

Report for 2005TX200B: A Decision Support system to Develop Sustainable Groundwater Management Policies for a Multi-county Single Aquifer System

Publications

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Report Follows

*A Decision Support System to Develop Sustainable Groundwater Management
Policies for a Multi-county Single Aquifer System*

Muthukumar Kuchanur

Abstract

In arid and semiarid regions, groundwater is one of the critical natural resource constraints. Groundwater, who owns it, and how a regulatory authority governs it remain issues of importance to all Texans and particularly for citizens in parts of the state that receive little rainfall and experiencing rapid population growth. The importance of sustainable groundwater management that incorporates ecological demands is being widely recognized in Texas. Groundwater conservation districts (GCD) are authorized by state law (Texas Water Code, Chapter 36) and are empowered to regulate groundwater resources within their boundaries. Many of these GCDs are striving to formulate sustainable groundwater policies that reconcile both the economical growth and the ecological demands of the region. Typically GCD's jurisdiction confines with the county boundaries and as such does not span the entire aquifer. In other words, the same aquifer is governed by different GCDs. In such instances, the policies formulated by one county will greatly influence the groundwater dynamics in the other counties because of the interconnected nature of the geological formations in this region.

Therefore, ad hoc groundwater management policies adopted by a single county (GCD) without systematic integration of science, management and cooperation among adjoining counties may act as a serious impediment for regional-scale sustainable growth. Hence the development of a strategic decision support framework to formulate and assess these sovereign yet highly interactive groundwater management policies is essential. In this research work a game theory based decision support framework is developed and the application of this framework is illustrated using case studies in South Texas.

Introduction

The major aquifers in Texas span over multiple counties but typically the jurisdiction of a GCD is confined within the county boundaries. In other words, the same aquifer is shared and governed by different GCDs. In such instances, the policies formulated by one county (GCD) will greatly influence the groundwater dynamics in the neighboring counties because of the interconnected nature of the geological formations of the aquifers. Consequently, the subjective preferences in the policy formulation of one GCD can persuade the policy decisions of the other neighboring GCDs in the region. Therefore, ad hoc groundwater management policies adopted by a single county (GCD) without systematic integration of science, management and cooperation among adjoining counties may act as a serious impediment for regional-scale sustainable growth. However, the uncertainty in management decisions resulting from disagreements between the GCDs needs to be effectively characterized to move towards sustainable groundwater management. Thus, the development of a strategic decision support framework that can guide the decision makers to assess these sovereign yet highly interactive groundwater management policies is essential.

Groundwater is a multi-dimensional resource and often is shared by stakeholders with competing objectives. When aquifers are shared and managed by different GCDs these competing objectives may become conflicts that are more difficult to resolve because of the inherent emotional nature of the problem coupled with administrative obstacles. The overall goal of this research is to develop a strategic framework that reconciles the impacts of externalities in policy decisions and in particular, to evaluate the effect of the groundwater management policies formulated by one county towards the other. Game theoretic approaches are illustrated in this research as suitable tools to evaluate policy options and guide negotiations between decision makers towards adopting sustainable groundwater management policies.

Research objectives

This research is aimed at the formulation of a decision support framework for developing sustainable groundwater management policies. A common management goal for the GCDs is to fully analyze water resources for economic and social benefit. However, sustainable management of groundwater aquifers is a complex issue that requires a delicate balance between the abstractions and recharges. Particularly in semiarid coastal regions critical issues like baseflows to the bays, saltwater intrusions craft the management of groundwater into a more exigent framework.

In light of the discussion above, the objectives of this research include (1) To develop a transparent and pragmatic methodology that helps the GCDs and decision makers to evaluate sustainable regional groundwater management policies;(2)To develop and demonstrate game theoretic framework that reconciles the environmentally conservative options vs. economically beneficial pay offs and their interactions between different counties; (3) To use game theoretic approach coupled with simulation-optimization to develop payoff matrices and thus making it a transparent tool that facilitates stakeholder participation; (4) To incorporate subjective preferences, uncertainties in the utilities function, and risk behaviors of various players.

Methodology

Game theoretic framework

Game theory is a formal way to analyze strategic interaction among a group of rational players (or agents) who behave strategically. Game theory has been widely used in the areas of economics, social sciences, industrial management, logistics, military and political sciences and the literature is replete with the application of game theory in these areas (Thomas 1984; Wolters and Schuller, 1997; Burns and Gomolinska 2001; Bell and Cassir, 2002; Li et al., 2002; Tchangani, 2005). The bestowal of the Nobel price for John Nash in 1994 and in 2005 to Robert J. Aumann and Thomas C. Schelling for their works in game theory underscores the important role of game theory in resolving conflicts. Despite its applicability to characterize conflicts, game theory has not been widely used in groundwater management. Recently Loaiciga (2004) had implemented an analytical game theoretic formulation to explore the roles of cooperation and non-cooperation on the sustainable exploitation of groundwater. However, game theory has not been applied to analyze the policy interactions between groundwater management institutions and this study is an effort to illustrate the application of game theory in this arena.

The object of study in game theory is the game, which is a formal model of an interactive situation. Typically, a game can be defined as consisting of three elements:

- (i) Players denoted as i , and can vary from $(2, 3, \dots, N)$,
- (ii) Strategies chosen by a player (i) from a set of strategies S_i and
- (iii) Payoff to a player $P_i (A_i, O_i)$, where P_i is the payoff to player i when the player chooses a strategy A_i and the other players choose a strategy O_i

The players are assumed to be rational and choose strategies that maximize their expected pay offs. However, in situations involving resources that are shared by multiple decision makers/entities the payoff to a player's strategy (action) cannot be determined without taking into account the strategies chosen by other players. Thus, game theory acts as a tool to model the interaction between the players (decision makers) and helps the decision makers to analyze policy choices anticipating the strategies of the other players. Games are broadly classified as being (a) dual or plural, (b) finite or infinite, and (c) cooperative or non-cooperative (Owen, 1982). Briefly, if the number of players is two, it is a dual game or if the players are more than two it is a plural game (multi-player game). Depending on the possible number of strategies and moves the game can be classified as a finite or infinite game. In non-zero-sum games, the two players' payoffs are not directly opposed. In such cases, the games can be classified as cooperative (where communication, binding contracts and correlated strategies are allowed) and non-cooperative games (where no possibilities of commitment are allowed).

In the context of groundwater availability estimation, an aquifer can be considered a common resource that is shared by multiple GCDs. The GCDs can be construed as players. The GCDs consider and analyze various policy strategies that may include a gamut of choices. Some of the policy choices are: (i) A GCD may decide on a policy level either to develop or not to develop groundwater within their jurisdiction. (ii) In both of the above cases, the GCD may opt to have a conservative or a liberal policy. The extent and the nature of liberal or conservative behavior of a GCD largely hinges on the risk preferences and other geologic factors of their county. (iii) The GCDs may also decide either to honor or not to honor their neighbor's policies and the degree of this policy behavior may vary from honoring all or some of the neighbor's policies to not considering any of their policies/concerns at all. A schematic of the policy choices that are deliberated by GCDs at various stages of policy formulation is shown in Figure 1. If the neighboring county does not have a GCD (which is the case of some counties in the coastal bend region like Calhoun, Kleberg Counties) it can be broadly grouped into the category of a county not honoring their neighbor's policy. But in this particular case the reason for not honoring the neighbor's policy is that they do not have a GCD and consequently no management policy as such.

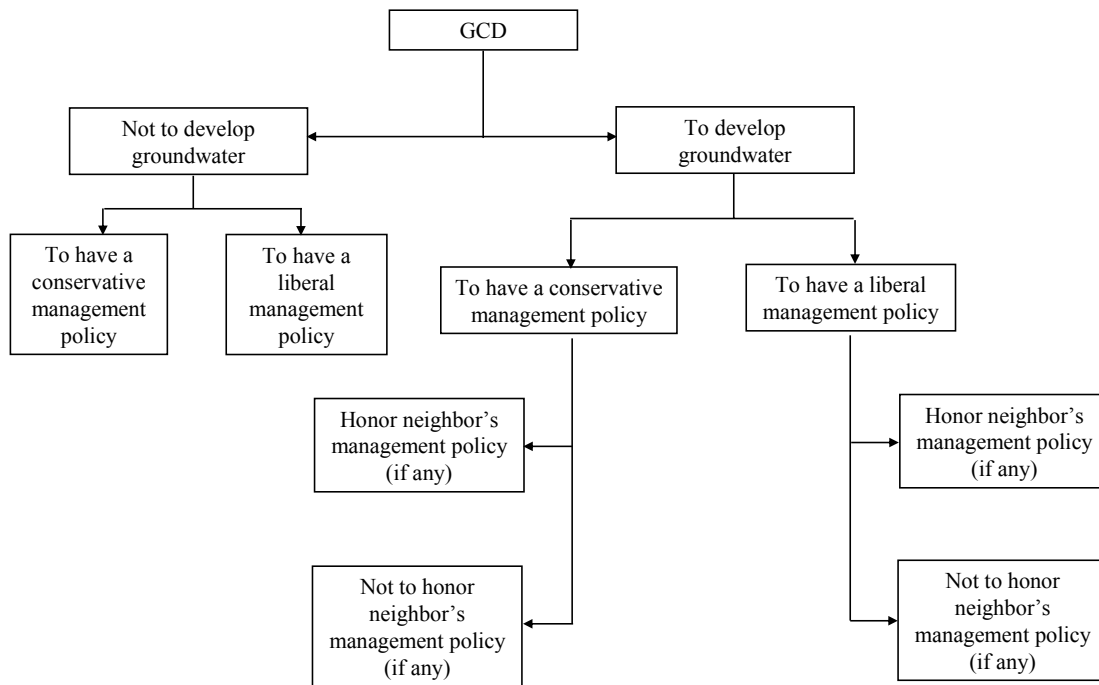


Figure 1: Overview of policy choices for a GCD

Groundwater is a shared resource and various stakeholders may have different payoffs or utilities that are dependent directly or indirectly on groundwater. The utility function reflects the interests and orientation of the GCDs. The utility function may be as simple as the amount of groundwater pumped from the aquifer to as complex as the ecosystem productivity in the bays and estuaries. GCDs are deemed to reconcile the environmentally conservative options vs. economically beneficial pay offs. Therefore, in this research a ratio of the amount of groundwater pumped (Q in ac-ft/year) over drawdown inducted by pumpage this Q (h in ft) is used as the payoff function. The amount of groundwater (Q) that can be pumped from the aquifer is an indicator of the economic benefits that can be derived and the drawdown inducted can be used as a surrogate measure of the environmental impacts and other negative externalities that include: (i) decreased water availability, (ii) decreased environmental flows, (iii) negative economic externalities linked with the drying of shallow wells, (iv) associated increase in the cost of pumpage with increased drawdowns (v) impacts potential saltwater intrusion and (vi) impacts of subsidence. Thus a ratio between Q and h acts as an adequate payoff function that reconciles both the positive and negative impacts associated with any policy formulation made by a GCD and is used in this research as such. Hence a higher value of this ratio indicates a higher payoff to the player.

Simulation – Optimization approaches can be used to calculate the payoff function as they have been demonstrated to be capable of incorporating the preferences of the stakeholders within a GCD (Uddameri et al., 2006; Uddameri and Kuchanur, 2006). The ratio of Q/h can be calculated using this approach. The payoffs need to be calculated for all the identified policy interactions

within GCDs and then can be summarized in a payoff matrix. A typical two player payoff matrix is shown in figure 2.

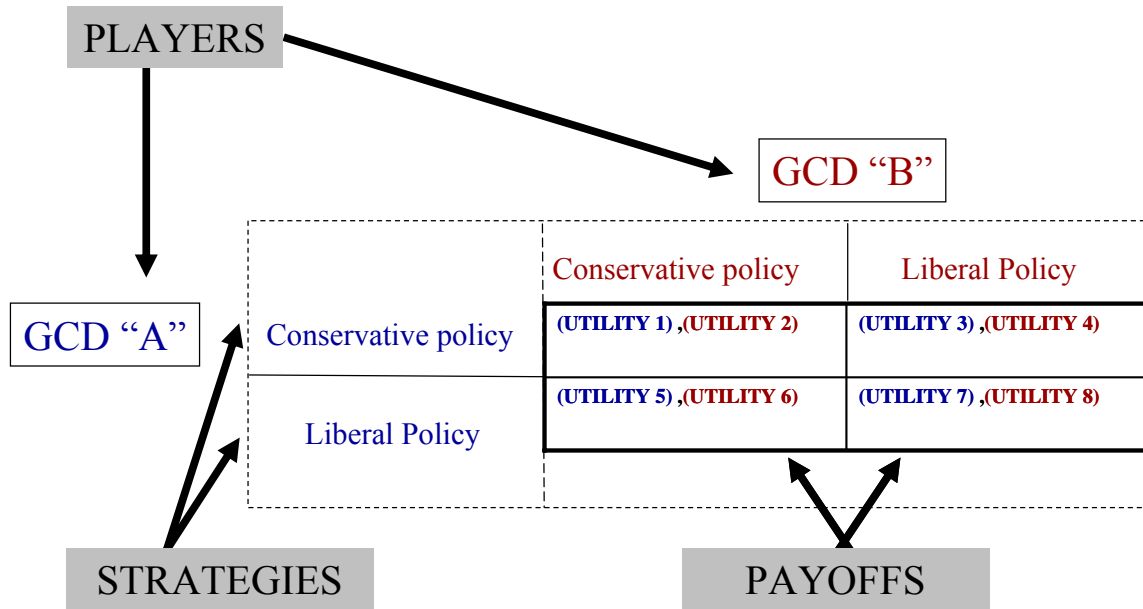


Figure 2: Schematic of a two player payoff matrix

It is important to note that this payoff can be easily modified to reflect the orientation and the risk preferences of a GCD by adequately incorporating weights for Q and h in the identified payoff ratio. In some cases the payoff function can also be an aggregate of sub-payoff functions.

Game-theory provides a simple yet effective tool that can model the interaction between various GCDs. It is concerned with identifying the impacts triggered by the actions of the neighboring GCDs while a GCD is formulating its own policy. This framework enables the GCDs to choose the best possible strategies (dominant and closer to optimality) given the potential policy choices by the neighboring GCDs. A strategy is dominant if it outperforms all other choices no matter what opposing players do. This approach helps the GCDs to identify the dominant strategies (both strict and weak) of theirs as well as their neighbors and provides the insight that both the GCD and their neighbors will try to adopt their dominant strategies. From a policy standpoint, this approach also emphasizes the fact that optimality can be relative and is generally difficult to define the best possible outcome for a GCD without considering the impacts of the externalities caused by their neighbors. The value of game-theoretic approaches lies in its ability to provide insights that are easy to discern by the decision makers. Based on the above discussion, a broad overview of the steps needed in the development of a game-theoretic tool is depicted in Figure 3.

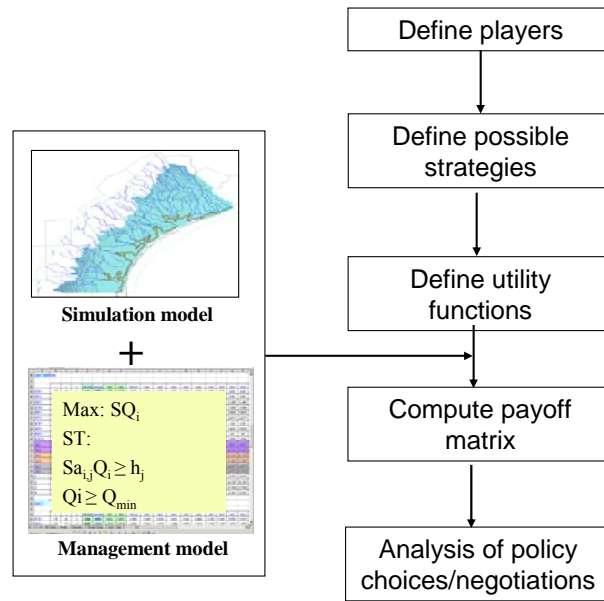


Figure 3: Overview of the development of a game theory based groundwater management tool

Three case studies illustrating the application of game theory based tools for groundwater management in the Gulf Coast aquifer of Texas are presented next.

Illustrative Case Studies:

Background:

The game-theoretic will be demonstrated in the coastal bend region of south-Texas particularly focusing on the three-county area of Bee, Goliad and Refugio. The groundwater flow gradient is typically from north-west to south-east towards the Gulf of Mexico (Chowdhury et al., 2004; Uddameri and Kuchanur 2006). The Mission river flows across all the three counties and is depicted in Figure 4

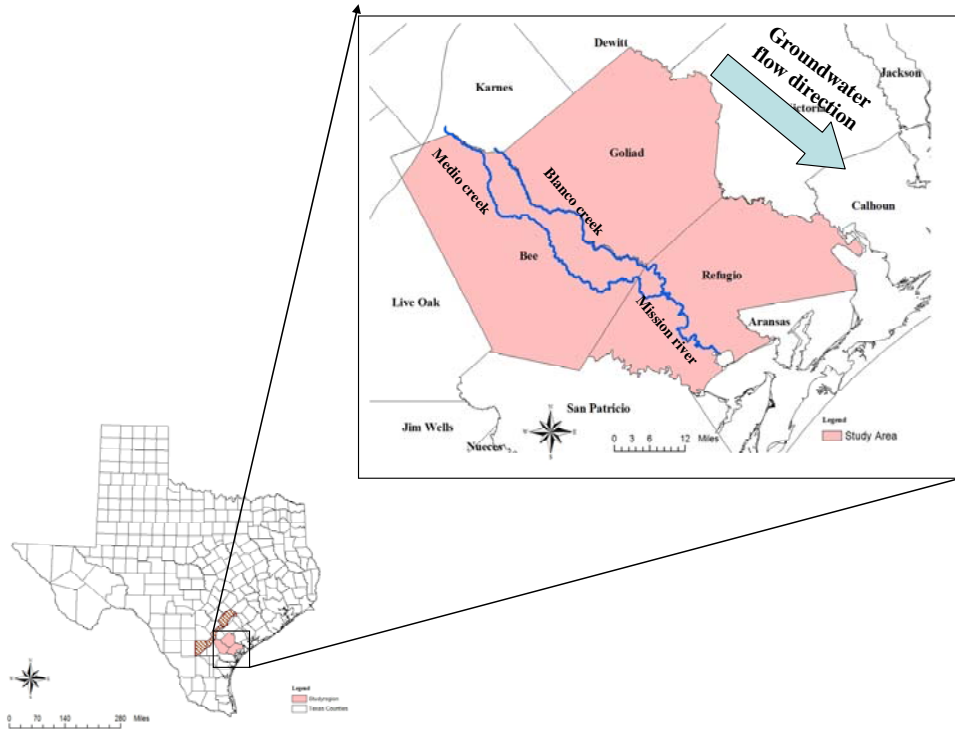


Figure 4: Three-County study region

The water demand in all the three counties almost remains constant for the next 50 years (TWDB, 2002). Municipal, irrigation and livestock are the major water demand categories. The sandy Evangeline aquifer outcrops in the Bee and Goliad Counties (Figure 3.4) and also acts as a major source of recharge for the deeper Evangeline aquifer underlying Refugio County. Refugio County consists of the outcrops of Chicot formation which is mainly the Beaumont and the upper Lissie sand deposits. The interconnected nature of the aquifer formation with all the relevant hydrologic processes in the region is depicted in Figure 5.

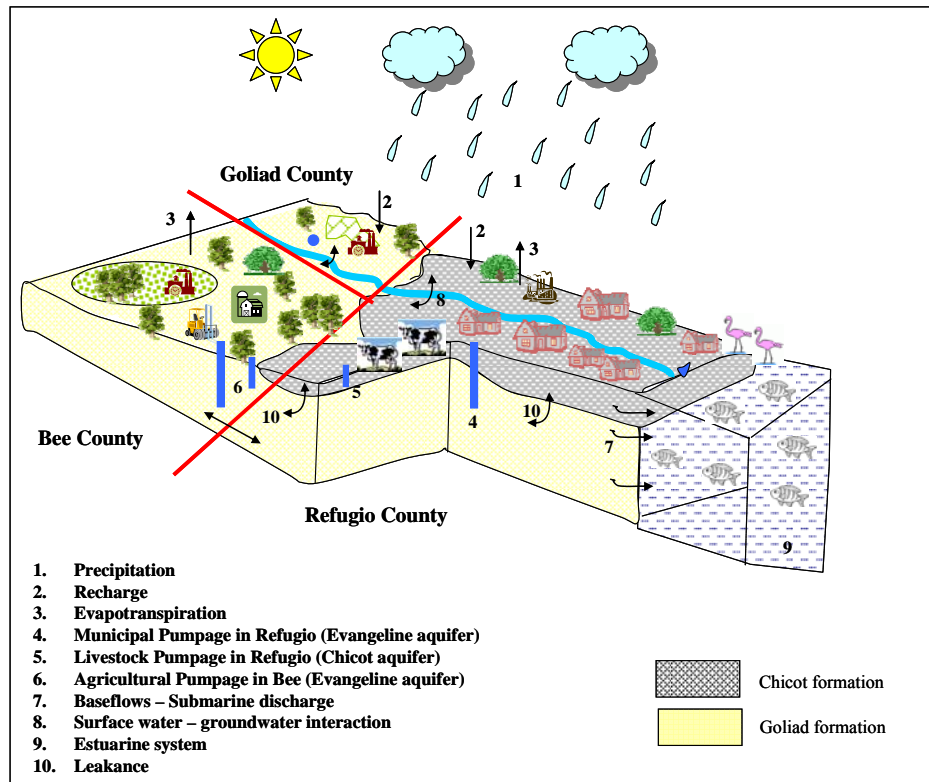


Figure 5: Depiction of aquifer formations and key hydrologic processes

Considering the hydrogeologic studies carried out by Mason (1963), the Evangeline aquifer is found to be more prolific and any future groundwater development projects may be located in the Evangeline formation. However, any such development in a larger scale even in any of one of these counties will affect the groundwater dynamics in the other two counties. For example, a proposed large scale development in the deeper Evangeline aquifer in Refugio County may affect the drawdowns at the wells located in the shallow Evangeline aquifer in Bee and Goliad Counties. Also, if the proposed groundwater development is carried out improperly it may affect the regional flow gradients and consequently the baseflows to the Mission river which flows through all the three counties. Though the aquifer is shared by all the three counties, the concerns and views of the stakeholders towards future large-scale need not be the same. For example, due to the proximity of Refugio to the coast, the foremost concerns for this GCD may be potential saltwater intrusion, reduction in baseflows to Copano and Aransas bays. These concerns may or may not be shared by Bee and Goliad GCD depending on their risk as well subjective preferences. But, understanding these interactions are crucial for regional-scale sustainable growth. The potential policy choices for these counties and the associated payoffs and interactions among them using game theory are presented next.

Players:

In this case study the defined players are (1) Bee GCD, (2) Goliad GCD and (3) Refugio GCD. The individual GCDs assumed to be rational and seek to play in a manner which maximizes their own payoffs. There are three illustrative two-player case studies presented in this research study. The players in these case studies are presented in Table 1

Table 1: Players in the case studies

Case Study	Players (GCDs) involved
Case study: I	Refugio and Goliad
Case study: II	Bee and Refugio
Case study: III	Bee and Goliad

Strategies:

A strategy is a complete plan of choices, one for each decision point of the player. Considering the infrastructure costs involved and administrative challenges in changing an implemented groundwater management the case studies assume that there is only one decision point for each player. Some of the strategies deliberated by a GCD are depicted in Figure 1. Based on these possible choices five games were designed for each of the case studies to obtain relevant insights on the interactive impacts of these potential management policies. For illustrative purposes, the GCDs are assumed to have 6 policy choices

- (i) To allow the development of groundwater within the jurisdiction of a GCD
- (ii) Not to allow the development of groundwater within the jurisdiction of a GCD
- (iii) Adopt a conservative management policy; where the allowed drawdown should not be greater than 5 ft.
- (iv) Adopt a liberal management policy; where the allowed drawdown should not be greater than 25 ft.
- (v) To honor the management of the neighbor(s).
- (vi) Not to honor the management of the neighbor(s).

A total of 15 games were designed incorporating a variety of combinations of the above strategies and is listed in Table 2.

Table 2: List of illustrative games

Game	Strategy
	Case study : I
	Players: Refugio and Goliad
1	Groundwater development is only in Refugio and Refugio decides not to honor Goliad's management policies
2	Groundwater development is only in Goliad and Goliad decides not to honor Goliad's management policies
3	Groundwater development is only in Refugio and Refugio decides to honor Goliad's management policies
4	Groundwater development is only in Goliad and Goliad decides to honor Goliad's management policies
5	Groundwater development is in both Refugio and Goliad. Both the counties decide to honor their neighbor's policies.
	Case study : II
	Players: Bee and Refugio
6	Groundwater development is only in Bee and Bee decides not to honor Refugio's management policies
7	Groundwater development is only in Refugio and Refugio decides not to honor Bee's management policies
8	Groundwater development is only in Bee and Bee decides to honor Refugio's management policies
9	Groundwater development is only in Refugio and Refugio decides to honor Bee's management policies
10	Groundwater development is in both Bee and Refugio. Both the counties decide to honor their neighbor's policies.
	Case study : II
	Players: Bee and Goliad
11	Groundwater development is only in Bee and Bee decides not to honor Goliad's management policies
12	Groundwater development is only in Goliad and Goliad decides not to honor Bee's management policies
13	Groundwater development is only in Bee and Bee decides to honor Goliad's management policies
14	Groundwater development is only in Goliad and Refugio decides to honor Bee's management policies
15	Groundwater development is in both Bee and Goliad. Both the counties decide to honor their neighbor's policies

Payoff matrix:

The ratio between the amount of groundwater pumped (Q in ac-ft/year) to the levels of average drawdown (h in ft) induced by this pumpage is defined in this case study as the payoff for the GCDs.

$$Payoff = \frac{\text{Total amount groundwater pumped from a GCD}(Q \text{ in ac - ft/year})}{\text{Average drawdown induced by } Q \text{ in the GCD (h in ft)}}$$

The simulation optimization approach as illustrated in Uddameri and Kuchanur 2006; and Uddameri et al., 2006 is used to calculate the amount of groundwater pumped and the average drawdowns in the monitoring locations.

The steady-state Central Gulf Coast aquifer Groundwater Availability Model developed by the TWDB (Chowdhury et al., 2004) was used as the simulation model. In order to calculate the payoff matrices and to model the strategic interactions between the three GCDs a management schematic was developed as depicted in Figures 6 and 7. A total of 51 well fields were selected with 17 in each county. Nine well fields (3 in each county) were located in the Chicot aquifer. The remaining 42 well fields (14 in each county) were located in the Evangeline formation. Monitoring locations were also identified to be uniformly located near the county boundaries in such a way to monitor the drawdowns near the boundaries. Ten monitoring wells were located in Chicot formation and 18 monitoring wells were located in Evangeline aquifer. The identified well fields are hypothetical and these are used only for the illustrative purposes of this dissertation.

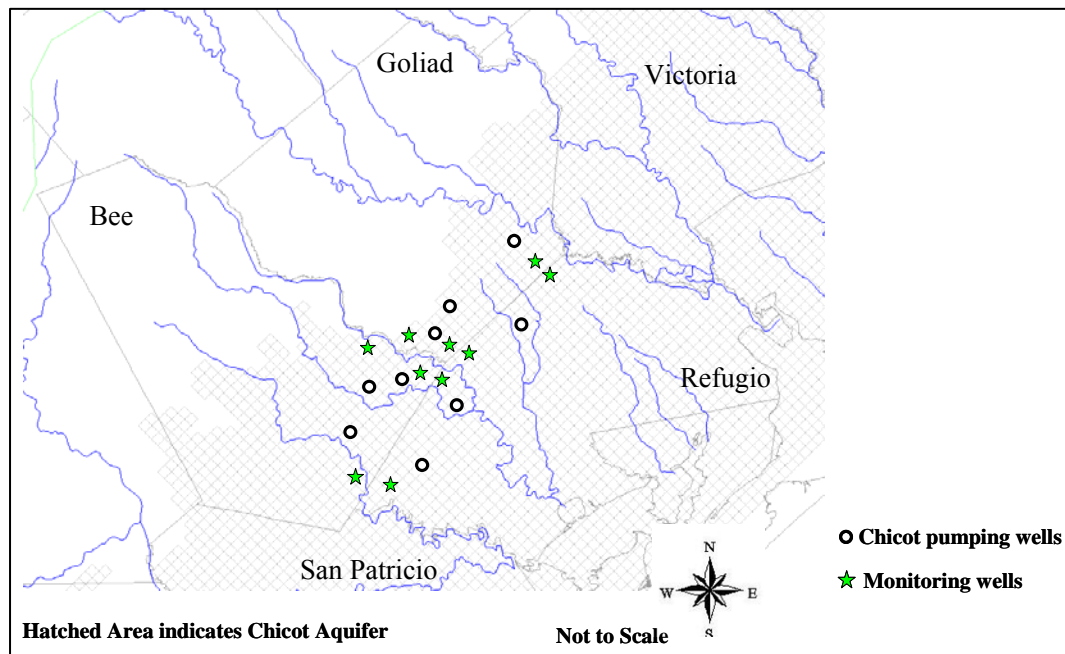


Figure 6: Location of pumping and monitoring wells in Chicot aquifer

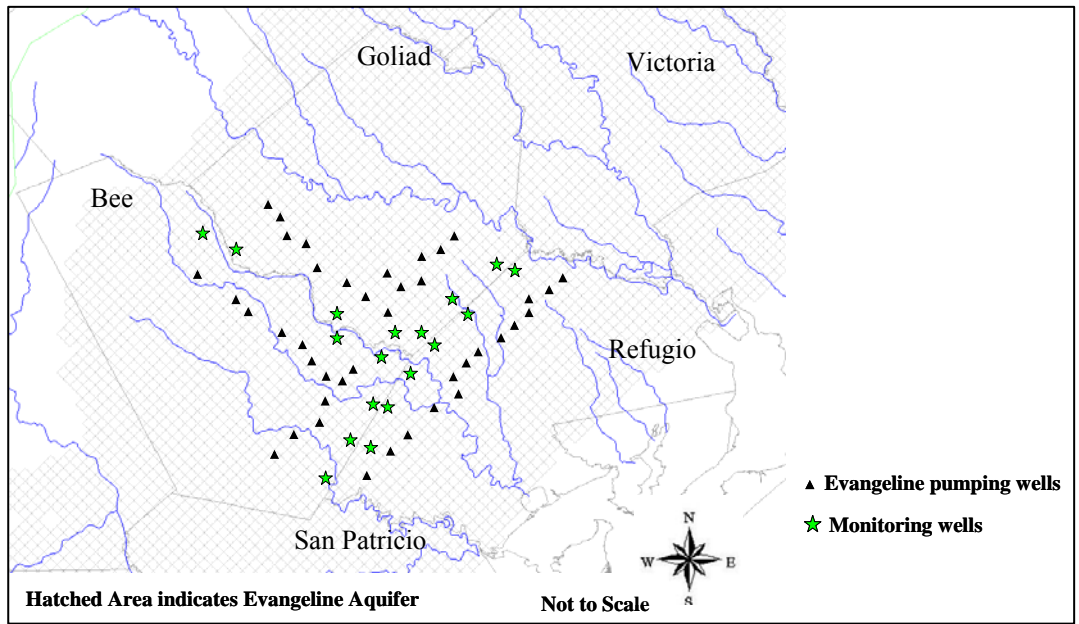


Figure 7: Location of pumping and monitoring wells in Evangeline aquifer

The optimization model used in the case studies is mathematically represented as:

$$Max : \sum_{i=1}^{i=17} Q_{i,GCD} \dots\dots\dots(1)$$

Where,

- i is the number of pumping wells
- GCD represents the pumping wells in Refugio for games 1,3,7,9
- GCD represents the pumping wells in Goliad for games 2, 4, 12, 14
- GCD represents the pumping wells in Bee for games 6, 8, 11, 13
- GCD represents the pumping wells in Refugio and Goliad for game 5
- GCD represents the pumping wells in Bee and Refugio for game 10
- GCD represents the pumping wells in Bee and Goliad for game 15

Subject to:

Constraints:

$$DD_{AP,1,k,GCD} \leq \Delta \quad \forall \text{ and } k = 1,2,3; \{GCD = Player1, Player2\} \dots\dots\dots(2)$$

$$DD_{AP,2,k,GCD} \leq \Delta \quad \forall \text{ and } k = 1,\dots,6; \{GCD = Player1, Player2\} \dots\dots\dots(3)$$

$$Q_{i,GCD} \geq Q_{min,i,GCD} \quad \forall i = 1,\dots,34; \{GCD = Player1, Player2\} \dots\dots\dots(4)$$

Equation (1) represents the objective of maximizing the amount of groundwater that can be safely pumped (Q) from the aquifer. The objective function includes only the pumpage from wells in a GCD where groundwater development is allowed. The drawdowns in the monitoring locations in Chicot and Evangeline formations are captured by equations 2 and 3 respectively. The value of Δ is 5 ft if the policy of the player is conservative or 25 ft in the case of a liberal management policy and Δ can also be changed to any value depending on the preferences of a GCD. If a player (GCD) decides to honor their neighbor’s policy then the constraints set by both

the players (player 1 and player 2) should be met. If the chosen strategy of a GCD is not to honor their neighbor's policy then, only the constraints set by the GCD need to be met. The players and the constraints to be included for each game are tabulated in Table 3. Equation 4 indicates the minimum total amount of pumpage from a county in order to render the calculation of payoff ratio as a non-zero numerical value.

Table 3: Objective functions and drawdown constraints

Game	Player 1	Player 2	Objective function (Equation 1) includes the pumping wells in:	Drawdown constraints (Equations 2 and 3) to be met for:
1	Refugio	Goliad	Refugio	Refugio
2	Refugio	Goliad	Goliad	Goliad
3	Refugio	Goliad	Refugio	Refugio and Goliad
4	Refugio	Goliad	Goliad	Refugio and Goliad
5	Refugio	Goliad	Refugio and Goliad	Refugio and Goliad
6	Bee	Refugio	Bee	Bee
7	Bee	Refugio	Refugio	Refugio
8	Bee	Refugio	Bee	Bee and Refugio
9	Bee	Refugio	Refugio	Bee and Refugio
10	Bee	Refugio	Bee and Refugio	Bee and Refugio
11	Bee	Goliad	Bee	Bee
12	Bee	Goliad	Goliad	Goliad
13	Bee	Goliad	Bee	Bee and Goliad
14	Bee	Goliad	Goliad	Bee and Goliad
15	Bee	Goliad	Bee and Goliad	Bee and Goliad

Finally, the payoff ratios are calculated using the simulation-optimization approach and are tabulated in a matrix format. The results from the illustrative case studies will be presented next.

Results:

Case study: I – Refugio Vs Goliad

The payoff matrices for case study I (Games 1 to 5) are represented in figure 8. Game 1 considers the scenario where the strategy of Refugio GCD is to allow the development of groundwater and the strategy of Goliad GCD is not to allow the development of groundwater. In this game Refugio GCD does not honor the policy preferences of Goliad. The payoff matrix for game 1 in figure 8 indicates the payoffs for Refugio and Goliad under both conservative (5 ft average drawdown) and liberal (25 ft average drawdown) management choices set by Refugio GCD. The payoffs for Refugio in this game are more than Goliad; however the payoff decreases when Refugio opts for a liberal management strategy. Further analysis indicated that though the amount of groundwater pumped increases, the ratio of this increase over the drawdown values make the payoff to decrease to a lower value than the payoff under the conservative management scenario. Similarly payoff matrices were also calculated for Games 2, 3, 4 and 5 and are summarized in figure 8.

Game	Payoff matrix																		
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5	<p style="text-align: center;">Goliad</p> <p style="text-align: center;">Pumping</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td colspan="2"></td> <td style="text-align: center;">Conservative policy</td> <td style="text-align: center;">Liberal Policy</td> </tr> <tr> <td rowspan="2" style="vertical-align: middle;">Refugio</td> <td style="text-align: center;">Conservative policy</td> <td style="text-align: center;">1531,6938</td> <td style="text-align: center;">50,8854</td> </tr> <tr> <td style="text-align: center;">Liberal Policy</td> <td style="text-align: center;">1471,6121</td> <td style="text-align: center;">1449,7149</td> </tr> <tr> <td colspan="2" style="text-align: center;">Pumping</td> <td colspan="2"></td> </tr> </table>						Conservative policy	Liberal Policy	Refugio	Conservative policy	1531,6938	50,8854	Liberal Policy	1471,6121	1449,7149	Pumping			
		Conservative policy	Liberal Policy																
Refugio	Conservative policy	1531,6938	50,8854																
	Liberal Policy	1471,6121	1449,7149																
Pumping																			

Figure 8: Payoff matrices for case study I

Games 1 to 4 are designed with an assumption that groundwater development is carried in only in one of the counties. The GCD that allowed the development of groundwater ended up with a higher payoff in all these cases. In reality this may not be the case, however these games will help the decision makers to identify the minimum-most and the maximum possible payoff that can be obtained for their GCD. Game 5 is a more realistic scenario where the groundwater development is in both the counties and the GCDs honor their neighbor's policies. From the payoff matrix, it can be noted that the payoffs for Refugio are lower than Goliad irrespective of their strategies; which is a reflection of the hydro-geologic conditions in this region. This is an example of a lop-sided game where the payoffs of one player completely outplay the other under all strategies considered.

Typically, the preferred strategy of a player is to choose a point of equilibrium. This point is called as Nash equilibrium or strategic equilibrium (Owen 1982, Nash, 1950). Nash equilibrium is a pair of strategies with the property that none of the players would be able to increase their payoffs by switching strategies unless others did. Nash equilibrium can be calculated by either a simple cell by cell inspection or using the iterated elimination of strictly dominated strategies. A strategy is dominated if there is some other strategy which always does better. For example in game 5, the conservative strategy of Goliad always yields a lesser payoff to Goliad irrespective of the strategies of Refugio. Therefore, the conservative strategy of Goliad is strictly dominated by the liberal strategy and it is eliminated. Then, the comparison of payoffs for Refugio's strategies indicate that the conservative strategy of Refugio is dominated by their liberal strategy. The eliminations carried out indicate that the pair of liberal policy of Goliad and liberal policy of Refugio turns out to be the equilibrium point in this game under the given conditions (Figure 9). In other words, this pair of strategies has the property that no player can unilaterally change their strategy and get a better payoff.

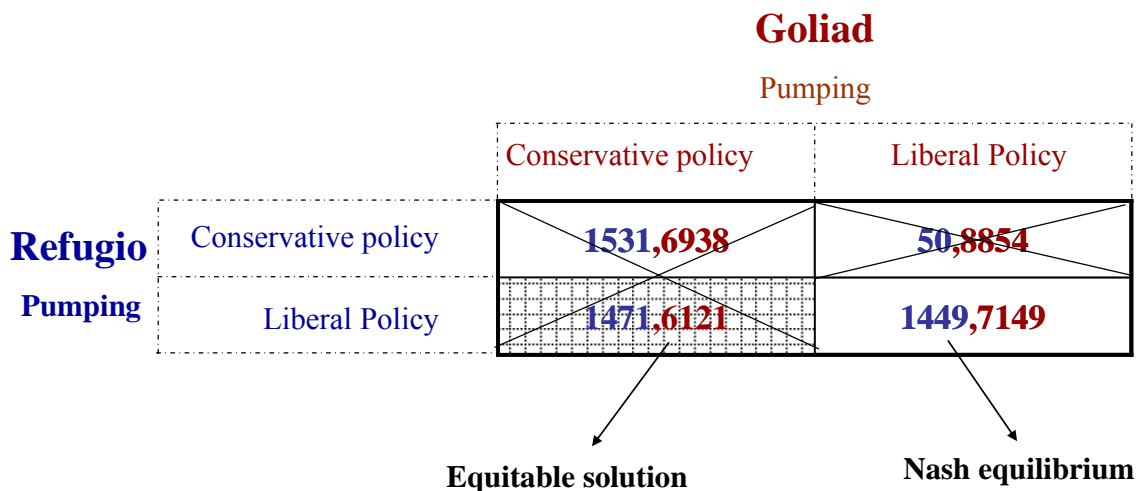


Figure 9: Nash equilibrium and equitable solution for game 5

Social/economic equity is an impact aspect of sustainable solutions. The equity of a solution can be computed by taking into account the difference in payoffs between the players for a particular strategy. A minimal difference in the payoffs between the players indicates a more equitable solution. An equitable payoff may also result in consensus and will reduce standoffs and possible legal disputes between the players. From figure 9, it can be noted that the difference in payoffs (surrogate for equity) is found to be minimal when Refugio adopts a liberal policy and Goliad a conservative policy. Thus, it can also be noted that equitable solution may be different from the equilibrium solution.

As can be seen from figure 9, the players do not achieve their maximum payoffs by adopting equilibrium or equitable solutions. There is a reduction in the payoff (also referred as regret) by not adopting the strategy that can yield the maximum most payoff. In this game, Goliad has a regret of 1705 (8854-7149) for adopting the equilibrium solution and a regret of 2733 (8854-6121) for adopting the equitable solution. Similarly, Refugio has regret of 82 (1531-1449) for adopting the equilibrium solution and a regret of 60 (1531-1471) for adopting the equitable solution. Thus, in the case of Goliad adopting an equitable solution generates a higher regret than adopting the equilibrium solution but in the case of Refugio agreeing to an equilibrium solution has a higher regret. This is also an indicator of the existing hydro-geologic conditions in these counties. As Goliad is located in the recharge region, adopting an equilibrium stance yields a higher payoff. However, the scenario of Refugio following a liberal strategy when Goliad adopting a conservative management strategy increases the payoffs of Refugio (as drawdown impacts in Refugio caused by Goliad are reduced) and thus yielding a more equitable solution. While moving from equilibrium to an equitable solution the net regret for Goliad is -1028 but the payoffs of Refugio increase by 22.

Case study: II – Bee Vs Refugio

The results of this case study are also observed to be similar to case study I and the results are depicted in figure 10. In games 6, 7, 8 and 9 the groundwater development is assumed to be only either in Bee or Refugio. The results from these games indicate the payoffs are always higher in the county where development is allowed under the given conditions. Game 10 is a more realistic scenario in which groundwater development is allowed in both the counties and the counties also decide to honor their neighbor's policies. From the payoff matrix of game 10, it can be inferred that this game is also a lop-sided game and the payoff of Bee always out-perform the payoff of Refugio GCD. A comparison between the payoffs of Refugio between games 5 and 10 indicates that Goliad's policy preferences have a higher impact on Refugio, when Refugio has a conservative strategy. But, When Refugio has a liberal management policy; the policy preferences of Bee GCD have a higher impact on the payoffs of Refugio.

Game	Payoff matrix																						
6	<table border="1"> <tr> <td colspan="2"></td> <td colspan="2" style="text-align: center;">Refugio</td> </tr> <tr> <td colspan="2"></td> <td colspan="2" style="text-align: center;">No Pumping/No Policy</td> </tr> <tr> <td rowspan="2" style="vertical-align: middle;">Bee Pumping</td> <td style="text-align: center;">Conservative policy</td> <td style="text-align: center;">(6273,74)</td> <td style="background-color: #cccccc;"></td> </tr> <tr> <td style="text-align: center;">Liberal Policy</td> <td style="text-align: center;">(6236,15)</td> <td style="background-color: #cccccc;"></td> </tr> </table>						Refugio				No Pumping/No Policy		Bee Pumping	Conservative policy	(6273,74)		Liberal Policy	(6236,15)					
		Refugio																					
		No Pumping/No Policy																					
Bee Pumping	Conservative policy	(6273,74)																					
	Liberal Policy	(6236,15)																					
7	<table border="1"> <tr> <td colspan="2"></td> <td colspan="2" style="text-align: center;">Refugio</td> </tr> <tr> <td colspan="2"></td> <td colspan="2" style="text-align: center;">Pumping</td> </tr> <tr> <td colspan="2"></td> <td style="text-align: center;">Conservative policy</td> <td style="text-align: center;">Liberal Policy</td> </tr> <tr> <td rowspan="2" style="vertical-align: middle;">Bee No Pumping/No Policy</td> <td></td> <td style="text-align: center;">(75 , 1675)</td> <td style="text-align: center;">(15, 1579)</td> </tr> <tr> <td></td> <td style="background-color: #cccccc;"></td> <td style="background-color: #cccccc;"></td> </tr> </table>						Refugio				Pumping				Conservative policy	Liberal Policy	Bee No Pumping/No Policy		(75 , 1675)	(15, 1579)			
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		Refugio																					
		No Pumping																					
		Conservative policy	Liberal Policy																				
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	Liberal Policy	10738,55	6236,15																				
9	<table border="1"> <tr> <td colspan="2"></td> <td colspan="2" style="text-align: center;">Refugio</td> </tr> <tr> <td colspan="2"></td> <td colspan="2" style="text-align: center;">Pumping</td> </tr> <tr> <td colspan="2"></td> <td style="text-align: center;">Conservative policy</td> <td style="text-align: center;">Liberal Policy</td> </tr> <tr> <td rowspan="2" style="vertical-align: middle;">Bee No Pumping</td> <td style="text-align: center;">Conservative policy</td> <td style="text-align: center;">75,1675</td> <td style="text-align: center;">55,1843</td> </tr> <tr> <td style="text-align: center;">Liberal Policy</td> <td style="text-align: center;">75,1675</td> <td style="text-align: center;">15,1579</td> </tr> </table>						Refugio				Pumping				Conservative policy	Liberal Policy	Bee No Pumping	Conservative policy	75,1675	55,1843	Liberal Policy	75,1675	15,1579
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		Conservative policy	Liberal Policy																				
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		Pumping																					
		Conservative policy	Liberal Policy																				
Bee Pumping	Conservative policy	6162,1087	6042,2013																				
	Liberal Policy	10466,853	6140,962																				

Figure 10: Payoff matrices for case study II

The Nash equilibrium for game 10 is calculated using the iterated elimination of strictly dominated strategies and is denoted in Figure 11. The conservative policy of Refugio is strictly dominated by its liberal policy. Therefore, the conservative strategy of Refugio is eliminated. Then, the payoffs of Bee were compared and the conservative policy choice is also eliminated. Thus, the Nash equilibrium in game 10 is also similar to game 5 and the both the GCDs are expected to choose a liberal management strategy to achieve equilibrium.

		Refugio	
		Pumping Conservative policy	Liberal Policy
Bee	Conservative policy	6162,1087	6042,2013
	Liberal Policy	10466,853	6140,962

Equitable solution

Nash equilibrium

Figure 11: Nash equilibrium and equitable solution for game 10

From figure 11, it can be noted that the difference in payoffs is found to be minimal when Refugio adopts a liberal policy and Bee a conservative policy. As can be seen from figure 11, Bee has a regret of 4326 (10466-6140) for adopting the equilibrium solution and a regret of 4424 (10466- 6042) for adopting the equitable solution. Similarly, Refugio has regret of 1051 (2013-962) for adopting the equilibrium solution. However, in this game the maximum payoff is incidentally associated with the equitable solution. Therefore, Refugio has zero regret for adopting the equitable solution.

In the case of Bee adopting an equitable solution generates a higher regret than adopting the equilibrium solution but in the case of Refugio agreeing to an equilibrium solution has a higher regret. While moving from equilibrium to an equitable solution the net regret for Goliad is -98 but the payoffs of Refugio increase more than twice the equilibrium payoff.

Case study: II – Bee Vs Goliad

This illustrative case is also designed with 5 games (games 11 to 15) and the payoff matrices are shown in figure 12. In games 11 and 13 the development of groundwater is only in Bee and the payoff matrices indicate that irrespective of Bee GCD’s policy behaviors towards Goliad, the payoffs of Bee tend to remain higher. Similarly, in games 12 and 14 the payoffs of Goliad are always higher. Game 15 which is a more realistic case, the payoff matrix indicate that the payoff of a GCD is higher if it has a liberal management policy and the neighboring GCD has a conservative policy. However the calculation of Nash equilibrium indicates that both the GCDs will tend to adopt liberal strategies under the given conditions.

Game	Payoff matrix																						
11	<table border="1"> <tr> <td colspan="2"></td> <td colspan="2" style="text-align: center;">Goliad</td> </tr> <tr> <td colspan="2"></td> <td colspan="2" style="text-align: center;">No Pumping/No Policy</td> </tr> <tr> <td rowspan="2" style="vertical-align: middle;">Bee Pumping</td> <td style="text-align: center;">Conservative policy</td> <td style="text-align: center;">(6273,102)</td> <td style="background-color: #cccccc;"></td> </tr> <tr> <td style="text-align: center;">Liberal Policy</td> <td style="text-align: center;">(6236 ,23)</td> <td style="background-color: #cccccc;"></td> </tr> </table>						Goliad				No Pumping/No Policy		Bee Pumping	Conservative policy	(6273,102)		Liberal Policy	(6236 ,23)					
		Goliad																					
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12	<table border="1"> <tr> <td colspan="2"></td> <td colspan="2" style="text-align: center;">Goliad</td> </tr> <tr> <td colspan="2"></td> <td colspan="2" style="text-align: center;">Pumping</td> </tr> <tr> <td colspan="2"></td> <td style="text-align: center;">Conservative policy</td> <td style="text-align: center;">Liberal Policy</td> </tr> <tr> <td rowspan="2" style="vertical-align: middle;">Bee No Pumping/No Policy</td> <td></td> <td style="text-align: center;">(198 , 8353)</td> <td style="text-align: center;">(40, 8698)</td> </tr> <tr> <td></td> <td style="background-color: #cccccc;"></td> <td style="background-color: #cccccc;"></td> </tr> </table>						Goliad				Pumping				Conservative policy	Liberal Policy	Bee No Pumping/No Policy		(198 , 8353)	(40, 8698)			
		Goliad																					
		Pumping																					
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		Goliad																					
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		Conservative policy	Liberal Policy																				
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	Liberal Policy	7190,3285	5517,6208																				

Figure 12: Payoff matrices for case study III

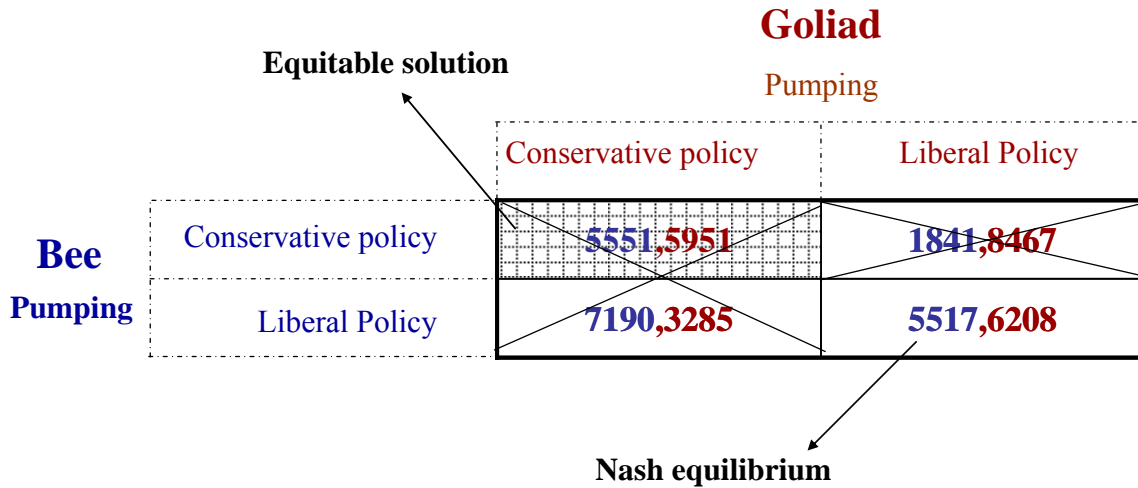


Figure 13: Nash equilibrium for game 15

The result from the Nash equilibrium (figure 13) indicates that for the calculated equilibrium the payoff of Goliad is higher than Bee. Further analysis on the differences between various choices in game 15 indicates that when both the counties have conservative policies, the difference in payoffs between them is less and thus an equitable solution can be realized. When Bee and Goliad decide to move from the equilibrium solution to a more equitable solution, the regret of Goliad is -257 but the payoffs of Bee increase by 34.

Discussion

In this study, a steady-state regional groundwater model (CGC – GAM) as the basis for development. As such, large-scale groundwater extractions at the management wells were assumed to occur at a continuous rate and the investments for the development of groundwater tend to be fairly large. Hence a change in policy of a GCD in time may face stiff resistance from the investors. Moreover, for long-term policy planning endeavors decision makers must work under the assumption that the groundwater users will utilize all of their permitted withdrawals within the allotted time. However, the same game theoretic approach can be extended to multi-stage games. Multi-stage incorporates the option for the GCDs to review and change their strategies if needed after the stipulated planning period (say 10 years). The locations of the identified pumping and monitoring well locations in this study are hypothetical and depending on the real world proposed pumpage projects and mutually agreed monitoring well locations, the results will vary for the GCDs.

The games illustrated in this study are two-player games and can be easily converted to multi-player games as warranted by the hydro-geologic condition. On the other hand, it is important to be borne in mind that the increase in the number of players also increases the complexity and hence affects the understanding and insights that can be obtained by decision makers. Similarly, the strategies of the players are limited to liberal and conservative choices in this study and they

can even be varied with a range of choices as indicated by the decision makers (like most conservative, most liberal... etc).

The defined payoff (Q/h) in this study is a simple yet effective surrogate to account for both the economic benefits and the associated environmental impacts. However, in real world scenarios the defined payoff may or may not be the same for all the GCDs. Therefore, different payoff calculations can be used to indicate the different management inclinations of the involved GCDs. The results from this study (figures 9, 11 and 13) indicate that the Nash equilibrium of all these case studies occurs when both the players adopt liberal management policies. This can be attributed to the defined payoff (Q/h) in this study, a different payoff say $Q/(2h)$ or decrease in baseflows may shift the equilibrium to some other policy choice. The highly sensitive nature of game-theoretic approaches with respect to this definition of payoffs should be adequately conveyed to the decision makers. This in-turn will help the GCDs to identify as well refine and quantify their management goals. It is also to be noted that when the players involved do not have the same defined payoffs; then the comparison of the payoffs between players have to be carried with adequate caution in considering the differences in the definitions of the computed payoffs.

Summary and conclusions

The objective of this research was to develop and illustrate a decision support framework to guide the decision makers to assess the sovereign yet highly interactive groundwater management policies. Game theoretic approaches were identified to adequately capture the strategic interactions between the policies of neighboring GCDs. As part of this endeavor three illustrative case studies were developed and 15 games were designed to demonstrate and assess the impacts of policy choices made by one county over the other. The first two case studies (Refugio Vs Goliad and Bee Vs Refugio) were found to be lop-sided and Refugio ended up with lower payoffs irrespective of the management policies of Goliad and Bee GCDs. This was also found to be a reflector of the hydro-geologic conditions, where Bee and Goliad are located in the recharge and Refugio in the discharge area. Refugio outplayed Goliad and Bee GCDs only in the games (1, 3, 7 and 9) where the development was assumed to occur only in Refugio and not in Bee or Goliad. The third case study (Bee Vs Goliad) was found to be an equal strength game where the chances to achieve an optimal or equitable payoff between the GCDs are possible.

The calculations of regret for each GCD indicate that Goliad always incurs regret (decrease in payoff) and Refugio always increases its payoffs when moving from equilibrium to an equitable solution. However, Bee incurs a regret in game 10 (Bee vs. Refugio) but also experiences some minimal gains in game 15 (Bee vs. Goliad) while adopting an equitable strategy instead of equilibrium. Thus, this analysis indicates that the equitable strategy cannot be achieved without some loss in payoffs for Goliad and Bee. As there is an associated reduction in payoffs for Bee and Goliad counties for adopting an equitable solution, it can be inferred from these case studies that Bee and Goliad may prefer to adopt equilibrium strategies and may neglect Refugio's preferences in formulating their management policies. However, it is vital for Refugio to negotiate with both Bee and Goliad to protect its interests. The results also provide an important insight that the gain in payoffs associated with a change in the policy of a GCD does not exactly balance the loss in the payoffs of the neighbor. Thus regional groundwater management is not an

exact zero sum game as the gain of one GCD does not translate into an equal loss for the neighboring GCD. Thus the results highlight the importance of joint planning and help the decision makers prioritize their negotiations and tradeoffs.

Game theoretic approaches when used in-tandem with reliable simulation models provides transparent easy-to-use decision support platform to analyze policy interactions. The application of this approach in an interactive mode can help the GCDs to initiate dialogues and move towards groundwater management strategies focused on regional-scale sustainability. Game theoretic tools provide valuable insights as well quantify the increase or decrease in the defined payoffs of a GCD while negotiating with their neighbors on policy tradeoffs.

Acknowledgements

The author would like to acknowledge the immense amount of support given by Dr. Uddameri throughout the research. The author would also like to thank the financial support provided by Texas Water Resources Institute (TWRI) and National Science Foundation (NSF – Grant HRD 0206259).

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