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Study of drift compression for heavy ion beams

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Abstract

The longitudinal dynamics of drift compression and pulse shaping for a space-charge-dominated heavy ion fusion beam is studied. A nonperiodic quadrupole lattice is designed for a beam undergoing drift compression, and an adiabatically matched solution is found for the transverse envelope equations in the nonperiodic lattice.

Keywords: Heavy ion fusion; Beam compression; Pulse shaping; Non-periodic lattice; Adiabatically-matched solution

1. INTRODUCTION

In the currently envisioned configurations for heavy ion fusion, it is necessary to longitudinally compress the beam bunches by a large factor after the acceleration phase (Haber, 1982). In this article, we first study the longitudinal dynamics of drift compression and pulse shaping using a one-dimensional warm-fluid model. A parabolic self-similar drift compression solution is given, and it is demonstrated that an arbitrary pulse shape can be shaped into a parabolic one by imposing an appropriate velocity distribution. Because the space-charge force increases as the beam is compressed, a larger focusing force is needed to confine the beam in the transverse direction. It is necessary to have a nonperiodic quadrupole lattice along the beam path. In this article, we also describe the design of such a focusing lattice, in which we search for *adiabatically matched* solutions of the transverse envelope equations. The following set of beam parameters typical of heavy ion fusion are used in the present study. We consider a Cs^+ beam with rest mass $m = 133\ m_p$, where m_p is the proton rest mass, kinetic energy $(\gamma - 1)mc^2 = 2.5$ GeV, and initial beam length $z_{b0} = 9.5$ m. Our goal is to compress the beam by a factor of 16, that is, to $z_{bf} = z_{b0}/16 = 0.6$ m.

2. LONGITUDINAL DYNAMICS

We use a one-dimensional warm-fluid model to describe the longitudinal dynamics of drift compression. For the longitudinal electric field, the conventional g-factor model is adopted with $eE_z = -(ge^2/\gamma^2)\partial\lambda/\partial z$ and $g = 2\ln(r_w/r_b)$. Here, e is the charge, $\lambda(t, z)$ is the line density, r_w is the wall radius, and r_b is the average beam radius. We also allow for an externally applied focusing force $F_z = -\kappa_z z$. In the beam frame, the warm-fluid equations for the line density $\lambda(t, z)$, longitudinal velocity $v_z(t, z)$, and longitudinal pressure $p_z(t, z)$ are given by

$$\frac{\partial\lambda}{\partial t} + \frac{\partial}{\partial z}(\lambda v_z) = 0, \tag{1}$$

$$\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} + \frac{e^2 g}{m\gamma^5}\frac{\partial\lambda}{\partial z} + \frac{\kappa_z z}{m\gamma^3} + \frac{r_b^2}{m\gamma^3\lambda}\frac{\partial p_z}{\partial z} = 0, \tag{2}$$

$$\frac{\partial p_z}{\partial t} + v_z \frac{\partial p_z}{\partial z} + 3p_z \frac{\partial v_z}{\partial z} = 0. \tag{3}$$

We treat g and r_b as constants for present purposes. Among all the self-similar solutions (Qin & Davidson, 2002) admitted by the nonlinear hyperbolic PDE system (1), (2), and (3), the parabolic self-similar solution is the most suitable for the purpose of drift compression, and it has the form of

$$\lambda(t,z) = \lambda_b(t)\left(1 - \frac{z^2}{z_b^2(t)}\right), \qquad v_z(t,z) = -v_{zb}(t)\frac{z}{z_b(t)}, \tag{4}$$

$$p_z(t,z) = p_{zb}(t)\left(1 - \frac{z^2}{z_b^2(t)}\right)^2, \qquad \frac{dz_b(t)}{dt} = -v_{zb}(t). \tag{5}$$

Following the derivation in Qin and Davidson (2002), we obtain the familiar longitudinal envelope equation

$$\frac{d^2 z_b}{ds^2} + \frac{\kappa_z}{m\gamma^3\beta^2c^2}z_b - K_l\frac{1}{z_b^2} - \varepsilon_l^2\frac{1}{z_b^3} = 0, \tag{6}$$

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where $s = \beta ct$ is the normalized time variable, $K_l \equiv 3N_b e^2 g/2m\gamma^5\beta^2c^2$ is the effective longitudinal self-field perveance, and $\varepsilon_l \equiv (4r_b^2 W/m\gamma^3\beta^2c^2)^{1/2}$ is the longitudinal emittance. From Eq. (6), the beam path length required for drift compression can be expressed as (J.J. Barnard, pers. comm.)

$$s_f = -\int_{z_{b0}}^{z_{bf}} \frac{dz_b}{\sqrt{z_{b0}'^2 - 2K_l\left(\frac{1}{z_b} - \frac{1}{z_{b0}}\right) - \varepsilon_l^2\left(\frac{1}{z_b^2} - \frac{1}{z_{b0}^2}\right)}}. \quad (7)$$

In the drift compression scheme considered in this article, the longitudinal emittance is taken to be $\varepsilon_l = 7.7 \times 10^{-6}$ m, and $K_l = 1.3 \times 10^{-4}$ m, corresponding to an average final current $\langle I_f \rangle = 2500$ A, $z_{bf} = 0.6$ m, and $g = 2.0$. Assuming $z_{b0}' = -0.025$, we obtain $s_f = 376$ m by evaluating the integral in Eq. (7), and $z_{bf}' = -0.0145$ from the first integral of Eq. (6). The axial beam size $z_b(s)$, obtained numerically from the longitudinal envelope equation (6), is plotted together with the velocity tilt $z_b'(s)$ in Figure 1. The parabolic self-similar drift compression solution described here requires the initial beam pulse shape to be parabolic. However, the beam pulse shape is generally not parabolic after the acceleration phase in practical accelerator applications. It is necessary to shape the beam pulse into a parabolic form in the upstream before imposing a velocity tilt. The pulse shaping problem can be posed as finding the initial velocity distribution $V(z) \equiv v_z(t = 0, z)$ such that a given initial pulse shape $\Lambda(z) \equiv \lambda(t = 0, z)$ evolves into a given final pulse shape $\Lambda_T(z) \equiv \lambda(t = T, z)$ at time $t = T$. For the heavy ion fusion beams currently considered, the pressure effects, external axial focusing, and the axial space-charge effects can be neglected in the upstream region. In this case, the fluid equations can be solved by integrating along characteristics or using Lagrangian coordinates. Following the derivation in Qin and Davidson (2002), the solution for $\lambda(t, z)$ is

$$\lambda(t, z) = \frac{\Lambda(\xi)}{1 + V'(\xi)t}, \quad (8)$$

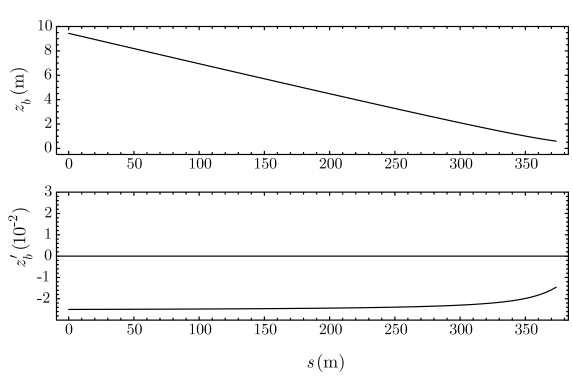

Fig. 1. Longitudinal drift compression of a heavy ion fusion beam.

where ξ is Lagrangian coordinates defined by $z = \xi + V(\xi)t$. For the pulse shaping problem, the final line density profile $\Lambda_T(z) \equiv \lambda(t = T, z)$ is specified. We therefore obtain

$$\Lambda_T(U)dU = \Lambda(\xi)d\xi, \quad (9)$$

where $U(\xi) \equiv \xi + V(\xi)T$, and $V(\xi)$ is determined by solving Eq. (9) for $U(\xi)$ for the given functional forms of $\Lambda_T(z)$ and $\Lambda(z)$, and with the appropriate boundary conditions. As an example, we consider the case where $\Lambda(z) = 1 - z^m$ $(0 \le z \le 1)$ and $\Lambda_T(z) = (1 - z^n)m(n + 1)/n(m + 1)$ $(0 \le z \le 1)$. Here, $m, n \neq -1$, and the coefficient $m(n + 1)/n(m + 1)$ in the expression for $\Lambda_T(z)$ assures the conservation of the total number of particles. Equation (9) can be integrated to give

$$\left[U(\xi) - \frac{U(\xi)^{n+1}}{n+1}\right]\frac{m(n+1)}{n(m+1)} = \xi - \frac{\xi^{m+1}}{m+1}. \quad (10)$$

The parabolic self-similar drift compression solution corresponds to $n = 2$. For large value of $m \gg 1$, $\Lambda(z)$ has a flat-top shape with a fast fall-off near the ends of the pulse. The solution of Eq. (10) then gives the initial velocity distribution $V(z)$ necessary to shape a flat-top bunched beam into a parabolic shape, which can be self-similarly compressed after imposing a linear velocity tilt. In Figure 2, $\Lambda_T(z) = (15/11)(1 - z^2)$ and $\Lambda(z) = 1 - z^{10}$, corresponding to $n = 2$ and $m = 10$, are plotted versus z, together with $V(z)$.

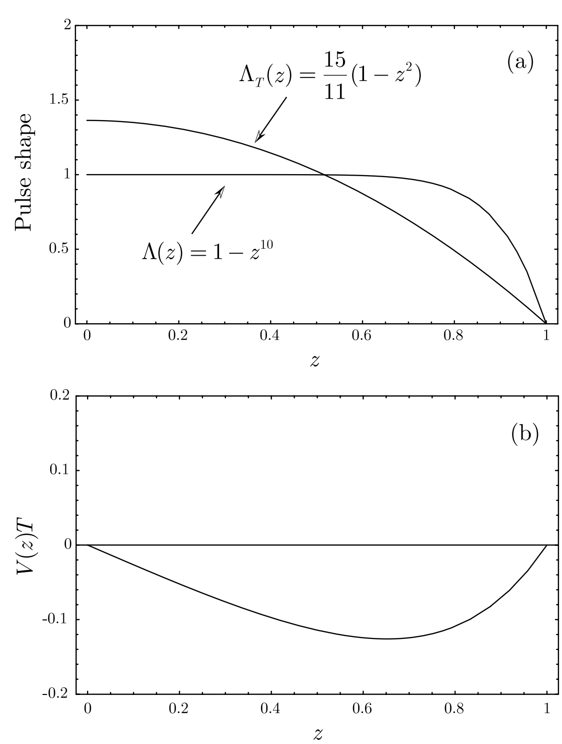

Fig. 2. Initial pulse shape $\Lambda(z) = 1 - z^{10}$ and final pulse shape $\Lambda_T(z) = (15/11)(1 - z^2)$ are plotted in (a). The initial velocity profile $V(z)$ is given by Eq. (10) and is plotted in (b).

3. TRANSVERSE DYNAMICS

For a long charge bunch, the transverse dynamics in a quadrupole lattice is described approximately by the transverse envelope equations

$$\frac{d^2a}{ds^2} + \kappa_q a - \frac{2K(s)}{a+b} - \frac{\varepsilon_x^2}{a^3} = 0, \tag{11}$$

$$\frac{d^2b}{ds^2} - \kappa_q b - \frac{2K(s)}{a+b} - \frac{\varepsilon_y^2}{b^3} = 0, \tag{12}$$

where $K(s) \equiv 2N_b e^2/m\gamma^3\beta^2c^2z_b(s)$ is the effective perveance. Because $K(s)$ is an increasing function of s, it is necessary to increase the strength of the quadrupole lattice coefficient $\kappa_q(s)$ along the beam path to reduce the expansion of the beam radius. Since the quadrupole lattice is not periodic, the concept of a *matched* beam is not well defined. However, if the the nonperiodicity is small, that is, if the quadrupole lattice changes slowly along the beam path, we can seek an *adiabatically matched* solution which, by definition, is locally matched everywhere. On the other hand, for the problem of drift compression, we describe the design of a nonperiodic lattice which provides the required control of beam radius when the beam is compressed, and equally importantly, minimizes the possibility of global mismatch. It is intuitive that a lattice, which keeps both the vacuum phase advance and depressed phase advance constant, is less likely to induce beam mismatch. Lee (1996) have derived the expressions for the vacuum phase advance σ_v and depressed phase advance σ for a periodic step-function (FODO) lattice given by

$$2(1-\cos\sigma_v) = \left(1-\frac{2\eta}{3}\right)\eta^2\left(\frac{B'}{[B\rho]}\right)^2 L^4, \tag{13}$$

$$\sigma^2 = 2(1-\cos\sigma_v) - K\left(\frac{2L}{\langle a\rangle}\right)^2. \tag{14}$$

Here, η is the filling factor, L is the lattice period, B' is field gradient of the magnets, and $\langle a\rangle$ is the average beam radius. Assuming $\eta \ll 1$, we obtain

$$\eta^2\left(\frac{B'}{[B\rho]}\right)^2 L^4 = const., \qquad K\left(\frac{2L}{\langle a\rangle}\right)^2 = const., \tag{15}$$

for constant vacuum phase advance and constant depressed phase advance. For the drift compression scheme considered here, $K_f/K_0 = 16$. If we allow $\langle a\rangle$ to increase by a factor of 2, that is, $\langle a\rangle_f/\langle a\rangle_0 = 2$, we obtain $L_f/L_0 = \frac{1}{2}$, and $(\eta B')_f/(\eta B')_0 = 4$. We determine $K(s)$ from the solution of the longitudinal envelope equation. The value of $\langle a\rangle$ is determined from the solutions to Eqs. (11) and (12). For the lattice design, we need to specify η, B', and L. If we choose $L_i = L_0\exp[-(s_i/s_f)\ln 2]$, and $B_i' = const.$, then from Eq. (15), $\eta_i = \eta_0\exp[(s_i/s_f)\ln 2]$, where $s_i = \sum_{j=0}^{i-1} L_j$. We also choose self-consistently the following system parameters: $\sigma_v = 72°$, $B_i' = 31.70$ T/m, $L_0 = 6.72$ m, and $\eta_0 = 0.036$. The focusing strength of each magnet is $\hat{\kappa} = 0.38\,\mathrm{m}^{-2}$. Let N denote the total number of quadrupole magnet sets. From $s_f = \sum_{j=0}^{N-1} L_j$, we obtain $N = 40$. The lattice design is illustrated in Figure 2 together with the solutions to Eqs. (11) and (12). After determining the nonperiodic lattice layout, we search iteratively for the adiabatically matched solutions to Eqs. (11) and (12). An adiabatically matched solution is plotted in Figure 3. It is adiabatically matched because the envelope is locally matched and contains no oscillations other than the local envelope oscillations. On the global scale, the beam radius increases monotonically. From the numerical solution shown in Figure 2, the average beam size increases by a factor of 2, which agrees with the design assumption. Currently, well-behaved adiabatically matched solutions are obtained by using an intuitive trial-and-error approach (J.J. Barnard, pers. comm.). A recently derived equation for the average beam envelope in nonperiodic lattices will provide a systematic understanding of the adiabatically matched solutions (Davidson & Qin, 2001).

4. CONCLUSIONS

In this article, we have studied the longitudinal dynamics of drift compression and pulse shaping for a space-charge-dominated heavy ion fusion beam using a one-dimensional warm-fluid model. A nonperiodic quadrupole lattice configuration has been designed for a beam undergoing drift com-

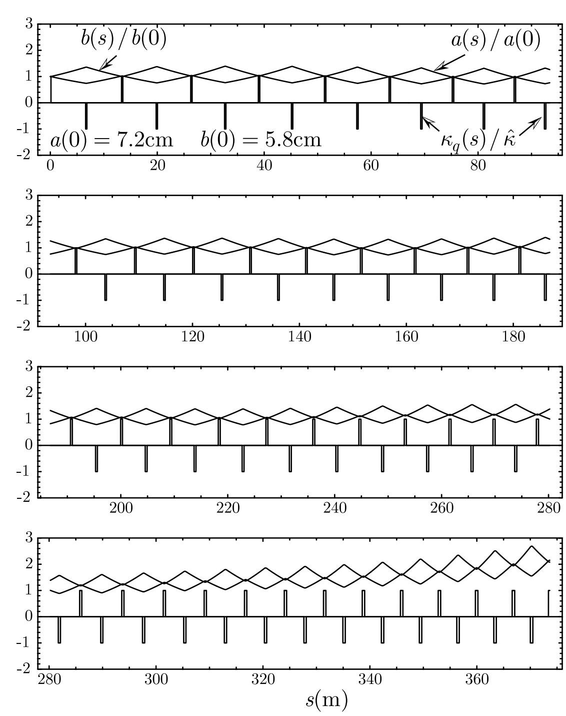

Fig. 3. Adiabatically matched envelope solutions in a nonperiodic lattice for a heavy ion fusion beam under drift compression.

pression with fixed vacuum phase advance and depressed phase advance. An adiabatically matched solution was found for the transverse envelope equations in the nonperiodic lattice.

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REFERENCES

Davidson, R.C. & Qin, H. (2001). *Physics of Intense Charged Particle Beams in High Energy Accelerators*. World Scientific, Singapore.

Haber, I. (1982). Simulation of the longitudinal dynamics of a cold bunched beam. *Proc. Symp. Accelerator Aspects of Heavy Ion Fusion*, Gesellschaft für schwerionen forschung, GSI-82-8, Darmstadt, W. Germany. 372.

Lee, E.P. (1996). *Phys. Plasmas* **52**, 115.

Qin, H. & Davidson, R.C. (2002). *Phys. Rev. Special Topics—Accelerators and Beams* **5**, 034401.