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CONSUMPTION TAXES AND ECONOMIC EFFICIENCY IN A STOCHASTIC OLG ECONOMY

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Abstract

Fundamental tax reform is examined in a heterogeneous overlapping-generations life-cycle model in which agents face idiosyncratic earnings shocks and uncertain life spans. Following Auerbach and Kotlikoff (1987), a lump-sum redistribution authority is used to examine efficiency gains over the transition path. A progressive income tax is replaced with a flat consumption tax (for example, a value-added tax or a national retail sales tax). If shocks are insurable (that is, no risk), this reform improves (interim) efficiency, a result consistent with the previous literature. But if, more realistically, shocks are uninsurable, this reform reduces efficiency, even though national wealth and output increase over the entire transition path. This efficiency loss, in part, stems from reduced intragenerational risk sharing that was previously provided by the progressive tax system. Social safety net programs can substitute for insurance to maintain efficiency along with growth effects.

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1 Introduction

The potential economic benefits from replacing the current income tax system with a flat (proportional) consumption tax system have generated a considerable amount of attention in recent years. Examples of a flat consumption tax include a value-added tax (VAT) in many European nations as well as a national retail sales tax that is gaining attention in the United States as a possible substitute tax base. Replacing the current income tax system with a revenue-neutral flat consumption tax would (i) flatten tax rates, (ii) tax consumption rather than wage and capital income, and (iii) eliminate all tax-base reductions (preferences) contained in current law. In all likelihood, this reform would significantly increase national saving and output over the long run (Altig and others, 2001), a result that we confirm. This paper examines whether this reform actually improves economic *efficiency.*

Judging the economic efficiency of a particular policy reform has always been important to economists, but it is important to remind ourselves why. Many different policy reforms can, for example, increase the welfare of people born in a long-run steady state, but those gains might simply represent losses to intermediate generations. (Indeed, this point has gained a considerable amount of attention in the recent debate on Social Security privatization.) If there is no economic gain after fully compensating intermediate generations who otherwise lose from reform, then judgments over a reform must be made purely on a philosophical *a priori* basis[1] or based on subjective intragenerational and intergenerational distribution choices.[2] To be sure, economists contribute in important ways to these debates, especially on distributional issues. For example, economists have estimated the impact that a particular tax system has on the distribution of income or wealth within generations (Auerbach and Hassett, 2001) or between generations (Auerbach, Gokhale, and Kotlikoff, 1994). Economists have also derived the implied optimal tax schedules under an *assumed* social welfare function that weights the utility of different people in a particular way (Mirrlees, 1971). But economists are not particularly better qualified in making philosophical or moral

[1] For example, the philosopher Thomas Hobbes argued that it is *a priori* “wrong” to tax estates on the basis that wealth was already taxed when earned. Others have argued that it is *a priori* “wrong” for the government to take money from one generation to give to another.

[2] For example, in terms of intragenerational distribution, the philosopher John Rawls argued that social welfare must be judged on the basis of the utility of the worse-off person in society.

judgments than noneconomists. Ultimately, what economists bring to the table in policy debates is our insight about *efficiency*, interpreted here in the Pareto sense.

The point of departure of our paper is that previous analyses of fundamental tax reform have not incorporated the *intragenerational* risk-sharing benefits of the current progressive income tax system. Our main finding is that this risk sharing is important for determining the efficiency changes associated with tax reform, even at modest levels of risk aversion. When idiosyncratic earnings shocks are assumed to be fully insurable so that each agent faces no risk, moving to a flat consumption tax increases efficiency, a common result in the literature. However, when wages are uninsurable, efficiency is actually reduced by moving to a flat consumption tax, even though national wealth and output increase over the entire transition path.[3] Social safety net programs can substitute for insurance to maintain efficiency along with growth effects.

1.1 The Ramsey Model

The simplest way to analyze the impact that a revenue-neutral tax reform has on economic efficiency is with the Ramsey infinite-horizon representative-agent model, which assumes that households are Ricardian. Since, in general, efficiency changes are always calculated relative to compensated changes, the presence of a single agent in the Ramsey model dramatically simplifies these calculations. In particular, calculating efficiency changes does not require redistributing resources across agents in order to compensate those who would otherwise lose from reform. As a result, one can often derive the most efficient long-run tax structure analytically (see the reviews by Judd, 1985, and Auerbach and Hines, 2001). But the Ramsey model is less suitable for capturing the intragenerational risk-sharing benefits of a progressive income tax system, which is the focus of the current paper.[4]

[3]This paper examines policy changes rather than attempting to derive optimal progressive tax schedules in the Mirrlees tradition where a social welfare function must be assumed. However, our results on the importance of risk sharing would also be relevant to that literature, which, thus far, has found little cause for progressive tax schedules. Although computational considerations limit our ability to derive optimal progressive tax schedules in our model, this extension could prove useful in the future as computers become more powerful.

[4]Well-known complications arise when attempting to model multiple infinite-horizon agents. If agents have identical time preferences (which, realistically, is a measure of zero), then there is an infinite number of wealth distributions compatible with a steady state. If, more realistically, agents have nonidentical time preferences, then the wealth distribution becomes trivial (one agent owns everything). Incorporating progressive tax rates creates additional problems.

1.2 The Stochastic OLG Model with Finite Horizons

Instead, this paper uses a calibrated overlapping-generations (OLG) life-cycle model with uninsurable idiosyncratic earnings shocks, uncertain life spans, and elastic labor supply to examine the efficiency gains associated with adopting a revenue-neutral flat consumption tax. The multiplicity of agents, though, complicates the efficiency calculations since tax reform redistributes resources across different households. To deal with this problem, we follow the pioneering work done by Auerbach and Kotlikoff (1987), who developed the first large-scale OLG model without uncertainty. Like theirs, our model incorporates a "Lump-Sum Redistribution Authority" (LSRA) that calculates the overall efficiency gains or losses of a policy change, by restoring the utility of the agents alive at the time of the reform to their prereform levels through lump-sum redistributions both across and within generations.

A heuristic technique for calculating efficiency gains that is more common in the literature would simply sum welfare changes across households (discounted if across time); a policy is then deemed to increase [decrease] efficiency if the net sum is positive [negative]. This latter approach implicitly assumes that lump-sum transfers are made across households in order to compensate the losers of policy reform with some of the gains of the winners. But this approach fails to capture the price effects associated with these lump-sum transfers.

1.3 Progressive Consumption Taxes

While flattening tax rates receives the bulk of the attention in the tax reform debate, it is important to note that consumption-based taxes can also be progressive. In particular, allowing firms to deduct their full investment expenses at the time of purchase from their tax payments ("full expensing") would effectively produce a consumption tax.[5] This approach would also allow for some progressivity through either a standard deduction, as in the "flat tax" plan (Hall and Rabushka, 1995), and/or progressive tax rates, as in the "X tax" plan (Bradford, 1986). The "flat tax" and "X tax" plans also protect housing wealth. By maintaining progressive tax rates, the "X tax," in particular, could increase efficiency in our model.

The current paper, however, focuses on a flat consumption tax, for several reasons. First, we are more interested in creating an understanding of the risk-sharing aspect of the current

[5] In fact, a flat consumption tax produces the same outcomes as a flat income tax with full expensing.

income system than in analyzing specific reform proposals. Second, the computation requirements of our model are already significant. In order to capture the differential tax treatment of various capital items under the “flat tax” and the “X tax,” we would have to include housing wealth and other types of capital as separate capital categories. While this addition is possible in deterministic models (Fullerton and Rogers, 1993; Jorgenson and Yun, 2001), adding housing wealth to our stochastic model would alone increase the computation time by two orders of magnitude, requiring about three months to solve a single simulation on a two-gigahertz Pentium IV computer. We, therefore, leave this innovation to future research. Third, as a practical manner, a national retail sales tax is gaining considerable attention in the United States. Later in the paper, we do, however, consider a simple progressive consumption tax in the form of a national sales tax with a rebate of the first $20,000 of consumption per household. This reform is similar to the “flat tax” proposal except that we don’t protect existing housing wealth; our version of the “flat tax,” therefore, creates more potential for efficiency gains.

1.4 Outline

The outline of our paper is as follows. Section 2 gives a brief primer on the economic factors associated with moving to a flat consumption tax using a simple two-period model. Section 3 outlines our large-scale model that we use to simulate the introduction of a revenue-neutral flat consumption tax. Section 4 summarizes the calibration of the baseline economy. Section 5 explains the Lump-Sum Redistribution Authority and shows policy experiments. Section 6 concludes the paper. The appendix describes the computational algorithm.

2 A Primer on Tax Reform

This section uses a simple partial-equilibrium model to present a brief overview of tax reform in an OLG economy. We first consider tax reform in a deterministic economy in which the prereform income tax system is linear. We then expand the discussion to nonlinear taxes, and then finally to uncertainty in the form of idiosyncratic wage shocks and life spans. We do not consider the effects of removing tax preferences; this issue and others are addressed in our larger-scale model, which is presented in the next section.

2.1 No Uncertainty, Linear Taxes

Consider a simple two-period model in which agents work during the first period and retire in the second period. During the first period, an agent born in time t earns pretax wages $w_{1,t}$, pays a wage tax at rate τ_t^w, consumes $c_{1,t}$, and saves the remainder as assets, $a_{2,t+1}$, in order to afford second-period consumption. During period two at time $t+1$, the agent's consumption, $c_{2,t+1}$, is equal to $a_{2,t+1}$ plus net interest paid at time $t+1$, r_{t+1}, after paying a capital income tax at rate τ_t^r. The agent's budget constraints, therefore, are as follows:

$$c_{1,t} + a_{2,t+1} = (1 - \tau_t^w)\, w_{1,t},$$

$$c_{2,t+1} = \left[1 + \left(1 - \tau_{t+1}^r\right) r_{t+1}\right] a_{2,t+1}.$$

A linear income tax, τ_t^y, is created by setting $\tau_t^r = \tau_t^w = \tau_t^y$. Assuming that there are no borrowing constraints (or, alternatively, $a_{2,t+1} \geq 0$), the household's lifetime budget constraint equals

$$c_{1,t} + \frac{c_{2,t+1}}{\left[1 + \left(1 - \tau_{t+1}^r\right) r_{t+1}\right]} = (1 - \tau_t^w)\, w_{1,t}. \tag{1}$$

We can rewrite equation (1) as

$$\frac{c_{1,t}}{1 - \tau_t^w} + \frac{c_{2,t+1}}{\left[1 + \left(1 - \tau_{t+1}^r\right) r_{t+1}\right] (1 - \tau_t^w)} = w_{1,t}$$

or

$$\left(1 + \tilde{\tau}_{1,t}^c\right) c_{1,t} + \frac{\left(1 + \tilde{\tau}_{2,t+1}^c\right) c_{2,t+1}}{(1 + r_{t+1})} = w_{1,t},$$

where $\tilde{\tau}_{1,t}^c \equiv \frac{1}{1-\tau_t^w} - 1$ and $\tilde{\tau}_{2,t+1}^c \equiv \frac{1+r_{t+1}}{\left[1+\left(1-\tau_{t+1}^r\right)r_{t+1}\right](1-\tau_t^w)} - 1$. Hence, a system of linear wage taxes, capital income taxes, and income taxes can be represented in terms of equivalent *age-indexed* effective consumption tax rates, $\tilde{\tau}^c$. (The tilde [~] superscript is used to denote *effective* rates.) Both tax systems collect the same *lifetime present value* of taxes from each agent and offer the same incentives. Notice that if tax rates are stationary then $\tilde{\tau}_2^c > \tilde{\tau}_1^c$ when $\tau_{t+1}^r > 0$, that is, a positive capital income tax increases the effective consumption tax rate over the life cycle. By the inverse-elasticity rule of the optimal tax literature, this increasing tax rate is a potential source of inefficiency unless the demand elasticity for second-period consumption was lower relative to first-period consumption, a result that is not an implication of most specifications of household preferences.

Government revenue each period finances a fixed level of spending, $\bar{G}$:

$$\tau_t^w w_{1,t} + \tau_t^r r_t a_{2,t} = \bar{G}, \tag{2}$$

where the population size is assumed to be stationary. Equation (2) could also be represented in terms of the effective consumption tax rates shown above. However, when analyzing a tax reform, which changes the present value of taxes paid by each generation, equation (2) is needed in its current form because second-period agents alive at the reform did not actually face the effective consumption tax $\tilde{\tau}_1^c$ during their first period of life.

2.1.1 A Revenue-Neutral Consumption Tax

Now suppose that the government introduces a consumption tax, τ_t^c, at time t to replace the income tax. The private budget constraints become

$$(1 + \tau_t^c)\, c_{1,t} + a_{2,t+1} = w_{1,t},$$

$$\left(1 + \tau_{t+1}^c\right) c_{2,t+1} = [1 + r_{t+1}]\, a_{2,t+1},$$

which produces the following lifetime budget constraint:

$$(1 + \tau_t^c)\, c_{1,t} + \frac{\left(1 + \tau_{t+1}^c\right) c_{2,t+1}}{(1 + r_{t+1})} = w_{1,t}.$$

The government's budget constraint equals

$$\tau_t^c \left(c_{1,t} + c_{2,t}\right) = \bar{G}.$$

This tax reform has two major effects. First, capital income is no longer taxed, eliminating the intertemporal price distortion in a stationary economy. Second, as explained below, this tax reform imposes a lump-sum tax on existing asset holders. We consider each of these effects below.

This decomposition between these two effects is convenient because it allows us to interpret a move from a linear income tax to a flat consumption tax in two steps: (i) replace a linear income tax with a linear *wage* tax, thereby removing the tax on capital income; and

(ii) replace this linear wage tax with a *consumption* tax, thereby imposing a lump-sum tax on existing assets. Although we regard the second reform as occurring immediately after the first reform, the age-asset profile in the initial income tax economy is the one that is relevant for determining the impact of the lump-sum tax on existing assets.[6]

The Nontaxation of Capital Income. Notice that in a stationary economy, this tax reform generates uniform consumption tax rates across the life cycle since capital income is no longer taxed. As a result, the intertemporal price distortion is removed, encouraging saving. If labor supply was also elastic, an increase in after-tax interest rates would encourage more labor supply and saving earlier in the life cycle since asset values would accumulate more quickly.

However, this good news does not come for free. The assets that have already been accumulated by generation $(t-1)$ at time t, which would have been taxed under the original income tax, will not be taxed after the reform. So generation $(t-1)$ receives a lump-sum transfer (negative tax) equal to $\tau_t^r r_t a_{2,t}$, which must be paid by future workers in the form of wage taxes in order to make up the lost revenue.[7] Because the wage base is smaller than the income base, tax rates increase. Since tax distortion increases with the square of the tax rate, a smaller tax base will produce more distortions. In the finite-horizons OLG model, these new distortions could, in theory, outweigh the gains associated with removing the intertemporal price distortion. In fact, in their simulation analysis using a multiple-period deterministic model with elastic labor supply, Auerbach and Kotlikoff (1987) find that replacing a linear income tax with a linear wage tax *reduces* long-run output and welfare.[8]

A Lump-Sum Tax on Existing Wealth. As first demonstrated by Summers (1981), adopting a consumption tax also imposes a lump-sum tax on older people. In particular, the con-

[6] In static models with one or two periods (and no bequests), one could alternatively distinguish between the tax reform's "substitution effect" and "income effect." However, this distinction is substantially more cumbersome in a model with more than two periods where some agents have accumulated wealth by the time of the reform and will also live for more than one additional period after the reform. See Gravelle (2002) for a detailed critique of intertemporal models.

[7] Our simple two-period model, though, somewhat exaggerates this point by taxing capital income only at the beginning of the second period. With multiple periods, asset holders will have already paid some taxes on capital income before the tax reform.

[8] In contrast, the optimal long-run tax rate on capital income is zero in the Ramsey model.

sumption by generation ($t-1$) in their second period of life is inelastic at time t since their consumption is based on previous saving. Hence, when the government changes the tax system at time t, these agents face a lump-sum tax equal to $\tau_t^c c_{2,t}$, which, under revenue neutrality, accrues as reduced taxes paid by future workers.[9]

In fact, the lump-sum tax on generation $(t-1)$ accrues as a reduced tax liability to *all* future generations. To demonstrate this fact, consider the clever expositional simplification used by Auerbach and Kotlikoff (1987, pp. 58 - 59): agents only consume during the second period of their lives (that is, $c_{1,t}=0$) and taxes on interest are zero ($\tau^r=0$) so that only wages are taxed. Under these assumptions, wage and consumption taxes are lump sum for *all* generations, allowing for easy illustration. Now suppose that the government switches tax bases from wages to consumption at time t. Generation $(t-1)$, which is in their second period of life when the tax reform occurs, is charged a wealth levy equal to $\bar{G}$. Clearly, they are worse off: they paid $\bar{G}$ during their first period of life under the previous wage tax, and now they must pay it again during their second period under the new consumption tax. Now consider generation t, which is in their first period of life when taxes are reformed. Under the former wage tax, generation t would have paid $\bar{G}$ during their first period of life. Instead, they now pay $\bar{G}$ in their second period, reducing the present value of their lifetime taxes by $\left(\bar{G}-\frac{\bar{G}}{1+r}\right)$, or $r\bar{G}/\left(1+r\right)$. Similarly, every future generation s ($s>t$) receives a present-value reduction in their tax liability equal to $r\bar{G}/\left(1+r\right)$, calculated with respect to their generation index, s. The present value sum of tax saving across all future generations, calculated at time t, therefore, equals $\sum\limits_{i=0}^{\infty}\frac{\left(r\bar{G}/(1+r)\right)}{(1+r)^i}=\bar{G}$. In other words, the present value of the tax reduction to future generations exactly equals the loss to the initial elderly.

This intergenerational transfer has a very powerful impact on long-run output and welfare. In their multiple-period simulation model, Auerbach and Kotlikoff find that replacing a linear income tax with a flat consumption tax increases long-run output and welfare. Why are the results so different in the AK model relative to the wage tax base discussed above? The reason is the wealth levy on the existing capital held by generation $(t-1)$ that occurs with a consumption tax but not with a wage tax; in fact, this wealth levy is the only differ-

[9]As noted in the previous section, a flat consumption tax produces the same outcomes as a flat income tax with full expensing. Under expensing, the lump-sum tax on existing wealth takes the form of a fall in Tobin's q, as old capital becomes less valuable relative to new capital.

ence between those two tax bases. In the case of a consumption tax, this wealth levy extracts enough resources to reduce future tax burdens, producing a long-run gain. In fact, *over* 100 percent of the long-run gain in the AK model stems from this wealth levy (Engen, Gravelle, and Smetters, 1997).

2.1.2 Efficiency Versus Redistribution

It is important, though, to distinguish between redistribution and efficiency. In the previous two-period example, replacing the linear wage tax with a consumption tax would produce a sizable long-run increase in the capital stock and output. But this entire gain comes off the backs of generation $(t-1)$. In other words, if generation $(t-1)$ were to receive a lump-sum rebate equal to $\bar{G}$ so that its utility is held fixed, future generations would no longer benefit from tax reform. Hence, the efficiency gain is exactly zero despite the large long-run gains. Recall, though, two key assumptions that we made: (i) agents lived for only two periods, and (ii) all taxes were effectively lump-sum for all generations. Not surprisingly, therefore, tax reform produces zero efficiency gains.

In a more realistic setting with more than two periods and with consumption in each period, replacing a linear *income* tax with a consumption tax would probably produce efficiency gains. To be sure, removing the tax on capital income alone has an unclear impact on efficiency since the benefit from removing the intertemporal price distortion must be balanced against the higher tax rate. However, the lump-sum tax is likely to lead to sizable efficiency gains inside a *multiperiod* model. The reason is that, after controlling for intergeneration redistribution, agents with assets who are alive at the time of the reform benefit from replacing some of their *own* future distorting taxes with the lump-sum taxes that they pay today in the form of a wealth levy.[10] Accounting for these different effects requires simulation analysis. Auerbach and Kotlikoff find that efficiency is increased by replacing a linear income tax with a linear consumption tax in a deterministic framework, a result that we verify later. Within the Ramsey model, many previous papers have also found positive gains from adopting a consumption tax (see the review in Stokey and Rebelo, 1995).

[10] We benefited from a helpful conversation with Alan Auerbach on this point.

2.2 No Uncertainty, Nonlinear Taxes

With nonlinear tax rates in the prereform economy, the agent's budget constraints are

$$c_{1,t} + a_{2,t+1} = w_{1,t} - T_t^w\left(w_{1,t}\right),$$

$$c_{2,t+1} = \left(1 + r_{t+1}\right) a_{2,t+1} - T_{t+1}^r\left(r_{t+1} a_{2,t+1}\right),$$

where the $T\left(\cdot\right)$ functions represent total taxes paid. The average wage tax rate equals $T_t^w\left(w_{1,t}\right)/w_{1,t}$, which is smaller than the marginal wage tax rate, $\partial T_t^w\left(w_{1,t}\right)/\partial w_{1,t}$. Similarly, the average and marginal tax rates on capital income are $T_{t+1}^r\left(r_{t+1}a_{2,t+1}\right)/\left(r_{t+1}a_{2,t+1}\right)$ and $\partial T_{t+1}^r\left(r_{t+1}a_{2,t+1}\right)/\partial\left(r_{t+1}a_{2,t+1}\right)$, respectively.

With a stationary population size, the government's budget constraint is

$$T_t^w\left(w_{1,t}\right) + T_{t+1}^r\left(r_{t+1}a_{2,t+1}\right) = \bar{G}.$$

The introduction of progressive income taxes into the prereform economy alters the impact of the intertemporal price and lump-sum tax effects described above. It also means that a shift to a consumption tax adds a third effect: flattening tax rates. The adoption of a flat consumption tax now has three steps: (i) adopt a revenue-neutral progressive wage tax by dropping taxes on capital income; (ii) move from a progressive wage tax to a progressive consumption tax, producing a lump-sum tax on existing assets;[11] and (iii) move from the progressive consumption tax to a flat consumption tax.

2.2.1 The Nontaxation of Capital Income

Much of the debate about whether to adopt a consumption tax focuses on removing distortions caused by progressive tax rates. Indeed, progressive tax rates tend to magnify intertemporal distortions. First, even with inelastic labor supply, saving decisions are distorted more with progressive taxes since the future marginal tax rate that a person faces on capital income is now directly affected by their saving decisions. Removing the tax on capital income, therefore, encourages even more saving when the prereform income tax is progressive. Second, allowing for elastic labor supply tends to enhance this result. Marginal income tax rates

[11] In practice, this could be achieved with full expensing, discussed in Section 1. An equivalent progressive VAT or sales tax could also be implemented, but would be substantially more cumbersome to administer.

tend to peak at middle age when labor productivity is high and after a fair amount of assets have been accumulated for retirement. So agents tend to shift their labor supply away from high-tax years in the middle years of life toward lower-tax years later in life. Since more labor income is earned later in life, less saving is needed earlier in life to smooth consumption. Removing the tax on capital income, therefore, would eliminate those distortions, which tend to be more significant when the prereform income tax is progressive.

As with the linear income tax considered before, however, one must also account for the government's budget constraint. Increasing wage tax rates to make up lost revenue now creates more distortions than in the linear tax case considered earlier. Distortions increase the most if the progressive wage tax schedule is increased in a progressive manner in order to protect the poor. But even if the additional tax burden is distributed in a proportional manner, distortions will rise faster relative to the linear case considered earlier.

2.2.2 A Lump-Sum Tax on Existing Wealth

As noted earlier, in a multiple-period model, the efficiency gains stemming from the lump-sum tax on existing wealth depend on the extent to which the lump-sum taxes replace the future distorting taxes of those alive at the time of the reform. The more assets held by younger and middle-aged workers at the time of the reform, the more likely the wealth levy would produce efficiency gains. Whether, for a given capital-output ratio,[12] these cohorts hold a larger share of capital under progressive taxes depends on the exact model parameters.[13] Still, the efficiency gains associated with moving to a consumption tax are likely to be much larger when the prereform income tax is progressive since this tax system is more distorting, increasing the value of the substitute lump-sum taxes.

[12] When comparing across models, one should always solve for the deep parameters that generate the same observable economy, including the capital-output ratio. In this way, you ensure that the predictions are not being generated by different calibrations.

[13] Younger workers tend to face lower tax rates under a progressive system, giving them more resources to save. But they also face increasing marginal tax rates in the future as their human capital returns increase, decreasing their incentive to save. Older people, except those who have accumulated lots of wealth, also have a few more resources to reinvest under a progressive tax system. But the intertemporal shift in their labor supply described earlier in the text tends to reduce their saving. Hence, the remaining share held by middle-aged workers also depends on the parameters.

2.2.3 Flattening Tax Rates

The effect of moving from a progressive consumption tax to a flat consumption tax produces two competing major effects (and a couple minor effects that we'll ignore). First, it removes an important price distortion across the life cycle. Specifically, consumption increases over the life cycle when the interest rate exceeds the rate of time preference.[14] As a result, marginal consumption tax rates also increase over the life cycle, similar to the pattern produced by a capital income tax considered earlier. A shift to a proportional tax, therefore, creates a uniform tax rate on consumption, removing this intertemporal distortion. Second, this reform gives many asset holders a lump-sum transfer (negative tax), reducing efficiency.

2.2.4 Total Efficiency Gains

On net, there are likely to be sizable efficiency gains from adopting a flat consumption tax when the original income tax system is progressive. Both Auerbach and Kotlikoff (1987), using the OLG model, as well as Jorgenson and Yun (2001), who use the Ramsey model described in Section 1, find large efficiency gains.

2.3 Wage and Life-Span Uncertainty, Nonlinear Taxes

Adding wage and life-span uncertainty influences all three of the effects of tax reform discussed above. We consider each in turn.

2.3.1 The Nontaxation of Capital Income

The addition of wage uncertainty tends to reduce the importance of price distortions over the life cycle. Even with linear tax rates and inelastic labor, agents will hedge their earnings uncertainty by saving in a precautionary manner.[15] As a result, household saving becomes less responsive to an increase in the after-tax interest rate following a shift to a consumption tax base.[16] To be sure, with elastic labor supply, the need to save in a precautionary manner is reduced somewhat since agents can, for example, work multiple low-paying jobs in order to

[14] This effect cannot happen in the infinite-horizon model since the interest rate equals the time preference rate in a steady state.

[15] Precautionary saving is positive if the third derivative of the agent's felicity function is positive, a condition which holds in our model.

[16] This point was emphasized in Engen and Gale (1996) and Engen, Gravelle, and Smetters (1997).

replace a former higher-paying job. But since utility is concave in leisure and the maximum leisure time is bounded, the ability to vary working hours cannot eliminate precautionary saving altogether. As a result, saving will be less sensitive to changes in the after-tax interest rate relative to the case without uncertainty. Labor supply also becomes less sensitive to changes in interest rates.

When fair annuities are not available, adding life-span uncertainty produces two competing effects. On the one hand, life-span uncertainty should lead to greater precautionary saving, which decreases the interest elasticity of saving. On the other hand, the horizon of agents is effective "longer" since a prudent agent will plan for a time period longer than average. This longer time period *might* enhance price sensitivity somewhat.[17] The presence of a Social Security system in our large-scale model (Section 3) will tend to reduce the importance of both of those effects.

2.3.2 A Lump-Sum Tax on Existing Wealth

While the lump-sum tax on existing wealth following the adoption of a consumption tax has a large impact on efficiency, the addition of uncertainty produces two competing effects. On the one hand, for a given capital-output ratio, the asset-age profile for the average person is relatively less "hump-shaped" due to greater precautionary saving both earlier in the life cycle (due to earnings uncertainty) and later in the life cycle (due to life-span uncertainty). Since labor supply is partly a self-insurance mechanism, people are also less likely in the prereform economy to take advantage of falling future marginal tax rates by postponing their labor supply, thereby generating more saving earlier in life. As a result, the wealth levy on the young is higher, reducing their future distorting taxes by more than without uncertainty.

[17] The interaction of life-span uncertainty with wage uncertainty, though, complicates matters. For example, if in the extreme, agents lived forever with certainty, we would be back in the infinite-horizons world where we would want to focus on a single agent. In this case, Aiyagari (1995) demonstrates that the optimal tax rate on capital income would actually be *positive* when the infinitely lived agent faces uninsurable indiosyncratic earnings shocks. Intuitively, elastic labor supply prevents the government from employing confiscatory wage taxes to replicate full insurance. As a result, precautionary saving drives the interest rate below the agent's rate of time preference, generating too much capital in the economy relative to the modified golden rule. A positive capital income tax brings the economy's level of capital back to the efficient level. The Aiyagari motive for a positive capital income tax rate, though, is not present in a stochastic finite-horizon OLG model—not even as an approximation—unless precautionary saving produces enough capital so that the economy becomes dynamically inefficient. Whereas dynamic inefficiency is guaranteed in Aiyagari's model (where the actual level of capital is compared against the *modified* golden rule level of capital), it is not in a finite-horizon OLG model (where the comparison is made with the *golden* rule level of capital).

On the other hand, the value of this lump-sum tax is not as large as without uncertainty due to the reduced importance of distortions, discussed above.

2.3.3 Flattening Tax Rates

When uncertainty is added to the model, flattening the tax rates reduces risk sharing within generations. The reason is that the original income tax system shares idiosyncratic earnings shocks. To some extent, even a linear tax shares risks since people with higher earnings realizations pay more taxes. But progressive taxes share those risks even more.

In fact, if labor supply was completely inelastic then all agents in identical states (that is, same assets; same current-period wage income; same age; and so on) would want to *fully* share future wage shocks. In this case, the optimal tax on consumption would be extremely progressive: consumption levels above *these* individuals' expectation (that is, state-contingent expectation) would be taxed at 100 percent while consumption below average would be taxed at a negative rate (subsidized) so that it equaled the expected outcome.

In reality, of course, the optimal progressive tax system will not fully share all future risks. First, labor supply is obviously not completely inelastic. In essence, agents with greater-than-expected wage realizations will distort their future labor supply in order to partially "renege" on the previous risk-sharing "agreement." Since this incentive is understood ex ante, the optimal tax schedule cannot fully share risks ex post. Second, in realistic tax systems, taxes paid by any agent in a given year depend only on the agent's current state and not on that agent's state in *previous* years. As a result, a progressive tax system will redistribute resources in the future across agents at *different* states today. With elastic labor supply, this "extra" redistribution is a source of (interim) inefficiency.[18]

2.3.4 Total Efficiency Gains

The remainder of this paper examines the importance of wage and life-span uncertainty when analyzing tax reform. Since closed-form solutions are not possible in assessing these differ-

[18] In our analysis, we focus on "interim" efficiency, where the expected remaining lifetime utility of living agents is calculated conditional on their current state at the time of reform, and the expected utility of future generations is calculated conditional on the initial state into which they are "born" as independent economic actors. If we instead measured expected utility across all possible states (the so-called "ex ante" position), our results regarding the importance of risk sharing would only be strengthened.

ent competing forces, we use simulation analysis to help determine the impact on efficiency from replacing a progressive income tax system with a flat consumption tax. The next section lays out the computation model that we use.

3 Model

The economy consists of three main sectors: heterogeneous households with elastic labor supply; a perfectly competitive representative firm with constant-returns-to-scale production technology; and a government with a full commitment technology.[19]

3.1 The Household Sector

Households are heterogeneous with respect to age i, working ability e_i (measured by its hourly wage), beginning-of-period wealth holding a_i, and average historical earnings b_i that is used to determine Social Security benefits. Every year, a large number (normalized to unity) of new households of age 20 enter into the economy.[20] A household of age i observes idiosyncratic working ability shock, e_i, at the beginning of each year and chooses its optimal consumption c_i, working hours h_i, and end-of-period wealth holding a_{i+1}, taking the government's policy schedule and a series of factor prices and the government's policy variables as given.[21] At the end of each year, a fraction of households die. Households are alive at most up to 109 years old, and the mortality rate at the end of age 109 is one. Tables 1 and 2 show the main variables and functions used in the household's problem.

3.1.1 The Household's Problem

Let $\mathbf{s}_i$ denote the individual state vector of an age i household, let $\mathbf{S}_t$ denote the aggregate state vector at the beginning of year t, and let $\mathbf{\Psi}_t$ denote the series of government policy rules known at the beginning of year t,

$$\mathbf{s}_i = (i, e_i, a_i, b_i)\,, \tag{3}$$

[19] As is standard in the optimal tax literature, we assume that the government can commit to future policies, thereby ignoring time-consistency issues.

[20] The population of this economy is normalized by the constant population growth rate ν.

[21] Because there are no aggregate shocks in the present model, the rational expectation of these policy variables and factor prices are actually the ones of perfect foresight. But, they still do not know their future working ability and mortality.

Table 1: Main Variables and Functions in the Model

Individual state: $\mathbf{s}_i = (i, e_i, a_i, b_i)$		
i	$\in I = \{20, ..., 109\}$	Age
e_i	$\in E = [e^{\min}, e^{\max}]$	Working ability (hourly wage) (a)
a_i	$\in A = [a^{\min}, a^{\max}]$	Beginning-of-period wealth (a)
b_i	$\in B = [b^{\min}, b^{\max}]$	Average historical earnings (AIME$\times$12) (a)
Aggregate state: $\mathbf{S}_t = (x_t(\mathbf{s}_i), W_{g,t})$		
$x_t(\mathbf{s}_i)$		Joint distribution of households (b)
$W_{g,t}$		Beginning-of-period government wealth (c)
Policy schedule and rule: $\mathbf{\Psi}_t = \{W_{g,s+1}, C_{g,s}, \tau_{I,s}(.), \tau_{P,s}(.), \tau_{C,s}, tr_{SS,s}(.)\}_{s=t}^{\infty}$		
$W_{g,s+1}$		End-of-period government wealth (c)
$C_{g,s}$		Government consumption (c)
$\tau_{I,s}(.)$		Federal income tax function (d)
$\tau_{P,s}(.)$		Payroll tax function (d)
$\tau_{C,s}$		Consumption tax rate
$tr_{SS,s}(.)$		Social Security benefit function (d)
Household decision rules: $\mathbf{d}(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t) = (c_i(.), h_i(.), a_{i+1}(.))$		
$c_i(.)$		Consumption (a)
$h_i(.)$		Working hours
$a_{i+1}(.)$		End-of-period wealth (a)
Main parameters and other variables		
β	$\in \mathbf{R}_+$	Time preference (e)
ϕ_i	$\in [0, 1]$	Survival rate at the end of age i
μ	$\in \mathbf{R}$	Labor augmenting productivity growth rate
ν	$\in \mathbf{R}$	Population growth rate
w_t	$\in \mathbf{R}_+$	Wage rate (1.0 in the baseline)
r_t	$\in \mathbf{R}$	Interest rate
q_t	$\in \mathbf{R}_+$	Bequests per surviving working-age household (a)

(a) These variables are adjusted by the steady-state (per capita) economic growth rate.

(b) The measure of households is adjusted by the steady-state population growth rate.

(c) The government's net wealth and most aggregate variables (shown below) are adjusted by the steady-state economic growth rate and population growth rate.

(d) The arguments of these functions are adjusted by the steady-state economic growth rate. Time invariant tax and benefit functions imply that the actual schedules are adjusted so that there is no real bracket creep whenever the economy is on the balanced growth path.

(e) The time preference parameter is adjusted by the steady-state economic growth rate. The adjustment depends on the specification of utility function.

Table 2: Other Aggregate Variables in the Model

W_t	$\in \mathbf{R}_+$	National wealth
L_t	$\in \mathbf{R}_+$	Total labor supply
K_t	$\in \mathbf{R}_+$	Capital stock
Y_t	$\in \mathbf{R}_+$	Gross national product
$T_{I,t}$	$\in \mathbf{R}$	Federal income tax revenue
$T_{P,t}$	$\in \mathbf{R}$	Federal payroll tax revenue
$T_{C,t}$	$\in \mathbf{R}$	Federal consumption tax revenue
$Tr_{SS,t}$	$\in \mathbf{R}$	Total OASDI benefits

Note: All aggregate variables are adjusted by the steady-state economic growth and population growth; that is, these variables in the model stay at the same level when the economy is on the balanced growth path.

$$\mathbf{S}_t = (x_t(.), W_{g,t}), \tag{4}$$

$$\mathbf{\Psi}_t = \{W_{g,s+1}, C_{g,s}, \tau_{I,s}(.), \tau_{P,s}(.), \tau_{C,s}, tr_{SS,s}(.)\}_{s=t}^{\infty}. \tag{5}$$

Then, the value function of a household is

$$v(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t) = \max_{c_i, h_i, a_{i+1}} u_i(c_i, h_i) + \beta\,\phi_i E\left[v(\mathbf{s}_{i+1}, \mathbf{S}_{t+1}; \mathbf{\Psi}_{t+1}) \mid e_i\right] \tag{6}$$

subject to

$$\begin{aligned} a_{i+1} = \frac{1}{1+\mu}\{&w_t e_i h_i + (1+r_t)a_i - \tau_{I,t}(w_t e_i h_i, r_t a_i, tr_{SS,t}(i, b_i)) \\ &-\tau_{P,t}(w_t e_i h_i) + tr_{SS,t}(i, b_i) - (1+\tau_{C,t})\,c_i\} \geq a^{\min}, \end{aligned} \tag{7}$$

and $a_{20} = 0,\ a_{110} \geq 0.$[22]

Let $\pi_{i,i+1}(e_{i+1} \mid e_i)$ be the conditional probability for the age $i+1$ working ability being e_{i+1} when the age i working ability is e_i.[23] Then,

$$E\left[v(\mathbf{s}_{i+1}, \mathbf{S}_{t+1}; \mathbf{\Psi}_{t+1}) \mid e_i\right] = \int_E v(\mathbf{s}_{i+1}, \mathbf{S}_{t+1}; \mathbf{\Psi}_{t+1})\,\pi_{i,i+1}(e_{i+1} \mid e_i)\,\mathrm{d}\,e_{i+1}. \tag{8}$$

At the beginning of the next period, the individual state, the aggregate state, and the government policy rules become

$$\mathbf{s}_{i+1} = (i+1, e_{i+1}, a_{i+1} + q_t, b_{i+1}) \quad \text{with} \quad \pi_{i,i+1}(e_{i+1} \mid e_i), \tag{9}$$

[22]Alternatively, we can use $\mathbf{\Psi}_t$ for $\mathbf{\Psi}_{t+1}$ on the right-hand side of the objective function because $\mathbf{\Psi}_t$ includes the information of $\mathbf{\Psi}_{t+1}$.

[23]Since e_{i+1} is a random variable with conditional probability distribution $\pi_{i,i+1}(e_{i+1} \mid e_i)$, $\mathbf{s}_{i+1} = (i+1, e_{i+1}, a_{i+1}, b_{i+1})$ is a random vector. The present paper uses e_i as a realized number and $\mathbf{s}_i$ as a realized vector.

$$\mathbf{S}_{t+1} = \left(x_{t+1}\left(.\right), W_{g,t+1}\right), \tag{10}$$

$$\boldsymbol{\Psi}_{t+1} = \left\{W_{g,s+1}, C_{g,s}, \tau_{I,s}\left(.\right), \tau_{P,s}\left(.\right), \tau_{C,s}, tr_{SS,s}\left(.\right)\right\}_{s=t+1}^{\infty}, \tag{11}$$

where $W_{g,t+1}$ is determined by the government budget constraint, and b_i follows

$$b_{i+1} = \begin{cases} 0 & \text{if } i \leq 24 \\ \frac{1}{i-24}\{(i-25)b_i \frac{w_t}{w_{t-1}} + \min\left(w_t e_i h_i/2, weh_t^{\max}\right)\} & \text{if } 25 \leq i \leq 59 \\ (1+\mu)^{-1} b_i & \text{if } i \geq 60, \end{cases} \tag{12}$$

where $weh_t^{\max}$ is the threshold, which is \$80,400 in 2001. For simplicity, we assume that the highest 35 years of earnings correspond to those years of age between 25 and 59.[24]

The decision rule of an age i household in year t is a function of its individual state $\mathbf{s}_i$, the aggregate state $\mathbf{S}_t$, and the government policy rules $\boldsymbol{\Psi}_t$, and is shown as

$$\mathbf{d}\left(\mathbf{s}_i, \mathbf{S}_t; \boldsymbol{\Psi}_t\right) = \left\{c_i\left(\mathbf{s}_i, \mathbf{S}_t; \boldsymbol{\Psi}_t\right), h_i\left(\mathbf{s}_i, \mathbf{S}_t; \boldsymbol{\Psi}_t\right), a_{i+1}\left(\mathbf{s}_i, \mathbf{S}_t; \boldsymbol{\Psi}_t\right)\right\}. \tag{13}$$

3.2 The Measure of Households

Let $x_t\left(\mathbf{s}_i\right)$ denote the measure of households, and let $X_t\left(\mathbf{s}_i\right)$ be the corresponding cumulative measure. The measure of households is adjusted by the population growth rate. The population of age 20 households is normalized to be unity in the baseline economy on the balanced growth path, that is,

$$\int_E \mathrm{d}X_t\left(20, e_{20}, 0, 0\right) = 1. \tag{14}$$

Let $\mathbf{1}_{[a=y]}$ be an indicator function that returns 1 if $a = y$ and 0 if $a \neq y$. Then, the law of motion of the measure of households is, for $i \in I = \{20, ..., 109\}$,

$$x_{t+1}\left(\mathbf{s}_{i+1}\right) = \frac{\phi_i}{1+\nu}\int_{E\times A\times B} \mathbf{1}_{[a_{i+1}=a_{i+1}(\mathbf{s}_i,\mathbf{S}_t;\boldsymbol{\Psi}_t)+q_t]} \tag{15}$$
$$\times \mathbf{1}_{[b_{i+1}=b_{i+1}(w_t e_i h_i(\mathbf{s}_i,\mathbf{S}_t;\boldsymbol{\Psi}),b_i)]}\, \pi_{i,i+1}(e_{i+1}\,|\,e_i)\,\mathrm{d}X_t\left(\mathbf{s}_i\right).$$

For simplicity, accidental bequests due to uncertain life span are captured by the government and distributed equally to all surviving working-age households in a lump-sum manner. Nishiyama (2002) presents a four-period model with altruistically motivated bequests;

[24] Social Security benefits in the United States are computed on the basis of the highest 35 years of earnings, adding an additional state variable to our model. Earnings before age 60 are wage indexed, and earnings after age 60 are price indexed. The approximation of AIME by the average historical earnings follows previous Social Security literature, for example, Huggett and Ventura (1999) and Di Nardi and others (1999).

incorporating altruism into the current model in a realistic manner would make the model computationally intractable at current computer speeds. The accidental bequests per household at the end of year t is

$$q_t = q(\mathbf{S}_t; \mathbf{\Psi}_t) = \frac{\sum_{i=20}^{109} (1-\phi_i) \int_{E\times A\times B} a_{i+1}(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t) \, \mathrm{d}X_t\,(\mathbf{s}_i)}{\sum_{i=20}^{64} \phi_i \int_{E\times A\times B} \, \mathrm{d}X_t\,(\mathbf{s}_i)}. \tag{16}$$

The steady-state condition is

$$\mathbf{S}_{t+1} = \mathbf{S}_t \tag{17}$$

for all t and $\mathbf{s}_i \in I \times E \times A \times B$.

3.3 The Firms' Problem

National wealth W_t is the sum of total private wealth and government net wealth $W_{g,t}$. Total labor supply L_t is measured in efficiency units.

$$W_t = W(\mathbf{S}_t) = \sum_{i=20}^{109} \int_{E\times A\times B} a_i \, \mathrm{d}X_t\,(\mathbf{s}_i) + W_{g,t}, \tag{18}$$

$$L_t = L(\mathbf{S}_t; \mathbf{\Psi}_t) = \sum_{i=20}^{109} \int_{E\times A\times B} e_i \, h_i(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}_t) \, \mathrm{d}X_t\,(\mathbf{s}_i)\,. \tag{19}$$

There are a large number of perfectly competitive firms in this economy. In a closed economy, the capital stock is equal to national wealth, that is,

$$K_t = W_t, \tag{20}$$

and gross national product Y_t is determined by a constant-returns-to-scale production function,

$$Y_t = F(K_t, L_t). \tag{21}$$

The profit-maximizing condition of the firm is

$$r_t + \delta = F_K(K_t, L_t), \tag{22}$$

$$w_t\,(1 + \tau'_{P,t}) = F_L(K_t, L_t), \tag{23}$$

where δ is the depreciation rate of capital and $\tau'_{P,t}$ is the marginal payroll tax rate.[25]

[25] U.S. payroll taxes are divided equally between firms and employees. While the incidence of the tax does not depend on this division, our model explicitly includes the division for calibration purposes. In doing so, we ignore the small fraction of the representative firm's workforce whose wages exceed the payroll tax ceiling. However, the ceiling is enforced on the worker's share, as shown earlier.

3.4 The Government's Policy Rules

Government tax revenue consists of federal income tax $T_{I,t}$, payroll tax for Social Security $T_{P,t}$, and consumption tax $T_{C,t}$. These revenues equal:

$$\begin{aligned} T_{I,t} &= T_I(\mathbf{S}_t;\mathbf{\Psi}_t) \\ &= \sum_{i=20}^{109}\int_{E\times A\times B}\tau_{I,t}\left(w_t e_i h_i(\mathbf{s}_i,\mathbf{S}_t;\mathbf{\Psi}_t), r_t a_i, tr_{SS,t}\left(i,b_i\right)\right)\,\mathrm{d}X_t\left(\mathbf{s}_i\right), \end{aligned} \tag{24}$$

$$T_{P,t} = T_P(\mathbf{S}_t;\mathbf{\Psi}_t) = 2\times\sum_{i=20}^{109}\int_{E\times A\times B}\tau_{p,t}\left(w_t e_i h_i(\mathbf{s}_i,\mathbf{S}_t;\mathbf{\Psi}_t)\right)\,\mathrm{d}X_t\left(\mathbf{s}_i\right), \tag{25}$$

$$T_{C,t} = T_C(\mathbf{S}_t;\mathbf{\Psi}_t) = \sum_{i=20}^{109}\int_{E\times A\times B}\tau_{C,t}c_i(\mathbf{s}_i,\mathbf{S}_t;\mathbf{\Psi}_t)\,\mathrm{d}X_t\left(\mathbf{s}_i\right).$$

Total Social Security benefits $Tr_{SS,t}$ equals

$$Tr_{SS,t} = Tr_{SS}(\mathbf{S}_t;\mathbf{\Psi}_t) = \sum_{i=20}^{109}\int_{E\times A\times B}tr_{SS,t}\left(i,b_i\right)\,\mathrm{d}X_t\left(\mathbf{s}_i\right). \tag{26}$$

The law of motion of the government wealth (normalized by productivity growth and population growth) is

$$\begin{aligned} W_{g,t+1} &= W_g(\mathbf{S}_t;\mathbf{\Psi}_t) \\ &= \frac{1}{\left(1+\mu\right)\left(1+\nu\right)}\left\{\left(1+r_t\right)W_{g,t} + \left(T_{I,t}+T_{P,t}+T_{C,t}\right) - Tr_{SS,t} - C_{g,t}\right\}, \end{aligned} \tag{27}$$

where $C_{g,t}$ is government consumption.

3.5 Recursive Competitive Equilibrium

Definition **Recursive Competitive Equilibrium (Steady State):** Let $\mathbf{s}_i = (i, e_i, a_i, b_i)$ be the individual state of households and $\mathbf{\Psi}$ be the time-invariant government policy rules,

$$\mathbf{\Psi} = \left\{W_g, C_g, \tau_I(.), \tau_P(.), \tau_C, tr_{SS}(.)\right\}.$$

Factor prices (r, w); accidental bequests q; the policy variables $(W_g, C_g, \tau_C, tr_{LS})$; the parameters φ of policy functions $(\tau_I(.), \tau_P(.), tr_{SS}(.))$; the value function of households, $v\left(\mathbf{s}_i;\mathbf{\Psi}\right)$; the decision rule of households,

$$\mathbf{d}(\mathbf{s}_i;\mathbf{\Psi}) = \{c_i(\mathbf{s}_i;\mathbf{\Psi}), h_i(\mathbf{s}_i;\mathbf{\Psi}), a_{i+1}(\mathbf{s}_i;\mathbf{\Psi})\};$$

and the measure of households, $x\,(\mathbf{s}_i)$, are in a steady-state recursive competitive equilibrium if, in every period, each household solves the utility maximization problem (3) – (7) taking $\mathbf{\Psi}$ as given; the firm solves the profit maximization problem, and the capital and labor markets clear, that is, (18) – (23) hold; the government policy rules satisfy (24) – (27); the goods market clears; and the measure of households is constant that is, (17) holds.

Definition **Recursive Competitive Equilibrium (Equilibrium Transition Path):** Let $\mathrm{s}_i = (i, e_i, a_i, b_i)$ be the individual state of households, $\mathbf{S}_t = (x_t(\mathbf{s}_i), W_{g,t})$ be the aggregate state of the economy, and $\mathbf{\Psi}_t$ be the government policy rules known at the beginning of year t,

$$\mathbf{\Psi}_t = \{W_{g,s+1}, C_{g,s}, \tau_{I,s}(.), \tau_{P,s}(.), \tau_C, tr_{SS,s}(.)\}_{s=t}^{\infty}.$$

A series of factor prices, accidental bequests, the policy variables, and the parameters of policy functions,

$$\mathbf{\Omega} =\{r_s, w_s, q_s, W_{g,s+1}, C_{g,s}, \tau_{C,s}, \varphi_s\}_{s=t}^{\infty};$$

the value function of households,$\{v\,(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty}$; the decision rule of households,

$$\{\mathbf{d}(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty} = \{c_i(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s), h_i(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s), a_{i+1}(\mathbf{s}_i, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty};$$

and a series of the measure of households, $\{x_s(\mathbf{s}_i)\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, in every period $s = t, ..., \infty$, each household solves the utility maximization problem (3) – (7) taking $\mathbf{\Psi}_t$ as given; the firm solves the profit maximization problem, and the capital and labor markets clear, that is, (18) – (23) hold; the government policy rules satisfy (24) – (27); and the goods market clears.

4 Calibration

Table 3 summarizes the parameter choices. For the baseline economy on a balanced growth path, the degree of time preference β is chosen so that the capital-output ratio is 2.8; total factor productivity A is chosen so that the wage rate equals unity, and the share parameter of consumption α is chosen so that the average annual working hours of married couples

Table 3: Parameters

Time preference parameter	β	0.986
Share parameter for consumption	α	0.473
Coefficient of relative risk aversion	γ	2.0
Capital share of output	θ	0.32
Depreciation rate of capital stock	δ	0.046
Long-term real growth rate	μ	0.018
Population growth rate	ν	0.010
Total factor productivity	A	0.983

between the ages of 20 and 64 are consistent with U.S. data. As explained below, a Cobb-Douglas-CRRA utility function and a Cobb-Douglas production function are used for the calibration.[26]

The following sections describe the choice of functional forms and parameter values, the choice of four target variables and values.

4.1 Households

Utility Function. Like the recent important paper by Conesa and Krueger (1999) that focuses on Social Security reform, our model has elastic labor supply. We use the following Cobb-Douglas utility function with constant relative risk aversion (CRRA), which is compatible with the existence of a steady state:

$$u(c_i, h_i) = \frac{\left\{((1+n_i/2)^{-\zeta}\, c_i)^{\alpha}(h_i^{\max} - h_i)^{1-\alpha}\right\}^{1-\gamma}}{1-\gamma},$$

where γ is the coefficient of relative risk aversion, n_i is the number of dependent children, ζ is the consumption adjustment parameter, and $h_i^{\max}$ is the maximum working hours.[27] The coefficient of relative risk aversion is assumed to be 2.0. The numbers of dependent children by age cohorts are calculated from the Panel Study of Income Dynamics (PSID) 1993 Family Data (see Table 4). The consumption adjustment parameter is assumed to be 0.6.

The annual working hours in the model are the sum of the working hours of a husband

[26] The calibration basically followed that of a four-period altruism model in Nishiyama (2002) but extended it significantly because the present model is a 90-period model.

[27] In this setting, the growth-adjusted β becomes $\beta(1+\mu)^{\alpha(1-\gamma)}$, which is 0.977 in the calibration.

Table 4: Number of People Under 18 Years of Age in a Married Household

Age cohorts	Number of people under age 18	Age cohorts	Number of people under age 18
20-24	0.895	45-49	1.011
25-29	1.149	50-54	0.445
30-34	1.617	55-59	0.188
35-39	1.905	60-64	0.094
40-44	1.649	65-plus	0.000*

Source: Authors' calculations from the Panel Study of Income Dynamics (PSID) 1993 Family Data.

*The number 0.000 for ages 65-plus is an assumption and not from PSID data.

and a wife. The average working hours of married households between ages 20 and 64 are 3,368 hours in the 1998 Survey of Consumer Finances (SCF). The maximum working hours are set to be 8,760, which is simply calculated from two persons times 12 hours times 365 days. In this calibration, the parameter α is chosen to be 0.473 so that average working hours of age 20 and age 64 become 3,368 hours in the steady-state baseline economy.

Working Ability. The working ability in this calibration corresponds to the hourly wage (labor income per hour) of each household in the 1998 SCF. The average hourly wage of a married couple (family members #1 and #2 in SCF) used for the calibration is calculated by

$$\text{Hourly Wage} = \frac{\text{Regular and Additional Salaries (\#1 + \#2) + Welfare or Assistance}}{\text{Working Hours (\#1 + \#2)}}.$$

To capture the earnings risk a household is exposed to more precisely, unemployment or worker's compensation, TANF, food stamps, and other forms of welfare or assistance are added to the salaries before calculating the hourly wage. Table 5 shows the eight discrete levels of working abilities of five-year age cohorts.[28] Taking a five-year moving average of these numbers, we obtain the working ability of each age cohort. According to Bureau of Labor Statistics data, the average hourly earnings of production workers have increased by 16.7 percent during the years from 1997 to 2001. In the calibration, the numbers in the table are multiplied by 1.167 to convert the hourly wages in 1997 into those in 2001.

[28] Here, the hourly wage of a household that works less than 520 hours (10 hours a week per couple) is assumed to be zero. In the real economy, some households have fairly high working ability but choose not to work (for example, because of schooling). One observation of the age 20-24 cohort, which has an hourly wage of $193.01, is ignored.

Table 5: Working Abilities of a Household (in U.S. Dollars per Hour)

	Percentile	Age cohorts					
		20-24	25-29	30-34	35-39	40-44	45-49
e^1	0-20th	3.83	5.42	5.42	6.93	6.12	6.59
e^2	20-40th	7.07	8.64	9.76	11.28	11.36	12.70
e^3	40-60th	8.68	10.91	13.46	15.01	15.59	17.22
e^4	60-80th	10.67	14.01	18.08	19.96	22.09	23.22
e^5	80-90th	14.05	17.52	27.17	25.27	30.89	31.58
e^6	90-95th	18.20	22.48	33.71	33.38	48.59	44.31
e^7	95-99th	28.43	32.64	54.11	52.16	76.13	86.50
e^8	99-100th	36.81	46.09	167.15	186.47	221.34	301.99
	Percentile	Age cohorts					
		50-54	55-59	60-64	65-69	70-74	75-79
e^1	0-20th	5.48	3.52	0.00	0.00	0.00	0.00
e^2	20-40th	11.53	10.06	4.54	0.00	0.00	0.00
e^3	40-60th	16.16	14.26	11.18	2.82	0.00	0.00
e^4	60-80th	23.44	21.28	18.16	10.37	1.81	0.00
e^5	80-90th	32.14	30.93	28.56	19.48	12.57	0.00
e^6	90-95th	43.01	44.10	59.36	27.68	29.03	1.96
e^7	95-99th	78.61	85.29	96.22	59.34	64.91	14.25
e^8	99-100th	314.59	379.44	421.55	299.25	195.73	146.14

Source: Authors' calculations from 1998 SCF data.

Markov Transition Matrix. The Markov transition matrix, Γ, of working ability is calculated from the hourly wage of people ages 30-39 in 1991 in the PSID individual data. To make the working ability process more persistent, the matrix is calculated as the transition from the average of years 1989 and 1990 to the average of years 1990 and 1991.

$$\Gamma = \begin{pmatrix} 0.7674 & 0.2049 & 0.0183 & 0.0045 & 0.0049 & 0.0000 & 0.0000 & 0.0000 \\ 0.1810 & 0.6033 & 0.1844 & 0.0129 & 0.0000 & 0.0086 & 0.0046 & 0.0052 \\ 0.0388 & 0.1517 & 0.6768 & 0.1220 & 0.0011 & 0.0046 & 0.0050 & 0.0000 \\ 0.0126 & 0.0361 & 0.1039 & 0.7210 & 0.0980 & 0.0139 & 0.0145 & 0.0000 \\ 0.0000 & 0.0081 & 0.0332 & 0.2360 & 0.6306 & 0.0676 & 0.0145 & 0.0100 \\ 0.0000 & 0.0000 & 0.0000 & 0.0582 & 0.3224 & 0.5303 & 0.0891 & 0.0000 \\ 0.0007 & 0.0000 & 0.0000 & 0.0354 & 0.0000 & 0.2827 & 0.6433 & 0.0379 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.3553 & 0.6447 \end{pmatrix},$$

Table 6: Survival Rates in the United States (Weighted Average of Males and Females of Each Age)

Age	Survival Rate	Age	Survival Rate	Age	Survival Rate	Age	Survival Rate	Age	Survival Rate
20	0.999113	40	0.997978	60	0.989365	80	0.938048	100	0.676941
21	0.999066	41	0.997820	61	0.988361	81	0.931804	101	0.658846
22	0.999037	42	0.997654	62	0.987195	82	0.924980	102	0.639629
23	0.999028	43	0.997465	63	0.985840	83	0.917566	103	0.619216
24	0.999032	44	0.997267	64	0.984324	84	0.909481	104	0.597532
25	0.999043	45	0.997044	65	0.982631	85	0.900623	105	0.574495
26	0.999049	46	0.996797	66	0.980851	86	0.890904	106	0.550021
27	0.999041	47	0.996534	67	0.979101	87	0.880258	107	0.524022
28	0.999014	48	0.996258	68	0.977433	88	0.868650	108	0.496402
29	0.998970	49	0.995960	69	0.975763	89	0.856070	109	0.467066
30	0.998919	50	0.995626	70	0.973892	90	0.842518		
31	0.998865	51	0.995247	71	0.971745	91	0.828007		
32	0.998804	52	0.994823	72	0.969406	92	0.812554		
33	0.998735	53	0.994352	73	0.966856	93	0.796181		
34	0.998660	54	0.993826	74	0.964033	94	0.778913		
35	0.998573	55	0.993231	75	0.960839	95	0.761457		
36	0.998475	56	0.992570	76	0.957219	96	0.744011		
37	0.998368	57	0.991857	77	0.953175	97	0.726790		
38	0.998250	58	0.991094	78	0.948673	98	0.710031		
39	0.998122	59	0.990263	79	0.943665	99	0.693980		

Source: Authors' calculations from the Social Security Bulletin: Annual Statistical Supplement (2001). In the calibration, the survival rate at the end of age 109 is set to zero.

where $\Gamma(j,k) = \pi(e_{i+1} = e_{i+1}^k \,|\, e_i = e_i^j)$.

Population Growth and Mortality. The population growth rate ν is assumed to be 1.0 percent per year. The survival rates ϕ_i at the end of age $i = \{20, ..., 109\}$ are the weighted average of males and females in 1998 from Social Security Administration data (2001). The survival rates at the end of age 109 are replaced by zero.

4.2 The Firm

Production Function. Production takes the Cobb-Douglas form,

$$F(K_t, L_t) = A_t K_t^{\theta} L_t^{1-\theta}.$$

To compute GNP, we use the sum of working hours in efficiency units as total labor supply L_t. The capital share of output θ is chosen by

$$\theta = 1 - \frac{\text{Compensation of Employees } + (1-\theta) \times \text{Proprietors' Income}}{\text{National Income } + \text{ Consumption of Fixed Capital}}.$$

The average of θ in 1996-1998 is 0.32. The annual growth rate μ is assumed to be 1.8 percent. The annual population growth rate ν is assumed to be 1.0 percent. Total factor productivity A is chosen to be 0.983 so that the wage per unit of efficient labor is normalized to be unity.

Fixed Capital and Private Wealth. The fixed capital K_t for the calibration is obtained by "fixed reproducible tangible wealth" minus "durable goods owned by consumers" in the Survey of Current Business (1997). In 1990-1996, fixed capital accounted for 89.7 percent of fixed reproducible tangible wealth, and the capital-GDP ratio is approximately 2.8.

To connect the total private wealth with the fixed capital, it is assumed that all of the private capital is owned by households and that part of the government-owned fixed capital is effectively owned by households in the form of government bonds.

$$\begin{aligned}\text{Private Wealth (Excluding Durables)} &= \text{Fixed Capital} \\ &- \text{Government-Owned Fixed Capital} \\ &+ \text{Government Bonds Owned by Private Sector.}\end{aligned}$$

In the model, fixed capital is the sum of private wealth (excluding durables) and net government wealth. Based on the data from 1990 to 1996, net government wealth in the baseline economy is assumed to be 6.5 percent of total private wealth.

The Depreciation Rate of Fixed Capital. The depreciation rate of fixed capital δ is chosen by

$$\delta = \frac{\text{Total Gross Investment}}{\text{Fixed Capital}} - \mu - \nu.$$

In 1997-2000, gross private domestic investment accounted for, on average, 17.5 percent of GDP, and gross government investment (federal and state) accounted for 3.2 percent of GDP. When the capital-output ratio is 2.8, the ratio of gross investment to fixed capital is 7.4

percent. Subtracting the productivity and population growth rates, the annual depreciation rate is assumed to be 4.6 percent.

4.3 Taxes and Transfers

Income Taxes. For federal income tax, the model uses the following tax function in Gouveia and Strauss (1994),

$$\text{Federal Income Tax } = \phi_0 \left(y - (y^{-\phi_1} + \phi_2)^{-1/\phi_1} \right) \times \phi_{adj},$$

where y is the taxable income (in thousands of dollars) of a household, which includes the taxable portion of Social Security benefits. In 2001, the standard deduction for a married household was \$7,600, and the exemption was \$2,900 per person. When the parameters are $\phi_0 = 0.41$, $\phi_1 = 0.85$, $\phi_2 = 0.015$, and $\phi_{adj} = 1.0$, this function replicates the statutory income tax schedule. But because of itemized deductions, the effective tax rate of high-income households is much lower.[29] Since in 2000 the ratio of total private income tax to nominal GDP was 0.102, ϕ_{adj} is assumed to be 0.604 so that income tax revenue is 10.2 percent of GDP in the steady-state equilibrium.

In addition to federal income tax, a 4.0 percent state tax is assumed for an income (excluding Social Security benefits) above the same standard deduction and exemptions.

Social Security. The tax rate levied on both employers and employees for Old-Age, Survivors, and Disability Insurance (OASDI) is 6.2 percent, and the tax rate for Medicare (HI) is 1.45 percent. In 2001, employee compensation above \$80,400 was not taxable for OASDI. So, the firm's profit-maximization problem becomes

$$w \times (1 + \text{Marginal Payroll Tax Rate}) = AF_L(K, L),$$

where the marginal payroll tax rate is 0.0765 (equal to 0.062 + 0.0145).

Social Security benefits are based on each worker's average indexed monthly earnings (AIME), $b_i/12$, and the replacement rate schedule in the United States. The replacement rates are 90 percent for the first \$561, 32 percent for amounts between \$561 and \$3,381, and 15 percent for amounts above \$3,381.

[29] See Gouveia and Strauss (1994) for effective federal tax rates.

The benefits received by retired workers consisted of 69 percent of total OASDI benefits in December 2000.[30] The calibration simply assumes that each elderly household receives other benefits—those for spouses, children, and disabled workers—proportionally. Benefits are multiplied by 1.543 so that total OASDI benefits are equal to OASDI tax revenue in the baseline economy.

5 Policy Experiments

We now consider the effects of reducing the marginal income tax rates by 10 percent proportionately and raising the consumption tax rate to replace the lost revenue. For this policy change, we will show four different results—with or without the Lump-Sum Redistribution Authority, and with or without wage uncertainty (life-span uncertainty is operative in all the experiments). But before reporting these results, we first explain the construction of the LSRA, followed by a discussion of our construction of the representative-agent OLG economy without wage uncertainty.

5.1 The Lump-Sum Redistribution Authority

We measure the welfare gain or loss from a policy change using a Lump-Sum Redistribution Authority, following Auerbach and Kotlikoff (1987), who used an LSRA in their representative-agent OLG model. We extend their analysis to a stochastic OLG model with heterogeneous agents.

Suppose that a new policy is announced at the beginning of period 1. The LSRA first makes lump-sum transfers (taxes if negative) to *each living* household to bring their remaining expected lifetime utilities back to their levels in the baseline economy. Since the welfare gain or loss depends on each household's own state, these lump-sum transfers (taxes)[31] are shown as $tr_{R_1}(\mathbf{s}_i)$ where $\mathbf{s}_i = (i, e_i, a_i, b_i)$ such that

$$v\left(i, e_i, a_i + tr_{R_1}(\mathbf{s}_i), b_i, \mathbf{S}_1; \mathbf{\Psi}_1\right) = v\left(\mathbf{s}_i, \mathbf{S}_0; \mathbf{\Psi}_0\right).$$

Next, the LSRA makes lump-sum transfers (taxes) to *each future* household (that is, newborn households in periods 2, 3, ...) to make them as well off in the baseline economy, conditional

[30] The number is from Social Security Administration (2001), Table 5.A4.

[31] These lump-sum transfers are negative of the compensating variations in wealth for current households.

on their initial state as independent economic actors. These transfers (taxes) are shown as $tr_{R_2}(\mathbf{s}_{20}, t)$ such that, for $t = 2, 3, ...,$

$$v\left(20, e_i, a_i + tr_{R_2}(\mathbf{s}_{20}, t), b_i, \mathbf{S}_t; \mathbf{\Psi}_t\right) = v\left(\mathbf{s}_{20}, \mathbf{S}_0; \mathbf{\Psi}_0\right).$$

Finally, the LSRA makes additional lump-sum transfers (taxes) to the newborn households in periods 2, 3, ..., so that the net present value across all the LSRA transfers at the beginning of period 1 is zero. We assume these additional transfers are uniform on a growth-adjusted basis. When the steady-state growth rate is μ, the additional transfers are shown as tr_{R_3}, where

$$\sum_{\mathbf{s}_i} tr_{R_1}(\mathbf{s}_i)\, x_1(\mathbf{s}_i) + \sum_{t=2}^{\infty} \frac{(1+\mu)^{t-1} \sum_{\mathbf{s}_{20}} \left\{ tr_{R_2}(\mathbf{s}_{20}, t) + tr_{R_3} \right\} x_t(\mathbf{s}_{20})}{\prod_{k=1}^{t-1} (1 + r_k)} = 0.$$

When $tr_{R_3} > 0$, all of the current households would be as well off as the baseline economy and all of the future households would be strictly better off; hence, the new policy is Pareto improving after lump-sum redistributions. When $tr_{R_3} < 0$, the alternative policy is Pareto inferior after lump-sum redistributions.

The wealth held by the LSRA (normalized by the productivity growth and population growth), $\{W_a\}_{t=1}^{\infty}$, is derived as

$$\begin{aligned}
W_{a,1} &= 0, \\
W_{a,2} &= \frac{-\sum_{\mathbf{s}_i} tr_{R_1}(\mathbf{s}_i)\, x_1(\mathbf{s}_i)}{(1+\mu)(1+\nu)}, \\
W_{a,t} &= \frac{(1 + r_{t-1}) W_{a,t-1} - \sum_{\mathbf{s}_{20}} \left\{ tr_{R_2}(\mathbf{s}_{20}, t) + tr_{R_3} \right\} x_t(\mathbf{s}_{20})}{(1+\mu)(1+\nu)}
\end{aligned}$$

for $t = 3, 4, ..., \infty$. National wealth W_t is the sum of total private wealth, government net wealth $W_{g,t}$, and the LSRA wealth $W_{a,t}$. So, equation (18) is replaced with

$$W_t = \sum_{i=20}^{109} \int_{E \times A \times B} a_i \, \mathrm{d}X_t(\mathbf{s}_i) + W_{g,t} + W_{a,t},$$

and the government policy rule is defined as

$$\mathbf{\Psi}_t = \left\{ W_{g,s+1}, W_{a,s+1}, C_{g,s}, \tau_{I,s}(.), \tau_{P,s}(.), \tau_C, tr_{SS,s}(.) \right\}_{s=t}^{\infty}.$$

Table 7: Working Abilities of a Representative Household (In U.S. Dollars per Hour)

Age Cohorts	20-24	25-29	30-34	35-39	40-44	45-49
e^1	9.87	12.44	17.58	18.78	21.81	23.80
Age Cohorts	50-54	55-59	60-64	65-69	70-74	75-79
e^1	22.98	22.33	20.67	11.34	7.62	2.13

Source: Authors' calculations from 1998 SCF data.

5.2 A Deterministic OLG Economy with Representative Agents

In order to investigate the importance of idiosyncratic wages, we also consider a deterministic version of our model. We assume that the working ability schedule of the representative household is that shown in Table 7. Those numbers are the weighted average of the values shown in Table 5. The representative household does not receive any working ability shocks, but there is still lifetime uncertainty.

Similar to the stochastic OLG economy with heterogeneous agents, the model is calibrated to the U.S. economy so that the capital-output ratio is 2.8 and average working hours (per couple) of working age households (ages 20-64) are 3,368 hours. Also, the wage rate (per efficiency unit of labor) is normalized to unity. The obtained parameters are as follows:

Table 8: Parameters of the Representative Agent Model

β	α	A
1.019	0.431	0.983

5.3 A 10 Percent Income Tax Cut with an Increase in a Consumption Tax

In the baseline economy, the federal income tax function is

$$0.41 \times \left(y - (y^{-0.85} + 0.015)^{-1/0.85}\right) \times \phi_{adj},$$

and the adjustment factor ϕ_{adj} is set to 0.604. In this policy experiment, this ϕ_{adj} is lowered by 10 percent to 0.544. That change reduces marginal income tax rates proportionately by 10 percent, keeping deductions and exemptions unchanged.

From the government budget constraint (27), the consumption tax revenue needed to finance the tax cut is

$$T_{C,t} = (1+\mu)(1+\nu) W_{g,t+1} - (1+r_t) W_{g,t} - (T_{I,t} + T_{P,t}) + Tr_{SS,t} + C_{g,t}.$$

When the aggregate private consumption is

$$C = \sum_{i=20}^{109} \int_{E \times A \times B} c_i(\mathbf{s}_i, \mathbf{S}_t; \mathbf{\Psi}) \, \mathrm{d}X_t(\mathbf{s}_i),$$

and a balanced budget is assumed, that is, $W_{g,t+1} = W_{g,t}$, the consumption tax rate $\tau_{C,t}$ is

$$\tau_{C,t} = \{((1+\mu)(1+\nu) - (1+r_t)) W_{g,t} - (T_{I,t} + T_{P,t}) + Tr_{SS,t} + C_{g,t}\} / C.$$

5.3.1 In a Stochastic Economy with Heterogeneous Households

Table 9 shows the results of the tax change in a stochastic OLG economy with heterogeneous households. (Tables 9 through 17 and Figures 1 through 3 are included at the end of this paper.)

Without the Lump-Sum Redistribution Authority. In the long run, national wealth, which is equal to capital stock in a closed economy, increases by 2.9 percent from the baseline economy, labor supply increases by 0.6 percent, and GNP increases by 1.3 percent (see the bottom panel). The interest rate decreases by 0.18 percentage points and the wage rate increases by 0.7 percent. To keep the government wealth (debt) at the same level as that of the baseline, the consumption tax rate would have to be raised by 1.3 percentage points in the long run if the income tax rates were reduced by 10 percent. Although GNP would increase throughout the transition path, private consumption would decrease in the short run. In the first period, a simple ad hoc measure of "social welfare," computed as a population-weighted average of the compensating variations of people alive at the time of the reform, would decline by 1.54 percent of national wealth. When this measure is applied to new households who enter the economy in year 2 and thereafter, they are worse off in the short run but better off in the long run. (See also Figure 1.)

With the Lump-Sum Redistribution Authority. To evaluate the overall efficiency effect of the tax change, the top panel of Table 9 shows the results with the LSRA. In the first year, there is no welfare change because the LSRA compensates for all of the welfare gains or losses of the current households. In year 2 and thereafter, new households are worse off by 0.02 percent in total of national wealth. With the LSRA, the increase in private wealth is slightly larger than that without the LSRA. But, national wealth would increase only by 2.1 percent because of the debt of the LSRA. Government wealth (including the LSRA) would be reduced by 2.8 percent as a percentage of GNP in the long run.

Welfare Results by Cohorts. Table 10 shows the welfare changes faced by agents in a particular birth cohort and the (stochastic) income class they are in at the time of the reform. This disaggregation provides the most meaningful set of welfare measures because it does not require assuming a social welfare measure that aggregates compensating variations across households. The numbers are shown in units of $1,000.[32] Without the LSRA, most of the current households are worse off due to the lump-sum tax on existing wealth, especially retired households with the most assets. One important exception is the high-working-ability households who would benefit from the much lower income tax rates. Future households tend to gain from the tax change. But this gain is not enough to compensate the loss of current households. To see this fact, notice that after the LSRA is turned on, current households gain exactly $0 by construction, but all future households are worse off by $3,200, which represents a one-time loss per household at the beginning of age 20.[33] (See also Figure 3(a).) Keep in mind that these numbers correspond to replacing only 10 percent of the progressive income tax system; full replacement would produce even larger losses.

5.3.2 In a Deterministic Economy with Representative Households

In order to estimate the importance of earnings uncertainty, Table 11 shows the result of the same tax reform within the deterministic version of our OLG economy.

Without the Lump-Sum Redistribution Authority. In the long run, national wealth would increase by 3.7 percent from the baseline economy. This increase is larger than that in a sto-

[32] Unlike in the Auerbach and Kotlikoff model, our reported values cannot be expressed as a share of remaining full lifetime income since that value is stochastic in our model.

[33] All dollar values in the model are growth-adjusted. A household born in year t is actually worse off by $3,200$\times(1+\mu)^{t-1}$, where μ is the per capita growth rate in the baseline economy.

chastic and heterogeneous economy. While precautionary saving increases in the stochastic version of our model as the risk sharing in the progressive income tax system is reduced, overall saving is more responsive to changes in after-tax interest rates in our deterministic model with representative households. Labor supply is also more responsive in the deterministic model, increasing by 1.5 percent in the long run, causing GNP to increase by 2.2 percent. The consumption tax rate increases by 0.8 percentage points in the long run. Contrary to the stochastic and heterogeneous economy, private consumption would increase throughout the transition path. So does our ad hoc social welfare measure, described earlier, which averages across compensating variations in wealth. (See also Figure 2.) The bottom panel of Table 10 shows that, without the LSRA, older households lose several thousand dollars from the reform while younger households gain. Households born in the long run gain over $22,000.

With the Lump-Sum Redistribution Authority. The top panel of Table 11 shows the results with the operative LSRA. Welfare of current households is unchanged by construction. Although private wealth would increase by only 2.6 percent in the long run, total national wealth would increase by 5.1 percent. The reason is that, in this version of the model, younger households at the time of the reform gain more than the older households lose. (See Figure 3 (b).) Holding their remaining lifetime utility fixed at the baseline level, therefore, nets the LSRA a positive level of resources in the short run. As a result, future households are better off by 0.26-0.27 percent of national wealth under our ad hoc social welfare measure. The bottom panel of Table10 shows the disaggregated results by birth cohort. Again, by construction of the LSRA, households alive at the time of the reform gain $0. But future households are *better* off by $25,100 per household, a remarkable difference relative to the stochastic case considered earlier.

5.3.3 In a Small Open Economy

Table 12 shows the results of the tax change in a small open economy, in which the interest rate and the wage rate are kept at the same levels as those in the baseline economy. As with the closed economy case, this case assumes a stochastic OLG economy with heterogeneous households.

Both with and without the LSRA, the long-run increase in national wealth is larger, and the long-run increase in labor supply is smaller, than in a closed economy. The increase in consumption is slightly smaller in the long run, as is the welfare gain of the newborn households.

Without the LSRA, the average welfare loss of current households is smaller than it would be in a closed economy. After the first several years, the interest rate in a small open economy is higher, which attenuates the welfare loss of elderly households. The one-time loss of current households totals 0.56 percent of national wealth.

With the LSRA, the welfare loss of current households is $0 by construction; future households will lose a relatively modest $1,000. (See Table 13 and Figure 3(c).)

5.3.4 A Progressive Consumption Tax

Table 14 shows the results of the tax change if a progressive consumption tax is introduced instead of the proportional consumption tax considered earlier. In this case, a flat consumption tax is levied on each household's annual consumption above $20,000. To keep the government's wealth at the same level as in the baseline, the consumption tax rate will have to be higher by 1.9 percent in the long run.

Both with and without the LSRA, the long-run increase in national wealth is smaller than it would be in the case of a proportional consumption tax. At 1.84 percent of national wealth, the one-time welfare loss of current households is larger because the consumption tax rate is higher.

With the LSRA, however, the welfare loss of future households will be $2,800, which is smaller than the $3,200 loss in the proportional tax case without a deduction. The main reason for the smaller loss is that the $20,000 deduction produces some progressivity in the tax system, which has a positive risk-sharing effect. (See Table 15 and Figure 3(d).) However, future generations still face a loss, in part because a deduction is not as powerful a risk-sharing device as the progressive marginal tax rate schedule found in the prereform system. For many households, income (and, hence, consumption) typically fluctuates above the deduction level, for two reasons. First, unemployment leads to a drawing down of previous assets in order to support a level of consumption above the deduction. Second, unemployment insurance income (which is included in our income measure) provides some income as well. For those households, therefore, a modest decline in normal labor income produces only a small reduction in their average tax rates and no reduction in their marginal tax rates.

5.3.5 When Households Are More Risk Averse

Table 16 shows the results of the tax change when the coefficient of relative risk aversion, gamma, equals 4.0 instead of 2.0.

Without the LSRA, the long-run increase in national wealth is larger because the precautionary savings of households increase. Although other aggregate variables—labor supply, GDP, and consumption—are roughly the same, the welfare gain of future households is slightly smaller than the loss in the economy when the coefficient equals 2.0 because those households face higher risk after the tax change. The welfare loss of current households is larger, for the same reason.

With the LSRA, future households will face a one-time loss of $5,500 from the tax change, which is larger than the loss when the coefficient is 2.0. (See Table 17 and Figure 3(e).)

6 Concluding Remarks

Tax reform has been analyzed in numerous papers. The calculation of the actual efficiency gains or losses stemming from tax reform, however, has been previously limited to models that assume an infinite-horizon representative agent, making it difficult to analyze the risk-sharing benefits of progressive taxation in the prereform system. This paper presents a new finite-horizon OLG model with idiosyncratic earnings and longevity risks. Following the deterministic OLG model developed by Auerbach and Kotlikoff (1987), we construct a Lump-Sum Redistribution Authority that is used to fix the remaining lifetime expected utilities of agents alive at the time of the tax reform at their prereform level. The LSRA allows us to report actual efficiency gains associated with tax reform.

Our results point to the importance of incorporating the risk-sharing aspects of the prereform tax system into the analysis. With idiosyncratic shocks operative, tax reform produced a $3,200 loss to future households under our benchmark paramater settings and model form.

But when these shocks were turned off and the model was recalibrated to match the same initial economy, future households *gained* \$25,100. Those results (at least the direction of the gains) were robust to a fairly wide range of parameter and model assumptions.

The remarkable difference between the model with and without shocks has two sources. First, the presence of idiosyncratic shocks reduces the intertemporal savings response to a postreform increase in the after-tax interest rate. Second, the progressive nature of the prereform tax system—which often leads to larger efficiency gains after tax reform in models without risk—reduces the benefit of moving to a flat tax in the presence of idiosyncratic risks. The reason is that fundamental tax reform removes an important source of risk sharing that was present in the prereform progressive tax system.

Appendix

A The Discretization of the State Space

The state of a household is $\mathbf{s}_i = (i, e_i, a_i, b_i) \in I \times E \times A \times B$, where $I = \{20, ..., 109\}$, $E = [e^{\min}, e^{\max}]$, $A = \left[a^{\min}, a^{\max}\right]$, and $B = [b^{\min}, b^{\max}]$. To compute an equilibrium, the state space of a household is discretized as $\widehat{\mathbf{s}}_i \in I \times \hat{E} \times \hat{A} \times \hat{B}$, where $\hat{E} = \{e^1, e^2, ..., e^{N_e}\}$, $\hat{A} = \{a^1, a^2, ..., a^{N_a}\}$, and $\hat{B} = \{b^1, b^2, ..., b^{N_b}\}$.

For all these discrete points, we compute the optimal decision of households, $\mathbf{d}(\widehat{\mathbf{s}}_i, \mathbf{S}_t; \mathbf{\Psi}_t)$ $= (c_i\,(.)\,, h_i\,(.)\,, a_{i+1}\,(.)) \in (0, c^{\max}] \times [0, h_i^{\max}] \times A$, and the marginal values, $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_i, \mathbf{S}_t; \mathbf{\Psi}_t)$, given the expected factor prices and policy variables.

To find the optimal end-of-period wealth, we use the Euler equation and bilinear interpolation (with respect to a and b) of marginal value functions at the beginning of the next period. In this paper, N_e, N_a, and N_b are 8, 60, and 10, respectively. Since there are 90 different ages, the total number of discrete states is 432,000.

B A Steady-State Equilibrium

The algorithm to compute a steady-state equilibrium is as follows. Let $\mathbf{\Psi}$ denote the time-invariant government policy rules $\mathbf{\Psi} = (W_g, C_g, \tau_I(.), \tau_P(.), \tau_C, tr_{SS}(.))$.

1. Set the initial values of factor prices (r^0, w^0), accidental bequests q^0, the policy variables (W_g^0, C_g^0, τ_C^0), and the parameters φ^0 of policy functions $(\tau_I(.), \tau_P(.), tr_{SS}(.))$ if these are determined endogenously.[34]

2. Given $\mathbf{\Omega}^0 = (r^0, w^0, q^0, W_g^0, C_g^0, \tau_C^0, \varphi^0)$, find the decision rule of a household $\mathbf{d}(\widehat{\mathbf{s}}_i; \mathbf{\Psi}, \mathbf{\Omega}^0)$ for all $\widehat{\mathbf{s}}_i \in I \times \hat{E} \times \hat{A} \times \hat{B}$.[35]

   (a) For age $i = 109$, find the decision rule $\mathbf{d}(\widehat{\mathbf{s}}_{109}; \mathbf{\Psi}, \mathbf{\Omega}^0)$. Since the survival rate $\phi_{109} = 0$, the end-of-period wealth $a_{i+1}(\widehat{\mathbf{s}}_{109}; .) = 0$ for all $\widehat{\mathbf{s}}_{109}$. Compute consumption and working hours $(c_i(\widehat{\mathbf{s}}_{109}; .),\ h_i(\widehat{\mathbf{s}}_{109}; .))$ and, then, the marginal values $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_{109}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ for all $\widehat{\mathbf{s}}_{109}$.

   (b) For age $i = 108, ..., 20$, find the decision rule $\mathbf{d}(\widehat{\mathbf{s}}_i; \mathbf{\Psi}, \mathbf{\Omega}^0)$ and $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_i; \mathbf{\Psi}, \mathbf{\Omega}^0)$ for all $\widehat{\mathbf{s}}_i$, using $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_{i+1}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ recursively.

      i. Set the initial guess of $a_{i+1}^0(\widehat{\mathbf{s}}_i; .)$.

      ii. Given $a_{i+1}^0(\widehat{\mathbf{s}}_i; .)$, compute $(c_i(\widehat{\mathbf{s}}_i; .),\ h_i(\widehat{\mathbf{s}}_i; .))$. Plug these into the Euler equation with $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_{i+1}; \mathbf{\Psi}, \mathbf{\Omega}^0)$.

      iii. If the Euler error is sufficiently small, then stop. Otherwise, update $a_{i+1}^0(\widehat{\mathbf{s}}_i; .)$ and return to Step ii.

3. Find the steady-state measure of households $x(\widehat{\mathbf{s}}_i; \mathbf{\Omega}^0)$ using the decision rule obtained in Step 2. This computation is done forward from age 20 to age 109. Repeat this step to iterate q for q^1.

4. Compute new factor prices (r^1, w^1), the policy variables (W_g^1, C_g^1), and the parameters φ^1 of policy functions.

5. Compare $\mathbf{\Omega}^1 = (r^1, w^1, q^1, W_g^1, C_g^1, \tau_C^1, \varphi^1)$ with $\mathbf{\Omega}^0$. If the difference is sufficiently small, then stop. Otherwise, update $\mathbf{\Omega}^0$ and return to Step 2.

[34] Actually, if we find the capital-labor ratio, both r and w are calculated from the given production function and depreciation rate.

[35] In the steady-state economy, the decision rule of a household $\mathbf{d}(\widehat{\mathbf{s}}_i; \mathbf{\Psi}, \mathbf{\Omega}^0)$ is not a function of the aggregate state of economy $\widehat{\mathbf{S}} = (x(\widehat{\mathbf{s}}_i), W_g)$. The measure of household $x(\widehat{\mathbf{s}}_i)$ is determined uniquely by the steady-state condition, and the government's wealth W_g is determined by the policy rule $\mathbf{\Psi}$.

C An Equilibrium Transition Path

Let's assume that the economy is in the initial steady state in period 0, and that the new policy schedule $\mathbf{\Psi}_1$, which was not expected in period 0, is announced at the beginning of period 1, where $\mathbf{\Psi}_1 = \{W_{g,t+1}, C_{g,t}, \tau_{I,t}(.), \tau_{P,t}(.), \tau_{C,t}, tr_{SS,t}(.)\}_{t=1}^{\infty}$. Let $\widehat{\mathbf{S}}_1 = (x_1(\widehat{\mathbf{s}}_i), W_{g,1})$ be the aggregate state of the economy at the beginning of period 1. The state of the economy $\widehat{\mathbf{S}}_1$ is usually equal to that of the initial steady state. The algorithm to compute a transition path to a new steady-state equilibrium (thereafter, final steady-state equilibrium) is as follows.

1. Choose a sufficiently large number, T, such that the economy is said to reach the new steady state within T periods.[36] Set the initial guess, $\{\mathbf{\Omega}_t^0\}_{t=1}^T$, on factor prices (r_t^0, w_t^0), accidental bequests q_t^0, the policy variables $(W_{g,t+1}^0, C_{g,t}^0, \tau_{C,t}^0)$, and the parameters φ_t^0 of policy functions for $t = 1, 2, ..., T$.

2. Given $\mathbf{\Omega}_T^0 = (r_T^0, w_T^0, q_T^0, W_{g,T}^0, C_{g,T}^0, \tau_{C,t}^0, \varphi_T^0)$, find the final steady-state decision rule $\mathbf{d}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_T; \mathbf{\Psi}_T; \mathbf{\Omega}_T^0)$ and compute marginal values $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_T; \mathbf{\Psi}_T; \mathbf{\Omega}_T^0)$ for all $\widehat{\mathbf{s}}_i \in I \times \widehat{E} \times \widehat{A} \times \widehat{B}$. (See the algorithm for a steady-state equilibrium.)

3. For period $t = T-1, T-2, ..., 1$, based on the guess, $\mathbf{\Omega}_t^0$, find backward the decision rule $\mathbf{d}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t^0)$ and marginal values $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t^0)$ for all $\widehat{\mathbf{s}}_i \in I \times \widehat{E} \times \widehat{A} \times \widehat{B}$, using the next-period marginal values $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_{t+1}; \mathbf{\Psi}_{t+1}; \mathbf{\Omega}_{t+1}^0)$ recursively.

   (a) For age $i = 109$, find the decision rule $\mathbf{d}(\widehat{\mathbf{s}}_{109}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t^0)$ and compute the marginal values $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_{109}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t^0)$ for all $\widehat{\mathbf{s}}_{109}$.

   (b) For age $i = 108, ..., 20$, find the decision rule $\mathbf{d}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t^0)$ and compute $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t^0)$ for all $\widehat{\mathbf{s}}_i$, using $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_{i+1}, \widehat{\mathbf{S}}_{t+1}; \mathbf{\Psi}_{t+1}; \mathbf{\Omega}_{t+1}^0)$ previously computed.[37]

      i. Set the initial guess of $a_{i+1}^0(\widehat{\mathbf{s}}_i; .)$.

      ii. Given $a_{i+1}^0(\widehat{\mathbf{s}}_i; .)$, compute $(c_i(\widehat{\mathbf{s}}_i; .),\ h_i(\widehat{\mathbf{s}}_i; .))$. Plug these into the Euler equation with $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_{i+1}, \widehat{\mathbf{S}}_{t+1}; \mathbf{\Psi}_{t+1}; \mathbf{\Omega}_{t+1}^0)$.

[36] For this to be the case, the government's policy rule has to be time-invariant sufficiently before period T, that is, $\mathbf{\Psi}_s = \mathbf{\Psi}_T$ for $1 \leq s < T$.

[37] Note that this step does not use $\frac{\partial}{\partial a} v(\widehat{\mathbf{s}}_{i+1}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t^0)$ recursively.

iii. If the Euler error is sufficiently small, then stop. Otherwise, update $a^0_{i+1}(\widehat{\mathbf{s}}_i;.)$ and return to Step ii.

4. For period $t = 1, 2, ..., T-1$, compute forward $\mathbf{\Omega}^1_t = (r^1_t, w^1_t, q^1_t, W^1_{g,t+1}, C^1_{g,t}, \tau^1_{C,t}, \varphi^1_t)$ and the measure of households $x_{t+1}(\widehat{\mathbf{s}}_i)$, using the decision rule $\mathbf{d}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}^0_t)$ obtained in Step 3 and using the state of economy $\widehat{\mathbf{S}}_t = (x_t(\widehat{\mathbf{s}}_i), W_{g,t})$ recursively.

5. Compare $\{\mathbf{\Omega}^1_t\}^T_{t=1}$ with $\{\mathbf{\Omega}^0_t\}^T_{t=1}$. If the difference is sufficiently small, then stop. Otherwise, update $\{\mathbf{\Omega}^0_t\}^T_{t=1}$ and return to Step 2. (If the final steady-state equilibrium is known, return to Step 3 instead.)

D The Lump-Sum Redistribution Authority

When the Lump-Sum Redistribution Authority (LSRA) is assumed, the following computation is added to the iteration process for an equilibrium transition path.

1. [Step 1 in the previous subsection "An Equilibrium Transition Path" (hereafter ETP)] Add the wealth held by LSRA $\{W^0_{a,t}\}^T_{t=1}$ to $\{\mathbf{\Omega}^0_t\}^T_{t=1}$ and set the initial value.

2. [Steps 2 and 3 in ETP] For period $t = T, T-1, ..., 2$, compute the lump-sum transfers to newborn households (age 20) $tr_{R_2}(\widehat{\mathbf{s}}_{20}, t)$ to make those households as much better off as the baseline economy.

   (a) Set the initial value of lump-sum transfers $tr_{R_2}(\widehat{\mathbf{s}}_{20}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t)$ to newborn households.

   (b) Given $tr_{R_2}(\widehat{\mathbf{s}}_{20}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t)$, find the decision rule of newborn households $\mathbf{d}(\widehat{\mathbf{s}}_{20}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t)$.

   (c) Find the compensating variation in wealth $\Delta tr_{R_2}(\widehat{\mathbf{s}}_{20}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t)$ to make those households indifferent from the baseline economy. (Initial wealth of newborn households is assumed to be zero.) If the absolute value of $\Delta tr_{R_2}(\widehat{\mathbf{s}}_{20}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t)$ is sufficiently small, then go to Step (d). Otherwise, update $tr_{R_2}(\widehat{\mathbf{s}}_{20}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t)$ by adding $\Delta tr_{R_2}(\widehat{\mathbf{s}}_{20}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t)$ and return to Step (b).

(d) Given $tr_{R_2}(\widehat{\mathbf{s}}_{20}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t)$ and additional lump-sum transfers to newborns tr_{R_3}, find the decision rule of newborn households $\mathbf{d}(\widehat{\mathbf{s}}_{20}, \widehat{\mathbf{S}}_t; \mathbf{\Psi}_t; \mathbf{\Omega}_t)$.

3. [Step 3 in ETP] For period $t = 1$, compute the lump-sum transfers to current households (ages 20-109) $tr_{R_1}(\widehat{\mathbf{s}}_i)$ to make those households as much better off as the baseline economy.

   (a) Set the initial value of lump-sum transfers $tr_{R_1}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_1; \mathbf{\Psi}_1; \mathbf{\Omega}_1)$ to current households.

   (b) Given $tr_{R_1}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_1; \mathbf{\Psi}_1; \mathbf{\Omega}_1)$, find the decision rule of current households $\mathbf{d}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_1; \mathbf{\Psi}_1; \mathbf{\Omega}_1)$.

   (c) Find the compensating variation in wealth to make those households indifferent from the baseline economy. Compute $\Delta tr_{R_1}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_1; \mathbf{\Psi}_1; \mathbf{\Omega}_1)$ as the difference from current beginning-of-period wealth. If the absolute value of $\Delta tr_{R_1}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_1; \mathbf{\Psi}_1; \mathbf{\Omega}_1)$ is sufficiently small, then stop. Otherwise, update $tr_{R_1}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_1; \mathbf{\Psi}_1; \mathbf{\Omega}_1)$ by adding $\Delta tr_{R_1}(\widehat{\mathbf{s}}_i, \widehat{\mathbf{S}}_1; \mathbf{\Psi}_1; \mathbf{\Omega}_1)$ and return to Step (b).

4. [Before Step 4 in ETP] Compute an additional lump-sum transfer tr_{R_3} so that the net present value of all transfers becomes zero. Compute the wealth held by LSRA, $\{W^1_{a,t}\}_{t=1}^T$, which will be used to calculate national wealth.

5. [After Step 4 in ETP] Recompute tr_{R_3} and $\{W^1_{a,t}\}_{t=1}^T$ using new interest rates $\{r_t\}_{t=1}^T$. Compare $\{W^1_{a,t}\}_{t=1}^T$ with $\{W^0_{a,t}\}_{t=1}^T$. If the difference is sufficiently small, then stop. Otherwise, update $\{W^0_{a,t}\}_{t=1}^T$ and return to Step 2.

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Table 9: The Results of the Stochastic OLG Model with Heterogeneous Households (Closed Economy, Gamma = 2.0)

(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously)

	Year											
	1	2	3	5	10	15	20	30	40	60	80	120
(With the Lump-Sum Redistribution Authority)												
%ch(National Wealth)	0.0	0.2	0.3	0.7	1.2	1.6	1.8	2.0	2.1	2.1	2.1	2.1
%ch(Labor)	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
%ch(GNP)	0.5	0.6	0.6	0.7	0.8	0.9	1.0	1.0	1.1	1.1	1.1	1.1
%ch(Consumption)	0.1	0.0	0.0	0.2	0.5	0.6	0.8	0.9	0.9	1.0	1.0	1.0
%ch(Income Tax Rates)	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0
ch(Consumption Tax Rate%)	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
ch(Interest Rate%)	0.06	0.05	0.03	0.00	-0.05	-0.08	-0.09	-0.11	-0.12	-0.12	-0.12	-0.12
%ch(Wage Rate)	-0.2	-0.2	-0.1	0.0	0.2	0.3	0.4	0.5	0.5	0.5	0.5	0.5
%ch(Private Wealth)	0.0	1.1	1.3	1.7	2.3	2.7	3.0	3.2	3.3	3.3	3.3	3.3
ch(Government Wealth/GDP%)	0.0	-2.5	-2.5	-2.6	-2.7	-2.8	-2.8	-2.8	-2.8	-2.8	-2.8	-2.8
%ch(Accidental Bequests)	1.6	1.8	1.9	2.1	2.6	3.0	3.3	3.7	3.9	4.0	4.0	4.0
ch(Comp.Variation/Wealth%)	0.00	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
(Without the Lump-Sum Redistribution Authority)												
%ch(National Wealth)	0.0	0.3	0.6	1.0	1.8	2.2	2.5	2.8	2.8	2.9	2.9	2.9
%ch(Labor)	0.8	0.8	0.8	0.7	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
%ch(GNP)	0.6	0.6	0.7	0.8	1.0	1.1	1.2	1.3	1.3	1.3	1.3	1.3
%ch(Consumption)	-0.4	-0.3	-0.2	0.0	0.4	0.7	0.8	1.0	1.1	1.1	1.1	1.1
%ch(Income Tax Rates)	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0
ch(Consumption Tax Rate%)	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
ch(Interest Rate%)	0.07	0.04	0.02	-0.02	-0.09	-0.13	-0.15	-0.17	-0.17	-0.18	-0.18	-0.18
%ch(Wage Rate)	-0.3	-0.2	-0.1	0.1	0.4	0.5	0.6	0.7	0.7	0.7	0.7	0.7
%ch(Private Wealth)	0.0	0.3	0.6	1.1	1.9	2.4	2.7	3.0	3.0	3.1	3.1	3.1
ch(Government Wealth/GDP%)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
%ch(Accidental Bequests)	0.2	0.5	0.7	1.0	1.7	2.3	2.7	3.1	3.3	3.4	3.4	3.4
ch(Comp.Variation/Wealth%)	-1.54	-0.01	0.00	0.00	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02

Table 10: Welfare Changes in the Stochastic OLG Model and the Deterministic OLG Model (Closed Economy, Gamma = 2.0)
Compensating Variations in Wealth ($1,000 per Household)
(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously)

		The Year When Age 20 (Top) / the Age in Year 1 (Bottom)											
		-79	-59	-39	-29	-19	-9	1	11	21	41	61	81
		100	80	60	50	40	30	20	10	0	-20	-40	-60
(The Stochastic OLG Model with Heterogeneous Households, Closed Economy)													
(With the Lump-Sum Redistribution Authority)													
Before LSRA	e1	-0.6	-6.4	-5.4	-3.4	-2.3	-1.5	-0.8	-0.1	0.2	0.3	0.3	0.3
	e2			-6.6	-4.1	-2.5	-1.7	-1.3	0.0	0.5	0.8	0.8	0.8
	e3			-8.4	-4.7	-2.4	-1.5	-1.3	0.3	0.9	1.2	1.2	1.2
	e4			-10.4	-4.8	-1.5	-0.4	-0.9	1.0	1.7	2.1	2.1	2.1
	e5			-10.7	-4.0	0.2	1.3	-0.2	1.9	2.7	3.1	3.1	3.1
	e6			-7.2	-1.6	4.2	3.6	0.9	3.2	4.1	4.5	4.5	4.5
	e7			5.4	11.5	15.8	11.0	4.2	6.7	7.7	8.1	8.1	8.1
	e8			97.4	90.5	70.0	42.6	9.2	12.1	13.2	13.7	13.7	13.7
	Average	-0.6	-6.4	-6.4	-2.5	-0.2	0.2	-0.6	1.0	1.6	1.9	1.9	1.9
After LSRA	Average	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-3.2	-3.2	-3.2	-3.2	-3.2
(Without the Lump-Sum Redistribution Authority)													
Without LSRA	e1	-0.7	-7.5	-6.9	-4.6	-3.0	-2.0	-1.0	-0.1	0.2	0.4	0.4	0.4
	e2			-8.4	-5.5	-3.5	-2.3	-1.7	0.1	0.7	1.0	1.0	1.0
	e3			-10.6	-6.4	-3.8	-2.3	-1.7	0.4	1.1	1.4	1.4	1.4
	e4			-13.4	-7.3	-3.6	-1.8	-1.4	1.1	1.9	2.2	2.3	2.3
	e5			-14.6	-7.5	-2.8	-0.8	-0.9	1.8	2.8	3.1	3.2	3.2
	e6			-12.9	-7.0	-0.7	0.5	-0.1	2.9	3.9	4.3	4.3	4.3
	e7			-3.0	2.7	7.8	5.8	2.3	5.7	6.8	7.3	7.3	7.3
	e8			76.2	68.4	50.8	30.6	6.1	10.0	11.3	11.8	11.8	11.8
	Average	-0.7	-7.5	-9.3	-5.0	-2.3	-1.2	-1.1	0.9	1.7	1.9	2.0	2.0
(The Deterministic OLG Model with Representative Households, Closed Economy)													
(With the Lump-Sum Redistribution Authority)													
Before LSRA		-0.5	-3.8	-6.4	5.6	14.5	16.4	12.0	14.0	14.4	14.6	14.8	14.8
After LSRA		0.0	0.0	0.0	0.0	0.0	0.0	0.0	25.1	25.1	25.1	25.1	25.1
(Without the Lump-Sum Redistribution Authority)													
Without LSRA		-0.4	-3.2	-2.6	10.7	21.0	23.2	18.9	21.8	22.5	22.3	22.3	22.4

Table 11: The Results of the Deterministic OLG Model with Representative Households (Closed Economy, Gamma = 2.0)

(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously)

	Year											
	1	2	3	5	10	15	20	30	40	60	80	120
(With the Lump-Sum Redistribution Authority)												
%ch(National Wealth)	0.0	0.5	0.9	1.6	2.9	3.7	4.3	4.9	5.1	5.2	5.1	5.1
%ch(Labor)	2.4	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.5	1.5	1.5	1.5
%ch(GNP)	1.6	1.6	1.7	1.9	2.2	2.4	2.5	2.6	2.6	2.6	2.6	2.6
%ch(Consumption)	0.3	0.6	0.8	1.1	1.6	1.9	2.2	2.3	2.4	2.4	2.5	2.5
%ch(Income Tax Rates)	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0
ch(Consumption Tax Rate%)	0.9	0.9	0.9	0.9	0.8	0.7	0.7	0.8	0.8	0.8	0.8	0.8
ch(Interest Rate%)	0.19	0.13	0.09	0.03	-0.08	-0.15	-0.20	-0.25	-0.27	-0.28	-0.28	-0.28
%ch(Wage Rate)	-0.8	-0.5	-0.4	-0.1	0.3	0.6	0.8	1.0	1.1	1.2	1.2	1.2
%ch(Private Wealth)	0.0	-2.4	-2.0	-1.3	0.1	1.0	1.6	2.3	2.5	2.6	2.6	2.6
ch(Government Wealth/GDP%)	0.0	6.2	6.2	6.2	6.1	6.1	6.1	6.1	6.1	6.0	6.0	6.0
%ch(Accidental Bequests)	1.1	1.3	1.5	1.7	2.1	2.2	2.1	1.7	1.3	1.4	1.6	1.5
ch(Comp.Variation/Wealth%)	0.00	0.26	0.26	0.26	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
(Without the Lump-Sum Redistribution Authority)												
%ch(National Wealth)	0.0	0.4	0.7	1.4	2.4	3.0	3.4	3.7	3.7	3.7	3.7	3.7
%ch(Labor)	2.1	2.0	2.0	1.9	1.7	1.6	1.6	1.5	1.5	1.5	1.5	1.5
%ch(GNP)	1.4	1.5	1.6	1.7	1.9	2.1	2.1	2.2	2.2	2.2	2.2	2.2
%ch(Consumption)	0.5	0.6	0.8	1.0	1.5	1.9	2.0	2.2	2.3	2.3	2.3	2.3
%ch(Income Tax Rates)	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0
ch(Consumption Tax Rate%)	0.8	0.8	0.8	0.7	0.7	0.6	0.6	0.7	0.7	0.8	0.8	0.8
ch(Interest Rate%)	0.16	0.13	0.09	0.04	-0.05	-0.11	-0.14	-0.16	-0.16	-0.16	-0.16	-0.16
%ch(Wage Rate)	-0.7	-0.5	-0.4	-0.2	0.2	0.4	0.6	0.7	0.7	0.7	0.7	0.7
%ch(Private Wealth)	0.0	0.4	0.8	1.4	2.6	3.2	3.6	3.9	3.9	3.9	3.9	4.0
ch(Government Wealth/GDP%)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
%ch(Accidental Bequests)	0.3	0.5	0.8	1.2	2.0	2.7	3.2	4.0	4.2	4.1	4.1	4.1
ch(Comp.Variation/Wealth%)	4.89	0.19	0.19	0.20	0.21	0.22	0.22	0.22	0.22	0.22	0.22	0.22

Table 12: The Results of the Stochastic OLG Model with Heterogeneous Households (Small Open Economy, Gamma = 2.0)

(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously)

	Year											
	1	2	3	5	10	15	20	30	40	60	80	120
(With the Lump-Sum Redistribution Authority)												
%ch(National Wealth)	0.0	0.3	0.5	1.0	2.0	2.6	3.1	3.6	3.8	3.9	3.9	3.9
%ch(Labor)	1.1	1.1	1.0	0.8	0.6	0.5	0.4	0.3	0.3	0.3	0.3	0.3
%ch(GNP)	0.8	0.8	0.8	0.9	1.0	1.2	1.2	1.4	1.4	1.4	1.4	1.4
%ch(Consumption)	0.0	-0.1	0.0	0.1	0.3	0.5	0.6	0.8	0.9	1.0	1.0	1.0
%ch(Income Tax Rates)	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0
ch(Consumption Tax Rate%)	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
ch(Interest Rate%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
%ch(Wage Rate)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
%ch(Private Wealth)	0.0	0.7	1.0	1.5	2.5	3.3	3.8	4.3	4.6	4.7	4.7	4.7
ch(Government Wealth/GDP%)	0.0	-1.0	-1.1	-1.1	-1.2	-1.3	-1.3	-1.4	-1.4	-1.4	-1.4	-1.4
%ch(Accidental Bequests)	1.4	1.5	1.7	1.9	2.6	3.2	3.8	4.7	5.2	5.5	5.5	5.5
ch(Comp.Variation/Wealth%)	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
(Without the Lump-Sum Redistribution Authority)												
%ch(National Wealth)	0.0	0.4	0.7	1.3	2.4	3.2	3.7	4.3	4.5	4.6	4.6	4.6
%ch(Labor)	1.2	1.1	1.0	0.8	0.6	0.4	0.3	0.2	0.2	0.2	0.2	0.2
%ch(GNP)	0.8	0.9	0.9	1.0	1.2	1.3	1.4	1.5	1.6	1.6	1.6	1.6
%ch(Consumption)	-0.4	-0.3	-0.2	-0.1	0.2	0.5	0.6	0.9	1.0	1.0	1.0	1.0
%ch(Income Tax Rates)	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0
ch(Consumption Tax Rate%)	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
ch(Interest Rate%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
%ch(Wage Rate)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
%ch(Private Wealth)	0.0	0.4	0.7	1.4	2.6	3.4	4.0	4.6	4.8	4.9	4.9	4.9
ch(Government Wealth/GDP%)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
%ch(Accidental Bequests)	0.2	0.5	0.7	1.1	2.1	3.0	3.8	4.8	5.2	5.5	5.5	5.5
ch(Comp.Variation/Wealth%)	-0.56	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Table 13: Welfare Changes in the Stochastic OLG Model (Small Open Economy, Gamma = 2.0)

Compensating Variations in Wealth ($1,000 per Household)

(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously)

		The Year When Age 20 (Top) / the Age in Year 1 (Bottom)											
		-79	-59	-39	-29	-19	-9	1	11	21	41	61	81
		100	80	60	50	40	30	20	10	0	-20	-40	-60
		(The Stochastic OLG Model with Heterogeneous Households, Small Open Economy)											
(With the Lump-Sum Redistribution Authority)													
Before LSRA	e1	-0.6	-6.3	-5.0	-3.0	-2.0	-1.5	-0.8	-0.4	-0.2	0.1	0.2	0.2
	e2			-6.0	-3.3	-1.9	-1.5	-1.4	-0.7	-0.1	0.4	0.5	0.5
	e3			-7.3	-3.5	-1.3	-1.0	-1.4	-0.6	0.1	0.6	0.7	0.7
	e4			-8.3	-2.5	0.8	1.0	-0.7	0.2	0.9	1.4	1.6	1.6
	e5			-7.4	-0.2	3.9	4.1	0.3	1.3	2.0	2.6	2.7	2.8
	e6			-1.4	4.7	10.8	8.0	2.1	3.1	3.8	4.4	4.6	4.6
	e7			15.5	23.0	27.5	18.8	7.0	8.0	8.8	9.4	9.6	9.6
	e8			127.1	122.7	99.2	62.2	14.4	15.4	16.2	16.9	17.0	17.1
	Average	-0.6	-6.3	-4.2	-0.1	2.1	1.6	-0.3	0.5	1.0	1.5	1.6	1.6
After LSRA	Average	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	-1.0	-1.0	-1.0	-1.0
(Without the Lump-Sum Redistribution Authority)													
Without LSRA	e1	-0.6	-6.8	-5.6	-3.5	-2.4	-1.9	-1.0	-0.5	-0.2	0.1	0.2	0.2
	e2			-6.6	-3.8	-2.3	-1.9	-1.7	-0.8	-0.1	0.4	0.5	0.5
	e3			-8.0	-3.9	-1.7	-1.4	-1.7	-0.7	0.1	0.6	0.7	0.8
	e4			-9.0	-2.9	0.4	0.6	-1.1	0.1	0.8	1.4	1.6	1.6
	e5			-8.0	-0.6	3.5	3.7	0.0	1.2	2.0	2.6	2.7	2.8
	e6			-1.8	4.2	10.1	7.5	1.7	2.9	3.8	4.4	4.5	4.6
	e7			14.9	22.0	26.6	18.2	6.6	8.0	8.8	9.5	9.6	9.7
	e8			125.1	120.7	97.5	61.4	14.0	15.5	16.3	16.9	17.3	17.3
	Average	-0.6	-6.8	-4.9	-0.6	1.7	1.2	-0.6	0.4	1.0	1.5	1.6	1.7

Table 14: The Results of the Stochastic OLG Model with Heterogeneous Households (Closed Economy, Gamma = 2.0)

(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and a Progressive Consumption Tax Is Introduced Contemporaneously)

	Year											
	1	2	3	5	10	15	20	30	40	60	80	120
(With the Lump-Sum Redistribution Authority)												
%ch(National Wealth)	0.0	0.2	0.3	0.6	1.1	1.4	1.5	1.7	1.8	1.8	1.8	1.8
%ch(Labor)	0.5	0.5	0.4	0.4	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
%ch(GNP)	0.3	0.4	0.4	0.5	0.6	0.6	0.7	0.8	0.8	0.8	0.8	0.8
%ch(Consumption)	-0.3	-0.2	-0.2	0.0	0.2	0.4	0.5	0.6	0.6	0.6	0.6	0.6
%ch(Income Tax Rates)	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0
ch(Consumption Tax Rate%)	2.0	2.0	2.0	2.0	1.9	1.9	1.9	1.9	1.9	1.9	1.9	1.9
ch(Interest Rate%)	0.04	0.02	0.01	-0.02	-0.06	-0.08	-0.09	-0.11	-0.11	-0.11	-0.11	-0.11
%ch(Wage Rate)	-0.2	-0.1	0.0	0.1	0.2	0.3	0.4	0.4	0.5	0.5	0.5	0.5
%ch(Private Wealth)	0.0	1.3	1.5	1.8	2.3	2.6	2.8	3.0	3.1	3.1	3.1	3.1
ch(Government Wealth/GDP%)	0.0	-2.9	-2.9	-3.0	-3.1	-3.1	-3.2	-3.2	-3.2	-3.2	-3.2	-3.2
%ch(Accidental Bequests)	1.9	2.0	2.1	2.3	2.6	2.9	3.1	3.5	3.7	3.7	3.7	3.7
ch(Comp.Variation/Wealth%)	0.00	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
(Without the Lump-Sum Redistribution Authority)												
%ch(National Wealth)	0.0	0.3	0.5	0.9	1.6	2.0	2.3	2.5	2.6	2.6	2.6	2.6
%ch(Labor)	0.5	0.5	0.4	0.4	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
%ch(GNP)	0.4	0.4	0.5	0.5	0.7	0.8	0.9	1.0	1.0	1.0	1.0	1.0
%ch(Consumption)	-0.6	-0.5	-0.4	-0.2	0.1	0.4	0.5	0.7	0.7	0.7	0.7	0.7
%ch(Income Tax Rates)	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0
ch(Consumption Tax Rate%)	2.0	2.0	2.0	2.0	1.9	1.9	1.9	1.9	1.9	1.9	1.9	1.9
ch(Interest Rate%)	0.04	0.02	0.00	-0.04	-0.10	-0.13	-0.15	-0.17	-0.17	-0.17	-0.18	-0.17
%ch(Wage Rate)	-0.2	-0.1	0.0	0.2	0.4	0.5	0.6	0.7	0.7	0.7	0.7	0.7
%ch(Private Wealth)	0.0	0.3	0.5	0.9	1.7	2.1	2.4	2.7	2.7	2.8	2.8	2.8
ch(Government Wealth/GDP%)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
%ch(Accidental Bequests)	0.2	0.4	0.6	0.9	1.5	1.9	2.3	2.8	3.0	3.0	3.0	3.0
ch(Comp.Variation/Wealth%)	-1.84	0.00	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02

Table 15: Welfare Changes in the Stochastic OLG Model (Closed Economy, Gamma = 2.0)

Compensating Variations in Wealth ($1,000 per Household)

(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and a Progressive Consumption Tax Is Introduced Contemporaneously)

		The Year When Age 20 (Top) / the Age in Year 1 (Bottom)											
		-79	-59	-39	-29	-19	-9	1	11	21	41	61	81
		100	80	60	50	40	30	20	10	0	-20	-40	-60
		(The Stochastic OLG Model with Heterogeneous Households, Closed Economy)											
(With the Lump-Sum Redistribution Authority)													
Before LSRA	e1	-0.1	-6.7	-4.3	-1.2	-0.1	0.4	0.6	1.0	1.2	1.3	1.3	1.3
	e2			-5.8	-2.5	-0.7	0.2	0.8	1.8	2.2	2.4	2.4	2.4
	e3			-8.4	-4.2	-1.5	-0.1	0.8	2.0	2.5	2.8	2.8	2.8
	e4			-12.9	-6.9	-2.5	-0.2	0.9	2.3	2.9	3.3	3.3	3.3
	e5			-16.1	-8.6	-2.9	0.3	1.1	2.8	3.5	3.8	3.8	3.8
	e6			-17.5	-10.1	-2.0	1.0	1.7	3.5	4.3	4.6	4.7	4.6
	e7			-12.9	-4.0	4.7	5.7	3.8	5.9	6.7	7.2	7.2	7.2
	e8			50.1	52.5	44.1	30.1	7.3	9.7	10.7	11.2	11.2	11.1
	Average	-0.1	-6.7	-8.8	-4.0	-0.7	0.7	1.0	2.2	2.7	3.0	3.0	3.0
After LSRA	Average	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-2.8	-2.8	-2.8	-2.8	-2.8
(Without the Lump-Sum Redistribution Authority)													
Without LSRA	e1	-0.2	-7.9	-5.9	-2.5	-1.0	-0.2	0.3	1.0	1.3	1.5	1.5	1.5
	e2			-7.7	-4.2	-2.0	-0.7	0.3	1.7	2.3	2.6	2.6	2.6
	e3			-10.9	-6.3	-3.2	-1.2	0.1	2.0	2.7	3.1	3.1	3.1
	e4			-16.5	-9.9	-5.1	-2.0	0.1	2.2	3.0	3.4	3.5	3.5
	e5			-20.7	-12.8	-6.5	-2.4	0.1	2.5	3.4	3.8	3.9	3.9
	e6			-24.2	-16.5	-7.6	-2.7	0.3	2.9	3.9	4.4	4.4	4.4
	e7			-22.6	-13.9	-4.3	-0.1	1.4	4.5	5.6	6.1	6.1	6.2
	e8			27.7	29.2	23.8	17.1	3.5	7.1	8.3	8.8	8.9	9.0
	Average	-0.2	-7.9	-12.1	-6.9	-3.2	-1.0	0.3	2.0	2.7	3.0	3.1	3.1

Table 16: The Results of the Stochastic OLG Model with Heterogeneous Households (Closed Economy, Gamma = 4.0)

(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously)

	Year											
	1	2	3	5	10	15	20	30	40	60	80	120
(With the Lump-Sum Redistribution Authority)												
%ch(National Wealth)	0.0	0.1	0.2	0.5	0.9	1.2	1.4	1.7	1.8	1.8	1.9	1.8
%ch(Labor)	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
%ch(GNP)	0.4	0.5	0.5	0.5	0.7	0.8	0.8	0.9	1.0	1.0	1.0	1.0
%ch(Consumption)	0.2	0.1	0.1	0.2	0.4	0.6	0.7	0.8	0.9	0.9	0.9	0.9
%ch(Income Tax Rates)	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0
ch(Consumption Tax Rate%)	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
ch(Interest Rate%)	0.05	0.04	0.03	0.01	-0.03	-0.05	-0.07	-0.09	-0.09	-0.10	-0.10	-0.10
%ch(Wage Rate)	-0.2	-0.2	-0.1	0.0	0.1	0.2	0.3	0.4	0.4	0.4	0.4	0.4
%ch(Private Wealth)	0.0	1.3	1.5	1.7	2.3	2.6	2.9	3.2	3.3	3.4	3.4	3.4
ch(Government Wealth/GDP%)	0.0	-3.2	-3.2	-3.3	-3.4	-3.5	-3.5	-3.6	-3.6	-3.6	-3.6	-3.6
%ch(Accidental Bequests)	1.6	1.7	1.8	2.0	2.4	2.8	3.1	3.7	4.0	4.1	4.1	4.1
ch(Comp.Variation/Wealth%)	0.00	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
(Without the Lump-Sum Redistribution Authority)												
%ch(National Wealth)	0.0	0.3	0.5	0.9	1.7	2.2	2.5	2.9	3.1	3.1	3.2	3.2
%ch(Labor)	0.8	0.8	0.7	0.7	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
%ch(GNP)	0.5	0.6	0.7	0.8	1.0	1.1	1.2	1.3	1.4	1.4	1.4	1.4
%ch(Consumption)	-0.3	-0.2	-0.1	0.1	0.4	0.7	0.8	1.0	1.1	1.1	1.2	1.2
%ch(Income Tax Rates)	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0	-10.0
ch(Consumption Tax Rate%)	1.4	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.3	1.3	1.2
ch(Interest Rate%)	0.06	0.04	0.02	-0.02	-0.08	-0.12	-0.15	-0.18	-0.19	-0.19	-0.19	-0.19
%ch(Wage Rate)	-0.3	-0.2	-0.1	0.1	0.3	0.5	0.6	0.7	0.8	0.8	0.8	0.8
%ch(Private Wealth)	0.0	0.3	0.5	0.9	1.8	2.3	2.7	3.1	3.3	3.3	3.4	3.4
ch(Government Wealth/GDP%)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
%ch(Accidental Bequests)	0.2	0.4	0.6	0.9	1.5	2.1	2.5	3.2	3.5	3.6	3.6	3.6
ch(Comp.Variation/Wealth%)	-1.91	-0.01	-0.01	-0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Table 17: Welfare Changes in the Stochastic OLG Model (Closed Economy, Gamma = 4.0)

Compensating Variations in Wealth ($1,000 per Household)

(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously)

		The Year When Age 20 (Top) / the Age in Year 1 (Bottom)											
		-79	-59	-39	-29	-19	-9	1	11	21	41	61	81
		100	80	60	50	40	30	20	10	0	-20	-40	-60
		(The Stochastic OLG Model with Heterogeneous Households, Closed Economy)											
(With the Lump-Sum Redistribution Authority)													
Before LSRA	e1	-0.8	-7.0	-5.3	-3.4	-2.8	-2.2	-0.8	-0.3	-0.1	0.0	0.0	0.0
	e2			-6.6	-4.3	-3.3	-2.9	-2.0	-0.8	-0.2	0.2	0.3	0.3
	e3			-8.3	-5.1	-3.5	-2.9	-2.3	-0.9	-0.1	0.4	0.5	0.5
	e4			-10.0	-5.5	-3.0	-2.2	-2.2	-0.5	0.4	1.0	1.0	1.0
	e5			-10.2	-4.8	-1.6	-0.7	-1.8	0.1	1.1	1.7	1.8	1.8
	e6			-5.5	-1.8	2.2	1.4	-0.9	1.2	2.3	2.9	2.9	2.9
	e7			8.1	11.1	12.6	7.6	1.9	4.2	5.4	6.1	6.1	6.1
	e8			99.4	86.8	62.7	35.3	6.5	9.1	10.3	11.1	11.1	11.1
	Average	-0.8	-7.0	-6.0	-2.9	-1.4	-1.4	-1.5	-0.2	0.5	1.0	1.0	1.0
After LSRA	Average	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-5.5	-5.5	-5.5	-5.5	-5.5
(Without the Lump-Sum Redistribution Authority)													
Without LSRA	e1	-0.9	-8.8	-7.3	-4.9	-3.6	-2.6	-0.8	-0.2	0.1	0.2	0.3	0.3
	e2			-8.9	-6.0	-4.4	-3.2	-2.2	-0.4	0.5	1.0	1.0	1.1
	e3			-11.2	-7.1	-4.8	-3.4	-2.4	-0.2	0.8	1.4	1.4	1.5
	e4			-14.2	-8.5	-5.0	-3.3	-2.3	0.2	1.4	2.0	2.1	2.1
	e5			-15.6	-9.0	-4.7	-2.6	-2.1	0.8	2.1	2.8	2.9	2.9
	e6			-13.5	-8.6	-3.0	-1.5	-1.4	1.7	3.1	3.8	3.9	4.0
	e7			-3.5	-0.1	3.5	2.7	0.5	3.9	5.5	6.3	6.4	6.4
	e8			70.1	57.5	39.6	22.3	3.7	7.6	9.3	10.2	10.4	10.4
	Average	-0.9	-8.8	-10.0	-6.1	-3.6	-2.5	-1.8	0.3	1.2	1.8	1.8	1.8

Figure 1: The Results of the Stochastic OLG Model with Heterogeneous Households (Closed Economy)
(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously, Gamma = 2.0)

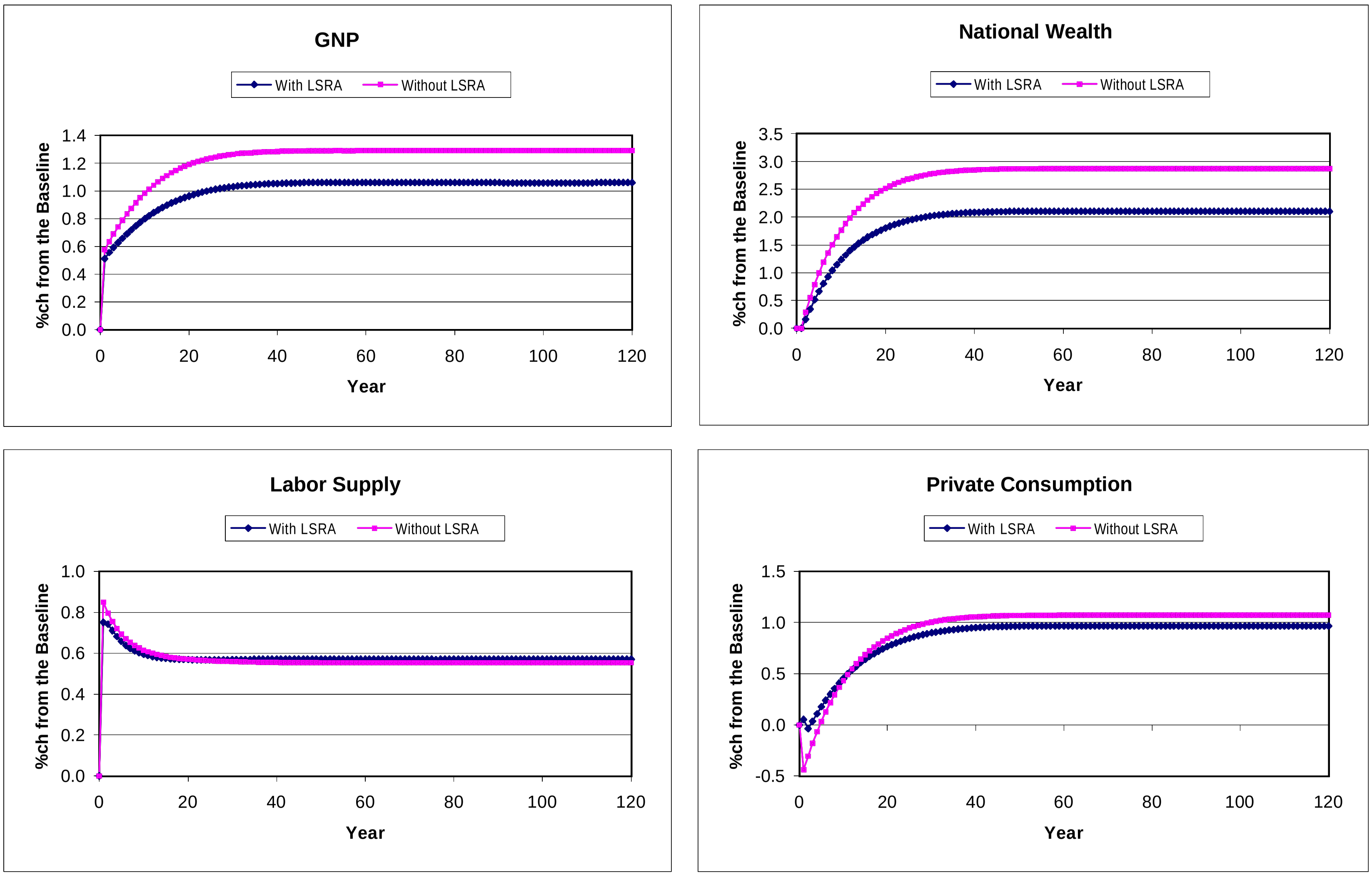


Figure 1 (Cont.): The Results of the Stochastic OLG Model with Heterogeneous Households (Closed Economy)
(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously, Gamma = 2.0)

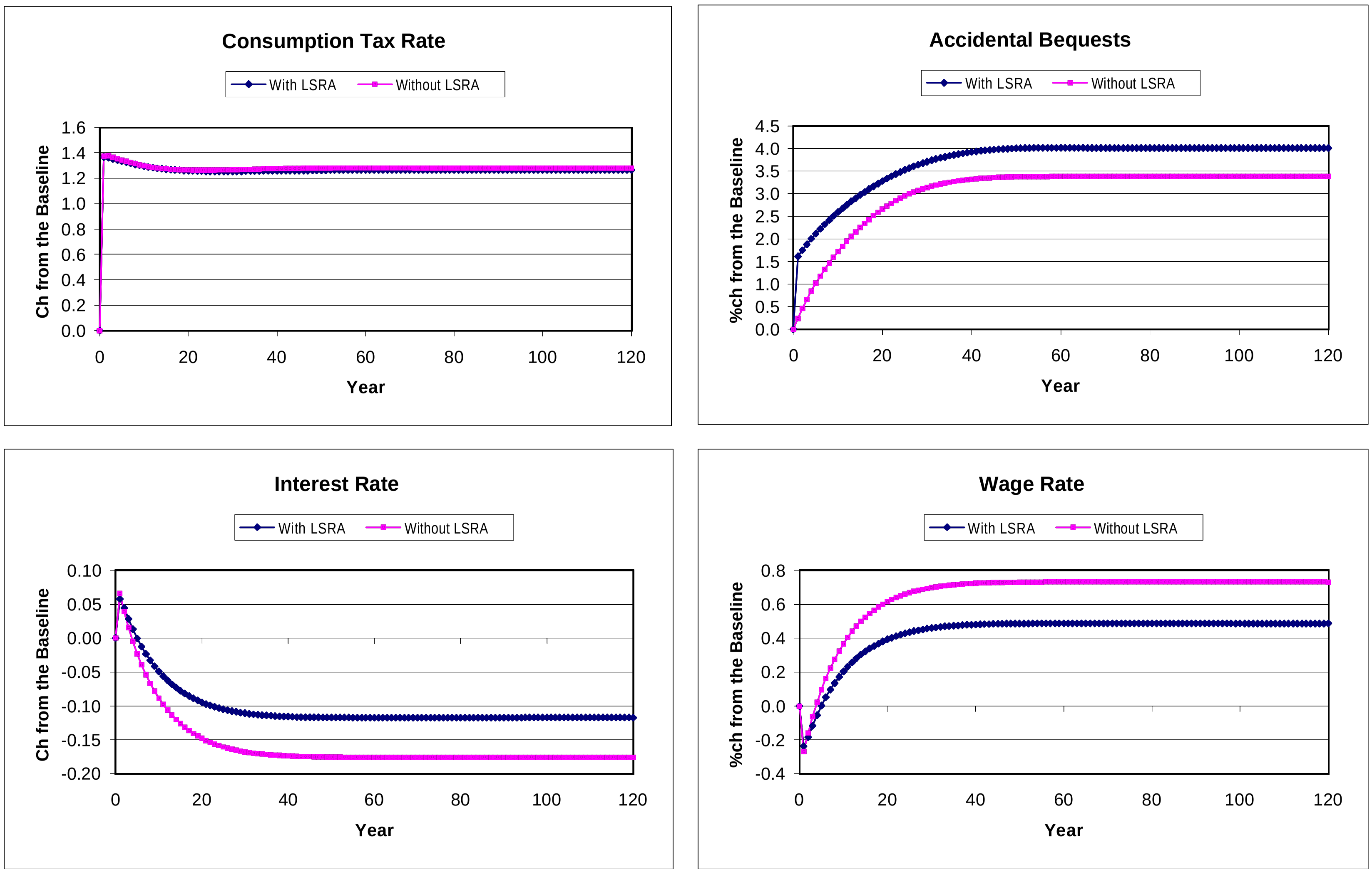


Figure 2: The Results of the Deterministic OLG Model with Representative Households

(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously, Gamma = 2.0)

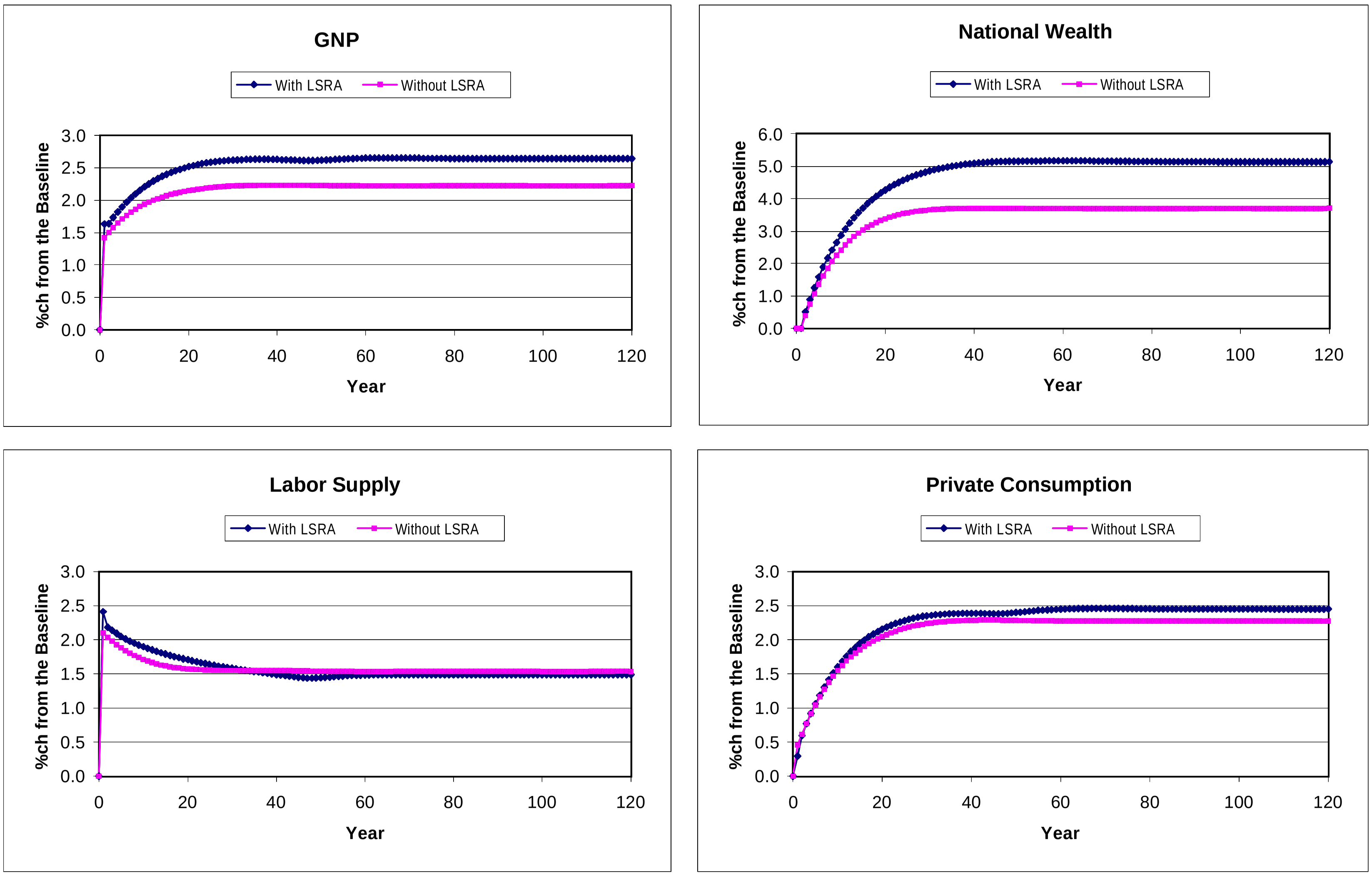
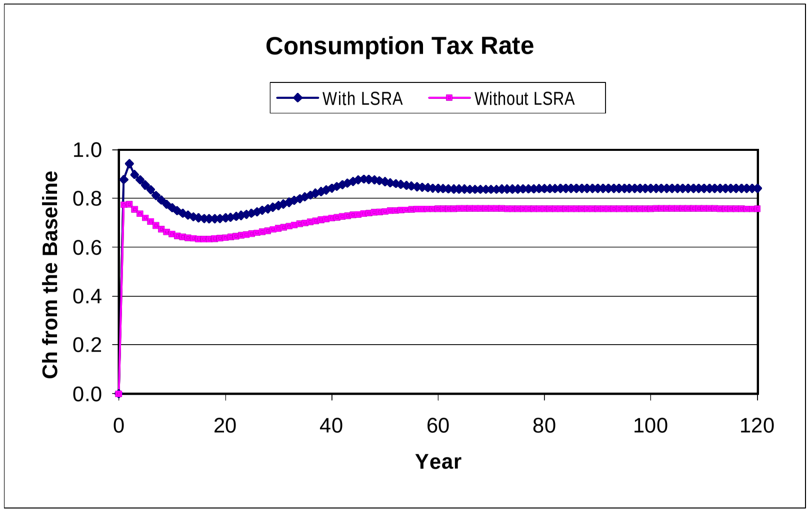
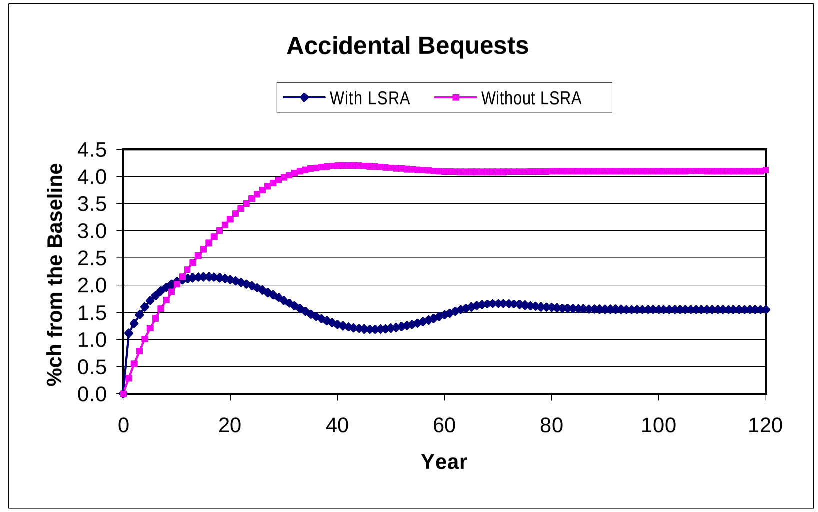
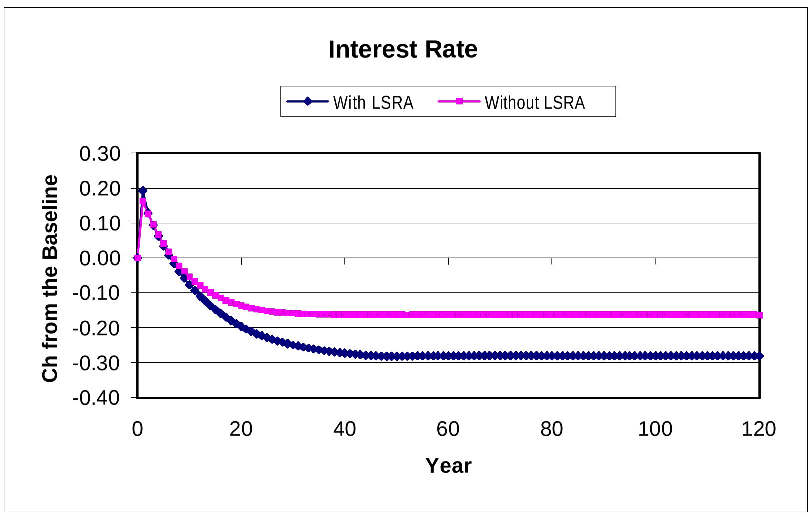


Figure 2 (Cont.): The Results of the Deterministic OLG Model with Representative Households

(When the Income Tax Rates Are Reduced by 10 Percent Proportionately, and the Consumption Tax Rate Is Raised Contemporaneously, Gamma = 2.0)







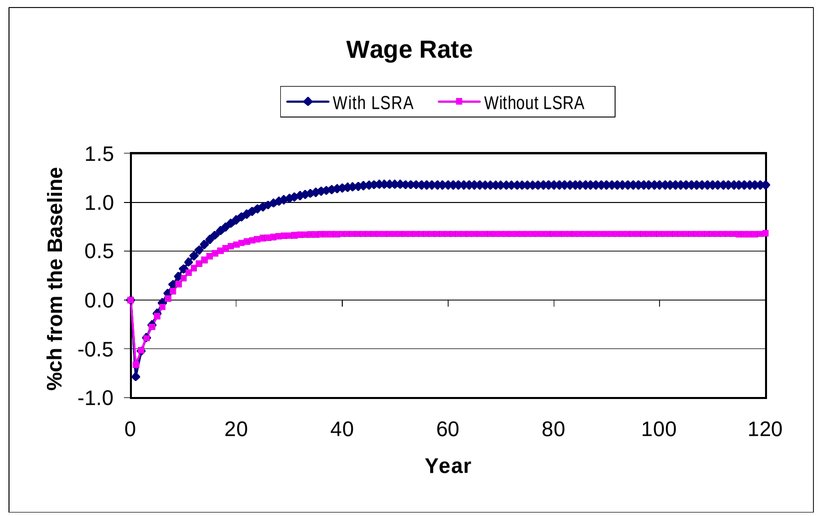
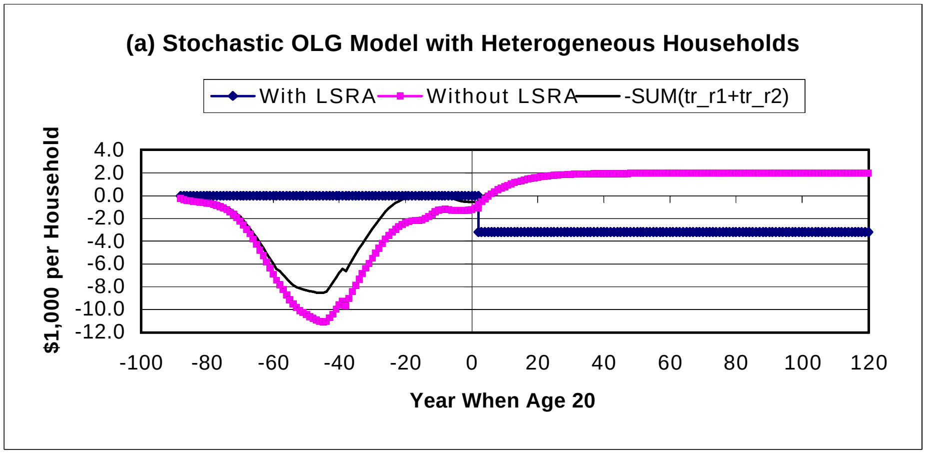
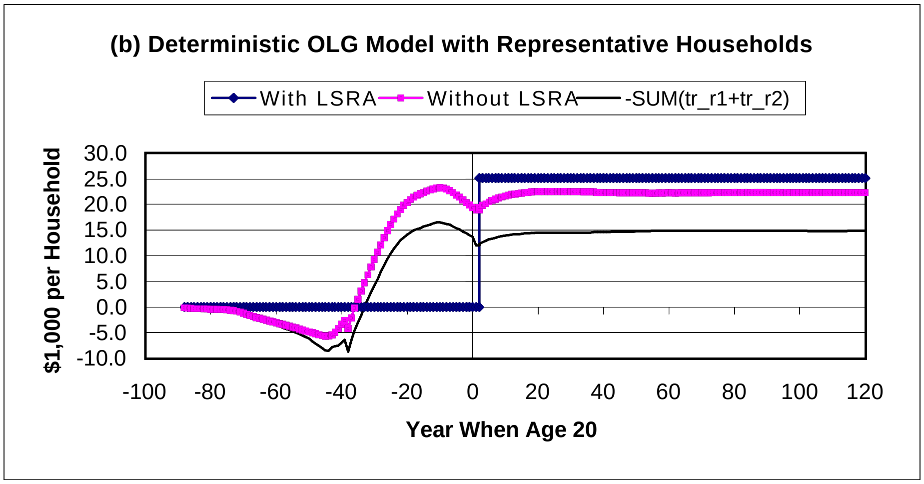


Figure 3: The Welfare Gains/Losses of Households Under Different Models and Assumptions





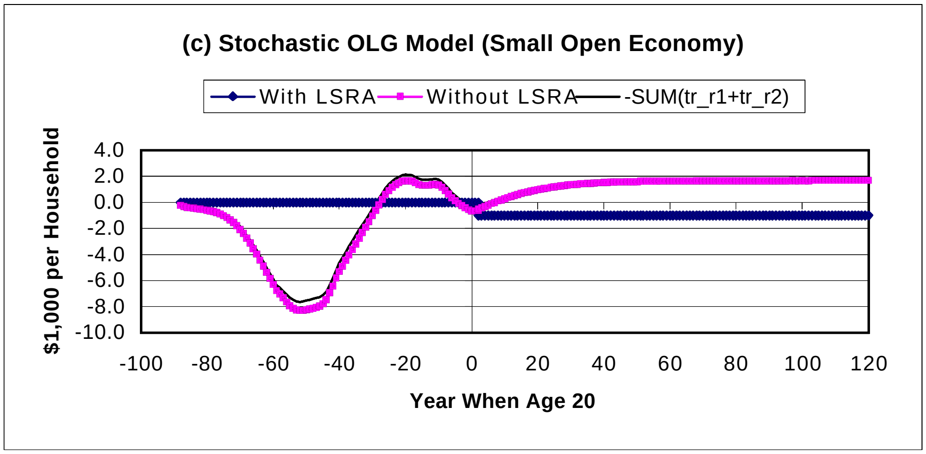


Figure 3 (Cont.): The Welfare Gains/Losses of Households Under Different Models and Assumptions