.0008968	0.0012061
34.0	-0.0000250	0.0005356	0.0011233	0.0011483
36.0	0.0002341	0.0007103	0.0012908	0.0010567
38.0	0.0005376	0.0009531	0.0015286	0.0009910
40.0	0.0009369	0.0012945	0.0018703	0.0009334
42.0	0.0013438	0.0017154	0.0022985	0.0009547
44.0	0.0017052	0.0021586	0.0027690	0.0010637
46.0	0.0020442	0.0025520	0.0032133	0.0011691
48.0	0.0022834	0.0028464	0.0035794	0.0012960
50.0	0.0023800	0.0030407	0.0038491	0.0014691

Figure 5. Difference (meters) between the centroid range correction at 1064 and 532 nm for circular polarization averaged over 16 orientations.



Microrad	Minimum	Average	Maximum	Max - Min
0.0	0.0084000	0.0084000	0.0084000	0.0000000
2.0	0.0087900	0.0091961	0.0093448	0.0005548
4.0	0.0097100	0.0109287	0.0120030	0.0022930
6.0	0.0101100	0.0128548	0.0164500	0.0063400
8.0	0.0085900	0.0124440	0.0187800	0.0101900
10.0	0.0054755	0.0081801	0.0126400	0.0071645
12.0	0.0027182	0.0031977	0.0040131	0.0012949
14.0	-0.0021326	-0.0000024	0.0012882	0.0034208
16.0	-0.0053700	-0.0014137	0.0009200	0.0062900
18.0	-0.0070400	-0.0016526	0.0012300	0.0082700
20.0	-0.0074800	-0.0012532	0.0017200	0.0092000
22.0	-0.0068400	-0.0006982	0.0019979	0.0088379
24.0	-0.0056319	-0.0003131	0.0019414	0.0075733
26.0	-0.0044826	-0.0000935	0.0016996	0.0061822
28.0	-0.0032696	0.0001595	0.0015283	0.0047979
30.0	-0.0020019	0.0005420	0.0015908	0.0035927
32.0	-0.0008422	0.0010101	0.0018338	0.0026760
34.0	0.0000249	0.0014318	0.0021385	0.0021136
36.0	0.0004381	0.0016572	0.0022838	0.0018457
38.0	0.0003831	0.0016450	0.0023445	0.0019613
40.0	0.0000402	0.0014793	0.0022924	0.0022522
42.0	-0.0003699	0.0013329	0.0022890	0.0026589
44.0	-0.0005633	0.0013867	0.0024991	0.0030624
46.0	-0.0003746	0.0017443	0.0030960	0.0034707
48.0	0.0001867	0.0023974	0.0039685	0.0037817
50.0	0.0010753	0.0032708	0.0050434	0.0039681

Figure 6. Difference (meters) between the centroid range correction at 847 and 423.5 nm for linear polarization averaged over 16 orientations.



Microrad	Minimum	Average	Maximum	Max - Min
0.0	0.0058900	0.0058900	0.0058900	0.0000000
2.0	0.0059800	0.0061890	0.0062805	0.0003005
4.0	0.0061800	0.0068690	0.0074800	0.0013000
6.0	0.0062400	0.0078688	0.0096900	0.0034500
8.0	0.0057700	0.0088255	0.0129700	0.0072000
10.0	0.0045200	0.0089707	0.0162600	0.0117400
12.0	0.0027000	0.0074038	0.0162300	0.0135300
14.0	0.0008582	0.0042991	0.0110200	0.0101618
16.0	-0.0005332	0.0011547	0.0045600	0.0050932
18.0	-0.0015610	-0.0010706	-0.0001778	0.0013832
20.0	-0.0034176	-0.0023016	-0.0015100	0.0019076
22.0	-0.0051100	-0.0027421	-0.0012900	0.0038200
24.0	-0.0059300	-0.0026057	-0.0007820	0.0051480
26.0	-0.0059700	-0.0020939	-0.0001628	0.0058072
28.0	-0.0054100	-0.0014419	0.0004223	0.0058323
30.0	-0.0046250	-0.0008678	0.0008392	0.0054642
32.0	-0.0039564	-0.0004575	0.0010744	0.0050308
34.0	-0.0033257	-0.0001730	0.0012010	0.0045267
36.0	-0.0026522	0.0001027	0.0013023	0.0039545
38.0	-0.0018890	0.0004489	0.0014623	0.0033513
40.0	-0.0010865	0.0008829	0.0017219	0.0028084
42.0	-0.0003166	0.0013762	0.0021404	0.0024570
44.0	0.0003381	0.0018714	0.0026083	0.0022702
46.0	0.0007916	0.0022975	0.0030562	0.0022646
48.0	0.0010227	0.0026102	0.0034504	0.0024276
50.0	0.0010601	0.0028136	0.0038679	0.0028078

Figure 7. Difference (meters) between the centroid range correction at 1064 and 532 nm for linear polarization averaged over 16 orientations.

7. Summary of range correction differences

Table 2 below summarizes the difference in the range correction between pairs of wavelengths. The first column is the transmitted polarization. Columns 2 and 3 are the wavelength pair. Column 4 is the average difference in the range correction (Wavelength 1 minus Wavelength 2) in the interval 32 to 38 microradians velocity aberration. Column 4 is the variation of the range difference around a circle of constant velocity aberration in the interval 32 to 38 microradians.

As an example, let us use Figure 7 above which is the data summarized in the last row of Table 2. In the third column of Figure 7 labeled 'Average', the average range difference at 32 microradians is -.0004547 m, and the average difference at 38 microradians is +.0004489 m. In the fifth column of Figure 7, the variation at 32 microradians is .0050308 m, and the variation at 38 microradians is .0033513 m.

In Table 1 at the beginning of this paper we see that the optical path length in the cube corner is always longer at shorter wavelengths. From this we would expect that the apparent reflection point would be closer to the center of the satellite at shorter wavelengths and the range correction would be smaller. We would expect the range correction at longer wavelengths minus the range correction at shorter wavelengths to always be positive. In fact this is not always the case. Figure 7 which is summarized in the last row of Table 2 below is unusual in that the range correction at the longer wavelength minus the range correction at the shorter wavelength is often negative.

Polarization	**Wavelength 1 (nm)**	**Wavelength 2 (nm)**	**Average (mm)**	**Max-Min (mm)**
Circular	847	423.5	1.4 to 1.9	0.7 to 1.0
Circular	1064	532	0.3 to 1.0	1.2 to 1.0
Linear	847	423.5	1.0 to 1.6	2.7 to 2.0
Linear	1064	532	-.4 to +.4	5.0 to 3.3

Table 2. Summary of range correction differences from Figures 4, 5, 6 and 7.

8. Symmetry of the range correction.

At a single orientation the range correction matrix is irregular due to the limited number of active cube corners. For circular polarization there is no physical mechanism to cause any asymmetry in the range correction. When the range correction is averaged over a number of orientations the pattern for circular polarization approaches circular symmetry. For linear polarization, the centroid range correction matrix averaged over a number of orientations has a "dumbbell" shape.

9. Lageos spin rate.

Lageos-2 still has a reasonably rapid spin rate. Each normal point should be an average over many orientations. The systematic effect should be removed from each normal point by applying the average range correction as a function of velocity aberration.

Lageos-1 has a very slow rotation rate. The only averaging will be from the change in orientation due to the observing geometry. The shape of the pass can be distorted due to the variations with satellite orientation. If the average range correction is applied to the data as a function of velocity aberration, the effect on the shape of the pass should be removed when many passes are averaged.

10. Coated vs uncoated cube corners.

If the cube corners are coated the diffraction pattern is independent of polarization. The range correction for a spherical satellite such as Starlette will be a function only of the wavelength and magnitude of the velocity aberration with no azimuthal dependence. If the cube corners are uncoated and linear polarization is used the phase changes caused by total internal reflection interact with the dihedral angle offset to create a "dumbbell" shape of the diffraction pattern. If there is no dihedral angle offset, or circular polarization is used, there is no azimuthal dependence for uncoated cubes on a spherical satellite.

11. Conclusions.

The average range correction for Lageos at longer wavelengths is typically about 1.5 millimeters greater than at shorter wavelengths. However, the difference can be as high as 5 millimeters and can sometimes be negative. Since the dispersion may be a factor of 12 to 15, the difference in the range correction will cause an error in the atmospheric correction that is larger than the uncertainty in the present atmospheric correction models (approximately one centimeter). Applying the average range correction to the data as a function of velocity aberration should remove systematic effects in any analysis that averages over many points and passes.