How to Construct Double-Block-Length Hash Functions *

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Abstract

In this article, it is discussed how to construct a compression function with $2n$-bit output using a component function with n-bit output. The component function is either a smaller compression function or a block cipher. Some constructions are presented which compose collision-resistant hash functions: Any collision-finding attack on them is at most as efficient as the birthday attack in the random oracle model or in the ideal cipher model. A new security notion is also introduced, which we call indistinguishability in the iteration, with a construction satisfying the notion.

1 Introduction

A cryptographic hash function is a function which maps an input of arbitrary length to an output of fixed length. It satisfies preimage resistance, second-preimage resistance and collision resistance. It is one of the most important primitives in cryptography [20]. For simplicity, a cryptographic hash function is called a hash function in this article.

A hash function usually consists of iteration of a compression function with fixed input/output length. This type of hash function is called an iterated hash function. There has been an interest in constructing a compression function from component functions with smaller output length. Many schemes have been presented following the approach [4, 10, 12, 14, 15, 16, 18, 21]. It is typical for constructions using block ciphers. For example, suppose that AES is used for construction. The block length of AES is 128 bits, and a hash function with 128-bit output is no longer secure against the birthday attack. Thus, it is desired to construct a compression function with larger output length.

In this article, we study how to construct a compression function with $2n$-bit output using a component function with n-bit output. A hash function with such a compression function is called a double-block-length (DBL) hash function (as opposed to a single-block-length (SBL) hash function, where the compression function has n-bit output). The component function may be either a block cipher or a smaller compression function.

We first discuss constructions using a smaller compression function. We focus on the constructions formalized by Nandi [23]. In his formalization, the compression function is of the form $F(x) = (f(x), f(p(x)))$, where f is a component compression function and p is a permutation such that both p and p^{-1} are easy to compute. We show that any collision-finding attack on a hash function with the compression function F is at most as efficient as the birthday attack if f is a random oracle and p satisfies some properties. Our properties for p are easy to be satisfied; for example, they are satisfied by the permutation $p(x) = x \oplus c$, where $\oplus$ is bit-wise addition and c is a non-zero constant.

Similar results are in fact already obtained by Nandi [22], whose analysis actually applies to a broader range of hash functions than our analysis. However, our results are sharper. We give a significantly better upper bound on the probability of finding a collision as a function of the number of queries made by the adversary.

A new security notion for a compression function is also introduced, which we call indistinguishability in the iteration. It is really weaker than the notion proposed in [5]. However, it may be valuable in practice. Loosely speaking, a compression function

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$F(x) = (f(x), f(p(x)))$ where f is a random oracle is called indistinguishable in the iteration if F cannot be distinguished from a random oracle in the iterated hash function. We give sufficient conditions on p for F to be indistinguishable in the iteration.

Second, we discuss constructions using a block cipher. A compression function composed of a block cipher is presented and its collision resistance is analyzed. We show that any collision-finding attack on a hash function composed of the compression function is at most as efficient as the birthday attack if the block cipher used is ideal. A block cipher is ideal if it is assumed to be a keyed invertible random permutation. The compression function presented in this article is quite simple but has not been explicitly discussed previously. We also present some other similar constructions.

In [10], it is shown that a collision-resistant hash function can be easily composed of a compression function using two distinct block ciphers. It is well-known that two distinct block ciphers can be obtained from a block cipher by fixing, for example, one key bit by 0 and 1. However, it is preferable in practice that fixing key bits is avoided. Moreover, fixing one bit may not be sufficient and more bits may be required to be fixed. Our new construction does not involve any fixing of key bits by constants.

The technique in [3] is used in the security proofs in this article. However, the analysis is more complicated than the one in [3] since the relation of two component-compression-function/block-cipher calls in a compression function need to be taken into account.

The rest of this article is organized as follows. Section 2 includes notations, definitions and a brief review of the related works. Section 3 discusses compression functions composed of a smaller compression function, including the results on collision resistance and our new notion of indistinguishability in the iteration. Section 4 exhibits block-cipher-based compression functions whose associated hash functions have optimal collision resistance. Section 5 gives a concluding remark which mentions a recent collision attack on the scheme in Sect. 4.

2 Preliminaries

2.1 Iterated Hash Function

A hash function $H : \{0,1\}^* \to \{0,1\}^\ell$ usually consists of a compression function $F : \{0,1\}^\ell \times \{0,1\}^{\ell'} \to \{0,1\}^\ell$ and a fixed initial value $h_0 \in \{0,1\}^\ell$. An input m is divided into the ℓ'-bit blocks $m_1, m_2, \ldots, m_l$. Then, $h_i = F(h_{i-1}, m_i)$ is computed successively for $1 \le i \le l$ and $h_l = H(m)$. H is called an iterated hash function.

Before being divided into the blocks, unambiguous padding is applied to the input. The length of the padded input is a multiple of ℓ'. In this article, we do not assume Merkle-Damgård strengthening [6, 21] for padding in the security analysis.

2.2 Random Oracle Model and Ideal Cipher Model

2.2.1 Random Oracle Model

Let $\boldsymbol{F}_{n',n} = \{f \mid f : \{0,1\}^{n'} \to \{0,1\}^n\}$. In the random oracle model, the function f is assumed to be randomly selected from $\boldsymbol{F}_{n',n}$. The computation of f is simulated by the following oracle.

The oracle f first receives an input x_i as a query. Then, it returns a randomly selected output y_i if the query has never been asked before. It keeps a table of pairs of queries and replies, and it returns the same reply to the same query.

2.2.2 Ideal Cipher Model

A block cipher with the block length n and the key length κ is called an (n,κ) block cipher. Let $e : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$ be an (n,κ) block cipher. Then, $e(k,\cdot)$ is a permutation for every $k \in \{0,1\}^\kappa$, and it is easy to compute both $e(k,\cdot)$ and $e(k,\cdot)^{-1}$.

Let $\boldsymbol{B}_{n,\kappa}$ be the set of all (n,κ) block ciphers. In the ideal cipher model, e is assumed to be randomly selected from $\boldsymbol{B}_{n,\kappa}$. The encryption e and the decryption e^{-1} are simulated by the following two oracles.

The encryption oracle e first receives a pair of a key and a plaintext as a query. Then, it returns a randomly selected ciphertext. On the other hand, the decryption oracle e^{-1} first receives a pair of a key and a ciphertext as a query. Then, it returns a randomly selected plaintext. The oracles e and e^{-1}

share a table of triplets of keys, plaintexts and ciphertexts, which are produced by the queries and the corresponding replies. Referring to the table, they select a reply to a new query under the restriction that $e(k, \cdot)$ is a permutation for every k. They also add the triplet produced by the query and the reply to the table.

2.3 DBL Hash Function

An iterated hash function whose compression function is composed of a block cipher is called a single-block-length (SBL) hash function if its output length is equal to the block length of the block cipher. It is called a double-block-length (DBL) hash function if its output length is twice larger than the block length.

Let F be a compression function composed of a block cipher. For an iterated hash function composed of F, the rate r defined below is often used as a measure of efficiency:

$$r = \frac{|m_i|}{(\text{the number of block-cipher calls in } F) \times n} \ .$$

In this article, we also call an iterated hash function a DBL hash function if its compression function F is composed of a smaller compression function f and its output length is twice larger than the output length of f.

2.4 Related Work

Knudsen and Preneel studied the schemes to construct secure compression functions with longer outputs from secure ones based on error-correcting codes [14, 15, 16]. It is an open question whether optimally collision-resistant compression functions are constructed by their schemes. A hash/compression function is optimally collision-resistant if any attack to find its collision is at most as efficient as the birthday attack.

Our work is largely motivated by the recent works by Lucks [19] and Nandi [23]. Nandi generalized the results by Lucks and by Hirose [10]. He discussed how to construct DBL hash functions and presented optimally collision-resistant ones. However, their security analysis is not so sharp as ours, which is mentioned in Sect. 1.

Coron, Dodis, Malinaud and Puniya [5] discussed how to construct a random oracle with arbitrary input length given a random oracle with fixed input length.

As is reviewed in the following, there are many papers on hash functions composed of block ciphers.

Preneel, Govaerts and Vandewalle [26] discussed the security of SBL hash functions against several generic attacks. They considered SBL hash functions with compression functions represented by $h_i = e(k, x) \oplus z$, where e is an (n, n) block cipher, $k, x, z \in \{h_{i-1}, m_i, h_{i-1} \oplus m_i, c\}$ and c is a constant. They concluded that 12 out of $64 (= 4^3)$ hash functions are secure against the attacks. However, they did not provide any formal proofs.

Black, Rogaway and Shrimpton [3] presented a detailed investigation of provable security of SBL hash functions given in [26] in the ideal cipher model. The most important result in their paper is that 20 hash functions including the 12 mentioned above is optimally collision-resistant.

Knudsen, Lai and Preneel [17] discussed the insecurity of DBL hash functions with the rate 1 composed of (n, n) block ciphers. Hohl, Lai, Meier and Waldvogel [12] discussed the security of compression functions of DBL hash functions with the rate 1/2. On the other hand, the security of DBL hash functions with the rate 1 composed of $(n, 2n)$ block ciphers was discussed by Satoh, Haga and Kurosawa [27] and by Hattori, Hirose and Yoshida [8]. These works presented no construction for DBL hash functions with optimal collision resistance.

Many schemes with the rates less than 1 were also presented. Merkle [21] presented three DBL hash functions composed of DES with the rates at most 0.276. They are optimally collision-resistant in the ideal cipher model. MDC-2 and MDC-4 [4] are also DBL hash functions composed of DES with the rates 1/2 and 1/4, respectively. Lai and Massey proposed the tandem/abreast Davies-Meyer [18]. They consist of an $(n, 2n)$ block cipher and their rates are 1/2. It is an open question whether the four schemes are optimally collision-resistant or not.

Hirose [10] presented a large class of DBL hash functions with the rate 1/2, which are composed of $(n, 2n)$ block ciphers. They were shown to be optimally collision-resistant in the ideal cipher model. However, his construction requires two independent block ciphers, which makes the results less attractive.

Nandi, Lee, Sakurai and Lee [24] also proposed an interesting construction with the rate 2/3. However,

they are not optimally collision-resistant. Knudsen and Muller [13] presented some attacks against it and illustrated its weaknesses, none of which contradicts the security proof in [24].

Black, Cochran and Shrimpton [2] showed that it is impossible to construct a highly efficient block-cipher-based hash function provably secure in the ideal cipher model. A block-cipher-based hash function is highly efficient if it makes exactly one block-cipher call for each message block and all block-cipher calls use a single key.

Gauravaram, Millan and May proposed a new approach based on iterated halving technique to design rate-1 hash functions that can be instantiated with any secure 128-bit block cipher reduced to half the number of rounds [7].

3 DBL Hash Function in the Random Oracle Model

3.1 Compression Function

In this section, we consider the DBL hash functions with compression functions given in the following definition.

Definition 1 Let $F : \{0,1\}^{2n} \times \{0,1\}^b \to \{0,1\}^{2n}$ be a compression function such that $(g_i, h_i) = F(g_{i-1}, h_{i-1}, m_i)$, where $g_i, h_i \in \{0,1\}^n$ and $m_i \in \{0,1\}^b$. F consists of $f : \{0,1\}^{2n} \times \{0,1\}^b \to \{0,1\}^n$ and a permutation $p : \{0,1\}^{2n+b} \to \{0,1\}^{2n+b}$ as follows:

$$\begin{cases} g_i = F_{\mathrm{U}}(g_{i-1}, h_{i-1}, m_i) = f(g_{i-1}, h_{i-1}, m_i) \\ h_i = F_{\mathrm{L}}(g_{i-1}, h_{i-1}, m_i) = f(p(g_{i-1}, h_{i-1}, m_i)) \end{cases} .$$

p satisfies the following properties:

- It is easy to compute both p and p^{-1},
- $p(p(\cdot))$ is an identity permutation, and
- p has no fixed points, that is, $p(g_{i-1}, h_{i-1}, m_i) \neq (g_{i-1}, h_{i-1}, m_i)$ for any (g_{i-1}, h_{i-1}, m_i).

3.2 Collision Resistance

We will analyze the collision resistance of DBL hash functions composed of F under the assumption that f is a random oracle.

Two queries to the oracle f are required to compute the output of F for an input. For this compression function, a query to f for F_{U} or F_{L} uniquely determines the query to f for the other since p is a permutation. Moreover, for every $w \in \{0,1\}^{2n+b}$, $f(w)$ and $f(p(w))$ are only used to compute $F(w)$ and $F(p(w))$, and $w \neq p(w)$ from the properties of p in Definition 1. Thus, it is reasonable to assume that a pair of queries w and $p(w)$ to f are asked at a time.

Definition 2 A pair of distinct inputs w, w' to F are called a matching pair if $w' = p(w)$. Otherwise, they are called a non-matching pair.

Notice that $w' = p(w)$ iff $w = p(w')$ since $p(p(\cdot))$ is an identity permutation.

3.2.1 Definition

Insecurity is quantified by success probability of an optimal resource-bounded adversary. The resource is the number of the queries to f in the random oracle model.

For a set S, let $z \xleftarrow{\mathrm{r}} S$ represent random sampling from S under the uniform distribution. For a probabilistic algorithm $\mathcal{M}$, let $z \xleftarrow{\mathrm{r}} \mathcal{M}$ mean that z is an output of $\mathcal{M}$ and its distribution is based on the random choices of $\mathcal{M}$.

Let H be a DBL hash function composed of a compression function F in Definition 1. The following experiment $\mathtt{FindColHF}(\mathcal{A}, H)$ is introduced to quantify the collision resistance of H. The adversary $\mathcal{A}$ with the oracle f is a collision-finding algorithm of H.

$\mathtt{FindColHF}(\mathcal{A}, H)$
  $f \xleftarrow{\mathrm{r}} \boldsymbol{F}_{2n+b,n}$;
  $(m, m') \xleftarrow{\mathrm{r}} \mathcal{A}^f$;
  `if` $m \neq m' \wedge H(m) = H(m')$ `return` 1;
  `else return` 0;

$\mathtt{FindColHF}(\mathcal{A}, H)$ returns 1 iff $\mathcal{A}$ finds a collision. Let $\mathbf{Adv}^{\mathrm{coll}}_H(\mathcal{A})$ be the probability that $\mathtt{FindColHF}(\mathcal{A}, H)$ returns 1. The probability is taken over the uniform distribution on $\boldsymbol{F}_{2n+b,n}$ and random choices of $\mathcal{A}$.

Definition 3 For $q \geq 1$, let

$$\mathbf{Adv}^{\mathrm{coll}}_H(q) = \max_{\mathcal{A}} \left\{ \mathbf{Adv}^{\mathrm{coll}}_H(\mathcal{A}) \right\} ,$$

where $\mathcal{A}$ makes at most q pairs of queries to f in total.

Without loss of generality, it is assumed that $\mathcal{A}$ does not ask the same query twice. $\mathcal{A}$ can keep pairs of queries and their corresponding answers by himself.

3.2.2 Analysis

In this section, we show the collision resistance of hash functions composed of F in Definition 1. We first present some lemmas which are used to prove the collision resistance.

Lemma 1 *Let H be a hash function composed of a compression function F specified in Definition 1 and an initial value (g_0, h_0). Let $\mathcal{A}$ be a collision-finding algorithm for H with the oracle f. $\mathcal{A}$ asks q pairs of queries to f in total. Then, there exists an algorithm $\mathcal{B}$ with the oracle f which succeeds in finding*

1. *a colliding pair of non-matching inputs for F,*
2. *a colliding pair of matching inputs for F, or*
3. *a preimage of (g_0, h_0) for F*

with the probability $\mathbf{Adv}_H^{\mathrm{coll}}(\mathcal{A})$. $\mathcal{B}$ asks q pairs of queries to f in total.

Proof. $\mathcal{B}$ first runs $\mathcal{A}$. Suppose that $\mathcal{A}$ finds a colliding pair m, m' for H. Then, it is easy to see that $\mathcal{B}$ finds a colliding pair of inputs for F or a preimage of (g_0, h_0) for F by tracking the computation of $H(m)$ and $H(m')$ backwards. The colliding pair is either matching or non-matching. During the process, $\mathcal{B}$ needs no other queries than those made by $\mathcal{A}$. □

The following three lemmas give upper bounds of the success probabilities of the events listed in Lemma 1.

Lemma 2 *Let F be a compression function specified in Definition 1. Let $\mathcal{B}_{\mathrm{c}}$ be an optimal algorithm to find a colliding pair of non-matching inputs for F. Suppose that $\mathcal{B}_{\mathrm{c}}$ asks q pairs of queries to f in total. Then, the success probability of $\mathcal{B}_{\mathrm{c}}$ is at most $q(q-1)/2^{2n}$.*

Proof. For $1 \le j \le q$, let w_j and $p(w_j)$ be the j-th pair of queries made by $\mathcal{B}_{\mathrm{c}}$. For $2 \le j \le q$, let C_j be the event that $\mathcal{B}_{\mathrm{c}}$ finds a colliding pair of non-matching inputs for F with the j-th pair of queries. Namely, it is the event that

$$(f(w_j), f(p(w_j))) = (f(w_{j'}), f(p(w_{j'}))) \text{ or } (f(p(w_{j'})), f(w_{j'}))$$

for some $j' < j$. Since both $f(w_j)$ and $f(p(w_j))$ are randomly selected by the oracle,

$$\Pr[\mathsf{C}_j] \le \frac{2(j-1)}{2^{2n}} \ .$$

Let C be the event that $\mathcal{B}_{\mathrm{c}}$ finds a colliding pair of non-matching inputs. Then,

$$\Pr[\mathsf{C}] = \Pr[\mathsf{C}_2 \vee \mathsf{C}_3 \vee \cdots \vee \mathsf{C}_q] \le \sum_{j=2}^{q} \Pr[\mathsf{C}_j] \le \frac{q\,(q-1)}{2^{2n}} \ .$$

□

Lemma 3 *Let F be a compression function specified in Definition 1. Let $\mathcal{B}'_{\mathrm{c}}$ be an optimal algorithm to find a colliding pair of matching inputs for F. Suppose that $\mathcal{B}'_{\mathrm{c}}$ asks q pairs of queries to f in total. Then, the success probability of $\mathcal{B}'_{\mathrm{c}}$ is at most $q/2^n$.*

Proof. For $1 \le j \le q$, let $\mathsf{C}^{\mathrm{m}}_j$ be the event that $\mathcal{B}'_{\mathrm{c}}$ finds a colliding pair of matching inputs for F with the j-th pair of queries, that is, $f(w_j) = f(p(w_j))$. Thus,

$$\Pr[\mathsf{C}^{\mathrm{m}}_j] = \frac{1}{2^n} \ .$$

Let C^{m} be the event that $\mathcal{B}'_{\mathrm{c}}$ finds a colliding pair of matching inputs for F. Then,

$$\Pr[\mathsf{C}^{\mathrm{m}}] = \Pr[\mathsf{C}^{\mathrm{m}}_1 \vee \mathsf{C}^{\mathrm{m}}_2 \vee \cdots \vee \mathsf{C}^{\mathrm{m}}_q] \le \sum_{j=1}^{q} \Pr[\mathsf{C}^{\mathrm{m}}_j] = \frac{q}{2^n} \ .$$

□

Lemma 4 *Let F be a compression function specified in Definition 1. Let $\mathcal{B}_{\mathrm{p}}$ be an optimal algorithm to find a preimage of a given output (g, h) for F, where $g, h \in \{0,1\}^n$. Suppose that $\mathcal{B}_{\mathrm{p}}$ asks q pairs of queries to f in total. Then, the success probability of $\mathcal{B}_{\mathrm{p}}$ is at most $2q/2^{2n}$.*

Proof. For $1 \le j \le q$, let P_j be the event that $\mathcal{B}_{\rm p}$ finds a preimage of (g,h) for F with the j-th pair of queries. Namely, it is the event that $(f(w_j), f(p(w_j))) = (g,h)$ or (h,g). Thus,

$$\Pr[\mathsf{P}_j] \le \frac{2}{2^{2n}} \ .$$

Let P be the event that $\mathcal{B}_{\rm p}$ finds a preimage of (g,h) for F. Then,

$$\Pr[\mathsf{P}] = \Pr[\mathsf{P}_1 \vee \mathsf{P}_2 \vee \cdots \vee \mathsf{P}_q] \le \sum_{j=1}^{q} \Pr[\mathsf{P}_j] \le \frac{2\,q}{2^{2n}} \ .$$

□

The following theorem is obvious from the above lemmas.

Theorem 1 *Let H be a hash function composed of a compression function F specified in Definition 1 and an initial value (g_0, h_0). Then,*

$$\mathbf{Adv}^{\rm coll}_{H}(q) \le \frac{q\,(q+1)}{2^{2n}} + \frac{q}{2^n} \ .$$

Proof. For Lemma 1, suppose that $\mathcal{A}$ is an optimal collision-finding algorithm for H. Then, from Lemmas 2, 3, and 4,

$$\begin{aligned}\mathbf{Adv}^{\rm coll}_{H}(\mathcal{A}) &= \mathbf{Adv}^{\rm coll}_{H}(q) \\ &\le \frac{q\,(q-1)}{2^{2n}} + \frac{q}{2^n} + \frac{2\,q}{2^{2n}} \\ &\le \frac{q\,(q+1)}{2^{2n}} + \frac{q}{2^n} \ .\end{aligned}$$

□

Theorem 1 is valid as long as its upper bound is less than 1.

From Theorem 1, any constant probability of success in finding a collision for H requires $\Omega(2^n)$ queries. The upper bound of Theorem 1 is optimal up to a constant factor. However, we can go further. A better bound can be obtained with more restricted permutations given below.

Definition 4 Let F be a compression function specified in Definition 1. Moreover, the permutation p is represented by $p(g,h,m) = (p_{\rm cv}(g,h), p_{\rm m}(m))$, where $p_{\rm cv} : \{0,1\}^{2n} \to \{0,1\}^{2n}$ and $p_{\rm m} : \{0,1\}^{b} \to \{0,1\}^{b}$. $p_{\rm cv}$ has no fixed points and $p_{\rm cv}(g,h) \ne (h,g)$ for any (g,h).

Example 1 Here is an example of the permutation p satisfying the conditions given in Definition 4:

$$p(g,h,m) = (g \oplus c_1, h \oplus c_2, m \oplus c_3) \ ,$$

where c_1, c_2 and c_3 are constants in $\{0,1\}^n$, and $c_1 \ne c_2$.

We first present some lemmas similar to those given above.

Lemma 5 *Let H be a hash function composed of a compression function F specified in Definition 4 and an initial value (g_0, h_0). Let $\mathcal{A}$ be a collision-finding algorithm for H with the oracle f. $\mathcal{A}$ asks q pairs of queries to f in total. Then, there exists an algorithm $\mathcal{B}$ with the oracle f which succeeds in finding*

1. *a colliding pair of non-matching inputs for F,*
2. *a pair of non-matching inputs w and w' for F such that $F(w) = p_{\rm cv}(F(w'))$,*
3. *a preimage of (g_0, h_0) for F, or*
4. *a preimage of $p_{\rm cv}(g_0, h_0)$ for F*

with the probability $\mathbf{Adv}^{\rm coll}_{H}(\mathcal{A})$. $\mathcal{B}$ asks q pairs of queries to f in total.

Proof. $\mathcal{B}$ first runs $\mathcal{A}$. Suppose that $\mathcal{A}$ finds a colliding pair m, m' for H. Then, it is easy to see that $\mathcal{B}$ finds a colliding pair of inputs for F or a preimage of (g_0, h_0) for F by tracking the computation of $H(m)$ and $H(m')$ backwards. The colliding pair is either matching or non-matching. During the process, $\mathcal{B}$ needs no other queries than those made by $\mathcal{A}$.

Suppose that a colliding pair of matching inputs are obtained for F from the collision of H found by $\mathcal{A}$. Let (g,h,m) and (g',h',m') be the colliding pair. Then, $(g,h) = p_{\rm cv}(g',h')$ (and $(g',h') = p_{\rm cv}(g,h)$). (g,h) and (g',h') are both outputs of F, or at most one of them is the initial value (g_0,h_0) of H since $(g,h) \ne (g',h')$. Thus, a pair of inputs w and w' are also found for F from the collision of H such that $F(w) = p_{\rm cv}(F(w'))$ or $F(w) = p_{\rm cv}(g_0,h_0)$.

Suppose that $(g,h) = F(w)$ and $(g',h') = F(w')$. Then, a pair of w and w' are non-matching since $(g,h) = p_{\rm cv}(g',h') \ne (h',g')$. □

As in the proof of Theorem 1, the following lemmas give upper bounds of the success probabilities of the events listed in Lemma 5.

Lemma 6 *Let F be a compression function specified in Definition 4. Let $\mathcal{B}_{\rm c}$ be an optimal algorithm to find a colliding pair of non-matching inputs for F. Suppose that $\mathcal{B}_{\rm c}$ asks q pairs of queries to f in total. Then, the success probability of $\mathcal{B}_{\rm c}$ is at most $q(q-1)/2^{2n}$.*

Proof. Omitted. It is similar to that of Lemma 2. □

Lemma 7 *Let F be a compression function specified in Definition 4. Let $\mathcal{B}'_{\rm c}$ be an optimal algorithm to find a pair of non-matching inputs w and w' for F such that $F(w) = p_{\rm cv}(F(w'))$. Suppose that $\mathcal{B}'_{\rm c}$ asks q pairs of queries to f in total. Then, the success probability of $\mathcal{B}'_{\rm c}$ is at most $2q(q-1)/2^{2n}$.*

Proof. For $2 \leq j \leq q$, let C'_j be the event that $\mathcal{B}'_{\rm c}$ finds a pair of non-matching inputs w and w' such that $F(w) = p_{\rm cv}(F(w'))$ with the j-th pair of queries w_j and $p(w_j)$. Namely, it is the event that

$$F(w_j) = p_{\rm cv}(F(w_{j'})) \text{ or } p_{\rm cv}(F(p(w_{j'})))$$

or

$$F(p(w_j)) = p_{\rm cv}(F(w_{j'})) \text{ or } p_{\rm cv}(F(p(w_{j'})))$$

for some $j' < j$. Thus,

$$\Pr[\mathsf{C}'_j] \leq \frac{4(j-1)}{2^{2n}} \ .$$

Let C' be the event that $\mathcal{B}'_{\rm c}$ finds a pair of non-matching inputs w and w' such that $F(w) = p_{\rm cv}(F(w'))$. Then,

$$\Pr[\mathsf{C}'] \leq \sum_{j=2}^{q} \Pr[\mathsf{C}'_j] \leq \sum_{j=2}^{q} \frac{4(j-1)}{2^{2n}} = \frac{2q(q-1)}{2^{2n}} \ .$$

□

Lemma 8 *Let F be a compression function specified in Definition 4. Let $\mathcal{B}_{\rm p}$ be an optimal algorithm to find a preimage of a given output (g, h) for F, where $g, h \in \{0,1\}^n$. Suppose that $\mathcal{B}_{\rm p}$ asks q pairs of queries to f in total. Then, the success probability of $\mathcal{B}_{\rm p}$ is at most $2q/2^{2n}$.*

Proof. Omitted. It is similar to that of Lemma 4. □

Theorem 2 *Let H be a hash function composed of a compression function F specified in Definition 4 and an initial value (g_0, h_0). Then,*

$$\mathbf{Adv}^{\rm coll}_H(q) \leq \frac{3\,q^2+q}{2^{2n}} \leq \left(\frac{q}{2^{n-1}}\right)^2 \ .$$

Proof. For Lemma 5, suppose that $\mathcal{A}$ is an optimal collision-finding algorithm for H which asks q pairs of queries to f in total. Then, from Lemmas 6, 7 and 8,

$$\begin{aligned} \mathbf{Adv}^{\rm coll}_H(q) &= \mathbf{Adv}^{\rm coll}_H(\mathcal{A}) \\ &\leq \frac{q\,(q-1)}{2^{2n}} + \frac{2\,q\,(q-1)}{2^{2n}} + \frac{2\,q}{2^{2n}} + \frac{2\,q}{2^{2n}} \\ &\leq \frac{3\,q^2+q}{2^{2n}} \leq \left(\frac{q}{2^{n-1}}\right)^2 \ . \end{aligned}$$

□

Theorem 2 is also valid as long as its upper bound is less than 1.

Remark 1 For $q < 2^{n-1}$, Theorem 2 gives a smaller upper bound than Theorem 1. Their difference is significant. The upper bound of Theorem 2 is at most $(q/2^{n-1})^2$. On the other hand, the upper bound of Theorem 1 is about $q/2^n$ if $q/2^n \ll 1$. For example, let $n = 128$ and $q = 2^{80}$. Then, the upper bound of Theorem 1 is about 2^{-48}, while the upper bound of Theorem 2 is less than 2^{-94}.

Remark 2 Contrasting Lemma 1 and Lemma 5, it is easy to see that the upper bound of Theorem 2 is obtained based not solely on the security of the compression function but on its iteration.

Remark 3 Suppose that we use the Merkle-Damgård strengthening for padding. Then, in the proofs of Theorems 1 and 2, we need not consider the event that a preimage of the initial value of the hash function is found for the compression function. However, the probability of this event is negligible compared to that of collision for the compression function.

3.3 Indistinguishability in the Iteration

We introduce a new security notion which is called indistinguishability in the iteration.

3.3.1 Definition

Let F be a compression function specified in Definition 1. The following experiment $\texttt{DistinguishCF}(\mathcal{A}, F)$ is introduced to quantify the indistinguishability in the iteration of F. The adversary $\mathcal{A}$ is a distinguishing algorithm of F. $\mathcal{A}$ has an oracle $\mathcal{O}$. In this experiment, a randomly chosen bit $d \in \{0,1\}$ is given to $\mathcal{O}$ first. If $d = 1$, then $\mathcal{O}$ chooses $f \in \boldsymbol{F}_{2n+b,n}$ randomly in advance. Then, $\mathcal{O}$ returns $F(w) = (f(w), f(p(w)))$ to each query w from $\mathcal{A}$. If $d = 0$, then $\mathcal{O}$ chooses $R \in \boldsymbol{F}_{2n+b,2n}$ randomly in advance. Then, $\mathcal{O}$ returns $R(w)$ to each query w from $\mathcal{A}$. $\mathcal{A}$ makes a chosen message attack and tries to tell whether $\mathcal{O}$ uses F or R. However, $\mathcal{A}$ is only allowed to select his j-th query $w_j = (w_j^{(1)}, w_j^{(2)}, w_j^{(3)})$ from the set of $(w^{(1)}, w^{(2)}, w^{(3)})$'s such that

$$(w^{(1)}, w^{(2)}) \in \bigcup_{\ell=0}^{j-1} (v_\ell^{(1)}, v_\ell^{(2)}) \wedge w^{(3)} \in \{0,1\}^b \ ,$$

where $(v_\ell^{(1)}, v_\ell^{(2)})$ is $\mathcal{O}$'s answer to the ℓ-th query for $\ell \geq 1$ and $(v_0^{(1)}, v_0^{(2)})$ is some fixed initial value of a hash function H. Thus, F is assumed to be used only in the iteration of H.

$\texttt{DistinguishCF}(\mathcal{A}, F)$
$d \xleftarrow{\mathrm{r}} \{0,1\}$;
$d' \xleftarrow{\mathrm{r}} \mathcal{A}^{\mathcal{O}(d)}$;
`if` $d = d'$ `return 1; else return 0;`

Let $\mathbf{Succ}_F^{\text{ind-it}}(\mathcal{A})$ be the probability that $\texttt{DistinguishCF}(\mathcal{A}, F)$ returns 1. Without loss of generality, it can be assumed that $\mathbf{Succ}_F^{\text{ind-it}}(\mathcal{A}) \geq 1/2$ because the probability that $d = d'$ is $1/2$ even if $\mathcal{A}$ chooses d' randomly. It can also be assumed that $\mathcal{A}$ does not ask the same query twice. Let

$$\mathbf{Adv}_F^{\text{ind-it}}(\mathcal{A}) \stackrel{\text{def}}{=} \mathbf{Succ}_F^{\text{ind-it}}(\mathcal{A}) - 1/2 \ .$$

Definition 5 For $q \geq 1$, let

$$\mathbf{Adv}_F^{\text{ind-it}}(q) = \max_{\mathcal{A}} \left\{ \mathbf{Adv}_F^{\text{ind-it}}(\mathcal{A}) \right\} \ ,$$

where $\mathcal{A}$ makes at most q queries to $\mathcal{O}$.

As long as $\mathbf{Adv}_F^{\text{ind-it}}(q)$ is small enough, the compression function F behaves like a random function in the iterated hash function. The following theorem presents an upper bound on $\mathbf{Adv}_F^{\text{ind-it}}(q)$ with additional restriction on the permutation p.

Theorem 3 *Let F be a compression function specified in Definition 1. Suppose that the permutation p is represented by $p(g,h,m) = (p_{\text{cv}}(g,h), p_{\text{m}}(m))$, where $p_{\text{cv}} : \{0,1\}^{2n} \to \{0,1\}^{2n}$ and $p_{\text{m}} : \{0,1\}^b \to \{0,1\}^b$. Suppose that p_{cv} has no fixed points. Then,*

$$\mathbf{Adv}_F^{\text{ind-it}}(q) \leq \frac{1}{2} \left(\frac{q}{2^n} \right)^2 \ .$$

Proof. Let $\mathcal{A}$ be an optimal distinguishing algorithm for F which makes q queries to $\mathcal{O}$. Let w_j be the j-th query by $\mathcal{A}$ and $T = \{w_j \,|\, 1 \leq j \leq q\} \cap \{p(w_j) \,|\, 1 \leq j \leq q\}$.

Suppose that $d = 1$. Then, $\mathcal{O}$ returns $F(w_j) = (f(w_j), f(p(w_j)))$ for w_j. If $T = \phi$, then F is completely indistinguishable from R. It is because each one of $f(w_j)$ and $f(p(w_j))$ for $1 \leq j \leq q$ appears only once and it is chosen randomly by $\mathcal{O}$.

Let Empty be the event that $T = \phi$. Then,

$$\begin{aligned}
&\mathbf{Succ}_F^{\text{ind-it}}(\mathcal{A}) \\
&= \Pr[d = d'] \\
&= \Pr[d = d' \wedge \mathsf{Empty}] + \Pr[d = d' \wedge \neg\mathsf{Empty}] \\
&= \Pr[d = d' \,|\, \mathsf{Empty}] \Pr[\mathsf{Empty}] + \\
&\qquad \Pr[d = d' \,|\, \neg\mathsf{Empty}] \Pr[\neg\mathsf{Empty}] \\
&\leq \frac{1}{2} + \Pr[\neg\mathsf{Empty}] \ .
\end{aligned}$$

Let v_j be the initial value if $j = 0$ and the answer of $\mathcal{O}$ to the j-th query by $\mathcal{A}$ if $j \geq 1$. For $1 \leq j \leq q$, let C'_j be the event that $v_j \in \{p_{\text{cv}}(v_\ell) \,|\, 0 \leq \ell \leq j-1\}$. Then,

$$\Pr[\mathsf{C}'_j] \leq \frac{j}{2^{2n}} \ .$$

Thus,

$$\begin{aligned}
\Pr[\neg\mathsf{Empty}] &\leq \Pr[\mathsf{C}'_1 \vee \cdots \vee \mathsf{C}'_{q-1}] \\
&\leq \sum_{j=1}^{q-1} \Pr[\mathsf{C}'_j] \leq \frac{1}{2} \left(\frac{q}{2^n} \right)^2
\end{aligned}$$

which implies that $\mathbf{Adv}_F^{\text{ind-it}}(q) \leq (q/2^n)^2/2$. □

Theorem 3 is valid as long as its upper bound is less than 1.

4 DBL Hash Function in the Ideal Cipher Model

4.1 Compression Function

In this section, the collision resistance of a DBL hash function composed of a compression function using a block cipher is analyzed. The compression function specified in the following definition is considered.

Definition 6 Let $F : \{0,1\}^{2n} \times \{0,1\}^b \to \{0,1\}^{2n}$ be a compression function such that $(g_i, h_i) = F(g_{i-1}, h_{i-1}, m_i)$, where $g_i, h_i \in \{0,1\}^n$ and $m_i \in \{0,1\}^b$. F consists of an $(n, n+b)$ block cipher e as follows:

$$\begin{aligned} g_i &= F_{\mathrm{U}}(g_{i-1}, h_{i-1}, m_i) \\ &= e(h_{i-1} \| m_i, g_{i-1}) \oplus g_{i-1} \\ h_i &= F_{\mathrm{L}}(g_{i-1}, h_{i-1}, m_i) \\ &= e(h_{i-1} \| m_i, g_{i-1} \oplus c) \oplus g_{i-1} \oplus c \ , \end{aligned}$$

where $\|$ represents concatenation and $c \in \{0,1\}^n - \{0^n\}$ is a constant.

The compression function in Definition 6 is also shown in Fig. 1. It can be regarded as a variant of the compression function specified in Definition 4, where f and p are specified as follows:

$$\begin{aligned} f(g_{i-1}, h_{i-1}, m_i) &= e(h_{i-1} \| m_i, g_{i-1}) \oplus g_{i-1} \ , \\ p(g_{i-1}, h_{i-1}, m_i) &= (g_{i-1} \oplus c, h_{i-1}, m_i) \ . \end{aligned}$$

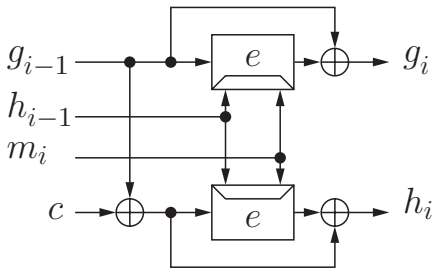

Figure 1: The compression function in Definition 6

F requires two invocations of e to produce an output. However, these two invocations need only one key scheduling of e. If F is implemented using the AES with 192-bit key-length, then $n = 128$, $b = 64$ and the rate is 1/4. If implemented using the AES with 256-bit key-length, then $n = b = 128$ and the rate is 1/2.

4.2 Collision Resistance

Let F be a compression function specified in Definition 6. Two queries to the oracles e and e^{-1} in total are required to compute the output of F for an input. It is apparent from Fig. 1 that a query to e or e^{-1} and the corresponding reply for F_{U} (F_{L}) uniquely determine the query to e for F_{L} (F_{U}). Moreover, these two queries are only used to compute the outputs of F for a matching pair of inputs. Thus, it is assumed that a pair of queries to e, e^{-1} required to compute an output of F are asked at a time.

4.2.1 Definition

The following experiment $\mathtt{FindColHF}(\mathcal{A}, H)$ is similar to the one in Sect. 3 except that the adversary $\mathcal{A}$ is a collision-finding algorithm with the oracles e, e^{-1}.

$\mathtt{FindColHF}(\mathcal{A}, H)$
  $e \xleftarrow{\mathrm{r}} \boldsymbol{B}_{n,n+b}$;
  $(m, m') \xleftarrow{\mathrm{r}} \mathcal{A}^{e,e^{-1}}$;
  `if` $m \neq m' \wedge H(m) = H(m')$ `return` 1;
  `else return` 0;

Let $\mathbf{Adv}_H^{\mathrm{coll}}(\mathcal{A})$ be the probability that $\mathtt{FindColHF}(\mathcal{A}, H)$ returns 1. The probability is taken over the uniform distribution on $\boldsymbol{B}_{n,n+b}$ and random choices of $\mathcal{A}$.

Definition 7 For $q \geq 1$, let

$$\mathbf{Adv}_H^{\mathrm{coll}}(q) = \max_{\mathcal{A}} \left\{ \mathbf{Adv}_H^{\mathrm{coll}}(\mathcal{A}) \right\} \ ,$$

where $\mathcal{A}$ makes at most q pairs of queries to e, e^{-1} in total.

Without loss of generality, it is assumed that $\mathcal{A}$ asks at most only once on a triplet of a key, a plaintext and a ciphertext obtained by a query and the corresponding reply.

4.2.2 Analysis

The following theorem shows the collision resistance of a hash function composed of F in Definition 6.

Theorem 4 *Let H be a hash function composed of the compression function F specified in Definition 6 and an initial value (g_0, h_0). Then, for every $1 \le q \le 2^{n-2}$,*

$$\mathbf{Adv}_H^{\mathrm{coll}}(q) \le \frac{3\,q^2+q}{2^{2(n-1)}} \le \left(\frac{q}{2^{n-2}}\right)^2 \quad .$$

Proof. Let $\mathcal{A}$ be a collision-finding algorithm of H which asks q pairs of queries to e, e^{-1} in total. Then, there exists an algorithm $\mathcal{B}$ with the oracles e, e^{-1} which succeeds in finding

1. a colliding pair of non-matching inputs for F,
2. a pair of non-matching inputs w and w' for F such that $F(w) = (F_{\mathrm{U}}(w') \oplus c, F_{\mathrm{L}}(w'))$,
3. a preimage of (g_0, h_0) for F, or
4. a preimage of $(g_0 \oplus c, h_0)$ for F

with the probability $\mathbf{Adv}_H^{\mathrm{coll}}(\mathcal{A})$. $\mathcal{B}$ asks q pairs of queries to e, e^{-1} in total.

Since $g_i = e(h_{i-1} \| m_i, g_{i-1}) \oplus g_{i-1}$, g_i depends both on the plaintext and the ciphertext of e. Either the plaintext or the ciphertext is fixed by a query and the other is determined randomly by the answer from the oracle. Thus, g_i is randomly determined by the answer. h_i is also randomly determined by the other answer.

Let $(k_j^1 \| k_j^2, x_j, y_j)$ and $(k_j^1 \| k_j^2, x_j \oplus c, z_j)$ represent the triplets of e obtained by the j-th pair of queries and the corresponding answers.

For (1): Let $\mathcal{B}_{\mathrm{c}}$ be an optimal algorithm to find a colliding pair of non-matching inputs for F. Suppose that $\mathcal{B}_{\mathrm{c}}$ asks q pairs of queries to e, e^{-1} in total.

For every $2 \le j \le q$, let C_j be the event that $\mathcal{B}_{\mathrm{c}}$ finds a colliding pair of non-matching inputs for F with the j-th pair of queries. Namely, it is the event that, for some $j' < j$,

$$\begin{aligned} &F(x_j, k_j^1, k_j^2) = \\ &\quad F(x_{j'}, k_{j'}^1, k_{j'}^2) \text{ or } F(x_{j'} \oplus c, k_{j'}^1, k_{j'}^2) \end{aligned}$$

or

$$\begin{aligned} &F(x_j \oplus c, k_j^1, k_j^2) = \\ &\quad F(x_{j'}, k_{j'}^1, k_{j'}^2) \text{ or } F(x_{j'} \oplus c, k_{j'}^1, k_{j'}^2) \ , \end{aligned}$$

which is equivalent to

$$\begin{aligned} &(y_j \oplus x_j, z_j \oplus x_j \oplus c) = \\ &\qquad (y_{j'} \oplus x_{j'}, z_{j'} \oplus x_{j'} \oplus c) \text{ or} \\ &\qquad (z_{j'} \oplus x_{j'} \oplus c, y_{j'} \oplus x_{j'}) \ . \end{aligned}$$

Thus,

$$\begin{aligned} \Pr[\mathsf{C}_j] &\le \frac{2(j-1)}{(2^n-(2j-2))(2^n-(2j-1))} \\ &\le \frac{2(j-1)}{(2^n-(2j-1))^2} \ . \end{aligned}$$

Let C be the event that $\mathcal{B}_{\mathrm{c}}$ finds a colliding pair of non-matching inputs for F. Then, for $1 \le q \le 2^{n-2}$,

$$\begin{aligned} \Pr[\mathsf{C}] &\le \sum_{j=2}^{q} \Pr[\mathsf{C}_j] \le \sum_{j=2}^{q} \frac{2(j-1)}{(2^n-(2j-1))^2} \\ &\le \sum_{j=2}^{q} \frac{2(j-1)}{2^{2(n-1)}} \le \frac{q(q-1)}{2^{2(n-1)}} \ . \end{aligned}$$

For (2): Let $\mathcal{B}'_{\mathrm{c}}$ be an optimal algorithm to find a pair of non-matching inputs w and w' for F such that $F(w) = (F_{\mathrm{U}}(w') \oplus c, F_{\mathrm{L}}(w'))$. Suppose that $\mathcal{B}'_{\mathrm{c}}$ asks q pairs of queries to e, e^{-1} in total.

Let C'_j be the event that $\mathcal{B}'_{\mathrm{c}}$ finds a pair of non-matching inputs for F such as given above with the j-th pair of queries. Namely, it is the event that, for some $j' < j$,

$$\begin{aligned} &F(x_j, k_j^1, k_j^2) = \\ &\quad (F_{\mathrm{U}}(x_{j'}, k_{j'}^1, k_{j'}^2) \oplus c, F_{\mathrm{L}}(x_{j'}, k_{j'}^1, k_{j'}^2)) \text{ or} \\ &\quad (F_{\mathrm{U}}(x_{j'} \oplus c, k_{j'}^1, k_{j'}^2) \oplus c, F_{\mathrm{L}}(x_{j'} \oplus c, k_{j'}^1, k_{j'}^2)) \ , \end{aligned}$$

or

$$\begin{aligned} &F(x_j \oplus c, k_j^1, k_j^2) = \\ &\quad (F_{\mathrm{U}}(x_{j'}, k_{j'}^1, k_{j'}^2) \oplus c, F_{\mathrm{L}}(x_{j'}, k_{j'}^1, k_{j'}^2)) \text{ or} \\ &\quad (F_{\mathrm{U}}(x_{j'} \oplus c, k_{j'}^1, k_{j'}^2) \oplus c, F_{\mathrm{L}}(x_{j'} \oplus c, k_{j'}^1, k_{j'}^2)) \ . \end{aligned}$$

It is equivalent to

$$\begin{aligned} &(y_j \oplus x_j, z_j \oplus x_j \oplus c) = \\ &\qquad (y_{j'} \oplus x_{j'} \oplus c, z_{j'} \oplus x_{j'} \oplus c) \ , \\ &\qquad (z_{j'} \oplus x_{j'}, y_{j'} \oplus x_{j'}) \ , \\ &\qquad (z_{j'} \oplus x_{j'} \oplus c, y_{j'} \oplus x_{j'} \oplus c) \text{ or} \\ &\qquad (y_{j'} \oplus x_{j'}, z_{j'} \oplus x_{j'}) \ . \end{aligned}$$

Thus,

$$\Pr[\mathsf{C}'_j] \leq \frac{4\,(j-1)}{(2^n-(2j-1))^2} \ .$$

Let C' be the event that $\mathcal{B}'_{\rm c}$ finds a pair of non-matching inputs w and w' for F such that $F(w) = (F_{\rm U}(w') \oplus c, F_{\rm L}(w'))$. Then, for $1 \leq q \leq 2^{n-2}$,

$$\Pr[\mathsf{C}'] \leq \sum_{j=1}^{q} \Pr[\mathsf{C}'_j] \leq \frac{2\,q\,(q-1)}{2^{2(n-1)}} \ .$$

For (3): Let $\mathcal{B}_{\rm p}$ be an optimal algorithm to find a preimage of (g_0, h_0) for F. Suppose that $\mathcal{B}_{\rm p}$ asks q pairs of queries to e, e^{-1} in total.

For $1 \leq j \leq q$, let P_j be the event that $\mathcal{B}_{\rm p}$ finds a preimage of (g_0, h_0) for F with the j-th pair of queries, that is, $F(x_j, k_j^1, k_j^2) = (g_0, h_0)$ or $F(x_j \oplus c, k_j^1, k_j^2) = (g_0, h_0)$. Thus,

$$\Pr[\mathsf{P}_j] \leq \frac{2}{(2^n-(2j-1))^2} \ .$$

Let P be the event that $\mathcal{B}_{\rm p}$ finds a preimage of (g_0, h_0) for F. Then, for $1 \leq q \leq 2^{n-2}$,

$$\Pr[\mathsf{P}] \leq \sum_{j=1}^{q} \Pr[\mathsf{P}_j] \leq \frac{2\,q}{2^{2(n-1)}} \ .$$

For (4): Let $\mathcal{B}'_{\rm p}$ be an optimal algorithm to find a preimage of $(g_0 \oplus c, h_0)$ for F. Suppose that $\mathcal{B}'_{\rm p}$ asks q pairs of queries to e, e^{-1} in total.

Let P' be the event that $\mathcal{B}'_{\rm p}$ finds a preimage of $(g_0 \oplus c, h_0)$ for F. Then, for $1 \leq q \leq 2^{n-2}$,

$$\Pr[\mathsf{P}'] \leq \frac{2\,q}{2^{2(n-1)}} \ .$$

Finally, suppose that $\mathcal{A}$ is an optimal collision-finder for H. Then, from the discussions so far,

$$\begin{aligned} \mathbf{Adv}_H^{\rm coll}(q) &= \mathbf{Adv}_H^{\rm coll}(\mathcal{A}) \\ &\leq \Pr[\mathsf{C}] + \Pr[\mathsf{C}'] + \Pr[\mathsf{P}] + \Pr[\mathsf{P}'] \\ &\leq \frac{3\,q^2+q}{2^{2(n-1)}} \leq \left(\frac{q}{2^{n-2}}\right)^2 \end{aligned}$$

for $1 \leq q \leq 2^{n-2}$. □

Remark 3 in Sect. 3 also holds for Theorem 4.

4.3 Other Schemes

In this section, we present some other schemes taking into consideration the construction with AES. For simplicity, AES with 192/256-bit key-length is called AES-192/256, respectively.

The compression function F given in Definition 6 can be composed of AES-192/256. It can be regarded as a function based on the Davies-Meyer scheme.

The following compression function can be composed of AES-256. It can be regarded as a function based on the Matyas-Meyer-Oseas scheme.

Let $F_1 : \{0,1\}^{2n} \times \{0,1\}^n \to \{0,1\}^{2n}$ be a compression function $(g_i, h_i) = F_1(g_{i-1}, h_{i-1}, m_i)$ such that

$$\begin{aligned} g_i &= e(g_{i-1} \| h_{i-1}, m_i) \oplus m_i \\ h_i &= e(g_{i-1} \| (h_{i-1} \oplus c), m_i) \oplus m_i \ , \end{aligned}$$

where $g_i, h_i, m_i \in \{0,1\}^n$ and $c \in \{0,1\}^n - \{0^n\}$ is a constant. F_1 is also given in Fig. 2.

The following compression function can be composed of AES-192. It can also be regarded as a function based on the Matyas-Meyer-Oseas scheme.

Let $F_2 : \{0,1\}^{2n} \times \{0,1\}^{n/2} \to \{0,1\}^{2n}$ be a compression function $(g_i, h_i) = F_2(g_{i-1}, h_{i-1}, m_i)$ such that

$$\begin{aligned} g_i &= e(g_{i-1}^{(2)} \| h_{i-1}, m_i \| g_{i-1}^{(1)}) \oplus (m_i \| g_{i-1}^{(1)}) \\ h_i &= e(g_{i-1}^{(2)} \| h_{i-1}, (m_i \| g_{i-1}^{(1)}) \oplus c) \oplus (m_i \| g_{i-1}^{(1)}) \oplus c \ , \end{aligned}$$

where $g_i, h_i \in \{0,1\}^n$, $g_i^{(1)}, g_i^{(2)}, m_i \in \{0,1\}^{n/2}$, $g_i = g_i^{(1)} \| g_i^{(2)}$, and $c \in \{0,1\}^n - \{0^n\}$ is a constant. F_2 is also given in Fig. 3.

Variants of F, F_1, F_2 are shown in Figures 4, 5, 6, respectively. They can be regarded as functions based on the Miyaguchi-Preneel scheme.

It is easy to obtain theorems similar to Theorem 4 for collision resistance of hash functions composed of compression functions presented here.

5 Concluding Remark

In this article, it has been discussed how to construct DBL hash functions with a smaller compression function or a block cipher.

Recently, Pramstaller and Rijmen presented a collision attack on the scheme in Sect. 4 with DESX as

an underlying block cipher [25]. Their result does not contradict Theorem 4. It is a warning that we should be careful when we choose an underlying block cipher. It also shows a limitation of the random oracle/ideal cipher model. Related topics are discussed in [1, 9].

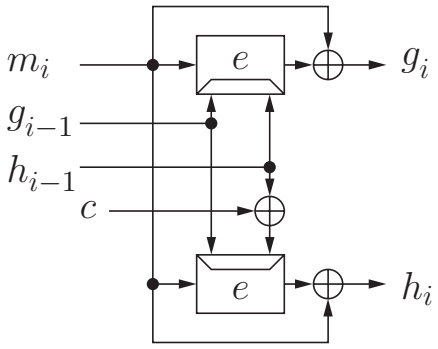

Figure 2: The compression function F_1

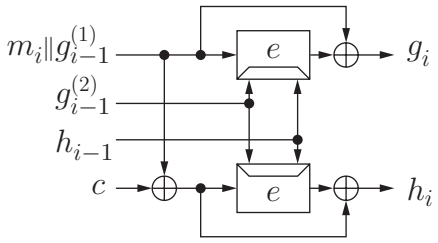

Figure 3: The compression function F_2

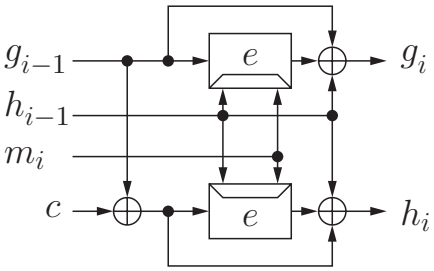

Figure 4: A variant of F

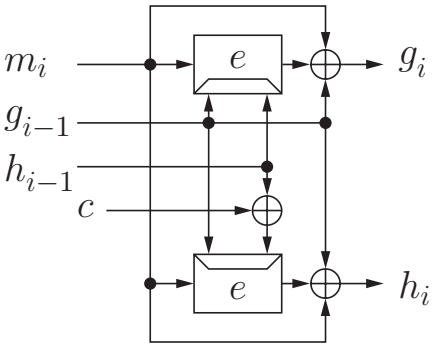

Figure 5: A variant of F_1

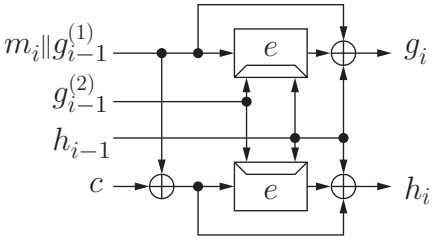

Figure 6: A variant of F_2

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