The Hagedorn Thermostat: a perfect particle reservoir

L. G. Moretto, K. A. Bugaev, J. B. Elliott and L. Phair
Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720

A system $\mathcal{H}$ with a Hagedorn-like mass spectrum imposes the same temperature to all emitted particles which are then in physical and chemical equilibrium with $\mathcal{H}$ and with each other. The coexistence between hadronic and partonic phases is thus completely characterized.

Hagedorn noted that the hadronic mass spectrum (level density) has the asymptotic ($m \to \infty$) form

$$\rho_{\mathcal{H}}(m) \approx \exp\left(m/T_{\mathcal{H}}\right), \tag{1}$$

m is the mass of the hadron and $T_{\mathcal{H}}$ is the parameter (temperature) controlling the the mass spectrum [1, 2].

The MIT bag model [3] produces the same behavior via a constant pressure B of the containing bag [4, 5]. The bag pressure B forces a constant temperature T_B and enthalpy density ϵ, thus the entropy is

$$S = \epsilon V/T_B = m/T_B, \tag{2}$$

V and m are the volume and mass of the bag respectively. This leads to a bag mass spectrum identical to Eq. (1) [4, 5]. This implies the lack of any bag surface energy.

In order to demonstrate how $\mathcal{H}$ imposes chemical equilibrium consider a vapor of $N \gg 1$ non-interacting particles of mass m coupled to $\mathcal{H}$. The microcanonical level density of the vapor with kinetic energy ε is

$$\rho_{\text{vapor}}(\varepsilon) = \frac{V^N}{N!\left(\frac{3}{2}N\right)!}\left(\frac{m\varepsilon}{2\pi}\right)^{\frac{3}{2}N}, \tag{3}$$

where V is is the volume. The microcanonical partition of the total system is

$$\begin{aligned}\rho_{\text{total}}(E,\varepsilon) &= \rho_{\mathcal{H}}(E-\varepsilon)\rho_{\text{vapor}}(\varepsilon)\\ &= \frac{V^N}{N!\left(\frac{3}{2}N\right)!}\left(\frac{m\varepsilon}{2\pi}\right)^{\frac{3}{2}N} e^{\frac{E-mN-\varepsilon}{T_{\mathcal{H}}}}.\end{aligned} \tag{4}$$

The distribution of the vapor is exactly canonical up to $\varepsilon_{max} = E$, if the particles are independently present, or $\varepsilon_{max} = E - mN$, if the particles are generated by $\mathcal{H}$.

The maximum of $\rho_{\text{total}}(E,\varepsilon)$ with respect to N at fixed V is given by

$$\frac{\partial \ln \rho_{\text{total}}(E,\varepsilon)}{\partial N} = -\frac{m}{T_{\mathcal{H}}} + \ln\left[\frac{V}{N}\left(\frac{mT_{\mathcal{H}}}{2\pi}\right)^{\frac{3}{2}}\right] = 0, \tag{5}$$

where $\varepsilon/N = 3T_{\mathcal{H}}/2$ was used for ε [6]. The most probable particle density of the vapor is *independent of* V:

$$\frac{N}{V} = \left(\frac{mT_{\mathcal{H}}}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m}{T_{\mathcal{H}}}} \equiv n_{\mathcal{H}}. \tag{6}$$

Equation (6) demonstrates that not only is $\mathcal{H}$ a perfect thermostat but also a perfect particle reservoir. Particles of different mass m will be in chemical equilibrium with each other automatically. At equilibrium, particles emitted from $\mathcal{H}$ form a saturated vapor at coexistence with $\mathcal{H}$ at temperature $T_{\mathcal{H}}$. This describes a first order phase transition (hadronic to partonic). Coexistence occurs at a single temperature fixed by the bag pressure [6].

Do the emitted particles need to remain in the proximity of $\mathcal{H}$ to insure equilibrium? Let us assume that $\mathcal{H}$ is a bag thick enough to absorb any given particle of the vapor striking it. Then, detailed balance requires that on average $\mathcal{H}$ radiates back the same particle. Under these conditions particles can be considered to be effectively emitted from the surface of $\mathcal{H}$. *Thus the relevant fluxes do not depend in any way upon the inner structure of* $\mathcal{H}$*, nor on the presence of the outer vapor.*

In fact, the results given by $\varepsilon/N = 3T_{\mathcal{H}}/2$ [6] and Eq. (6) show that the saturated vapor concentration depends only upon m and $T_{\mathcal{H}}$ as long as $\mathcal{H}$ is present. A decrease in the volume V does not increase the vapor concentration, but induces a condensation of the corresponding amount of energy out of the vapor and into $\mathcal{H}$. An increase in V keeps the vapor concentration constant via evaporation of the corresponding amount of energy out of $\mathcal{H}$ and into the vapor. This is reminiscent of liquid-vapor equilibrium at fixed temperature, except that here coexistence occurs at a single temperature $T_{\mathcal{H}}$, rather than over a range of temperatures as in ordinary fluids.

The bag wall is Janus faced: one side faces the partonic world and, aside from conserved charges, radiates a partonic black body radiation responsible for balancing the bag pressure; the other side faces the hadronic world and radiates a hadronic black body radiation, mostly pions. Both sides are at temperature $T_{\mathcal{H}}$. Despite the fact that this wall is an insurmountable horizon, with hadronic measurements such as bag size and total radiance we can infer some properties of the partonic world, e.g. the number of degrees of freedom [7].

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